Process-dependent transverse momentum distributions from lattice QCD

Bernhard Musch (Jefferson Lab)

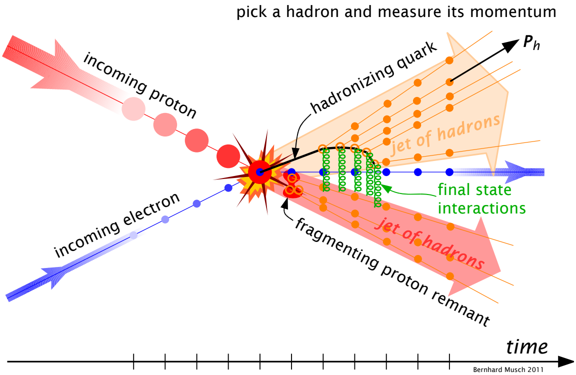
presenting work in collaboration with

Philipp Hägler (Johannes Gutenberg-Universität Mainz),
John Negele (MIT),
Alexei Prokudin (JLab),
Andreas Schäfer (Univ. Regensburg)

and using gauge configurations and propagators
form the MILC and LHP collaborations

Jefferson Lab
Thomas Jefferson National Accelerator Facility

$$\underbrace{l}_{\text{lepton}} + \underbrace{H(P)}_{\text{target hadron}} \longrightarrow l' + \underbrace{h(P_h)}_{\text{measured hadron}} + \underbrace{X}_{\text{rest}}$$



HERMES, COMPASS, JLab 6 GeV, JLab 12 GeV, . . ., EIC

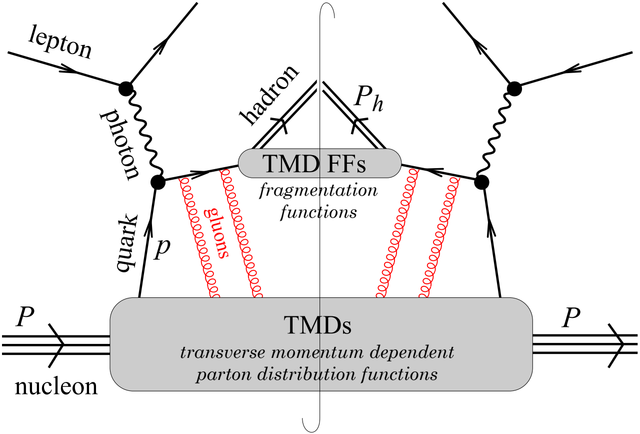
compare also to Drell-Yan, e.g., at RHIC

Asymmetries in P_h reveal motion of quarks in incoming hadron.
"Transverse momentum dependent parton distributions" (TMDs).

gluons ⇒ "final state interactions" → additional momentum exchange.
"Time-reversal odd TMDs" would not exist without this mechanism.

[Brodsky,Hwang,Schmidt PLB (2002)] [Ji,Yuan PLB (2003)] [Collins PLB (2002)]

$$\underbrace{l}_{\text{lepton}} + \underbrace{N(P)}_{\text{nucleon}} \longrightarrow l' + \underbrace{h(P_h)}_{\text{hadron}} + \underbrace{X}_{\text{rest}}$$



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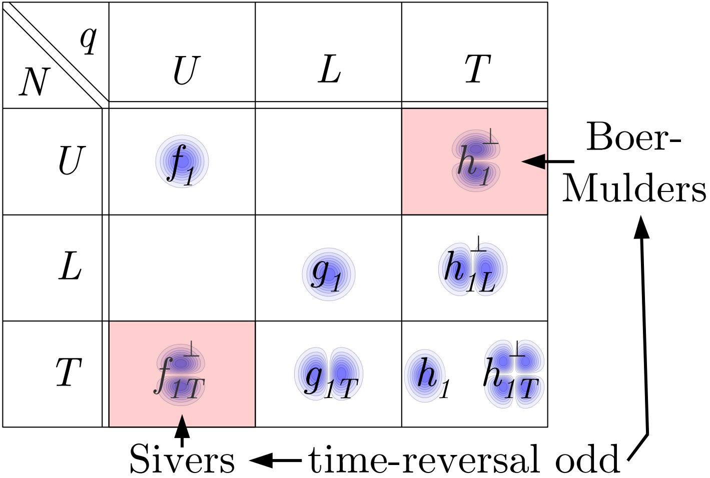
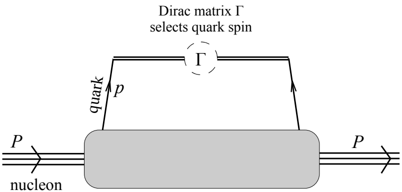

q
N
U
L
T
U
L
T
f_1
$h_1^\perp$
g_1
$h_{1L}^\perp$
$f_{1T}^\perp$
g_{1T}
h_1
$h_{1T}^\perp$
Boer-
Mulders
Sivers
time-reversal odd



light cone coordinates

$w^{\pm} = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$,
so $w = w^+ \hat{n}_+ + w^- \hat{n}_- + w_T$

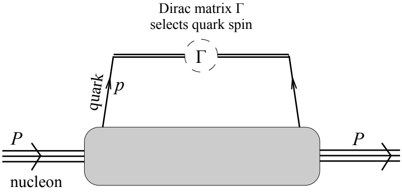
proton flies along z-axis:
P^+ large, $P_T = 0$

$$\Phi^{[\Gamma]} \equiv \text{“} \, \langle P,S|\, \bar{q}(p)\, \Gamma\, q(p)\, |P,S\rangle \, \text{”}$$

parametrization in terms of TMDs, example

$$\int dp^- \; \Phi^{[\gamma^+]}\Big|_{p^+=xP^+} = f_1(x,\boldsymbol{p}_T^2;\hat{\zeta},\ldots) - \frac{\boldsymbol{\epsilon}_{ij}\boldsymbol{p}_i\boldsymbol{S}_j}{m_N} f_{1T}^{\perp}(x,\boldsymbol{p}_T^2;\hat{\zeta},\ldots)$$

[Ralston, Soper NPB 1979], [Mulders, Tangerman NPB 1996]



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$$\Phi^{[\Gamma]} \equiv \frac{1}{2} \int \frac{d^4 b}{(2\pi)^4} e^{ip\cdot b} \frac{\overbrace{\langle P,S|\ \bar{q}(0)\,\Gamma\,\mathcal{U}[0,\infty v,\infty v+b,b]\,q(b)\ |P,S\rangle}^{\equiv \widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,v,\mu)}}{\underbrace{\widetilde{\mathcal{S}}(b^2,\ldots)}_{\text{soft factor}}}$$

parametrization in terms of TMDs, example

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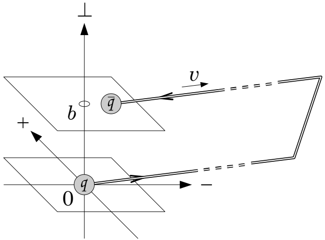

$$\hat{\zeta}^2 \equiv \frac{(P\cdot v)^2}{|P^2||v^2|} \qquad (\text{or } \zeta \equiv 4m_N^2\hat{\zeta}^2\ )$$

lightlike Wilson lines for $\hat{\zeta} \to \infty$

evolution eqns. for large $\hat{\zeta}$

[Collins Soper NPB (1981)], [Idilbi,Ji,Ma,Yuan PRD (2004)] [Aybat, Rogers (2011)] [Collins tbp]

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[Ralston, Soper NPB 1979], [Mulders, Tangerman NPB 1996]

$$\tilde{f}(x, \boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T} \; f(x, \boldsymbol{p}_T^2)$$

$$\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) \equiv n! \left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \; \tilde{f}(x, \boldsymbol{b}_T^2)$$

$$\Rightarrow \quad \tilde{f}^{(n)}(x, 0) = \int d^2\boldsymbol{p}_T \; \left(\frac{\boldsymbol{p}_T^2}{2m_N^2}\right)^n f(x, \boldsymbol{p}_T^2) \equiv f^{(n)}(x) \qquad (\boldsymbol{p}_T\text{-moment})$$

decomposition as in [Goeke,Metz,Schlegel PLB (2005)] but in Fourier-space

$$\frac{1}{2}\widetilde{\Phi}^{[\gamma^\mu]}_{\text{unsubtr.}}(b, P, S, v, \mu) = \langle P, S| \; \bar{q}(0)\, \gamma^\mu \, \mathcal{U}[0, \infty v, \infty v + b, b]\, q(b) \; |P, S\rangle$$

$$= P^\mu \, \widetilde{A}_2 - i m_N^2 b^\mu \, \widetilde{A}_3 - i m_N \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha S_\beta \, \widetilde{A}_{12}$$

$$+ \frac{m_N^2}{(v\cdot P)} v^\mu \, \widetilde{B}_1 + \frac{m_N}{v\cdot P}\epsilon^{\mu\nu\alpha\beta} P_\nu v_\alpha S_\beta \, \widetilde{B}_7 - \frac{i m_N^3}{v\cdot P}\epsilon^{\mu\nu\alpha\beta} b_\nu v_\alpha S_\beta \, \widetilde{B}_8$$

$$- \frac{m_N^3}{v\cdot P}(b\cdot S)\epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta \, \widetilde{B}_9 - \frac{i m_N^3}{(v\cdot P)^2}(v\cdot S)\epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta \widetilde{B}_{10}$$

$$\tilde{f}(x, \boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{p}_T} \; f(x, \boldsymbol{p}_T^2)$$

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$$= P^+ \underbrace{\left(\widetilde{A}_2 + R(\hat{\zeta}) \widetilde{B}_1 \right)}_{\widetilde{\mathcal{S}} \, \int\!\!\!\!\!\!\not{x} \; \tilde{f}_1^{(0)}} + i m_N P^+ \boldsymbol{\epsilon}_{ij} \boldsymbol{b}_i \boldsymbol{S}_j \underbrace{\left(\widetilde{A}_{12} - R(\hat{\zeta}) \widetilde{B}_8 \right)}_{\widetilde{\mathcal{S}} \, \int\!\!\!\!\!\!\not{x} \; \tilde{f}_{1T}^{\perp(1)}}$$

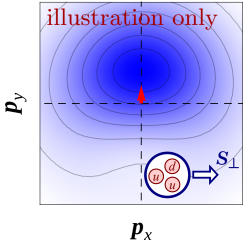
where
$$\int\!\!\!\!\!\!\not{x} \; \tilde{f}^{(n)} \equiv \int dx \; e^{-ix(b\cdot P)} \; \tilde{f}^{(n)}(x, \boldsymbol{b}_T^2),$$

$$R(\hat{\zeta}) \equiv 1 - \sqrt{1 + \hat{\zeta}^{-2}}, \qquad \text{note that} \quad \lim_{\hat{\zeta} \to \infty} R(\hat{\zeta}) = 0$$

unpolarized quark density in a transversely polarized nucleon

$$\rho_{TU}(x, \boldsymbol{p}_T, \boldsymbol{S}_T) = f_1(x, \boldsymbol{p}_T^2) - \frac{\epsilon_{ij} \boldsymbol{p}_i \boldsymbol{S}_j}{m_N} f_{1T}^{\perp}(x, \boldsymbol{p}_T^2) = \int dp^- \ \Phi^{[\gamma^+]}$$

$$\langle \boldsymbol{p}_y \rangle_{TU} \equiv \frac{\int dx \int d^2\boldsymbol{p}_T \ \boldsymbol{p}_y \ \rho_{TU}(x, \boldsymbol{p}_T, \boldsymbol{S}_T = (1,0))}{\int dx \int d^2\boldsymbol{p}_T \ \rho_{TU}(x, \boldsymbol{p}_T, \boldsymbol{S}_T = (1,0))} = m_N \frac{\int dx\ f_{1T}^{\perp(1)}(x)}{\int dx\ f_1^{(0)}(x)}$$

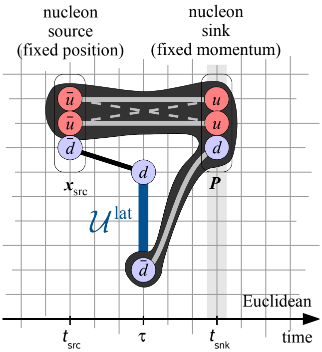


$\langle \boldsymbol{p}_y \rangle_{TU}$:= average quark momentum in transverse y-direction measured in a proton polarized in transverse x-direction.

"dipole moment", "shift"

"generalized" average transverse momentum shift

$$\langle \boldsymbol{p}_y \rangle_{TU}(|\boldsymbol{b}_T|) \equiv m_N \frac{\int dx\ \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)}{\int dx\ \tilde{f}_1^{(0)}(x, \boldsymbol{b}_T^2)} = m_N \frac{\not{S}\left(\widetilde{A}_{12} - R(\hat{\zeta})\widetilde{B}_8\right)}{\not{S}\left(\widetilde{A}_2 + R(\hat{\zeta})\widetilde{B}_8\right)}\Bigg|_{b \cdot P = 0}$$



We neglect disconnected contributions.

input from lattice collaborations

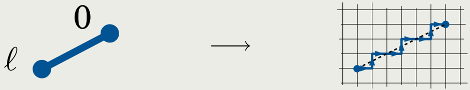
MILC lattices (staggered),
LHPC propagators (domain wall)

[Aubin et al. PRD ('04)] [Hägler et al. PRD ('08)]

first exploratory lattice studies

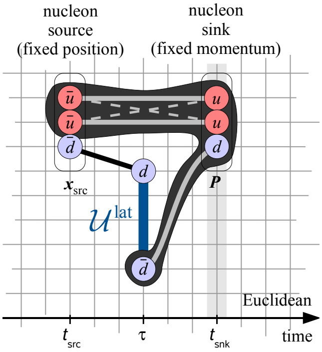
... employ(ed) a straight gauge link

[Hägler,BM, et al. EPL ('09) and arXiv:1011.1213]



⇒ No T-odd TMDs

⇒ probably only qualitatively related to TMDs for SIDIS and Drell-Yan



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now: staple-shaped links

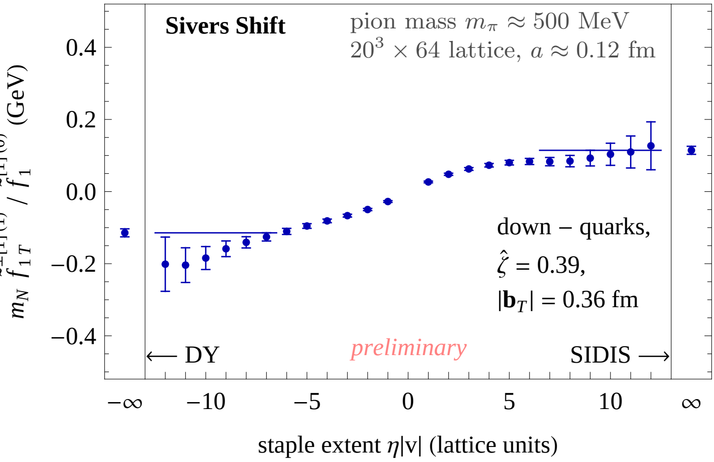
spacelike, finite length

⇒ look for plateau at large η

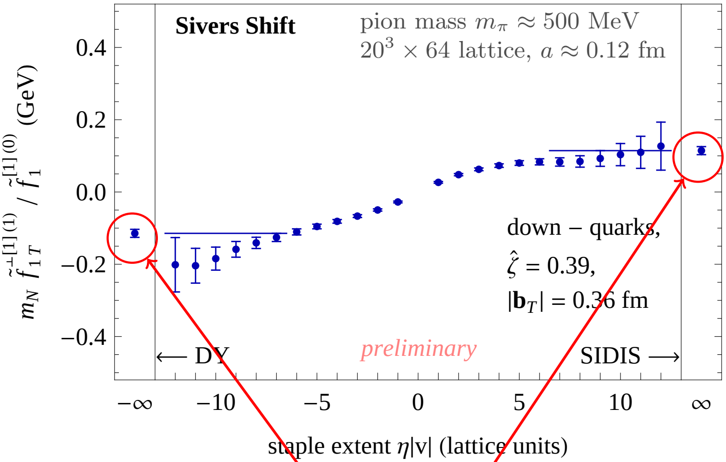
limitations: $\hat{\zeta}_{\max} = \frac{|\boldsymbol{P}_{\text{lat}}|}{m_N}$, $\sqrt{-b^2} \gtrsim 3a$

$$\langle \boldsymbol{p}_y \rangle_{TU}(|\boldsymbol{b}_T|;\zeta) \equiv m_N \frac{\int dx\, \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)}{\int dx\, \tilde{f}_1^{(0)}(x, \boldsymbol{b}_T^2)}$$

$$= \lim_{\eta\to\infty} \frac{\widetilde{A}_{12}^{\text{lat}}(-\boldsymbol{b}_T^2, 0, 0, \hat{\zeta}, \mu, \eta) - R(\hat{\zeta})\ \widetilde{B}_8^{\text{lat}}(-\boldsymbol{b}_T^2, 0, 0, \hat{\zeta}, \mu, \eta)}{\widetilde{A}_2^{\text{lat}}(-\boldsymbol{b}_T^2, 0, 0, \hat{\zeta}, \mu, \eta) + R(\hat{\zeta})\ \widetilde{B}_1^{\text{lat}}(-\boldsymbol{b}_T^2, 0, 0, \hat{\zeta}, \mu, \eta)}$$



$$m_N \frac{\int_{-1}^{1} dx\, \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)}{\int_{-1}^{1} dx\, \tilde{f}_1^{(0)}(x, \boldsymbol{b}_T^2)} = \frac{\widetilde{A}_{12} - R(\hat{\zeta})\, \widetilde{B}_8}{\widetilde{A}_2 + R(\hat{\zeta})\, \widetilde{B}_1}$$



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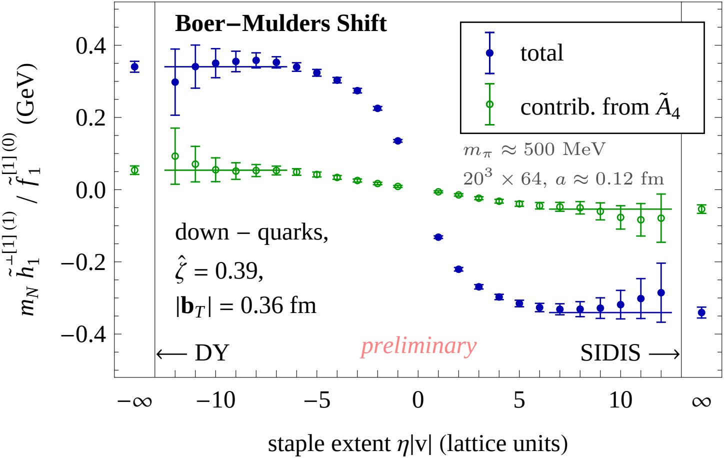
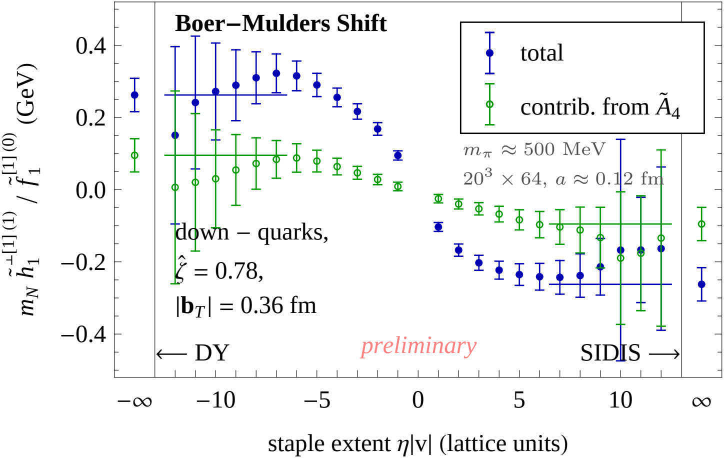
need improved asymptotic fit & more realistic errors



$$m_N \frac{\int_{-1}^{1} dx\, \tilde{h}_1^{\perp(1)}(x, \boldsymbol{b}_T^2)}{\int_{-1}^{1} dx\, \tilde{f}_1^{(0)}(x, \boldsymbol{b}_T^2)} = \frac{\widetilde{A}_4 - R(\hat{\zeta})\, \widetilde{B}_3}{\widetilde{A}_2 + R(\hat{\zeta})\, \widetilde{B}_1}$$

$R(\hat{\zeta}) \xrightarrow{\hat{\zeta}\to\infty} 0 \;\Rightarrow$ expect numerator dominated by $\widetilde{A}_4$ close to lighcone.
(We are still far from that region.)



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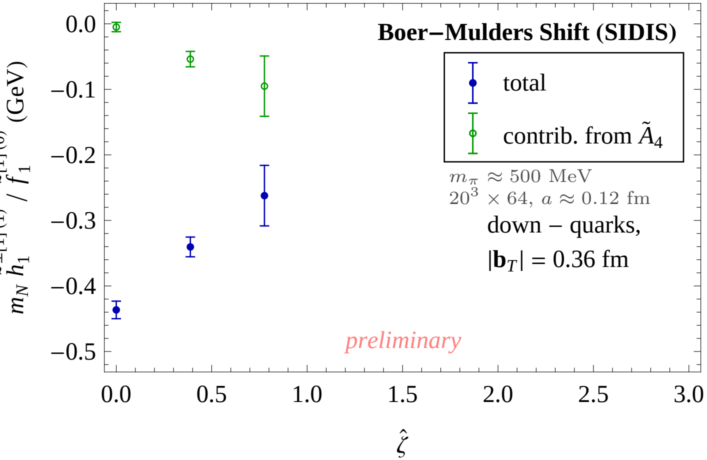

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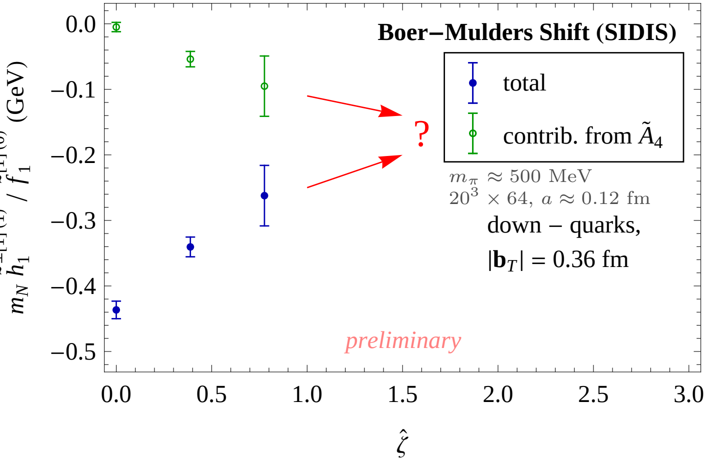

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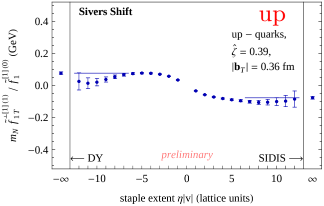
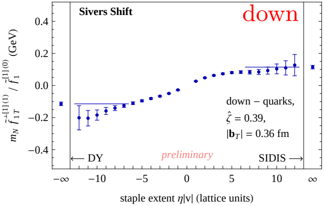
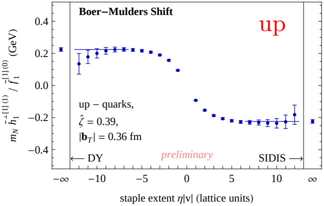
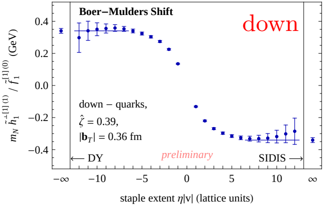




Signs compatible with phenomenology [Anselmino et. al. EPJA (2009)]:
$\langle \boldsymbol{p}_y \rangle_{TU} = m_N (\int dx\, f_{1T}^{\perp(1)}) / (\int dx\, f_1^{(0)}) = -48^{+30}_{-14}$ MeV (up), 113^{+45}_{-51} MeV (down)





COMPASS & HERMES data analysis [Barone, Melis, Prokudin PRD (2010)]:
$h_1^{\perp,u} / f_{1T}^{\perp,u} = 2.1 \pm 0.1, \quad h_1^{\perp,d} / f_{1T}^{\perp,d} = -1.1$ supporting Burkardt's mechanism [PRD (2005)] with κ_T from lattice [Göckeler et. al. PRL (2007)]

Lattice studies for TMDs as in SIDIS or Drell-Yan are possible

- for ratios of Fourier-transformed TMDs
- using space-like Wilson lines
  as in [AYBAT, ROGERS arXiv:1101.5057 (2011)]
  and J. Collins' book (to be published)

proof-of-concept results at $m_\pi \approx 500$ MeV, $|\boldsymbol{b}_T| = 0.36$ fm:

- Sivers: $m_N \int dx\, \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2) \;/\; \int dx\, \tilde{f}_1^{(0)}(x, \boldsymbol{b}_T^2)$:
  signs different for up and down quarks
- Boer-Mulders: $m_N \int dx\, \tilde{h}_1^{\perp(1)}(x, \boldsymbol{b}_T^2) \;/\; \int dx\, \tilde{f}_1^{(0)}(x, \boldsymbol{b}_T^2)$:
  same sign for up and down quarks

Major challenge:

- Currently relatively low Collins-Soper evolution parameter $\hat{\zeta}$.