Process-dependent transverse momentum distributions from Lattice QCD

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Certain single-spin asymmetries in semi-inclusive DIS (SIDIS) and the Drell-Yan process (DY) can be explained by transverse momentum dependent parton distribution functions (TMDs) that are predicted to differ in sign for SIDIS and DY. On the lattice, we can use non-local operators with U-shaped Wilson lines to study these TMDs, in particular the Sivers- and the Boer-Mulders function. We discuss the method, its limitations and preliminary results from an exploratory calculation using lattices generated by the MILC and LHP collaborations.

1 Introduction

Transverse momentum dependent parton distribution functions (TMDs) provide three-dimensional pictures of the momentum distribution of quarks inside a relativistic proton, see, e.g., chapter 2 of Ref. [1] for a recent review. TMDs enter as non-perturbative ingredients in the factorized cross section of processes like semi-inclusive deep inelastic scattering (SIDIS) or the Drell-Yan process (DY) at low transverse momentum. Due to initial state interactions (in DY) or final state interactions (in SIDIS) [2] whose theoretical explanation is deeply connected to the principle of gauge invariance, the operator definition of TMDs is to a certain extent process-dependent. This leads to the prediction that so-called naively time-reversal odd (T-odd) TMDs differ in sign for SIDIS and DY [3]. The T-odd distributions at leading twist are thought to be responsible for large single-spin asymmetries observed in experiment, see, e.g., [4]. Here we address them using lattice QCD.

In previous lattice studies of TMDs [5, 6], a simplified, "process-independent" operator geometry was chosen that does not strictly correspond to the definition of TMDs appearing in the description of SIDIS or DY, and that does not feature T-odd TMDs. Here we go beyond this simplification and show preliminary results obtained with a "process-dependent" operator geometry that may ultimately allow quantitative comparisons to experimental SIDIS or DY results.

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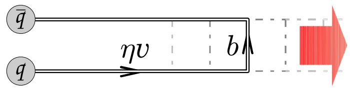

Figure 1: Geometry of the staple shaped gauge link. On the lattice, we compute the SIDIS/DY limits $\eta \to \pm\infty$ by increasing the extent of the staple step by step, as indicated by the dashed lines and the arrow.

The correlator that defines TMDs for SIDIS and DY can be written in general as

$$\Phi^{[\Gamma]} \equiv \frac{1}{2} \int \frac{d^4 b}{(2\pi)^4}\, e^{ip\cdot b}\, \frac{\overbrace{\langle P,S|\ \bar{q}(0)\,\Gamma\,\mathcal{U}[0,\eta v,\eta v+b,b]\,q(b)\ |P,S\rangle}^{\equiv\, \widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\eta v,\mu)}}{\widetilde{\mathcal{S}}(b^2,\ldots)} \tag{1}$$

where P and S are momentum and spin of the nucleon, p is the quark momentum, Γ is a Dirac matrix and μ is a renormalization scale. Note that b corresponds to the variable l in the notation of Refs. [5,6]. The precise definition of the soft factor $\widetilde{\mathcal{S}}$ varies in different theory frameworks, see, e.g., [7–9]. However, it cancels explicitly in the quantities for which we show results.

We use light cone coordinates and consider a fast nucleon, $P_\perp = 0$, $P^+ \gg M$. Integrating over the suppressed component p^- of the intrinsic quark momentum, the decomposition of the correlator [10,11] yields

$$\int dp^-\ \Phi^{[\gamma^+]} = f_1(x,\boldsymbol{p}_T^2;\hat{\zeta},\eta,\ldots) - \frac{\epsilon_{ij}\boldsymbol{p}_i\boldsymbol{S}_j}{m_N} f_{1T}^{\perp}(x,\boldsymbol{p}_T^2;\hat{\zeta},\eta,\ldots) \tag{2}$$

for a projection $\Gamma = \gamma^+$ on leading twist and for transverse nucleon polarization $\boldsymbol{S}_T$. Here f_1 and the Sivers function $f_{1T}^{\perp}$ [13] are the two TMDs that describe the corresponding distribution of quarks with respect to the longitudinal momentum fraction $x \equiv p^+/P^+$ and the transverse momentum $\boldsymbol{p}_T$ of the quark. The Wilson line $\mathcal{U}$ in Eq. (1) ensures gauge invariance and effectively represents gluon exchanges in initial or final state interactions. As illustrated in Fig. 1, it is composed of two parallel straight sections along the direction $v \approx \hat{n}_-$ and a gauge link bridging the (transverse) gap at the far ends. For SIDIS, the extent η of the staple is $+\infty$, while for DY the staple extends in the opposite direction, $\eta = -\infty$. The T-odd Sivers function $f_{1T}^{\perp}$ differs for SIDIS and DY, $f_{1T}^{\perp}(\eta{=}+\infty) = -f_{1T}^{\perp}(\eta{=}-\infty)$, while $f_1(\eta{=}+\infty) = f_1(\eta{=}-\infty)$ exhibits T-even behavior. Another leading-twist T-odd TMD is the Boer-Mulders function $h_1^{\perp}$ [14], which describes correlations in $\boldsymbol{p} \times \boldsymbol{s}$ of quarks polarized transversely along $\boldsymbol{s}_T$ in an unpolarized nucleon. Employing a direction v off the light cone $\hat{n}_-$ direction is one way to regularize rapidity divergences in the correlator, see, e.g., Refs. [7,15]. Taking v space-like [9,16,17] also opens up the possibility to perform lattice calculations. The TMDs obtained in this framework depend on an additional parameter, here introduced as a dimensionless quantity $\hat{\zeta} \equiv v\cdot P/\sqrt{|v^2|P^2}$. At large enough values of $\hat{\zeta}$, the $\hat{\zeta}$-dependence of TMDs can be obtained from evolution equations, see, e.g., Refs. [9,18].

Dependencies of the TMDs on further regularization or renormalization scales have been indicated by the dots and cancel in the quantities we consider.

2 Lattice calculations

In previous lattice studies of TMDs [5,6], a direct, straight gauge link $\mathcal{U}[0,b]$ was employed, corresponding to $\eta = 0$ in Eq. (1). We make use of the same lattice at $m_\pi \approx 500$ MeV [19] and the same techniques as in these earlier works, except that we now implement the staple-shaped operator geometry of Fig. 1. We also improve our statistics using the new arrangement of nucleon sources and coherent sinks of Ref. [20]. In essence, we calculate $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\eta v,\mu)$ directly for a large selection of lattice vectors $\boldsymbol{b}$, $\boldsymbol{P}$ and $\eta \boldsymbol{v}$. As before, we restrict the operator to have no extent in Euclidean time direction. Consequently, b and v can only have spatial components on the lattice. For a given lattice nucleon three-momentum $\boldsymbol{P}^{\text{lat}}$, the regularization parameter $\hat{\zeta}$ is thus limited by $\hat{\zeta} \leqslant |\boldsymbol{P}^{\text{lat}}|/M^2$. The translation of the results obtained in the lattice frame to the TMD language is established through a parametrization of $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}$ in terms of Lorentz-invariant amplitudes $\widetilde{A}_i$ and $\widetilde{B}_i$, analogously to Ref. [11] but in b-space. To be able to construct quantities where the soft factor cancels, we work with TMDs in Fourier space and their b-derivatives, see Ref. [12] for details. For a generic TMD f we define

$$\text{(3)}\qquad \tilde{f}(x,\boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\, f(x,\boldsymbol{p}_T^2), \qquad \tilde{f}^{(n)}(x,\boldsymbol{b}_T^2) \equiv n! \left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x,\boldsymbol{b}_T^2)$$

In the limit $\boldsymbol{b}_T^2 \to 0$, the latter correspond to the usual $\boldsymbol{p}_T$-moments:

$$\text{(4)}\qquad \tilde{f}^{(n)}(x,0) = \int d^2\boldsymbol{p}_T\, (\boldsymbol{p}_T^2/2M^2)^n f(x,\boldsymbol{p}_T^2) \equiv f^{(n)}(x).$$

Consider the quantity $\langle \boldsymbol{p}_y \rangle_{TU}(x) \equiv M\, f_{1T}^{\perp(1)}(x)/f_1^{(0)}(x)$, which has an interpretation as the average transverse momentum in transverse y-direction carried by the quarks inside a nucleon polarized in transverse x-direction. We now show that similar quantities are accessible on the lattice. Here we restrict ourselves to x-integrated TMDs $\tilde{f}^{[1](n)}(\boldsymbol{b}_T^2) \equiv \int_{-1}^{1} dx\, \tilde{f}^{[1](n)}(x,\boldsymbol{b}_T^2)$, which can be obtained from the amplitudes at $b\cdot P = 0$:

$$\text{(5)}\qquad \frac{1}{2}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}}\Bigg|_{\substack{b\cdot P=0\\ S=0}} = P^+ \underbrace{\left(\widetilde{A}_2 + R(\hat{\zeta})\widetilde{B}_1\right)}_{\widetilde{\mathcal{S}}(b^2,\ldots)\,\tilde{f}_1^{[1](0)}(\boldsymbol{b}_T^2)} - iMP^+\epsilon_{ij}\boldsymbol{b}_i\boldsymbol{S}_j \underbrace{\left(\widetilde{A}_{12} - R(\hat{\zeta})\widetilde{B}_8\right)}_{\widetilde{\mathcal{S}}(b^2,\ldots)\,\tilde{f}_{1T}^{\perp[1](1)}(\boldsymbol{b}_T^2)},$$

where $R(\hat{\zeta}) \equiv 1 - (1+\hat{\zeta}^{-2})^{1/2}$. We thus can construct a ratio which looks similar to the average momentum $\langle \boldsymbol{p}_y \rangle_{TU}(x)$, but is formed from x-integrated distributions and generalized to non-zero $\boldsymbol{b}_T$, in the following called the (generalized) Sivers shift:

$$\text{(6)}\qquad \langle \boldsymbol{p}_y \rangle_{TU}(|\boldsymbol{b}_T|) \equiv M\frac{\tilde{f}_{1T}^{\perp[1](1)}(\boldsymbol{b}_T^2)}{\tilde{f}_1^{[1](0)}(\boldsymbol{b}_T^2)} = -M\frac{\widetilde{A}_{12} - R(\hat{\zeta})\widetilde{B}_8}{\widetilde{A}_2 + R(\hat{\zeta})\widetilde{B}_1}\Bigg|_{b\cdot P\,=\,0}.$$

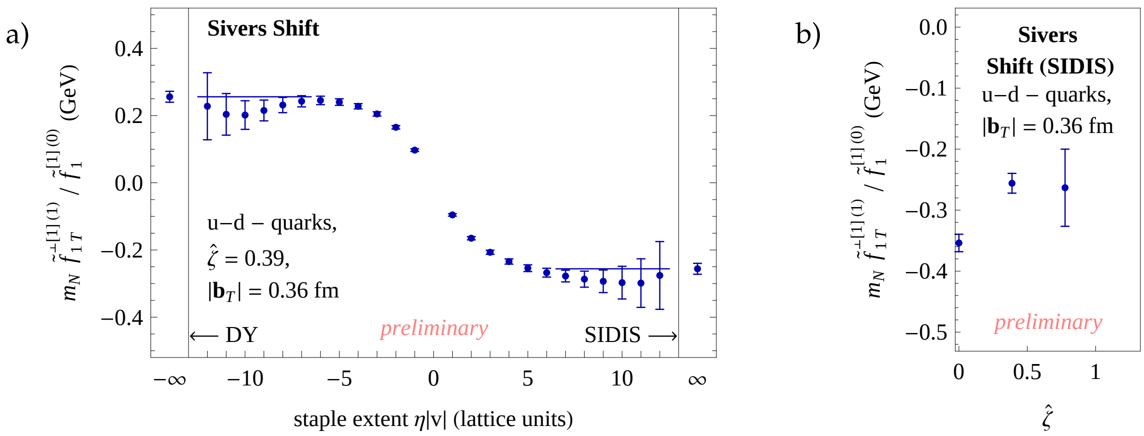

Figure 2: Generalized Sivers shift of up−down quarks (isovector) on the $20^3 \times 64$ lattice at a pion mass $m_\pi \approx 500$ MeV and a lattice spacing of $a \approx 0.12$ fm. a) results as a function of the staple extent η. A simple estimate of the SIDIS/DY values at $\eta \to \pm\infty$ is obtained from a fit of an odd but otherwise constant function to the data at $|\eta v| \geqslant 7a$. Potentially significant systematic uncertainties in this procedure have not been taken into account in this preliminary analysis. b) Extracted SIDIS results for several values of $\hat{\zeta}$.

Analogously, the "Boer-Mulders shift" can be constructed using $\tilde{h}_1^{\perp[1](1)}$ instead of $\tilde{f}_{1T}^{\perp[1](1)}$. The soft factor and multiplicative renormalization factors cancel in the above ratio. However, the dependence on the rapidity cutoff parameter $\hat{\zeta}$ (not shown in the arguments) survives. Figures 2 a) demonstrates how the SIDIS or DY Sivers shift can be read off from the plateau reached at large positive or negative η, respectively. The extraction of these asymptotic values is still preliminary and lacks an estimate of systematic errors. In Fig. 2 b), we plot the extracted SIDIS results as a function of $\hat{\zeta}$ and find indications of a strong $\hat{\zeta}$-dependence at the rather low values of $\hat{\zeta}$ presently accessible to us. A major future challenge is to generate statistically well-determined results at higher values of $\hat{\zeta}$ and to make contact with the $\hat{\zeta}$-evolution predicted by perturbative QCD.

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