Pion scattering and electro-production on nucleons in the resonance region in chiral quark models

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München | 16 June 2011

P11: EPJA **38** (2008) 271

P11: EPJA **42** (2009) 185

S11: EPJA **47** (2011) 61

Motivation for study of P_{11} and S_{11} resonances

- Large width of **** $\mathrm{P}_{11}(1440)$ "Roper"; existence of *** $\mathrm{P}_{11}(1710)$ unclear; difficult to identify directly (in cross-sections)
- Atypical behaviour of $\mathrm{Im}T_{\pi\mathrm{N}}$ in $J=I=1/2$ partial wave
- Level ordering (parity inversion) of $\mathrm{P}_{11}(1440)$ wrt. $\mathrm{S}_{11}(1535)$ on Lattice
- Many competing explanations of the Roper in models, e.g.

  q^3g hybrid ▷ Li, Burkert PRD **46** (1992) 70

  $qqqq\bar{q}$ admixtures ▷ Li, Riska PRC **74** (2006) 015202

  dynamical generation by $N+\sigma$ ▷ Krehl++ PRC **62** (2000) 025207

  ▷ Döring++ NPA **829** (2009) 170

- Two negative-parity resonances:

resonance	M [MeV]	Γ [MeV]	decays
S11(1535)	1535	150	πN 35–55 %, ηN 45–60 % $2\pi N < 10$ %, (πR)
S11(1650)	1655	165	πN 60–95 %, ηN 3–10 %, $K\Lambda$ 3–11 % $2\pi N$ 10–20 %, ρ 4–12 %, Δ 1–7 %

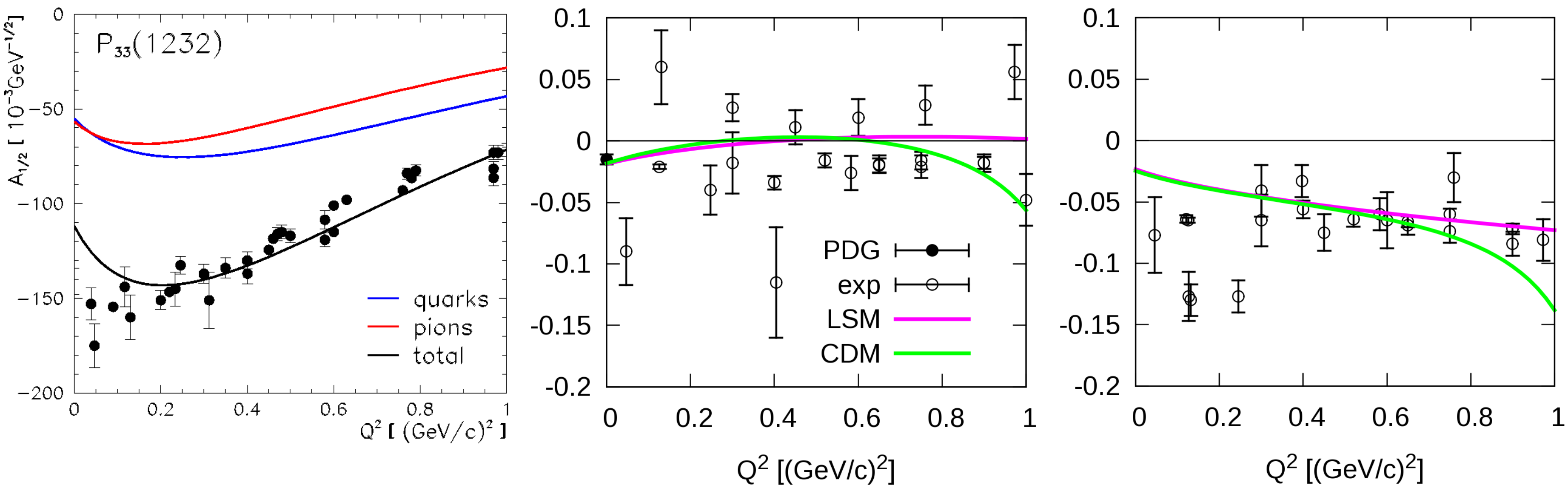
Present work

- A coupled-channels approach that includes many-body states of quarks and mesons in the scattering formalism
- Calculate scattering and electro-production amplitudes *within the same framework*
- Investigate whether quark+meson description is sufficient i.e. no exotic degrees of freedom involved
- Baryons treated as composite particles
  - $\longrightarrow$ coupling constants and cut-offs of form-factors computed from the underlying model, not fitted
  - $\longrightarrow$ smaller number of free parameters
- Physical resonances appear as linear combinations of bare resonances
- Bare quark-meson and quark-photon vertices are strongly modified by meson loops and mixing of resonances
- K-matrix real & symmetric $\longrightarrow$ S-matrix unitary

Reminder: $\Delta(1232)$ in quark models with pion cloud

Helicity and electro-production amplitudes for $\gamma^* N \to \Delta(1232) \to N\pi$

$A_{1/2}$ $\qquad$ E_{1+}/M_{1+} $\qquad$ S_{1+}/M_{1+}



- M_{1+} is (∼ 50 % pion cloud) + (∼ 50 % quarks)
- E_{1+} is (∼ 100 % pion cloud)

Golli++ PLB **373** (1996) 229

Coupled-channel K-matrix formalism

Our model

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The meson field linearly couples to the quark core; no meson self-interaction

$$H = H_{\mathrm{quark}} + \int \mathrm{d}k \sum_{lmt} \left\{ \omega_k\, a^\dagger_{lmt}(k) a_{lmt}(k) + \left[V_{lmt}(k) a_{lmt}(k) + V_{lmt}(k)^\dagger\, a^\dagger_{lmt}(k) \right] \right\}$$

$V_{lmt}(k)$ induces also radial excitations of the quark core, e.g. $1s \to 2s$, $1s \to 1p_{1/2}$, $1s \to 1p_{3/2}$, $\ldots$ transitions

For example: $V(k)$ from Cloudy Bag Model

$$V^{s\to s}_{1mt}(k) = \frac{1}{2f} \frac{k^2}{\sqrt{12\pi^2\omega_k}} \frac{\omega_{\mathrm{s}}}{\omega_{\mathrm{s}}-1} \frac{j_1(kR_{\mathrm{bag}})}{kR_{\mathrm{bag}}} \sum_{i=1}^{3} \sigma^i_m \tau^i_t \qquad (p\text{–wave pions})$$

$$V^{s\to 2s}_{1mt}(k) = \frac{1}{2f} \frac{k^2}{\sqrt{12\pi^2\omega_k}} \sqrt{\frac{\omega_{2\mathrm{s}}\omega_{\mathrm{s}}}{(\omega_{2\mathrm{s}}-1)(\omega_{\mathrm{s}}-1)}} \frac{j_1(kR_{\mathrm{bag}})}{kR_{\mathrm{bag}}} \sum_{i=1}^{3} \sigma^i_m \tau^i_t \qquad (p\text{–wave})$$

$$V^{s\to p_{1/2}}_{1=0,t}(k) = \frac{1}{2f} \frac{k^2}{\sqrt{4\pi^2\omega_k}} \sqrt{\frac{\omega_{\mathrm{p}_{1/2}}\omega_{\mathrm{s}}}{(\omega_{\mathrm{p}_{1/2}}+1)(\omega_{\mathrm{s}}-1)}} \frac{j_0(kR_{\mathrm{bag}})}{kR_{\mathrm{bag}}} \sum_{i=1}^{3} \tau^i_t \qquad (s\text{–wave})$$

$$V^{s\to p_{3/2}}_{2mt}(k) = \frac{1}{2f} \frac{k^2}{\sqrt{2\pi^2\omega_k}} \sqrt{\frac{\omega_{\mathrm{p}_{3/2}}\omega_{\mathrm{s}}}{(\omega_{\mathrm{p}_{3/2}}-2)(\omega_{\mathrm{s}}-1)}} \frac{j_2(kR_{\mathrm{bag}})}{kR_{\mathrm{bag}}} \sum_{i=1}^{3} \Sigma^i_{2m} \tau^i_t \qquad (d\text{–wave})$$

Constructing the K-matrix

Aim: include many-body states of quarks (and mesons) in the scattering formalism (Chew-Low type approach)

Construct the K-matrix in spin-isospin (JI) basis:

$$K^{JI}_{M'B'\,MB} = -\pi\sqrt{\frac{\omega_M E_B}{k_M W}}\,\langle \Psi^{MB}_{JI}(W)||V_{M'}(k)||\Psi_{B'}\rangle$$

dressed states

by using principal-value (PV) states

$$|\Psi^{MB}_{JI}(W)\rangle = \sqrt{\frac{\omega_M E_B}{k_M W}}\left\{\left[a^\dagger(k_M)|\Psi_B\rangle\right]^{JI} - \frac{\mathcal{P}}{H-W}\,[V(k_M)|\Psi_B\rangle]^{JI}\right\}$$

normalized as

$$\langle \Psi^{MB}(W)|\Psi^{M'B'}(W')\rangle = \delta(W-W')\delta_{MB,M'B'}(1+\mathbf{K}^2)_{MB,MB}$$

Ansatz for the channel PV states

free meson (defines the channel)

bare (genuine) baryons (3q)

$$|\Psi_{JI}^{MB}\rangle = \sqrt{\frac{\omega_M E_B}{k_M W}}\left\{[a^\dagger(k_M)|\tilde{\Psi}_B\rangle]^{JI} + \sum_{\mathcal{R}} c_{\mathcal{R}}^{MB}|\Phi_{\mathcal{R}}\rangle\right.$$

meson "clouds" with amplitudes χ

$$\left. + \sum_{M'B'}\int \frac{\mathrm{d}k\,\chi^{M'B'MB}(k,k_M)}{\omega_k + E_{B'}(k) - W}[a^\dagger(k)|\tilde{\Psi}_{B'}\rangle]^{JI}\right\}$$

Above the meson-baryon (MB) threshold:

$$K_{M'B'\,MB}(k,k_M) = \pi\sqrt{\frac{\omega_M E_B}{k_M W}}\sqrt{\frac{\omega_{M'}E_{B'}}{k_{M'}W}}\,\chi^{M'B'\,MB}(k,k_M)$$

2π decay through intermediate hadrons ($\Delta(1232)$, $N(1440)$; σ, ρ, ...), e.g.

$$\pi N \to B^* \to \pi\Delta \to \pi\pi N\,, \qquad \pi N \to B^* \to \sigma N \to (2\pi)N$$

Solve Lippmann-Schwinger eqs for χ; the solution has the form

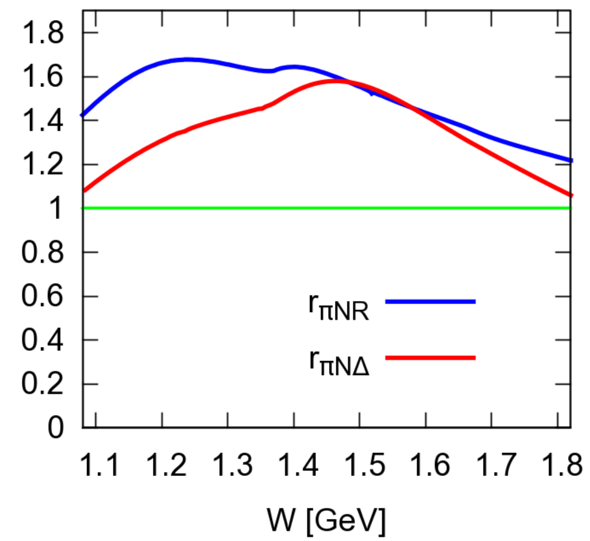
$$\chi^{M'B'MB}(k,k_M) = -\sum_{\mathcal{R}} c_{\mathcal{R}}^{MB}\,\underbrace{\mathcal{V}_{B'\mathcal{R}}^{M'}(k)}_{\textbf{dressed vertex}} + \underbrace{\mathcal{D}^{M'B'MB}(k,k_M)}_{\textbf{background part}}$$

Solving the coupled equations

The dressed vertices satisfy:

$$\mathcal{V}^{M}_{B\mathcal{R}}(k) = V^{M}_{B\mathcal{R}}(k) + \sum_{M'B'} \int \mathrm{d}k' \, \frac{\mathcal{K}^{MB\,M'B'}(k,k')\,\mathcal{V}^{M'}_{B'\mathcal{R}}(k')}{\omega'_k + E_{B'}(k') - W}$$

$$r_{MBR} = \frac{\mathcal{V}^{M}_{B\mathcal{R}}(k)}{V^{M}_{B\mathcal{R}}(k)}$$



Similarly, for the background part of the amplitude:

$$\begin{aligned}\mathcal{D}^{M'B'\,MB}(k,k_M) &= \mathcal{K}^{M'B'\,MB}(k,k_M) \\ &+ \sum_{M''B''} \int \mathrm{d}k' \, \frac{\mathcal{K}^{M'B'\,M''B''}(k,k')\mathcal{D}^{M''B''\,MB}(k',k_M)}{\omega'_k + E_{B''}(k') - W}\end{aligned}$$

The coefficients $c^{MB}_{\mathcal{R}'}$ of the quasi-bound states satisfy a set of equations:

$$\sum_{\mathcal{R}'} A_{\mathcal{R}\mathcal{R}'}(W)\, c^{MB}_{\mathcal{R}'}(W) = \mathcal{V}^{M}_{B\mathcal{R}}(k_M)$$

$$A_{\mathcal{R}\mathcal{R}'} = (W - M^0_{\mathcal{R}})\delta_{\mathcal{R}\mathcal{R}'} + \sum_{\mathcal{B}'} \int \mathrm{d}k \, \frac{\mathcal{V}^{M'}_{B'\mathcal{R}}(k)\, V^{M'}_{B'\mathcal{R}'}(k)}{\omega_k + E_{B'}(k) - W}$$

Calculating the K-matrix

To solve the set of equations, diagonalize A to obtain U, along with the poles of the K-matrix, and wave-function normalization Z:

$$UAU^T = \begin{bmatrix} Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}}) & 0 & 0 \\ 0 & Z_{\mathcal{R}'}(W)(W - M_{\mathcal{R}'}) & 0 \\ 0 & 0 & Z_{\mathcal{R}''}(W)(W - M_{\mathcal{R}''}) \end{bmatrix}$$

As a consequence, $\Phi_{\mathcal{R}}$ mix:

$$|\widetilde{\Phi}_{\mathcal{R}}\rangle = \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'}|\Phi_{\mathcal{R}}\rangle \qquad \widetilde{\mathcal{V}}_{B\mathcal{R}} = \frac{1}{\sqrt{Z_{\mathcal{R}}(W)}} \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} \mathcal{V}_{B\mathcal{R}'}$$

Solution for the K-matrix:

$$K_{MB,M'B'} = \pi \sqrt{\frac{\omega_M E_B}{k_M W}} \sqrt{\frac{\omega_{M'} E_{B'}}{k_{M'} W}} \left[\underbrace{\sum_{\mathcal{R}} \frac{\widetilde{\mathcal{V}}^{M}_{B\mathcal{R}} \widetilde{\mathcal{V}}^{M'}_{B'\mathcal{R}}}{(M_{\mathcal{R}} - W)}}_{\textbf{resonant}} + \underbrace{\mathcal{D}_{MB,M'B'}}_{\textbf{background}} \right]$$

Solution for the T matrix:

$$T_{MB,M'B'} = K_{MB,M'B'} + \mathrm{i} \sum_{M''K''} T_{MB,M''B''} K_{M''B'',M'B'}$$

Pion electro-production: including the γN channel

Only the strong $T_{MB,M'B'}$ appears on the RHS:

$$T_{MB,\gamma N} = K_{MB,\gamma N} + \mathrm{i} \sum_{M'K'} T_{MB,M'B'} K_{M'B',\gamma N}$$

$$K_{M'B',\gamma N} = -\pi \sqrt{\frac{\omega_\gamma E_N}{k_\gamma W}} \left\langle \Psi_{JI}^{M'B'} || V_\gamma || \Psi_N \right\rangle$$

Choosing a resonance, $\mathcal{R} = N^*$, the principal-value state can be split into the resonant and background parts. Then

$$\mathcal{M}_{MB\,\gamma N} = \underbrace{\sqrt{\frac{\omega_\gamma E_N^\gamma}{\omega_M E_B}} \frac{1}{\pi \mathcal{V}_{BN^*}} \overbrace{\langle \Psi_{N^*}^{(\mathrm{res})}(W) | \tilde{V}_\gamma | \Psi_N \rangle}^{\text{helicity amplitude}} T_{MB\,MB}}_{\mathcal{M}_{MB\,\gamma N}^{(\mathrm{res})}} + \mathcal{M}_{MB\,\gamma N}^{(\mathrm{bkg})}$$

The resonant state takes the form:

bare quark contrib

meson cloud contrib

$$|\Psi_{N^*}^{(\mathrm{res})}(W)\rangle = \frac{1}{\sqrt{Z_{N^*}}} \left\{ |\tilde{\Phi}_{N^*}\rangle - \sum_{MB} \int \frac{\mathrm{d}k \quad \widetilde{\mathcal{V}}_{BN^*}^{M}(k)}{\omega_k + E_B - W} [a^\dagger(k) |\Psi_B\rangle]^{JI} \right\}$$

Underlying quark model

Cloudy Bag Model extended to pseudo-scalar SU(3) octet

$$\mathcal{L}_{\mathrm{CBM}}^{(\mathrm{quark-meson})} = -\frac{\mathrm{i}}{2f}\,\overline{q}\gamma_5\lambda_a q\phi_a\delta\left(r - R_{\mathrm{bag}}\right)\,, \qquad a = 1,2,\ldots,8$$

Parameters:

$R_{\mathrm{bag}} = 0.83$ fm

$f_\pi = 76$ MeV

$f_K = 1.2\,f_\pi$

$f_\eta = f_\pi$ or $1.2\,f_\pi$

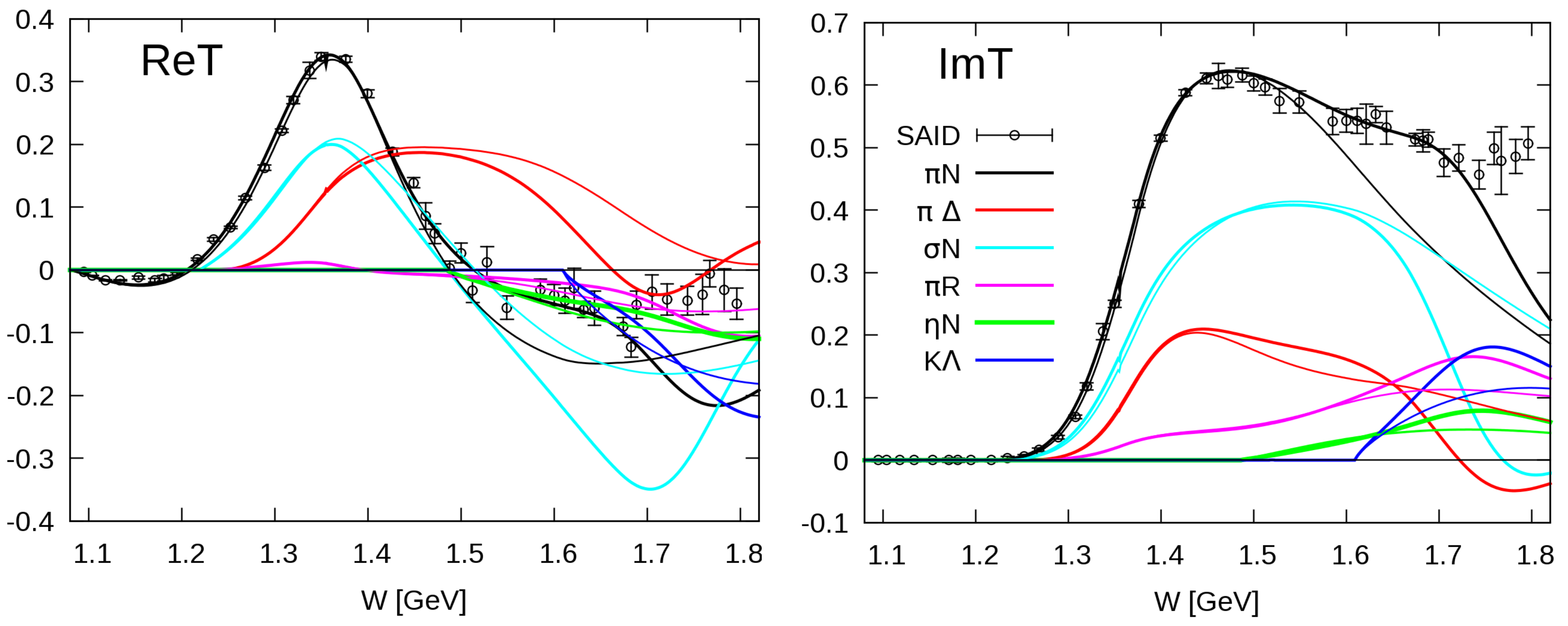
Similar results for $0.75\ \mathrm{fm} < R_{\mathrm{bag}} < 1.0\ \mathrm{fm}$

Free parameters: bare masses of the resonant states

Pion scattering in P11 partial wave $\pi N \to MB$

Channels: πN, $\pi\Delta$, σN, πR (preliminary: ηN, $K\Lambda$)

Parameters of the σN-channel: $g_{\sigma NR} = 1\,, \quad m_\sigma = 450$ MeV$\,, \quad \Gamma_\sigma = 550$ MeV



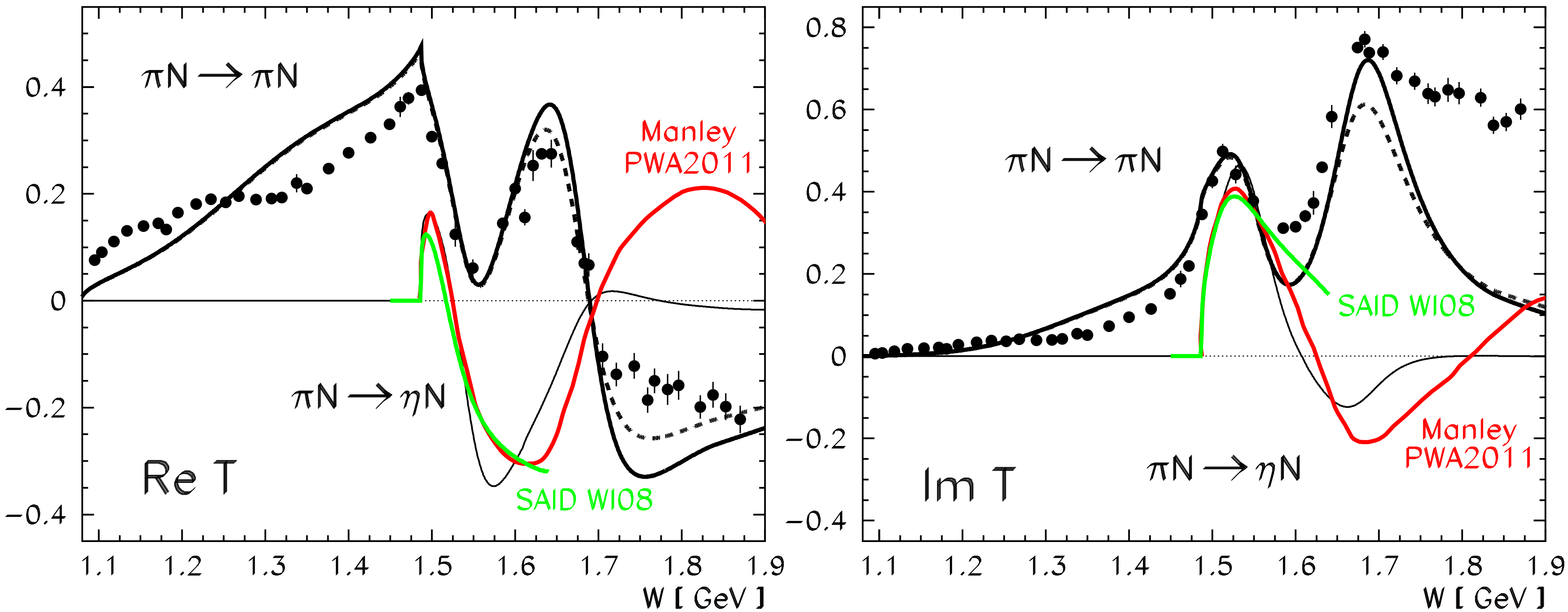
Thin lines: only $N(1440)$ included ($(1s)^2(2s)^1$)

Thick lines: $N(1710)$ added ($(1s)^1(2p)^2$) with $g_{\pi NN(1710)} = 0$

$$g_{\sigma NN(1710)} \approx g_{\sigma NN(1440)}$$

Pion scattering in S11 partial wave

$\pi N \to MB$



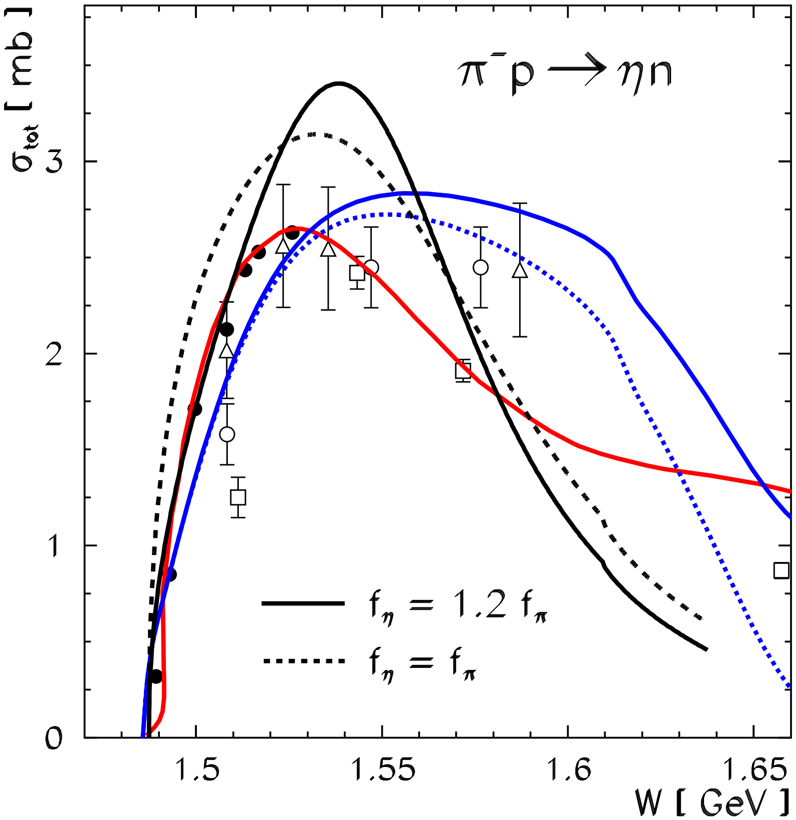
Allow single-quark excitations ($1s \to 1p_{1/2}$ and $1s \to 1p_{3/2}$)

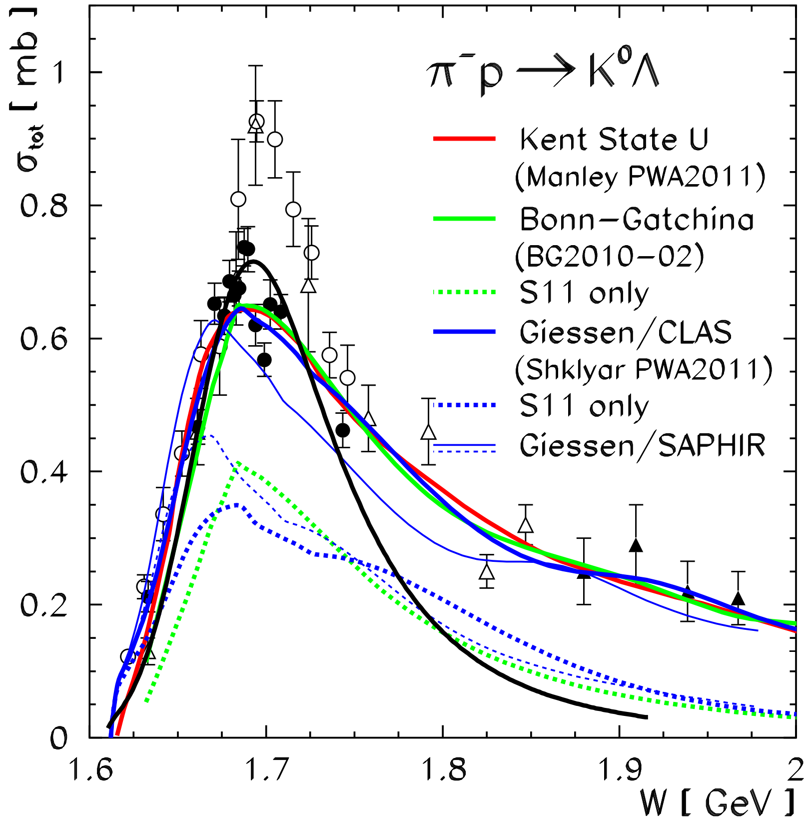
$$\begin{aligned} \Phi(1535) &= -\sin\vartheta_s |{}^4 8_{1/2}\rangle + \cos\vartheta_s |{}^2 8_{1/2}\rangle \\ \Phi(1650) &= \cos\vartheta_s |{}^4 8_{1/2}\rangle + \sin\vartheta_s |{}^2 8_{1/2}\rangle \end{aligned}$$

ϑ_s is a free parameter ($\approx -30°$)

Myhrer, Wroldsen / Z. Phys. C **25** (1984) 281

Pion scattering in S11 partial wave





S11 contribution dominates

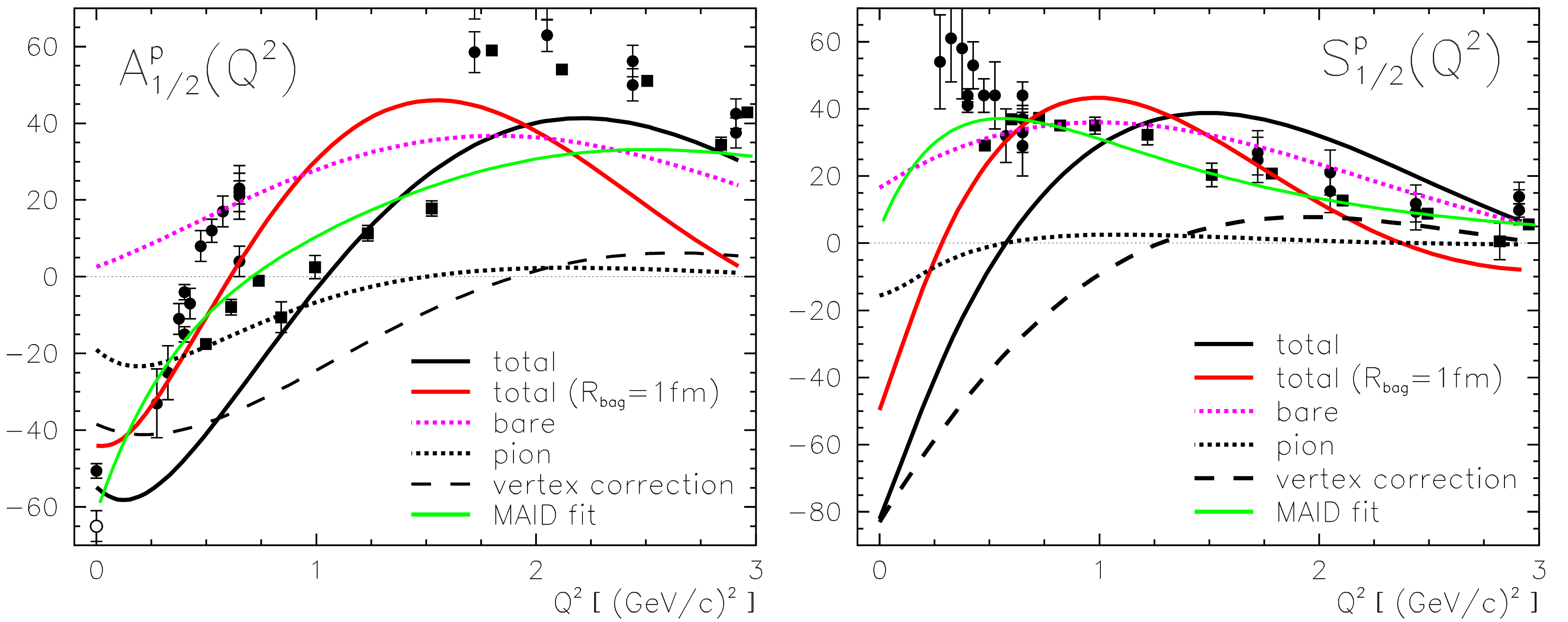
P11, P13 negligible

P11, P13 contributions sizeable

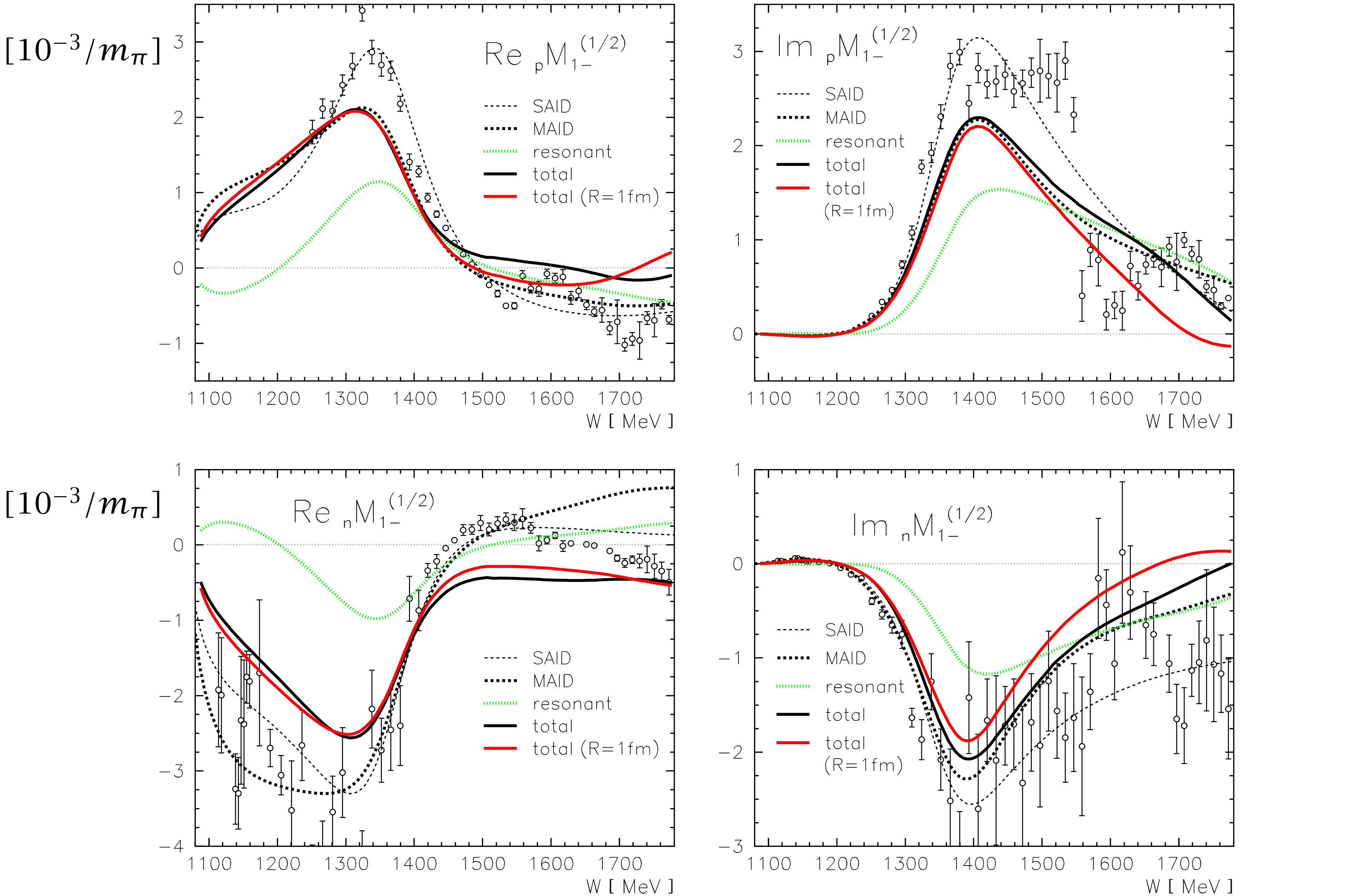
not yet included in our calculation

PWA results on S11 : P11 : P13 uncertain

Helicity amplitudes for $\gamma p \to N(1440)$ $[10^{-3}\,\mathrm{GeV}^{-1/2}]$



P11 transverse photo-production amplitudes $\gamma N \to N\pi^0$



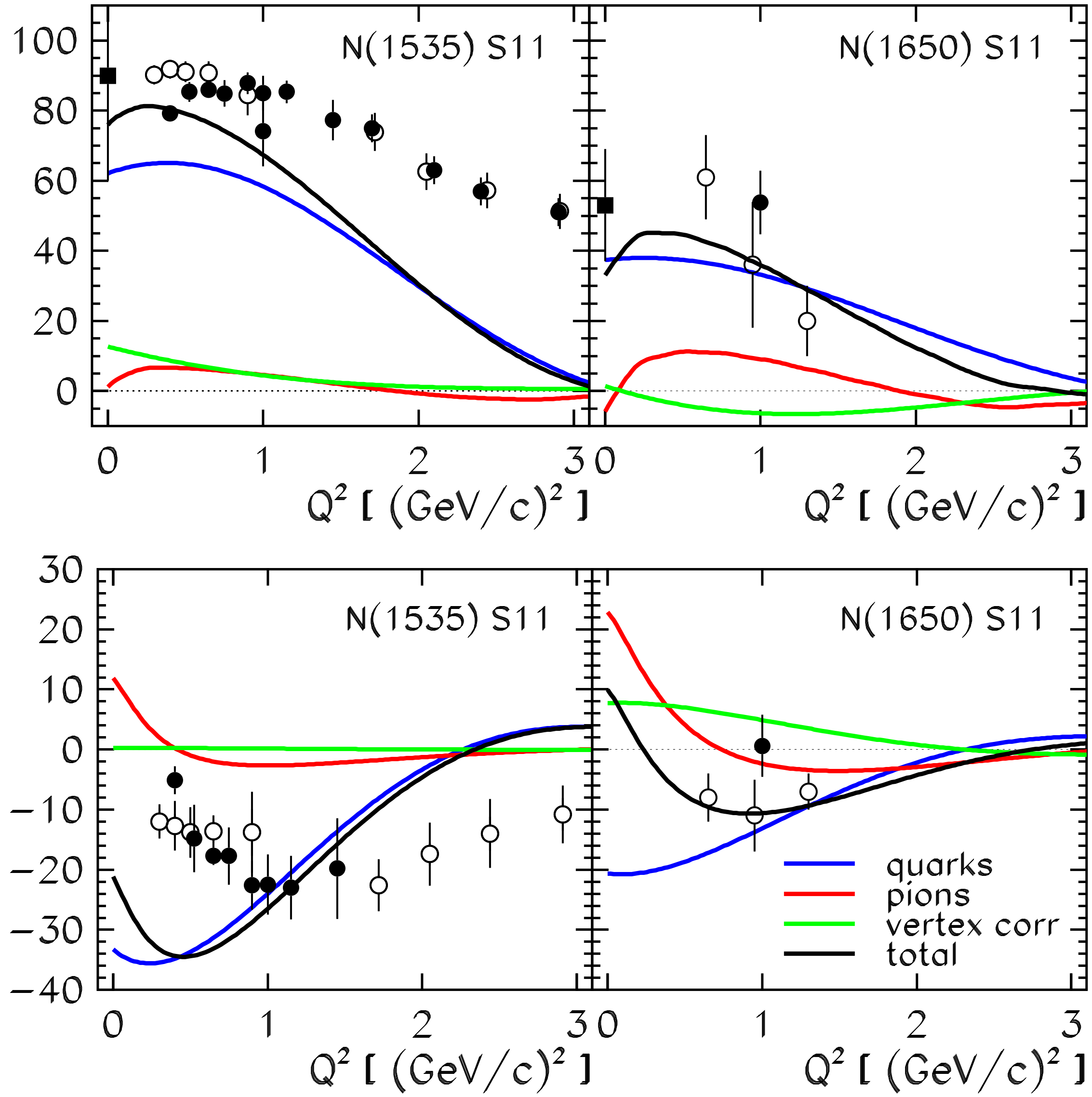
Helicity amplitudes $\gamma p \to S_{11}(1535), S_{11}(1650)$

$A_{1/2}(Q^2)$

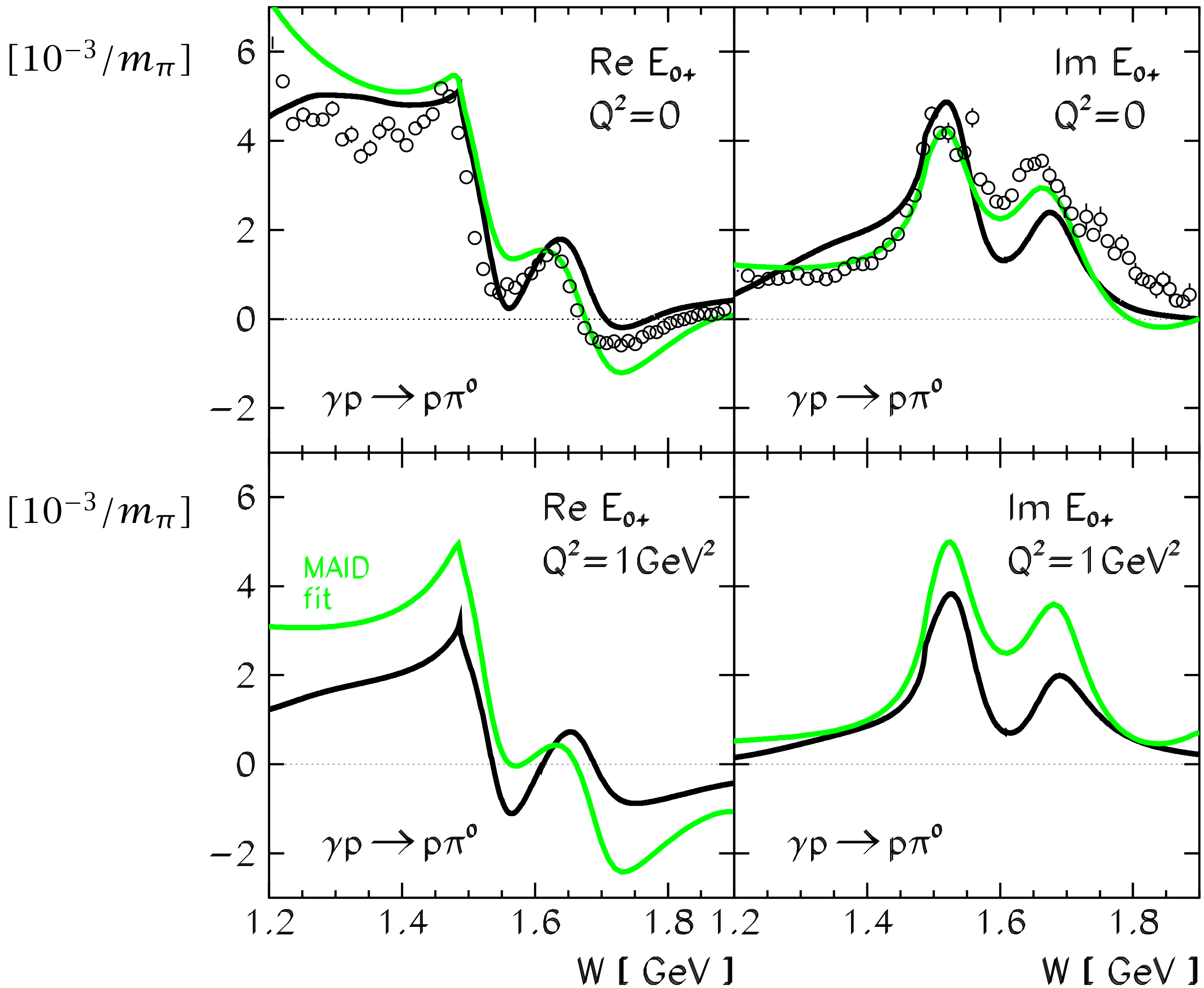
$[10^{-3}\,\text{GeV}^{-1/2}]$

$S_{1/2}(Q^2)$

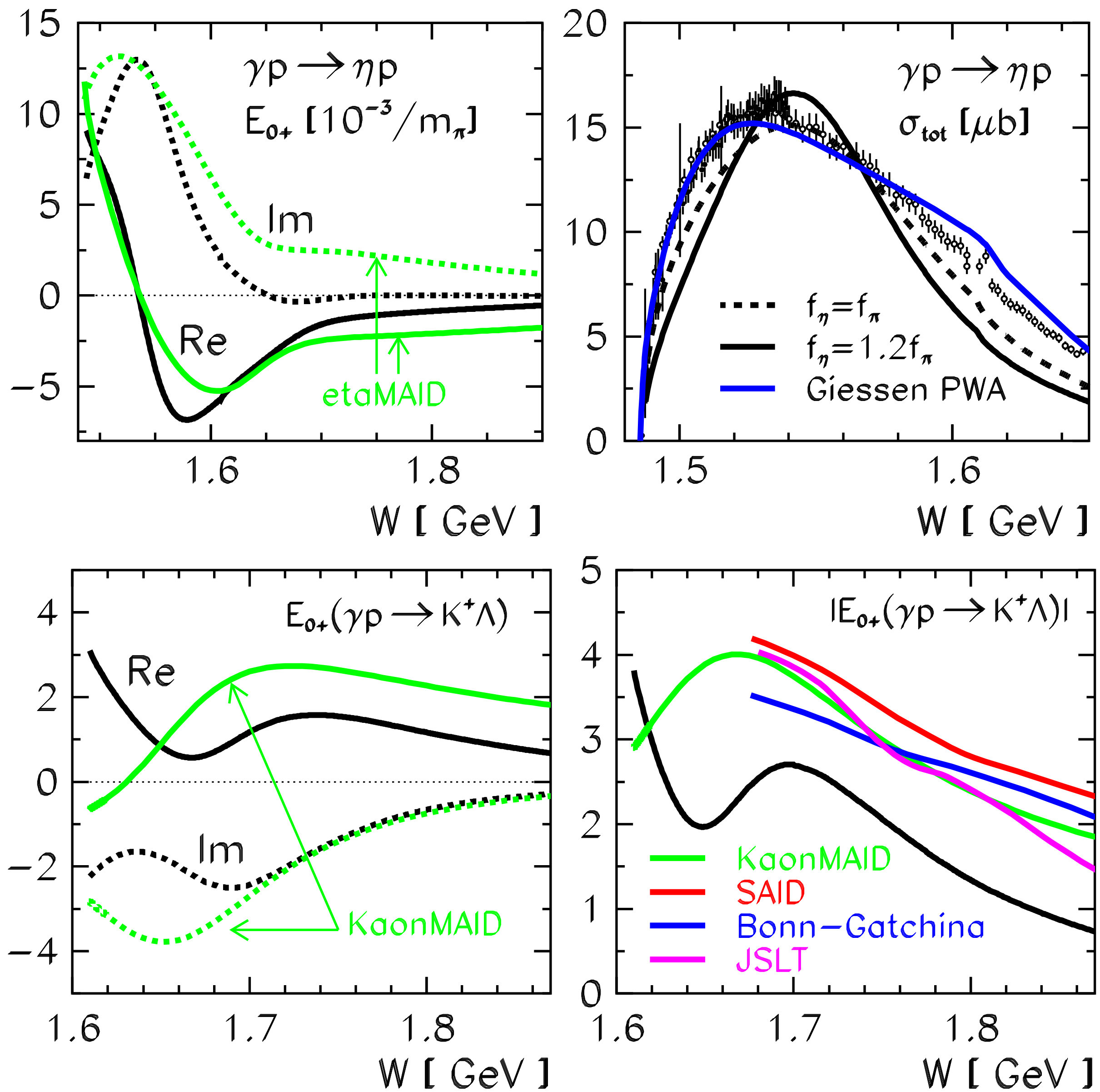
$[10^{-3}\,\text{GeV}^{-1/2}]$



S11 transverse amplitudes $\gamma p \to p\pi^0$



Eta, kaon photoproduction



work
in progress

Summary

- Using a single set of parameters we reproduce the main features of pion- and photon-induced production of π, η, and K mesons in P11 and S11 partial waves.
- Importance of the meson cloud:
  - it enhances the bare baryon-meson couplings;
  - it improves the behaviour of the helicity amplitudes at low Q^2.
- Enhancement of couplings stronger for P11 and P33 than in the case of S11 resonances which are dominated by quark-core contributions.

Spare slides

Lippmann-Schwinger equation for the K-matrix

$$\chi_{JT}^{NN}(k,k_0) = -\sum_B c_B^N(W) V_{NB}(k) + \mathcal{K}^{NN}(k,k_0) + \int \mathrm{d}k' \, \frac{\mathcal{K}^{NN}(k,k')\chi_{JT}^{NN}(k',k_0)}{\omega'_k + E_N(k') - W} + \int \mathrm{d}k' \, \frac{\mathcal{K}_{M_\Delta}^{N\Delta}(k,k')\hat{\chi}_{JT}^{\Delta N}(k',k_0)}{\omega'_k + E_\Delta(k') - W}$$

$$\hat{\chi}_{JT}^{\Delta\Delta}(k,k_1) = -\sum_B \hat{c}_B^\Delta(W,M) \, V_{\Delta B}^{M'}(k) + \mathcal{K}_{M'M}^{\Delta\Delta}(k,k_1) + \int \mathrm{d}k' \, \frac{\mathcal{K}_{M'M_\Delta}^{\Delta\Delta}(k,k')\hat{\chi}_{JT}^{\Delta\Delta}(k',k_1)}{\omega'_k + E_\Delta(k') - W} + \int \mathrm{d}k' \, \frac{\mathcal{K}_{M'}^{\Delta N}(k,k')\hat{\chi}_{JT}^{N\Delta}(k',k_1)}{\omega'_k + E_N(k') - W}$$

$$\hat{\chi}_{JT}^{\Delta N}(k,k_0) = -\sum_B c_B^N(W) V_{\Delta B}^{m}(k) + \mathcal{K}_M^{\Delta N}(k,k_0) + \int \mathrm{d}k' \, \frac{\mathcal{K}_M^{\Delta N}(k,k')\chi_{JT}^{NN}(k',k_0)}{\omega'_k + E_N(k') - W} + \int \mathrm{d}k' \, \frac{\mathcal{K}_{MM_\Delta}^{\Delta\Delta}(k,k')\hat{\chi}_{JT}^{\Delta N}(k',k_0)}{\omega'_k + E_\Delta(k') - W}$$

$$\hat{\chi}_{JT}^{N\Delta}(k,k_1) = -\sum_B \hat{c}_B^\Delta(W,M) V_{NB}(k) + \mathcal{K}_M^{N\Delta}(k,k_1) + \int \mathrm{d}k' \, \frac{\mathcal{K}_{M_\Delta}^{N\Delta}(k,k')\chi_{JT}^{\Delta\Delta}(k',k_1)}{\omega'_k + E_\Delta(k') - W} + \int \mathrm{d}k' \, \frac{\mathcal{K}^{NN}(k,k')\hat{\chi}_{JT}^{N\Delta}(k',k_1)}{\omega'_k + E_N(k') - W}$$

$$(W - M_B^0) c_B^N(W) = V_{NB}(k_0) + \int \mathrm{d}k \, \frac{\hat{\chi}_{JT}^{\Delta N}(k,k_0)\, V_{\Delta B}(k)}{\omega_k + E_\Delta(k) - W} + \int \mathrm{d}k \, \frac{\chi_{JT}^{NN}(k,k_0)\, V_{NB}(k)}{\omega_k + E_N(k) - W}$$

$$(W - M_B^0)\hat{c}_B^\Delta(W,M) = V_{\Delta B}(k_1) + \int \mathrm{d}k \, \frac{\chi_{JT}^{N\Delta}(k,k_1)\, V_{NB}(k)}{\omega_k + E_N(k) - W} + \int \mathrm{d}k \, \frac{\hat{\chi}_{JT}^{\Delta\Delta}(k,k_1)\, V_{\Delta B}(k)}{\omega_k + E_\Delta(k) - W}$$

Solving the Lippmann-Schwinger equation: separable kernels

$$\frac{1}{\omega_k + \omega'_k - \omega_0 + E_B(\bar{k}) - E_N(k_0)} \approx \frac{\omega_0 + E_B(\bar{k}) - E_N(k_0)}{(\omega_k + E_B(\bar{k}) - E_N(k_0))(\omega'_k + E_B(\bar{k}) - E_N(k_0))}$$

$$\bar{k}^2 \approx \langle(\boldsymbol{k}_0 + \boldsymbol{k}_1)^2\rangle \approx k_0^2 + k_1^2\,, \qquad E_B(\bar{k}) + E_N(k_0) - \omega_0 \approx 2M_B$$

$$\begin{aligned}
\mathcal{K}^{NN}(k,k') &= \sum_i f_{NN}^{B_i}\frac{M_{Bi}}{E_N}\,(\omega_0 + \varepsilon_i^N)\frac{\mathcal{V}_{B_iN}(k')\mathcal{V}_{B_iN}(k)}{(\omega'_k + \varepsilon_i^N)(\omega_k + \varepsilon_i^N)} \\
\mathcal{K}_M^{N\Delta}(k,k') &= \sum_i f_{N\Delta}^{B_i}\frac{M_{Bi}}{E}\,(\omega_1 + \varepsilon_i^N)\frac{\mathcal{V}_{B_iN}(k')\mathcal{V}_{B_i\Delta}(k)}{(\omega'_k + \varepsilon_i^N)(\omega_k + \varepsilon_i^\Delta(M))} = \mathcal{K}_M^{\Delta N}(k',k) \\
\mathcal{K}_{M'M}^{\Delta\Delta}(k,k') &= \sum_i f_{\Delta\Delta}^{B_i}\frac{M_{Bi}}{E'}\,(\omega'_1 + \varepsilon_i^\Delta(M))\frac{\mathcal{V}_{B_i\Delta}(k)}{(\omega_k + \varepsilon_i^\Delta(M))}\frac{\mathcal{V}_{B_i\Delta}(k')}{(\omega'_k + \varepsilon_i^\Delta(M'))}
\end{aligned}$$

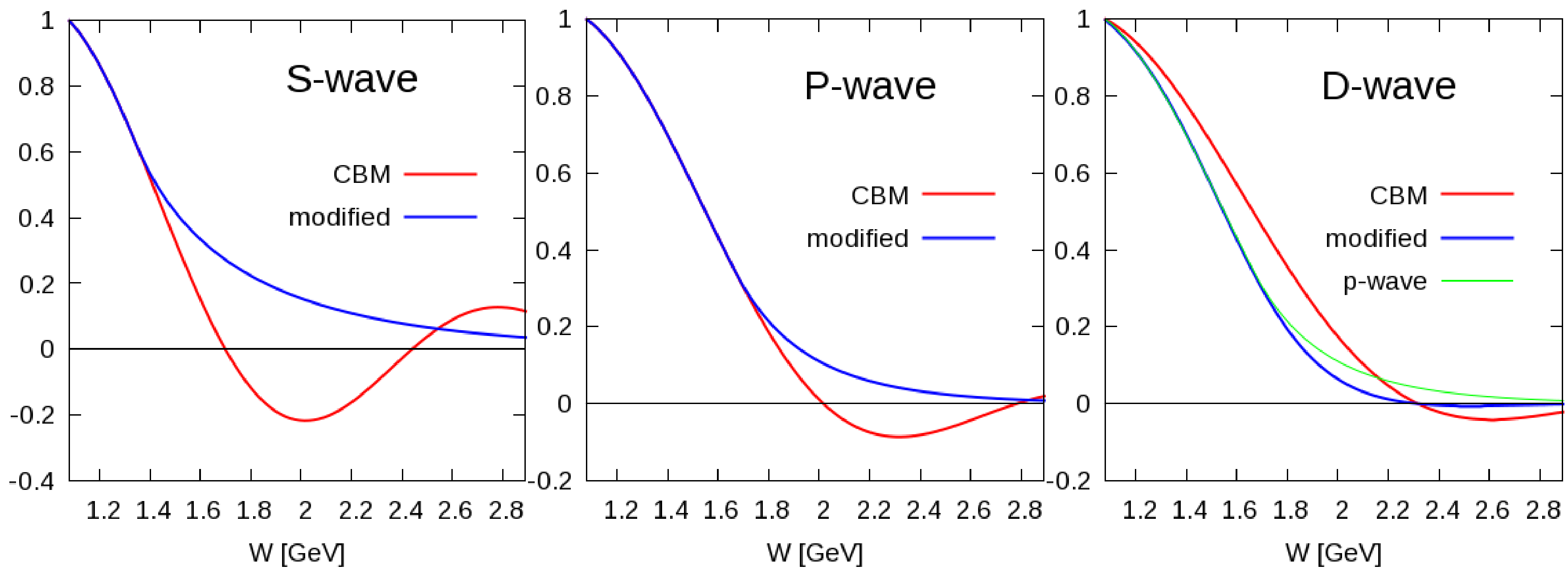
$$\varepsilon_i^N = \frac{M_{Bi}^2 - M_N^2 - m_\pi^2}{2E_N}\,, \qquad \varepsilon_i^\Delta(M) = \frac{M_{Bi}^2 - M^2 - m_\pi^2}{2E}\,,$$

Form factors of S, P and D-wave mesons-quark interaction

Determined by the bag radius $R_{\mathrm{bag}} = 0.83$ fm

Equivalent dipole momentum cut-off:

$$\Lambda_S = 510 \text{ MeV/c}, \qquad \Lambda_P = 550 \text{ MeV/c}, \qquad \Lambda_D = 550 \text{ MeV/c}$$



π-quark vertex: S, P, and D-wave pions

$$V^{\pi}_{l=0,t}(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\,\mathcal{P}_{sp}(i)$$

$$\begin{aligned} V^{\pi}_{1mt}(k) &= \frac{1}{2f_\pi}\frac{\omega_s}{(\omega_s-1)}\,\frac{1}{2\pi}\,\frac{1}{\sqrt{3}}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_1(kR)}{kR}\sum_{i=1}^{3}\tau_t(i) \\ &\times\left(\sigma_m(i)+\frac{\omega_{p_{1/2}}(\omega_s-1)}{\omega_s(\omega_{p_{1/2}}+1)}\,S^{[\frac{1}{2}]}_{1m}(i)+\frac{2\omega_{p_{3/2}}(\omega_s-1)}{5\omega_s(\omega_{p_{3/2}}-2)}\,S^{[\frac{3}{2}]}_{1m}(i)\right)\end{aligned}$$

$$V^{\pi}_{2mt}(k) = \frac{1}{2f_\pi}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\frac{\sqrt{2}}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\,\Sigma^{[\frac{1}{2}\frac{3}{2}]}_{2m}(i)$$

$$\mathcal{P}_{sp} = \sum_{m_j}|s\,m_j\rangle\langle p_{1/2}m_j| \qquad S^{[\frac{3}{2}]}_{1m} = \frac{\sqrt{15}}{2}\sum_{m_j m_j'} C^{\frac{3}{2}m_j}_{\frac{3}{2}m_j'1m}|p_{3/2}m_j\rangle\langle p_{3/2}m_j'|$$

$$S^{[\frac{1}{2}]}_{1m} = \sqrt{3}\sum_{m_j m_j'} C^{\frac{1}{2}m_j}_{\frac{1}{2}m_j'1m}|p_{1/2}m_j\rangle\langle p_{1/2}m_j'| \qquad \Sigma^{[\frac{1}{2}\frac{3}{2}]}_{2m} = \sum_{m_s m_j} C^{\frac{1}{2}m_s}_{\frac{3}{2}m_j2m}|s\,m_s\rangle\langle p_{3/2}m_j|$$

η-quark and K-quark vertex (S-wave)

$$V^{\eta}(k) = \frac{1}{2f_{\eta}}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_{i=1}^{3}\lambda_8(i)\,\mathcal{P}_{sp}(i)$$

$$V_t^{K}(k) = \frac{1}{2f_{K}}\sqrt{\frac{\omega_{p_{1/2}}\omega_s}{(\omega_{p_{1/2}}+1)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR} \times\sum_{i=1}^{3}(V_t(i)+U_t(i))\,\mathcal{P}_{sp}(i)$$

$$t = \pm\tfrac{1}{2},\ V_{\pm t} = (\lambda_4 \pm \mathrm{i}\lambda_5)/\sqrt{2}\ U_{\pm t} = (\lambda_6 \pm \mathrm{i}\lambda_7)/\sqrt{2}$$

$f_\eta = f_\pi$ or $f_\eta = 1.2\,f_\pi$

$f_K = 1.20\,f_\pi$.

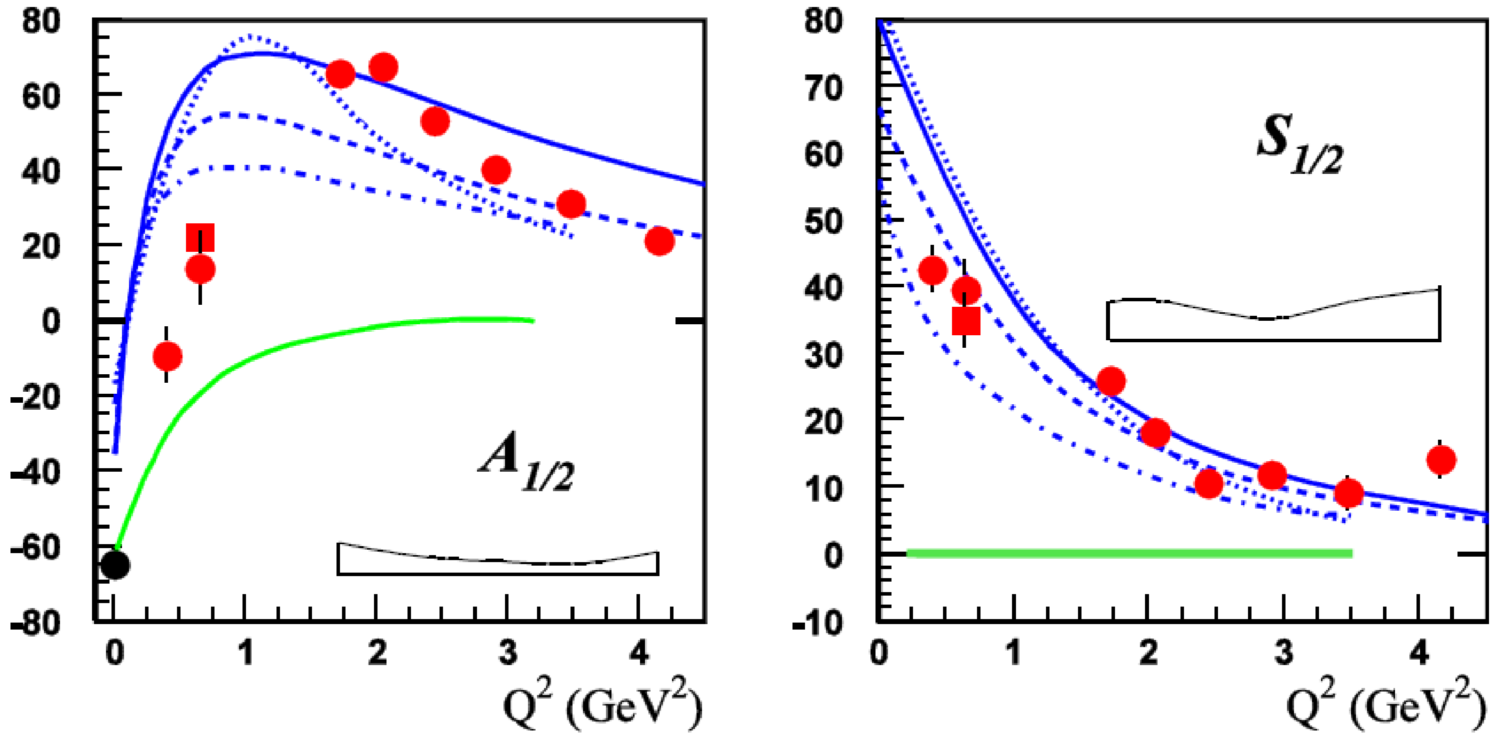
ρ-quark vertex ($S = \frac{1}{2}$, S-wave and $S = \frac{3}{2}$, D-wave)

$$
\begin{aligned}
V^{\rho}_{l=0mt}(k) &= \frac{1}{2f_\rho}\sqrt{\frac{\omega_s}{(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_0(kR)}{kR}\sum_i \tau_t(i) \\
&\quad\times\left(\frac{\sqrt{8}}{3}\sqrt{\frac{\omega_{p_{1/2}}}{\omega_{p_{1/2}}+1}}\,\Sigma^{[\frac{1}{2}]}_{1m} + 3\sqrt{\frac{\omega_{p_{3/2}}}{\omega_{p_{3/2}}-2}}\,\Sigma^{[\frac{1}{2}\frac{3}{2}]}_{1m}(i)\right) \\
V^{\rho}_{l=2mt}(k) &= \frac{1}{2f_\rho}\sqrt{\frac{\omega_{p_{3/2}}\omega_s}{(\omega_{p_{3/2}}-2)(\omega_s-1)}}\,\frac{1}{2\pi}\,\frac{1}{3}\,\frac{k^2}{\sqrt{\omega_k}}\,\frac{j_2(kR)}{kR}\sum_{i=1}^{3}\tau_t(i)\Sigma^{[\frac{1}{2}\frac{3}{2}]}_{1m}(i)
\end{aligned}
$$

$f_\rho = 200$ MeV

$$
\Sigma^{[\frac{1}{2}]}_{1m} = \sum_{m_s m_j} C^{\frac{1}{2}m_s}_{\frac{1}{2}m_j 1m}\,|s m_s\rangle\langle p_{1/2} m_j| \qquad\qquad \Sigma^{[\frac{1}{2}\frac{3}{2}]}_{1m} = \sum_{m_s m_j} C^{\frac{1}{2}m_s}_{\frac{3}{2}m_j 1m}\,|s m_s\rangle\langle p_{3/2} m_j|
$$

$P_{11}(1440)$ — q^3 or q^3g ?



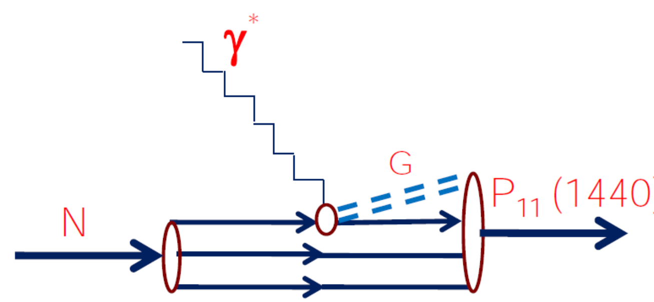
Weber / PRC **41** (1990) 2783

Capstick, Keister / PRD **51** (1995) 3598

Cardarelli++ / PLB **397** (1997) 13

Aznauryan / PRC **76** (2007) 025212

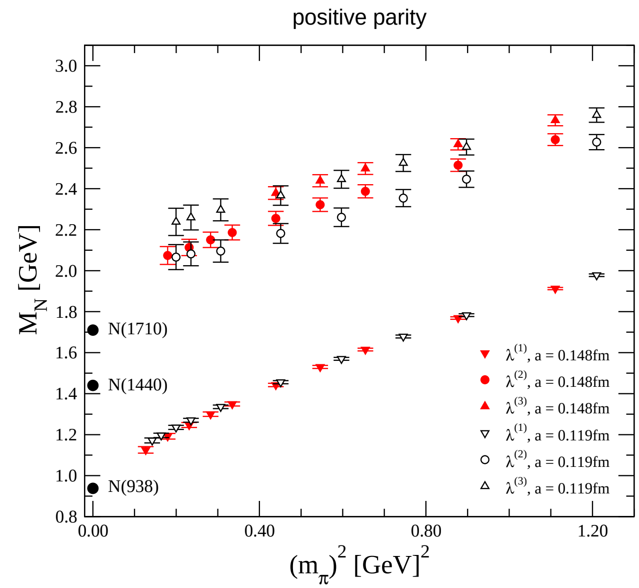
Li, Burkert, Li / PRD **46** (1992) 70



- **sign change** of $A_{1/2}$
- evidence for Roper as **radial excitation** of 3q
- **nonzero** $S_{1/2}$, hybrid q^3g picture **ruled out**

$P_{11}(1440)$ and $S_{11}(1535)$ on the Lattice

- close to chiral limit, effects of χSB important
- level ordering should change with m_q
  Heavy q: 1st radial above 1st orbital excitation
  chiral limit: reversed levels



Bern-Graz-Regensburg / PRD **70** (2004) 054502
PRD **74** (2006) 014504 ⟶

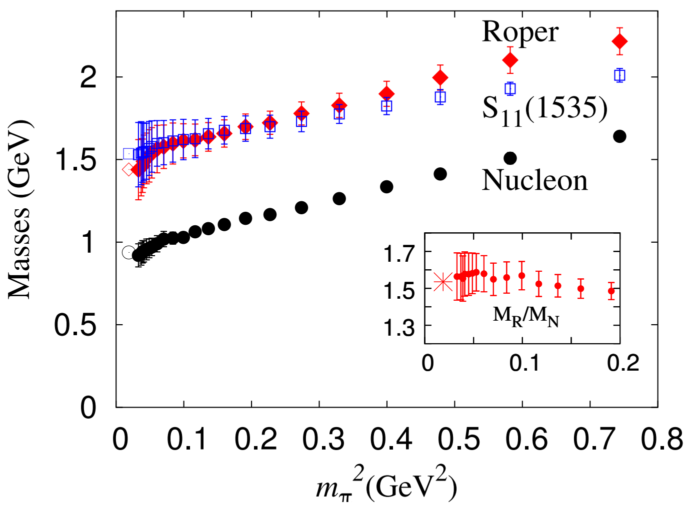
"... do not attempt a chiral extrapolation of our data ... numbers seem to approach the experimental data reasonably well"

"... the Roper's leading Fock component is a 3-quark state"

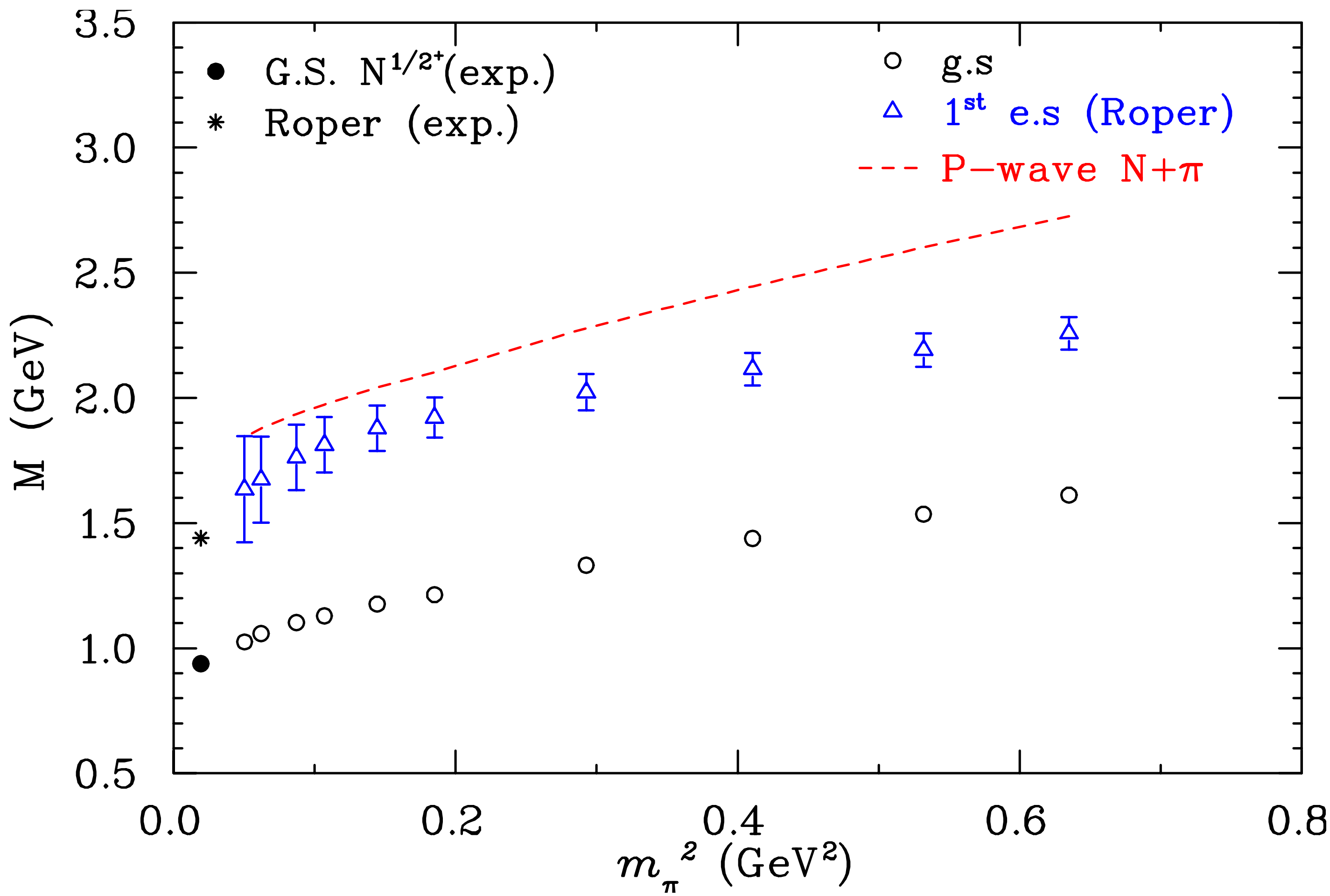
Kentucky / PLB **605** (2005) 137

"...+ and − parity excited states of the nucleon tend to cross over as the quark masses are taken to the chiral limit. Both results at the physical pion mass agree with the exp values ... seen for the first time in a lattice QCD calculation"

"...a successful description of the Roper resonance depends not so much on the use of the dynamical quarks ... most of the essential physics is captured by using light quarks"



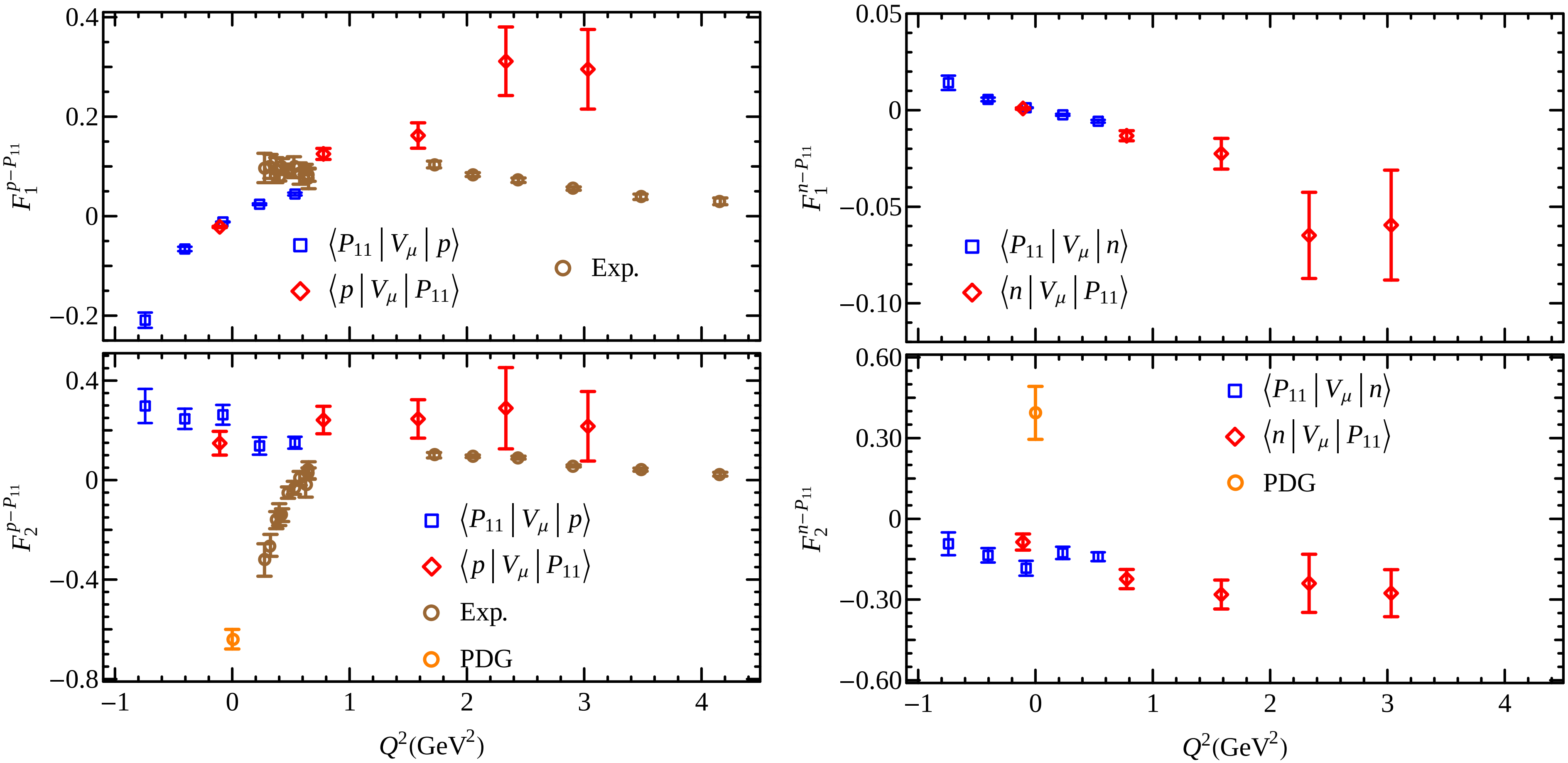
$P_{11}(1440)$ on the Lattice



Adelaide/JLab / PLB **679** (2009) 418 — quenched, FLIC fermion action

"A lower lying Roper state is observed that approaches the physical Roper state.
To the best of our knowledge, the first time this state has been identified
at light quark masses using a variational approach."

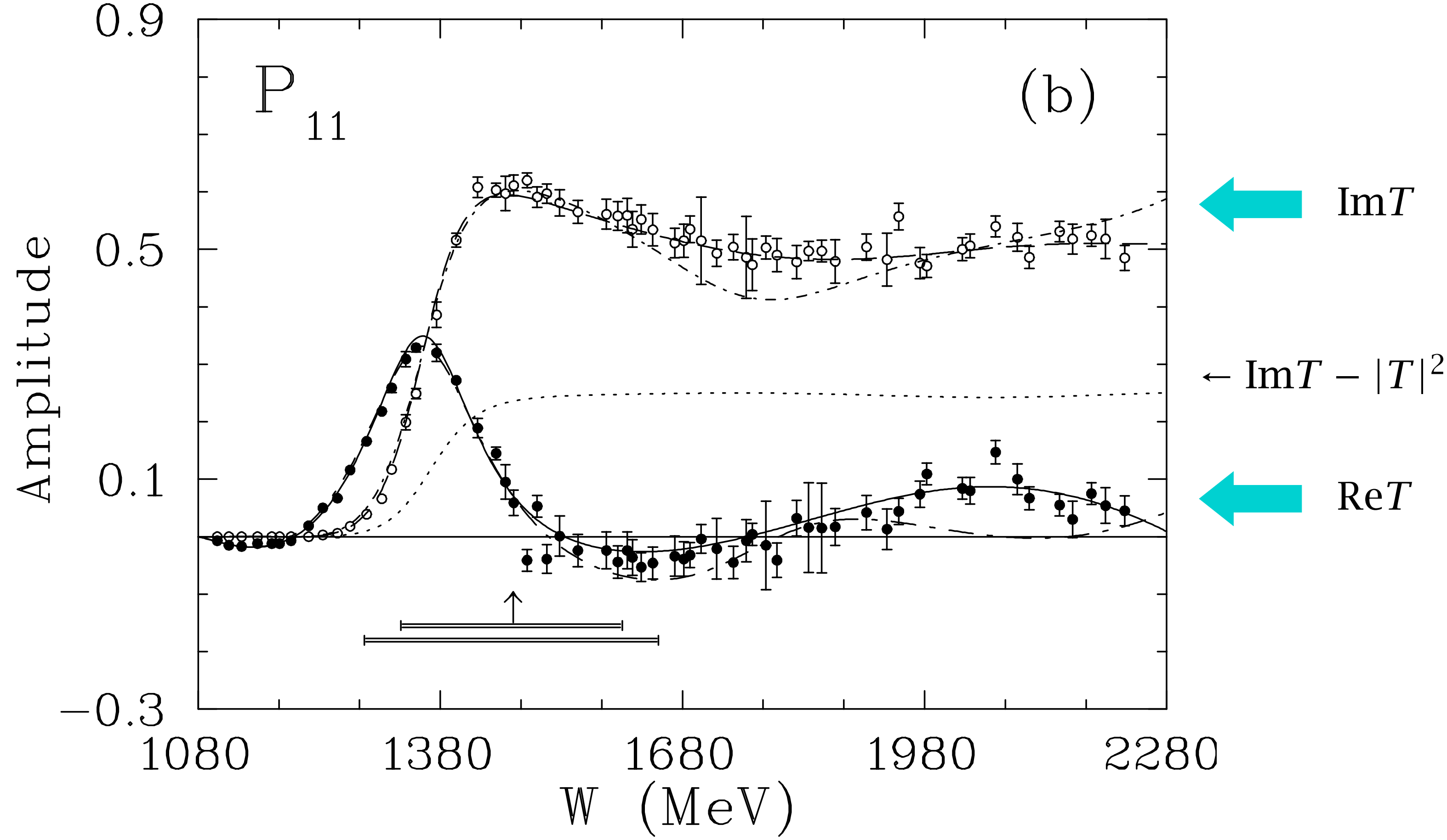
Lattice $N \to P_{11}(1440)$ EM transition form-factors



- quenched, $m_\pi = 720\,\mathrm{MeV}$ (!)
- "exploratory study"

Lin++ / PRD **78** (2008) 114508

SAID PWA of πN scattering in P_{11} channel



SAID FA02 $M_{\mathrm{BW}} = (1468 \pm 4.5)$ MeV, $\Gamma/2 = (180 \pm 13)$ MeV

$M_{\mathrm{pole}} = 1357 - i\,80$ MeV (I RS)

$1385 - i\,83$ MeV (II RS)

Ansaetze for the channel states

πN channel:

unmodified (free) π

bare baryons (3q)

$$|\Psi^{\pi N}_{JT}(W)\rangle = \sqrt{\frac{\omega_0 E_N(k_0)}{k_0 W}}\left\{\boxed{[a^\dagger_\pi(k_0)|\Psi_N(k_0)\rangle]^{JT}} + \sum_B c^N_B(W)|\Phi_B\rangle + \int \frac{\mathrm{d}k \quad \chi^{NN}_{JT}(k,k_0)}{\omega_k + E_N(k) - W}[a^\dagger_\pi(k)|\Psi_N(k)\rangle]^{JT} + \int \mathrm{d}M' \int \frac{\mathrm{d}k \ \chi^{\Delta N}_{JT}(k,k_0,M')}{\omega_k + E'(k) - W}[a^\dagger_\pi(k)|\widetilde{\Psi}_\Delta(M')\rangle]^{JT} + \int \frac{\mathrm{d}k \quad \chi^{mN}_{JT}(k,k_0)}{\omega_k + E_N(k) - W}[a^\dagger_m(k)|\Psi_N(k)\rangle]^{JT}\right\}$$

meson "clouds" with amplitudes χ

$\pi\Delta$ channel:

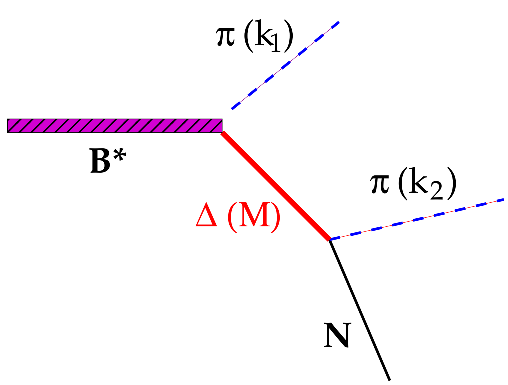
$$|\Psi^{\pi\Delta}_{JT}(W,M)\rangle = \sqrt{\frac{\omega_1 E(k_1)}{k_1 W}}\left\{\boxed{[a^\dagger_\pi(k_1)|\widetilde{\Psi}_\Delta(M)\rangle]^{JT}} + \sum_B c^\Delta_B(W,M)|\Phi_B\rangle + \int \frac{\mathrm{d}k \ \chi^{N\Delta}_{JT}(k,k_1,M)}{\omega_k + E_N(k) - W}[a^\dagger_\pi(k)|\Psi_N(k)\rangle]^{JT} + \int \mathrm{d}M' \int \frac{\mathrm{d}k \ \chi^{\Delta\Delta}_{JT}(k,k_1,M',M)}{\omega_k + E'(k) - W}[a^\dagger_\pi(k)|\widetilde{\Psi}_\Delta(M')\rangle]^{JT}\right\} + \ldots$$

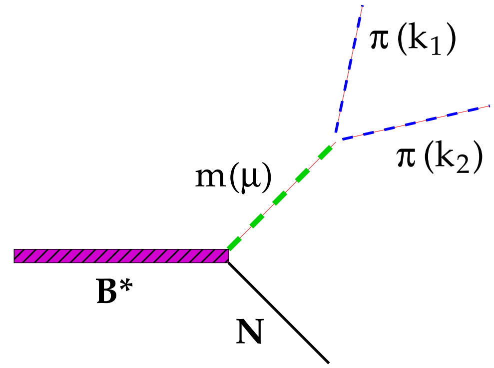
mN channel (e.g. $m = \sigma$):

$$|\Psi^{mN}_{JT}(W,\mu)\rangle = \sqrt{\frac{\omega_m E_N(k_m)}{k_m W}}\left\{\boxed{[a^\dagger_m(k_m)|\Psi_N(k_m)\rangle]^{JT}} + \sum_B c^m_B(W,\mu)|\Phi_B\rangle + \int \mathrm{d}\mu' \int \frac{\mathrm{d}k \quad \chi^{mm}_{JT}(k,k_m,\mu,\mu')}{\omega_k + E_N(k) - W}[a^\dagger_{m'}(k)|\Psi_N(k)\rangle]^{JT} + \int \frac{\mathrm{d}k \quad \chi^{Nm}_{JT}(k,k_m)}{\omega_k + E_N(k) - W}[a^\dagger_\pi(k)|\Psi_N(k)\rangle]^{JT}\right\} + \ldots$$

Kinematics of two-pion decay

$$\pi N \to B^* \to \pi\Delta \to \pi\pi N \qquad \pi \mathrm{N} \to B^* \to mN \to (2\pi)N$$





$$\omega_1 = W - E = \frac{W^2 - M^2 + m_\pi^2}{2W}, \qquad \omega_\mu = W - E_N = \frac{W^2 - M_N^2 + \mu^2}{2W},$$

$$k_1 = \sqrt{\omega_1^2 - m_\pi^2}\,, \qquad E = \sqrt{M^2 + k_1^2} \qquad k_\mu = \sqrt{\omega_\mu^2 - \mu^2} \qquad E_N = \sqrt{M_N^2 + k_\mu^2}\,.$$

The intermediate Δ state

$$\langle\tilde{\Psi}_\Delta(M)|\tilde{\Psi}_\Delta(M')\rangle = \delta(M - M')$$

$$|\tilde{\Psi}_\Delta(M)\rangle \approx \frac{K}{\sqrt{1+K^2}}\left\{|\Phi_\Delta\rangle - \int\frac{\mathrm{d}k\;\mathcal{V}_{N\Delta}(k)}{\omega_k + E_N(k) - M}\,[a^\dagger(k)|\Phi_N\rangle]^{\frac{3}{2}\frac{3}{2}} - \int\frac{\mathrm{d}k\;\mathcal{V}_{\Delta\Delta}(k)}{\omega_k + E_\Delta(k) - M}\,[a^\dagger(k)|\Phi_\Delta\rangle]^{\frac{3}{2}\frac{3}{2}}\right\}$$

$$\frac{K}{\sqrt{1+K^2}} \approx \frac{1}{\pi}\frac{(\frac{1}{2}\Gamma_\Delta)^2}{(M_\Delta - M)^2 + (\frac{1}{2}\Gamma_\Delta)^2}$$

Connection between K-matrix elements and pion aplitudes

$$
\begin{aligned}
\text{Above } \pi \text{ threshold :} \quad K_{\pi N \pi N}(W) &= \pi \frac{\omega_0 E_N(k_0)}{k_0 W} \chi_{JT}^{NN}(k_0, k_0) \\
\text{Above } 2\pi \text{ threshold :} \quad K_{\pi \Delta \pi N}(W, M) &= \pi \sqrt{\frac{\omega_0 E_N(k_0) \omega_1 E(k_1)}{k_0 k_1 W^2}} \chi_{JT}^{\Delta N}(k_1, k_0, M) \\
K_{\pi N \pi \Delta}(W, M) &= \pi \sqrt{\frac{\omega_0 E_N(k_0) \omega_1 E(k_1)}{k_0 k_1 W^2}} \chi_{JT}^{N\Delta}(k_0, k_1, M) \\
K_{\pi \Delta \pi \Delta}(W, M', M) &= \pi \sqrt{\frac{\omega_1 E(k_1) \omega_1' E(k_1')}{k_1 k_1' W^2}} \chi_{JT}^{\Delta\Delta}(k_1', k_1, M', M) \\
\text{Above } m(2\pi) \text{ threshold :} \quad K_{mNmN}(W) &= \pi \frac{\omega_m E_N(k_m)}{k_m W} \chi_{JT}^{mm}(k_m, k_m)
\end{aligned}
$$

The form of amplitudes χ

$$
\chi_{JT}^{NN} = -\sum_{\mathcal{R}} c_{\mathcal{R}}^{N}(W) \underbrace{\mathcal{V}_{N\mathcal{R}}(k)}_{\text{dressed vertex}} + \underbrace{\mathcal{D}^{NN}(k, k_0)}_{\text{background part}}
$$

$$
\chi_{JT}^{B'B} = -\sum_{\mathcal{R}} c_{\mathcal{R}}^{B}(W, M) \mathcal{V}_{B'\mathcal{R}}^{M'}(k) + \mathcal{D}_{M'M}^{B'B}(k, k_1) \qquad \text{etc.}
$$

$\langle \delta\Psi | H - E | \Psi \rangle \quad \Longrightarrow$

- **Lippmann-Schwinger equation for χ**

$$\mathcal{V}_{N\mathcal{R}} = V_{N\mathcal{R}} + \int \mathrm{d}k' \, \frac{\mathcal{K}^{NN}(k,k')\mathcal{V}_{N\mathcal{R}}(k')}{\omega_k' + E_N(k') - W} + \sum_{B'} \int \mathrm{d}k' \, \frac{\mathcal{K}^{NB'}_{M_{B'}}(k,k')\mathcal{V}^{M_{B'}}_{B'\mathcal{R}}(k')}{\omega_k' + E_{B'}(k') - W}$$

$$\mathcal{V}^{M}_{B\mathcal{R}} = V^{M}_{B\mathcal{R}} + \int \mathrm{d}k' \, \frac{\mathcal{K}^{BN}_{M}(k,k')\mathcal{V}_{N\mathcal{R}}(k')}{\omega_k' + E_N(k') - W} + \sum_{B'} \int \mathrm{d}k' \, \frac{\mathcal{K}^{BB'}_{MM_{B'}}(k,k')\mathcal{V}^{M_{B'}}_{B'\mathcal{R}}(k')}{\omega_k' + E_{B'}(k') - W}$$

- **System of linear eqs for coefficients $c^{H}_{\mathcal{R}'}$ of the bare 3q states** $(H \in \{\pi N\,,\ \pi B\,,\ \sigma B\,\})$

$$\sum_{\mathcal{R}'} A_{\mathcal{R}\mathcal{R}'}(W)\, c^{H}_{\mathcal{R}'}(W, m_H) = \mathcal{V}^{M}_{H\mathcal{R}}(k_H)$$

$$A_{\mathcal{R}\mathcal{R}'} = (W - M^0_{\mathcal{R}})\delta_{\mathcal{R}\mathcal{R}'} + \sum_{\mathcal{B}'} \int \mathrm{d}k \, \frac{\mathcal{V}^{M_{B'}}_{B'\mathcal{R}}(k) V^{M_{B'}}_{B'\mathcal{R}'}(k)}{\omega_k + E_{B'}(k) - W}$$

$\mathcal{R}$ **are not eigenstates of Hamiltonian and therefore they mix:** $|\widetilde{\Phi}_{\mathcal{R}}\rangle = \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'}|\Phi_{\mathcal{R}}\rangle$

Diagonalize A to obtain U**, the poles of the K matrix, and wave-function normalization** Z

$$UAU^T = D\,, \qquad D = \begin{bmatrix} Z_{\mathcal{R}}(W)(W - M_{\mathcal{R}}) & 0 & 0 \\ 0 & Z_{\mathcal{R}'}(W)(W - M_{\mathcal{R}'}) & 0 \\ 0 & 0 & Z_{\mathcal{R}''}(W)(W - M_{\mathcal{R}''}) \end{bmatrix}$$

Pion amplitudes pertaining to physical resonances H

$$\chi^{H'H} = \sum_{\mathcal{R}} \widetilde{\mathcal{V}}_{H\mathcal{R}} \frac{1}{Z_{\mathcal{R}}(W)(M_{\mathcal{R}} - W)} \widetilde{\mathcal{V}}_{H'\mathcal{R}}\,, \qquad \widetilde{\mathcal{V}}_{H\mathcal{R}} = \sum_{\mathcal{R}'} U_{\mathcal{R}\mathcal{R}'} \mathcal{V}_{H\mathcal{R}'}$$

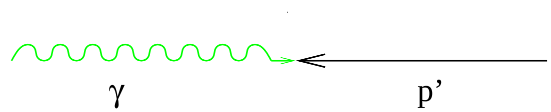
Solution for the K matrix

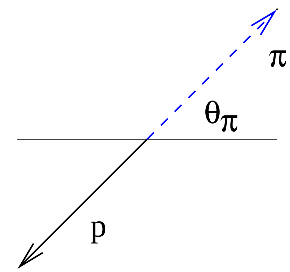
$$K_{HH'} = K_{HH'}(\text{resonant}) + K_{HH'}(\text{background}) = \pi \mathcal{N}_H \mathcal{N}_{H'} \left\{ \sum_{\mathcal{R}} \frac{\widetilde{\mathcal{V}}_{H\mathcal{R}} \widetilde{\mathcal{V}}_{H'\mathcal{R}}}{Z_{\mathcal{R}}(W)(M_{\mathcal{R}} - W)} + \mathcal{D}_{HH'} \right\}$$

T matrix and scattering matrix S
same structure for all channels

$$\begin{aligned} T &= K + \mathrm{i}\,TK \\ S &= I + 2\mathrm{i}\,T \end{aligned}$$

Pion electro-production





Formally, the K matrix acquires a new channel, γN

Because the EM interaction is considerably weaker than the strong interaction, we assume

$$K_{\gamma N\,\gamma N} \ll K_{\gamma N\,\pi N} \ll K_{\pi N\,\pi N}$$

(and similarly for other channels). The Heitler-like equation for the electro-production amplitudes then reduces to

$$\begin{aligned} T_{\gamma N\pi N}(W) &= K_{\gamma N\pi N}(W) + \mathrm{i}\Big[\, T_{\pi N\pi N}(W) K_{\gamma N\pi N}(W) + \overline{T}_{\pi N\pi\Delta}(W,\bar{M}) K_{\gamma N\pi\Delta}(W,\overline{M}) \\ &\qquad + \overline{T}_{\pi N m N}(W,\bar{\mu}) K_{\gamma N m N}(W,\overline{\mu}) \,\Big] \end{aligned}$$

The T matrix for electro-production is related to the electro-production amplitudes by

$$T^{(JT)}_{\gamma N\pi N} = \mathrm{i}\pi \frac{1}{\sqrt{2\pi}^{3}} \sum_m \sqrt{k_0 k_\gamma}\, \mathcal{M}_N(W, M_J, M_T, t, k_\gamma, \mu)\, Y_{1m}(\hat{r})\, C^{JM_J}_{\frac{1}{2}m_s 1m}\, C^{TM_T}_{\frac{1}{2}\frac{1}{2}1t}$$

$$\begin{aligned} \mathcal{M}_{\gamma N\pi N}(W) &= \mathcal{M}^{K}_{\gamma N\pi N}(W) + \mathrm{i}\Big[\, T_{\pi N\pi N}(W) \mathcal{M}^{K}_{\gamma N\pi N}(W) + \overline{T}_{\pi N\pi\Delta}(W,\bar{M}) \mathcal{M}^{K}_{\gamma N\pi\Delta}(W,\overline{M}) + \cdots \Big] \\ &= \mathcal{M}^{(\mathrm{res})}_{\gamma N\pi N}(W) + \mathcal{M}^{(\mathrm{bkg})}_{\gamma N\pi N}(W) \end{aligned}$$

Evaluation of matrix elements

The resonant part of amplitude for a chosen $\mathcal{R} = N^*$:

$$\mathcal{M}^{(\mathrm{res})}_{\gamma N\pi N} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{\omega_0 E_N}}\,\frac{1}{\pi \mathcal{V}_{NN^*}}\,\langle\Psi^{(\mathrm{res})}_{N^*}(W)|\tilde{V}_\gamma|\Psi_N\rangle\, T_{\pi N\pi N} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{\omega_0 E_N}}\,\frac{1}{\pi \mathcal{V}_{NN^*}}\,\underbrace{A(\gamma N \to N^*)}_{A_{N^*}}\, T_{\pi N\pi N}$$

The background part obeys the equation in which the resonance pole is absent:

$$\mathcal{M}^{(\mathrm{bkg})}_{\gamma N\pi N} = \mathcal{M}^{K\ (\mathrm{bkg})}_{\gamma N\pi N} + \mathrm{i}\left[T_{\pi N\pi N}\mathcal{M}^{K\ (\mathrm{bkg})}_{\gamma N\pi N} + \overline{T}_{\pi N\pi\Delta}\overline{\mathcal{M}}^{K\ (\mathrm{bkg})}_{\gamma N\pi\Delta} + \overline{T}_{\pi N m N}\overline{\mathcal{M}}^{K\ (\mathrm{bkg})}_{\gamma N m N}\right]$$

The **helicity amplitude** A_{N*} for the electro-excitation of the resonance is proportional to the transition electromagnetic form factor:

$$A_{N^*} \equiv \langle\Psi^{(\mathrm{res})}_{N^*}(W)|\tilde{V}_\gamma|\Psi_N\rangle$$

For example:

$$\mathcal{M}^{K}_{\gamma N\pi N}(W) = -\sqrt{\frac{\omega_\gamma}{k_0}}\langle\Psi^{\pi N}_{N^*}(W)|\tilde{V}_\gamma(\mu,\boldsymbol{k}_\gamma)|\Psi_N\rangle \qquad \tilde{V}_\gamma(\mu,\boldsymbol{k}_\gamma) = \frac{\mathrm{e}_0}{\sqrt{2\omega_\gamma}}\int \mathrm{d}\boldsymbol{r}\,\boldsymbol{\varepsilon}_\mu\cdot\boldsymbol{j}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\boldsymbol{k}_\gamma\cdot\boldsymbol{r}}$$

The resonant state takes the form:

$$|\Psi^{(\mathrm{res})}_{N^*}(W)\rangle = \frac{1}{\sqrt{Z_{N^*}}}\left\{|\Phi_{N^*}\rangle - \int\frac{\mathrm{d}k\,\widetilde{\mathcal{V}}_{NN^*}(k)}{\omega_k + E_N(k) - W}\,[a^\dagger(k)|\Psi_N\rangle]^{JT} - \ldots\right\}$$

Resonances and quark-model wave-functions

resonance	M [MeV]	Γ [MeV]	decays
S11(1535)	1535	150	πN 35–55 %, ηN 45–60 % $2\pi N < 10$ %, (πR)
S11(1650)	1655	165	πN 60–95 %, ηN 3–10 %, $K\Lambda$ 3–11 % $2\pi N$ 10–20 %, ρ 4–12 %, Δ 1–7 %

$$
\begin{aligned}
\Phi(1535) &= -\sin\vartheta_s |^4 8_{1/2}\rangle + \cos\vartheta_s |^2 8_{1/2}\rangle \\
&= c_A^1 \,|\,(1s)^2(1p_{3/2})^1\rangle + c_P^1 \,|\,(1s)^2(1p_{1/2})^1\rangle_1 + c_{P'}^1 \,|\,(1s)^2(1p_{1/2})^1\rangle_2
\end{aligned}
$$

$$
\begin{aligned}
\Phi(1650) &= \cos\vartheta_s |^4 8_{1/2}\rangle + \sin\vartheta_s |^2 8_{1/2}\rangle \\
&= c_A^2 \,|\,(1s)^2(1p_{3/2})^1\rangle + c_P^2 \,|\,(1s)^2(1p_{1/2})^1\rangle_1 + c_{P'}^2 \,|\,(1s)^2(1p_{1/2})^1\rangle_2
\end{aligned}
$$

$$
c_A^1 = \frac{1}{3}(2\cos\vartheta_s - \sin\vartheta_s)\,, \qquad c_A^2 = \frac{1}{3}(\cos\vartheta_s + 2\sin\vartheta_s)\,, \qquad \vartheta_s = -30^\circ
$$

Myhrer, Wroldsen / Z. Phys. C **25** (1984) 281

Separation of amplitudes into resonant and background parts

Because the K matrix elements contain poles, it convenient to separate the amplitudes as

$$\mathcal{M}_H^K = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, K_{NH}\, \langle \Psi_{N*}^{(\mathrm{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle + \mathcal{M}_H^{K\ (\mathrm{bkg})} \qquad H = \pi N, \pi\Delta, mN$$

$$\begin{aligned}\mathcal{M}_H^{K\ (\mathrm{bkg})} &= -\sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\{g(W)\, K_{NH}^{(\mathrm{bkg})}\, \langle \Psi_{N*}^{(\mathrm{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle \\ &\quad + \sqrt{\frac{\omega_H E_H}{k_H W}}\left[c_N^H \langle \Psi_{N*}^{(\mathrm{n.p.})}|\tilde{V}_\gamma|\Psi_N\rangle + \langle \Psi_{N*}^{H\ (\mathrm{non-res})}|\tilde{V}_\gamma|\Psi_N\rangle\right]\}\end{aligned}$$

The resonant part takes the form

$$\mathcal{M}_{\pi N}^{(\mathrm{res})} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, \langle \Psi_{N*}^{(\mathrm{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle\, T_{\pi N\pi N} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\, g(W)\, A_{N*}\, T_{\pi N\pi N}$$

The background part obeys the EQ in which the resonance pole is absent:

$$\mathcal{M}_{\pi N}^{(\mathrm{bkg})} = \mathcal{M}_{\pi N}^{K\ (\mathrm{bkg})} + \mathrm{i}[T_{\pi N\pi N}\mathcal{M}_{\pi N}^{K\ (\mathrm{bkg})} + \overline{T}_{\pi N\pi\Delta}\overline{\mathcal{M}}_{\pi\Delta}^{K\ (\mathrm{bkg})} + \overline{T}_{\pi N mN}\overline{\mathcal{M}}_{mN}^{K\ (\mathrm{bkg})}]$$

Phase shifts — P11, P33

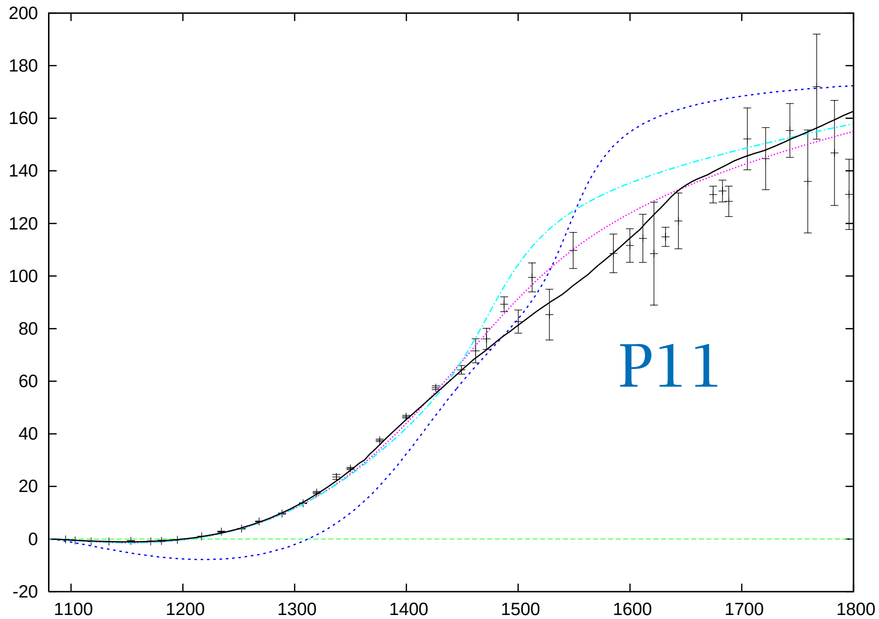
resonant term only, no background

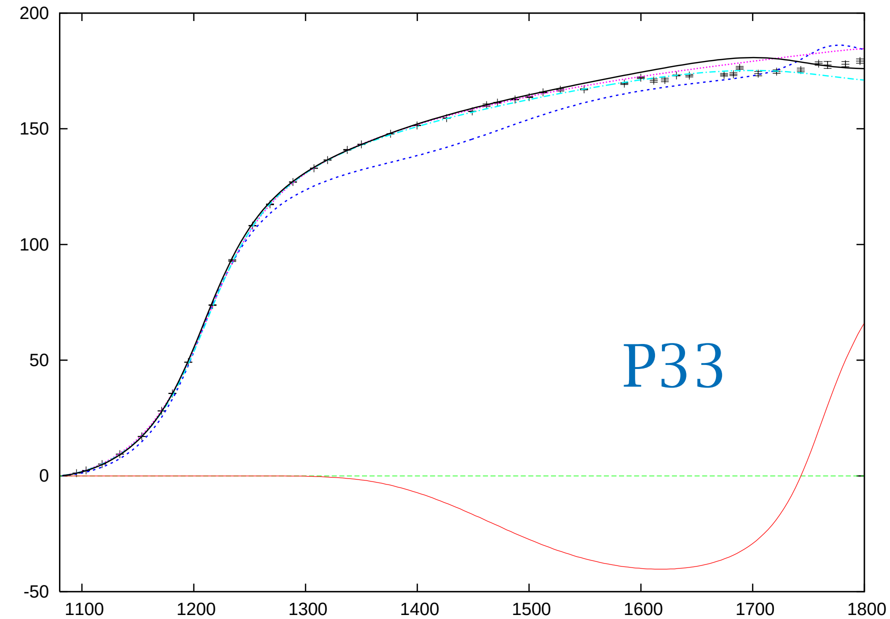
πN and $\pi\Delta$ channels

σN ($\sigma\Delta$) channel included :

Born approximation, $g_{\pi NR} = 1.68\; g_{\pi NR}^{\rm quark}$, $g_{\pi N\Delta} = 1.40\; g_{\pi N\Delta}^{\rm quark}$

Phase shift

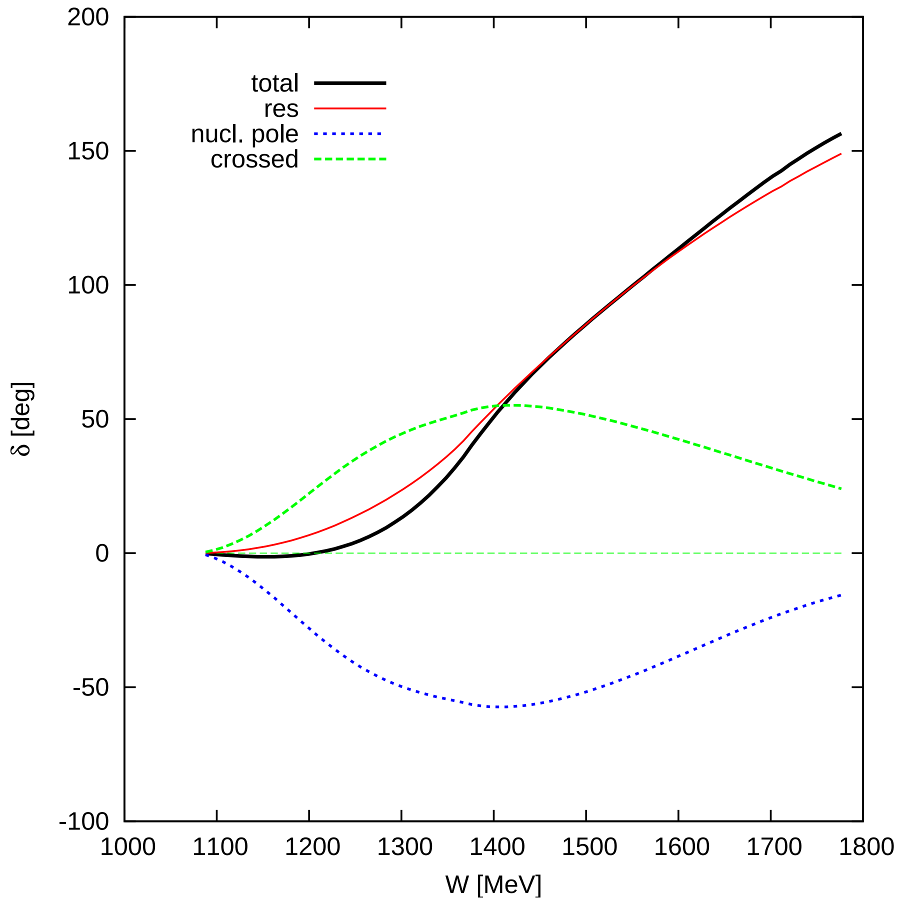


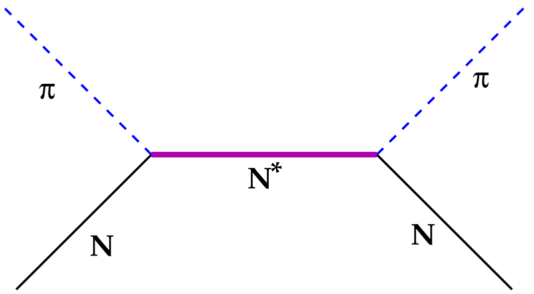


$$\frac{g_{\pi NR}}{g_{\pi NR}^{\rm quark}} = 1.0\,, \qquad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\rm quark}} = 1.0$$

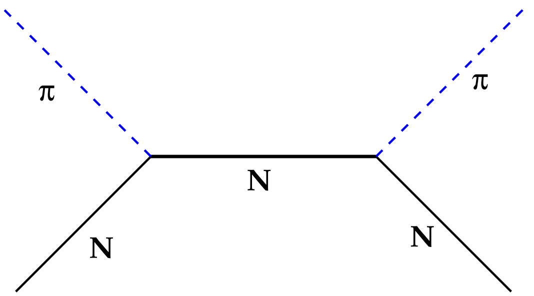
$$\frac{g_{\pi NR}}{g_{\pi NR}^{\rm quark}} = 1.0\,, \qquad \frac{g_{\pi N\Delta}}{g_{\pi N\Delta}^{\rm quark}} = 1.0$$

Resonant and background contributions to P11 phase shift

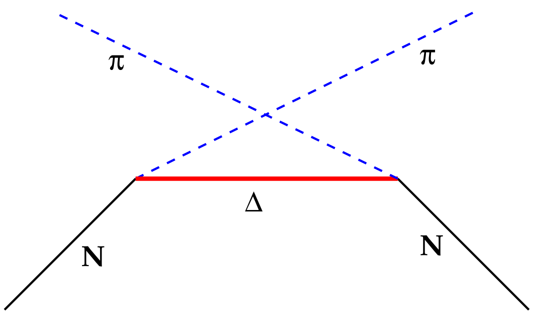




reson.

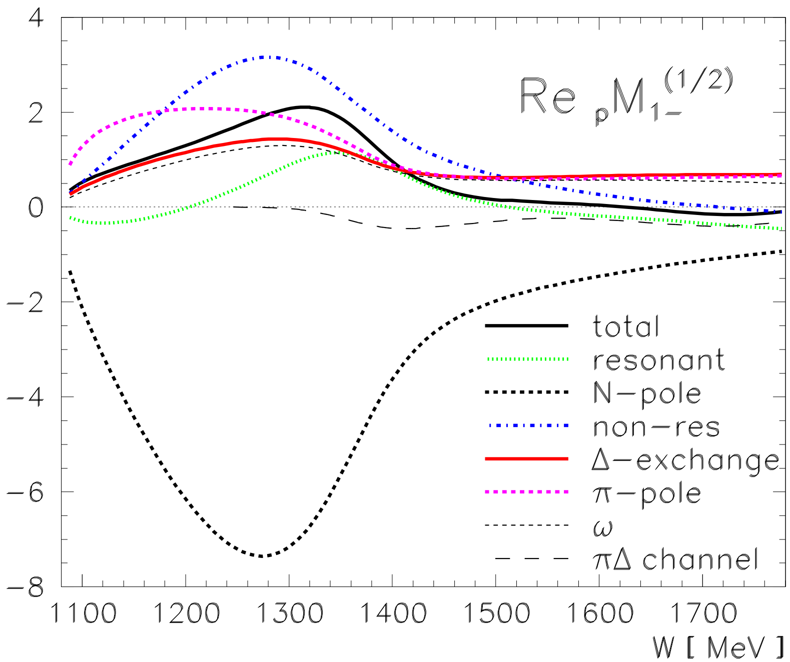


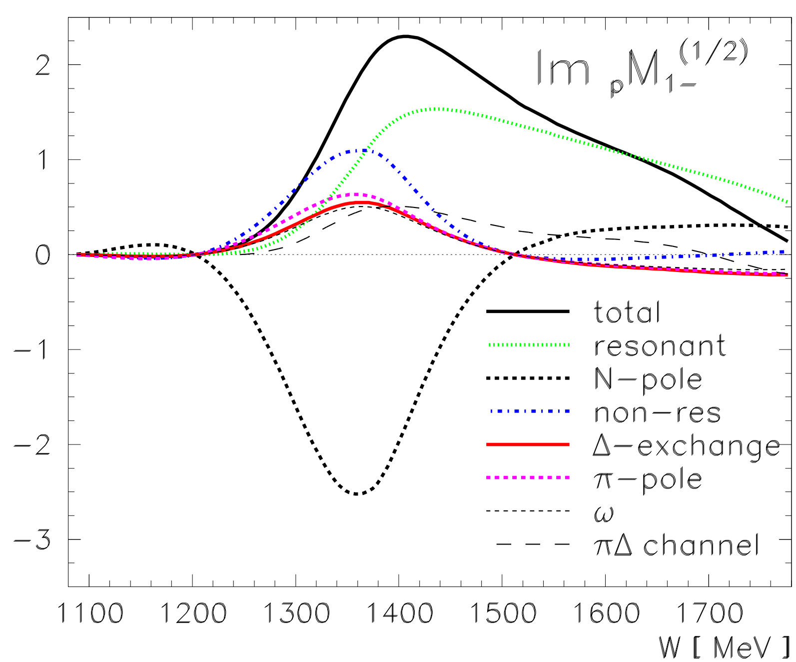
nucl. pol.

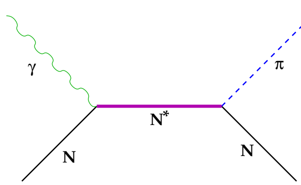


u-channel

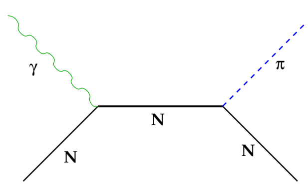
Contributions to the proton transverse amplitude



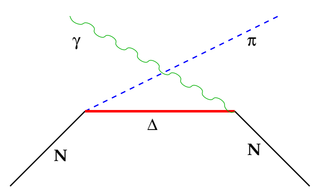




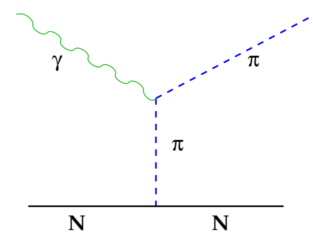
resonant



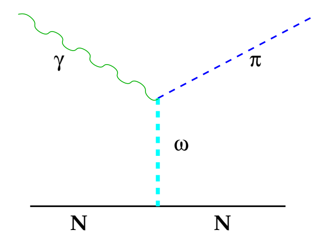
nucleon pole



crossed



pion pole



omega meson