Method of optimization of a wind pump

RATSIMBAZAFY [(1)], Solo RANDRIAMAHALEO [(2)], Adolphe A. RATIARISON [(3)]
[(1)] Ecole Normale Supérieure (ENS) - Université de Fianarantsoa – Madagascar
[Tél. : (+261)32 04 887 33]. [Mail : tsimba@email.com].
[(2)] Faculté des Sciences, Université de Fianarantsoa – Madagascar
[Tél. : (+261)75 901 56] [Mail : solo.randriamahaleo@gmail.com].
[(3)] Laboratoire de Rhéologie des Suspensions (LRS), Faculté des Sciences, Université d'Antananarivo – Madagascar [Tél. : (+261)32 04 266 58] [Mail: aaratiarison@yahoo.fr].

Abstract

The present work proposes a method to optimize the dimensions of different components of a wind pump: the support, the pump, the transmission mechanism of the energy from the rotor toward the pump and the rotor. The proposed pump is with piston for simple effect. The support is made from an assembly of wood. There are two fields of dimensions of the rod and the crank: the field where the hodograph of the speeds of piston present strong irregularity and the other where this hodograph varies less strongly. The results of modelling mechanism make it possible to separate these two fields. Laws, between the various characteristic kinematic magnitudes, are obtained. The mathematical model of the rotor is obtained by formulating the conditions of optimality of the aerodynamic forces, and, by equalizing the recoverable power from the wind, with the mechanical power necessary to start up the pump. The essential initial data are the speed of wind to starting the pump, the depth of the well and the densities of materials of the rotor. The resolution of the system of equations forming the model gives the forbidden measurements, and those optimal, of blades and the corresponding maximal debit of water. The wind motor can be armed by a special mechanism to obtain the working speed of an alternator to produce the electric energy.

Nomenclature

C_P : Local coefficient of power
V_C : Volume of pump,
D_M : Duration of the ascent,
S_1, S_2: Surfaces of the section of the tube of evacuation and the cylinder of the pump,
H : Depth of well,
V_1 : Speed of wind to the pump starting,
l_M : Length of hub.
ω_1 : Angular speed of the crank,
λ : Specific speed of the wind pump,
$\delta_C, \delta_1, \delta_3$: Densities of the hoop, the crank and the rod.
ρ, ρ_e, λ_M, σ : Densities of air, water, hub and blade,
$\Delta h_2, \Delta h_3$: Dips of the rod and the piston.

1. Introduction

The difference between the entering and outgoing kinetic energy of the rotor corresponds to the recovered mechanical energy. A wind pump can recover no more than 59% of the kinetic energy of wind (law of Betz, 1929). So, we propose a method to optimize the system to reach this maximum energy efficiency. The study concerns the entity formed by the wind pump with horizontal axis and the pump to forcing back, with simple effect, while looking in detail its parts: the support, the transmission mechanism, the blades, their number and their angle of wedging, and the pump itself.

Up to now, to the best of our knowledge, there are not deepened studies on the optimization of the wind pump, with weak speed of the wind, of the order of 3m/s again. The most frequent breakdown is the rupture of the rod; because of a too big effort to be developed by the device at certain time; that leds us to analyse the transmission mechanism, to formulate the mathematical model of this last, and of the wind one, to detect the source of the pain and to palliate it.

Mathematically, this work articulates therefore with the theory of control, with a view to master the usable energy optimization from wind. The sense of the optimal, or its rationality is in relation to the adequacy of the wind pump alloy "to horizontal axis and the pump to forcing back with simple effect" and the consistency of this non homogeneous system taken into consideration the materials, available locally, to compose the whole device. The determination of the different possible domains for the characteristic parameters of the system connects this approch of research to the mathematical theory of the viability [1].

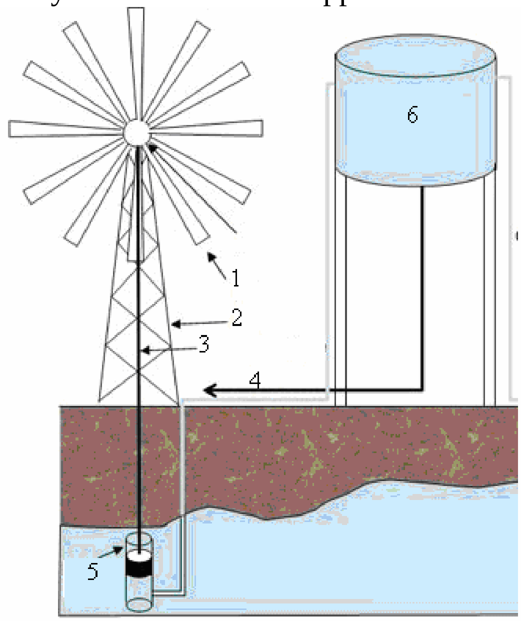


FIG. 1: *Wind pump with horizontal axis.*

FIG. 1:
1-blade
2- metallic pylon
3-bit
4-tube of evacuation
5-pump
6-tank

FIG. 2 :
a- trunk
b- timber
c- soil

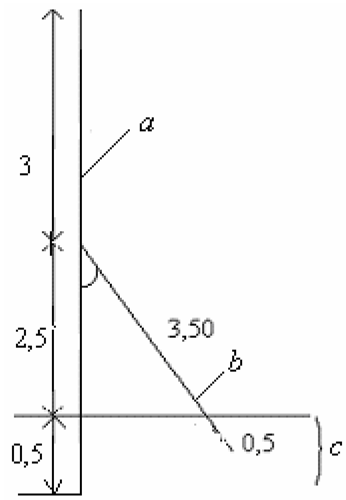


FIG. 2: *Schema of the wind pump support.*

2. Optimization of the wind pump

2.1. The support

Common practices use a metallic pylon as a support of wind pump, but it is expensive; furthermore, it can twist under the strokes of wind and the whirlwinds that are very frequent. To reduce the cost of the system, we use an assembly of wood that resists these aerodynamic efforts better (FIG. 2). A round and dry eucalyptus wood, of about 0,25m of diameter and 6m of length, acts as pillar. To limit the possible distortions, the trunk is planted vertically very close to the well, in a hole cemented of 0,5m of depth. Three timbers of 4m long lean on the trunk in a level of 2,5m from the ground. An angle of 120° is to separate every two timber. To protect woods from a fast rot, they need to be dried before the installation, and then covered of draining oil.

The structure undergoes periodic checking of its state. A replacement proves to be necessary if wood is decompositing. Like the support of the electric wires of the local electricity area network (JIRAMA[1]), the column can serve until five years.

2.2. The pump

A metallic pump of industrial manufacture is complex; it can get rusty and its fixing is difficult. The body of the proposed pump is made of the junction of the bodies of two bottles in plastic (recovered) glued tip to tip and every bottom of which is removed. The cylinder witch is obtained reaches more 0,25m of height and have an intern diameter D=0,06m. Two plastic disks are superposed to carry the valve of forcing back and to act as mobile piston that is cone-shaped bound with a head of bottle in plastic serving of fixing piece with the stem of

[1] 1 : Jiro sy Rano Malagasy

control (FIG. 3). Another double disk similar to the previous one is attached to the bottom of the cylinder, whose fixing is reinforced by bolts to nuts; this one carries the valve of stop. An impermeable transparent membrane, synthetic and flexible tube-shaped of a diameter slightly less than the one of the cylinder, joins the two disks by its two extremities. The whole is transparent and one can see at the time of the test before the installation, if the valves function well. The working conditions of the pump are shown in FIG. 4. The pressure of the water column that the piston supports is: $p_e = p_{atm} + g.\rho_e.[H - l_1(1-\cos\varphi_1)]$.

The debit Q is determined as follows: $Q = \frac{V_C}{T_C} = 2l_1\frac{\pi D^2}{4}\frac{\omega_1}{2\pi} = l_1\frac{D^2}{4}\frac{\lambda V_1}{x_2}$.

The users can move with the scale of Beaufort to value the speed of wind roughly. The V_N rated speeds and V_L limit correspond respectively to strengths 4 and 6 of that scale.

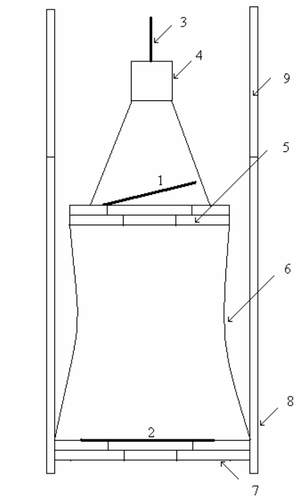


FIG. 3: *Diagram of the pump*

1- valve of repression,
2- valve of stop,
3- stem of order,
4- encapsulate the head of bottle,
5- mobile piston
6- impervious membrane,
7- stationary disk
8- body,
9- house of repression

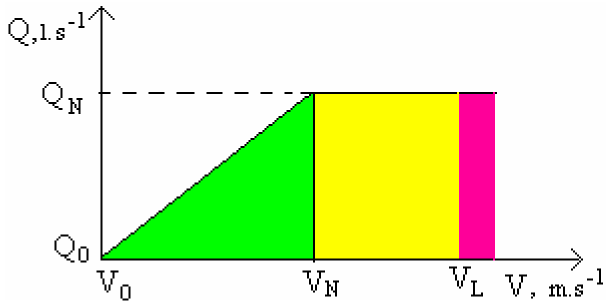


FIG. 4: *Domain of the wind speeds for the working of the pump.*

2.3. The transmission mechanism

In general, the used mechanism rod crank has a crank and a rod having very near lengths. However these neighbouring measurements provoke an enormous speed, therefore a big strength of inertia for the piston (case of the CAPR[2]), what requires an excessive energizing need for the rotor. To get a better mastery of the speed, we do the kinematical analysis of the two options of the mechanism rod crank (FIG. 5 and 6).

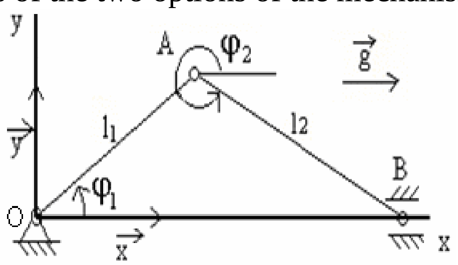


FIG. 5 : *Rod crank mechanism*

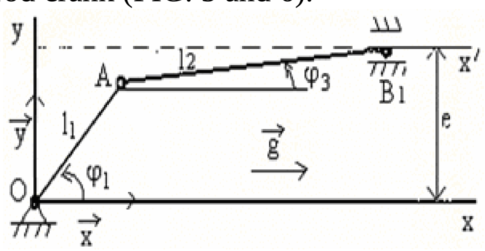


FIG. 6 : *Eccentric mechanism rod crank*

[2] Centre d'Apprentissage et de Promotion Rurale

Put: $0 \le \beta = e / l_1 \le 1$ and $1 < \gamma = l_2 / l_1$. The position of the piston is defined by $\overrightarrow{OB}_1 = \left[l_1 \cos\varphi_1 + \sqrt{l_2^2 - (e - l_1 \sin\varphi_1)^2} \right] \vec{x} + e\vec{y}$ its speed is: $\vec{V}(B_i) = -\omega_1 l_1 k_1(\varphi_1)\vec{x}$, with

$$k_1(\varphi_1) = \sin\varphi_1 - \frac{(\beta - \sin\varphi_1)\cos\varphi_1}{\left(\gamma^2 - (\beta - \sin\varphi_1)^2\right)^{\frac{1}{2}}} .$$

The acceleration of B_1 $(\beta \ne 0)$ to the lowest point of its course is θ bigger than the one of B $(\beta = 0)$, with $\theta = (1+\gamma).\left[(1+\gamma)^2 - \beta^2\right]^{-\frac{1}{2}}$.

When the rotation of the crank is uniform, the acceleration of the piston has the same form that the following expression:

$$\frac{\partial[k_1(\varphi_1)]}{\partial\varphi_1} = \cos\varphi_1 - \frac{\gamma^2\left[2.\sin^2\varphi_1 - \beta.\sin\varphi_1 - 1\right] + \sin\varphi_1.[\beta - \sin\varphi_1]^3}{\left[\gamma^2 - (\beta - \sin\varphi_1)^2\right]^{\frac{3}{2}}}$$

The positions of instantaneous stop $\varphi_{11}, \varphi_{12}$, the length D_M of the rise, the length D_{AM} of the accelerated phase of the rise and the length D_{AD} of the accelerated part of the descent are defined by:

$$\varphi_{11} = \arcsin[\beta/(\gamma+1)]$$

$$\varphi_{12} = \pi + \arcsin[\beta/(\gamma-1)]$$

with : $(\beta+1) < \gamma$

$$D_M = \frac{\varphi_{12} - \varphi_{11}}{360} \times 100$$

$$D_{AM} = \frac{\varphi_{1M} - \varphi_{11}}{D_M} \times 100$$

$$D_{AD} = \frac{\varphi_{1m} - \varphi_{12}}{T_C - (\varphi_{12} - \varphi_{11})} \times 100$$

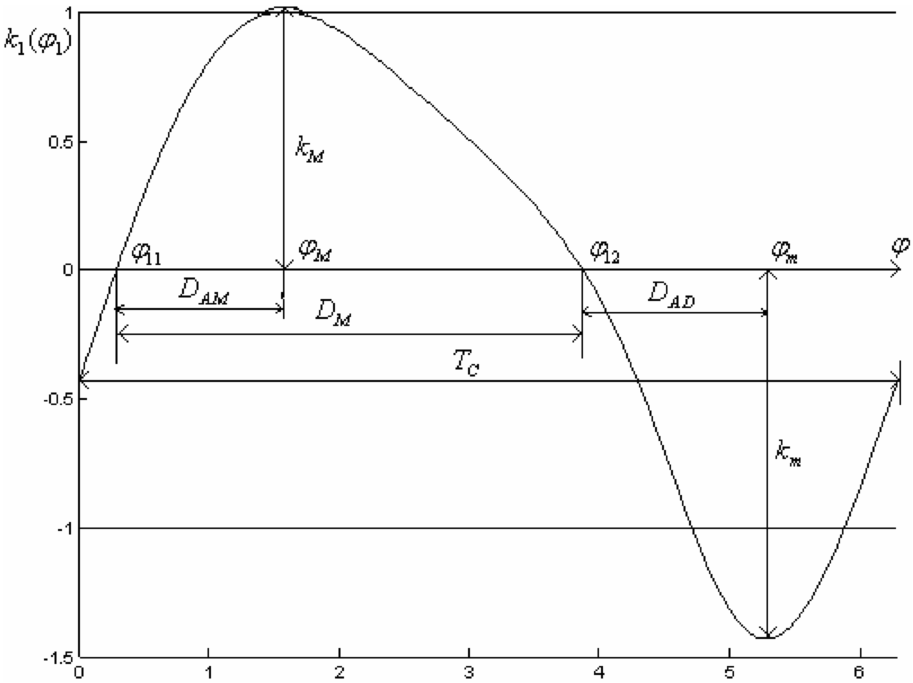


FIG. 7: *Graphic definitions* of $\varphi_{11}, \varphi_{12}, k_{1M}, k_{1m}, \varphi_{1M}, \varphi_{1m}, D_M, D_{AM}, D_{AD}, T_C$

The values of $\varphi_{11}, \varphi_{12}, k_{1M}, k_{1m}, \varphi_{1M}, \varphi_{1m}, D_M, D_{AM}, D_{AD}$ and θ are gotten for $0 \le \beta \le 1$ and $(\beta+1) < \gamma \le 8$ from the mathematical model (1) of the mechanism where the equations (e) and (f) permit to get $\varphi_{1M}, k_{1M}, \varphi_{1m}$ *and* k_{1m}:

$$\begin{cases} 0 \le \beta \le 1 \quad (a); \quad \beta + 1 < \gamma; \quad (b) \\ \varphi_{11} = \arcsin\left(\dfrac{\beta}{\gamma+1}\right); \quad (c) \\ \varphi_{12} = \pi + \arcsin\left(\dfrac{\beta}{\gamma-1}\right); \quad (d) \\ \cos\varphi_1 - \dfrac{\gamma^2\left[2.\sin^2\varphi_1 - \beta.\sin\varphi_1 - 1\right] + \sin\varphi_1.\left[\beta - \sin\varphi_1\right]^3}{\left[\gamma^2 - (\beta - \sin\varphi_1)^2\right]^{\frac{3}{2}}} = 0; \qquad (e) \\ k_1(\varphi_1) = \sin\varphi_1 - (\beta - \sin\varphi_1).\dfrac{\cos\varphi_1}{\sqrt{\gamma^2 - (\beta - \sin\varphi_1)^2}}; \qquad (f) \\ \theta = \dfrac{\gamma+1}{\sqrt{(\gamma+1)^2 - \beta^2}} \quad (g); \quad e = \beta\, l_1 \quad (h); \quad l_2 = \gamma\, l_1 \quad (i). \end{cases} \tag{1}$$

The numeric results give the curves of the FIG. 8 and the following laws:
- The mechanism as a system rod crank cannot exist for: $0 < \gamma \le \beta + 1$.
- In the defined zone by: $(1+\beta) < \gamma \le (1+1,42\beta)$, the slide of mechanisms acquires some speeds capable to reach big values. This zone presents a very elevated risk of blockage or rupture of the rod.
- The defined domain by: $(1+1,42\beta) < \gamma < (7,7431\beta^3 - 6,3512\,\beta^2 - 5,8573\,\beta + 6,8945)$ is supportable because the speeds of the slide there present no more of big irregularities; the speeds are not more critical, but they are not again sufficiently satisfactory for a wind pump.
- The measurements that verify:

$$\gamma \ge 7,7431\beta^3 - 6,3512\,\beta^2 - 5,8573\,\beta + 6,8945$$

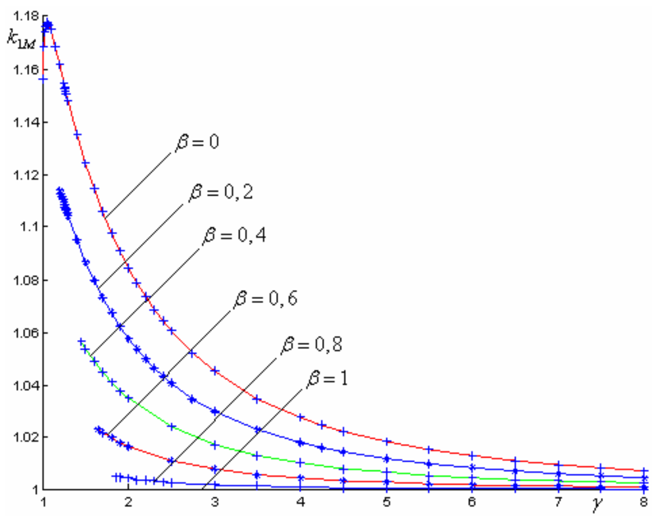
are optimal for a wind pump. The irregularity of speeds during the cycle is acceptable; they pass less 1% the one of the extremity of the crank. The mechanism having these measurements requires the least quantity of energy to move because of its simple structure, its reduced mass and its easiness of the lubrication of its links.
For the domain where the difference $(\gamma - \beta)$ is slightly superior to the unit, the speed of the slide is very big; the mechanisms having these measurements find their applications in the thermal motors; the piston behaves as the motor piece of the system. In these cases, the crank becomes the receiving piece of energy or brace.

We got the laws permitting to determine:
- the positions of instantaneous stop of the slide in below and top: φ_{11} *and* φ_{12} ;
- the report of the acceleration to the point φ_{11} : $\theta = (\cos\varphi_{11})^{-1}$;
- the positions of the crank for which the speed of the slide is maximal at the time of its displacements ascending and downward for $\gamma \ge 2,5$:

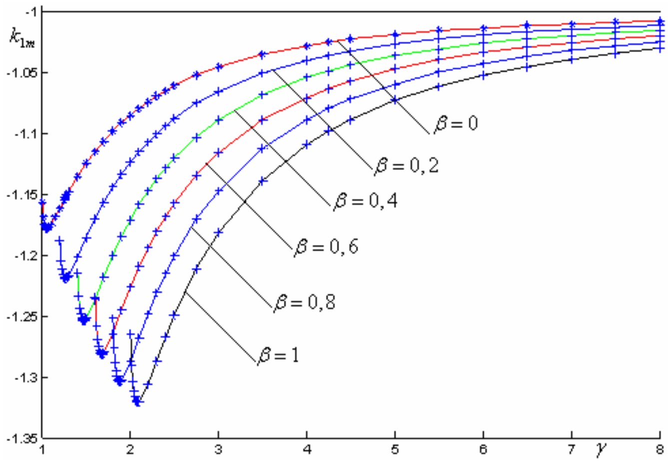
$$\varphi_{1M} = 90 + (4,6558\beta^2 + 23,2174\beta - 27,9129).\ \exp[-(0,0213\beta + 0,1793).\gamma]$$

$$\varphi_{1m} = 270 + (-4,3267\beta^2 + 24,4789\beta + 27,9109).\exp[(0,0190\beta - 0,1801)\gamma]$$

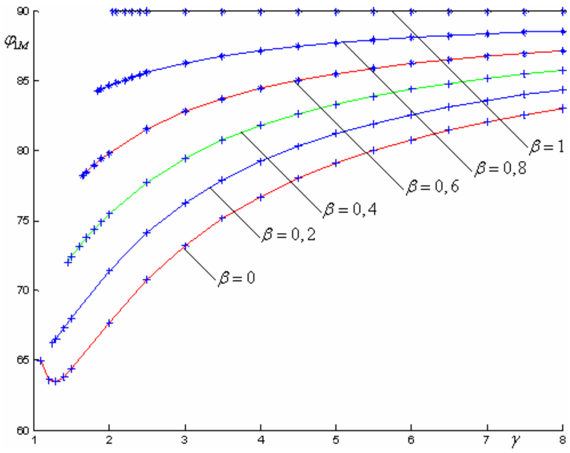
- the maximal values of φ_m who verify: $\gamma = 1,1343\,\beta + 1,2695$;



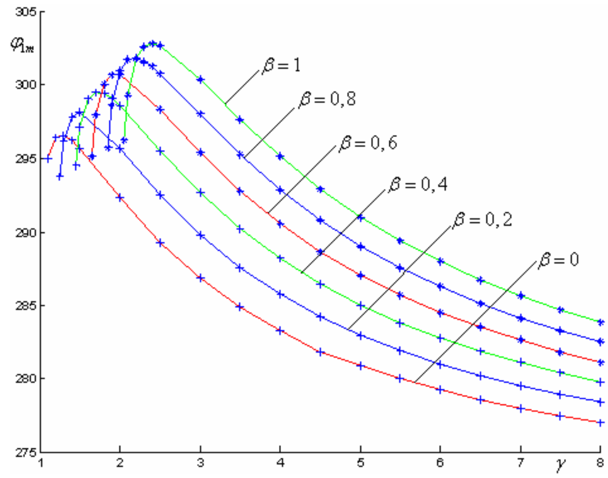
a) k_{1M} in function of γ



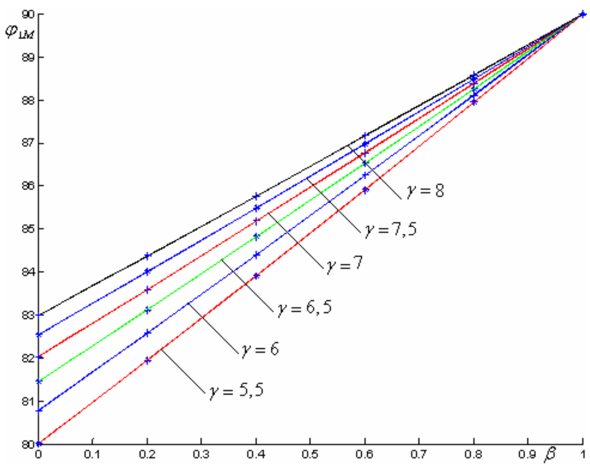
b) k_{1m} in function of γ



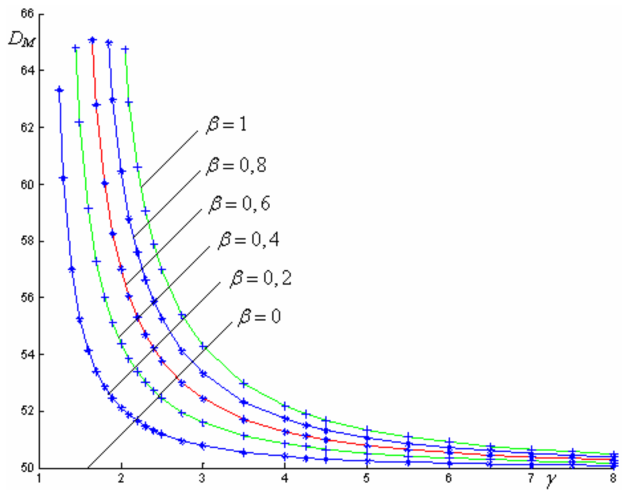
c) φ_{1M} in function of γ



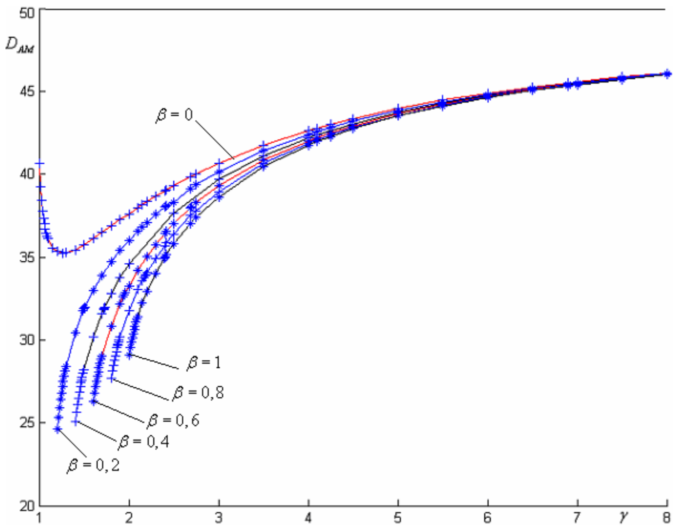
d) φ_{1m} in function of γ



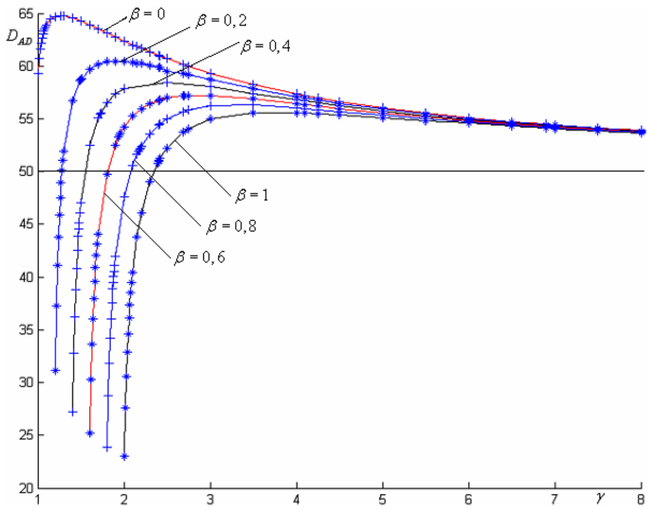
e) φ_{1M} in function of β



f) D_M in function of γ



g) D_{AM} in function of γ



h) D_{AD} in function of γ

FIG. 8: *Curves of variation of the characteristic parameters* $\varphi_{1M}, k_{1M}, \varphi_{1m}, k_{1m}, D_M, D_{AM}$ *and* D_{AD}

- the measurements of the mechanism for a report of speed fixed of the slide for $\gamma \geq 2,5$:

$$k_{1M} = 1 + (0,1074\beta^2 - 0,2363\beta + 0,1280).\exp[-(0,0363\beta + 0,3675)\gamma]$$

$$k_{1m} = -1 - (0,1308\beta^2 + 0,2543\beta + 0,1271).\exp[-(0,0030\beta + 0,3675)\gamma]$$

- the minimal values of k_m that verify: $\gamma = 1,0286\,\beta + 1,0624$;
- the length of the displacement of the ascent of the slide:

$$D_M = 50 + (2,5311\beta^2 + 11,9497\beta + 0,0561).\exp[-(0,0173\beta + 0,4234)\gamma] \; ;$$

- the length of the accelerated phase of the ascent of the slide:

$$D_{AM} = 50 - \left(-1,0792\beta^2 + 4,2004\beta + 14,0540\right).\exp\left((0,0095\beta^2 - 0,0325\beta - 0,1645).\gamma\right)$$

- the length of the accelerated phase of the descent of the slid:

$$D_{AD} = 50 + \left(-0,9861\beta^2 - 2,9854\beta + 12,9327\right).\exp\left((0,0182\beta^2 + 0,0214\beta - 0,1519).\gamma\right)$$

Global laws between k_{1M}, φ_{1M}, k_{1m} and φ_{1m} in the zone $2,5 \leq \gamma \leq 8$ are also gotten. They permit to determine a characteristic directly while knowing another; these laws are:

$$k_{1M} = 1 + 1,6.10^{-4}[90 - \varphi_{1M}]^2 \; ;$$

$$k_{1m} = -1 - (0,5\beta + 1,6).10^{-4}[\varphi_{1m} - 270]^2 \; ;$$

$$\varphi_{1m} = 270 + (2,3875\beta^3 - 1,2955\beta^2 + 1,9367\beta - 0,0016).[90 - \varphi_{1M}] \; ;$$

$$k_{1M} = 1 - (-2,0104\beta^3 + 4,3313\beta^2 - 3,4093\beta + 0,9989)[1 + k_{1m}] \; ;$$

$$k_{1M} = 1 + (-2,7104\beta^3 + 6,1280\beta^2 - 5,1628\beta + 1,6260).10^{-4}[\varphi_{1m} - 270]^2 \; ;$$

$$k_{1m} = -1 - (1282,1\beta^4 - 1397,8\beta^3 + 511,6\beta^2 - 47,5\beta).10^{-4}[90 - \varphi_{1M}]^2 .$$

These expressions show that the sizes $(\varphi_{11}, \varphi_{12}, k_{1M}, k_{1m}, \varphi_{1M}, \varphi_{1m}, D_M, D_{AM}, D_{AD}, \theta)$ stretch toward the values (0º, 180º, 1, -1, 90º, 270º, 50%, 50%, 50%, 1), feature of the cyclic symmetrical mechanism best adapted to the wind pump, when the length of the rod becomes more and more big for $7 \leq \gamma$ *and* $0 \leq \beta \leq 1$.

Remark

The adjustments with exponential functions are only pertinente for $\gamma \geq 2,5$. Exponential function can be transformed in limited development form. So, we can use rational fractional function as function of adjustment. The obtained accuracy is about 10^{-4}. We have the following expressions that are true for all the domain of parameters β and γ: ($0 \leq \beta \leq 1$ and $\gamma > \beta + 1$):

$$k_{1M} = 1 - \frac{0,3817\beta - 0,3407}{\gamma^2} - \frac{-0,2289\beta + 0,2165}{\gamma^5} ;$$

$$\mathrm{k}_{1\mathrm{m}} = -1 - \frac{0,4904\beta^2 + 0,9667\beta + 0,3670}{\gamma^2} + \frac{3,9156\beta^2 + 0,0973\beta + 0,2513}{\gamma^5} ;$$

$$\varphi_{1\mathrm{M}} = 90 - \frac{-54,9354\beta + 55,3980}{\gamma} + \frac{16,0507\beta^2 - 51,2298\beta + 34,6045}{\gamma^3} ;$$

$$\varphi_{1\mathrm{m}} = 270 + \frac{62,6855\beta + 54,1177}{\gamma} - \frac{127,1008\beta^2 + 87,0112\beta + 34,2747}{\gamma^3} ;$$

$$\mathrm{D}_{\mathrm{M}} = 50 + \frac{4,8688\beta + 0,1531}{\gamma} + \frac{127,9386\beta^2 + 80,4497\beta + 3,0423}{\gamma^4} ;$$

$$D_{AM} = 50 - \frac{-2,3257\beta^2 + 4,2824\beta + 29,9111}{\gamma} - \frac{60,8871\beta^2 + 21,0726\beta - 13,7592}{\gamma^4} ;$$

$$\mathrm{D}_{\mathrm{AD}} = 50 + \frac{2,7734\beta + 30,9877}{\gamma} - \frac{127,6077\beta^2 + 91,3563\beta + 23,9937}{\gamma^3} .$$

2.4. The rotor

Currently, some authors [2-3] use of the expression $P = 0,15\, D_r^{\,2} V_1^3$ to define the recoverable power. This formula is gotten while simplifying: $P = 0,5 C_p\, \rho \eta\, S\, V_1^3$; it only contains the diameter D_r of the rotor. The other elements decide by groping. We express this power according to the speed of the V_1 wind planned for the starting, the n number of blades and their measurements, the angle α of wedging and the radius x_2 of the rotor, where $\eta = \left(1 - \frac{1,39}{n}.\sin\alpha\right)^2$ and let's look for the expression of the necessary mechanical power for the functioning of the wind motor according to these different parameters, the depth of the well H and the debit Q of the pump. The configuration of the rotor is assimilated to a regular polygon formed by n sections ($with\ n \geq 3$) [4]. We get:

$$P = \frac{1}{2}\rho C_P n \left(1 - \frac{1,39}{n}\sin\alpha\right)^2 \cos\alpha\, .\left[1 - r_L\left(1 - r_R\right)\right] L x_2 V_1^3 \qquad (2)$$

where the frontal surface of the rotor is defined by: $S = S_{np} = n\, L\, x_2 \left[1 - r_L\left(1 - r_R\right)\right]\cos\alpha$,

with: $0 \leq r_L = \frac{l}{L} \leq 1$; $0 < \mathrm{r_R} = \frac{\mathrm{h}}{\mathrm{x}_2} < 1$; $0 < \alpha < \frac{\pi}{2}$. (3)

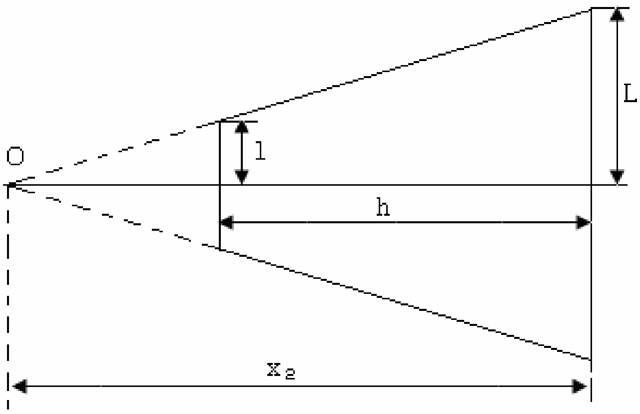


FIG. 9: *Trapezoidal blade*

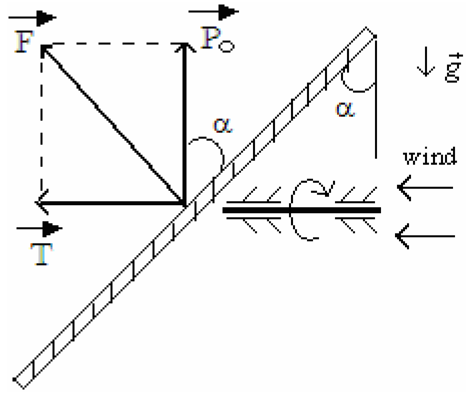


FIG. 10: *Profile of a plane blade in air.*

The triangles schematizing the blade (FIG. 9) in the same way summit O to the axis of rotation of the rotor, with bases *2l* and *2L* are similar; then $\frac{L}{x_2}=\frac{l}{x_2-h}$; that means: $r_L + r_R = 1$.

The expression (2) becomes:

$$P = \frac{1}{2}\rho\, C_P\, n\left(1-\frac{1,39}{n}\sin\alpha\right)^2 \cos\alpha\ .r_R\,(2-r_R)\, L\; x_2\, V_1^3 \text{, or}$$

$$P=\frac{1}{2}\rho C_P\, nK_{pp}L\, x_2\, V_1^3 \qquad (4)$$

where the factor $K_{pp}=\left(1-\frac{1,39}{n}\sin\alpha\right)^2\cos\alpha\; r_R(2-r_R)$ is optimal if we have:

$dK_{pp}=\frac{\partial K_{pp}}{\partial\alpha}d\alpha+\frac{\partial K_{pp}}{\partial r_R}dr_R=0$. The equation $\frac{\partial K_{pp}}{\partial r_R}=0$ has the solution: $\left(r_L\ ;r_R\right)=(0\,;1)$; but this couple corresponds to a triangular blade of height h equals to the radius x_2, what is impracticable because of the presence of the axis of rotation of the rotor and the hoop of fixing of the blades. An empty space in the centre of the rotor permits to reduce the harmful force $\vec{T}$ (FIG. 5). By this technical consideration, we limit the height of the blade to $h=0,9\,x_2$ and the half small basis *l=0,1L*. The radius of the hoop is $r_C=x_1=x_2-h=0,1x_2$. Thus, instead of the fictional solution $(0\,;1)$, the rational solution $(r_L\,;r_R)=(0,1\ ;\ 0,9)$ is taken like optimal condition for these two proportions. The trapeze has a centre of thrust situated to $x_G\approx 0,673\,x_2>(2/3)x_2$ of the axis of rotation. The equality between the frontal surface of the *n* blades and the swept surface by the radius x_2 is translated by:

$$n\,L\,x_2\,r_R(2-r_R)\cos\alpha=\pi x_2^2 r_R(2-r_R),$$

either: $nL\cos\alpha=\pi x_2$. This last equality expresses the optimal condition between parameters of the blade. The optimal condition of the angle of wedging α must get itself of the equation $\frac{\partial K_{pp}}{\partial\alpha}=0$. However this equation has not a root. To find the condition on α, we use the hypothesis on the force of lift $\vec{P}_0$, the proportional component to *sin* α of the aerodynamic strength $\vec{F}$. Indeed, the sum of all these forces creates the motor couple. However it is the recoverable power that turns into motor couple. It is here about a conversion of the same energizing size (principle of the thermodynamics). The ideal condition is to have a recoverable power and a force of lift at a time maximal (therefore a maximal motor couple also), that means:

$$\begin{cases} \left[1-\dfrac{1,39}{n}\sin\alpha\right]^2 \cos\alpha \to \max \\ \sin\alpha \to \max \end{cases} \qquad (5)$$

The system (5) is equivalent to the equation (6) that means the optimal condition of the couple (n,α) :

$$\left[1-\frac{1,39}{n}.\sin\alpha\right]^2 .\cos\alpha - \sin\alpha = 0 \qquad (6)$$

The formed system (6) and $(r_L \,;\, r_R) = (0{,}1 \,;\, 0{,}9)$ show the optimal condition for the recoverable power (2). The equality between the recoverable power P affected by the coefficient μ of energy dissipation and the total mechanical power E_m results in a polynomial equation following the explanatory variable x_2. In accordance with the convention in thermodynamics, P is counted positively for the wind pump, it is a quantity of stocked energy to the system, whereas E_m is counted negatively, an energy that the same system must spend to be able to function; what gives $P.\mu - E_m = 0$.

All mobile parts of the wind pump and water being in the tube of evacuation acquires kinetic energy and potential energy of weight [5-6]; the debit of the pump must in addition to the energy of pressure [7]. The sum of all these energies taken to their maximal value by unit of time constitute that is necessary at the time the motor phase of the pump, that means during one half period $(0,5T_C)$. Each of the n blades is stretched by a right metallic stem. The n stems are welded on a hoop that is fixed on the hub with the help of 4 stems. The kinetic energies of the rotor, the crank, the stem and the debit are respectively: $E_{C1} = \frac{1}{2} J_1 \omega_1^2$; $E_{Cm} = \frac{1}{2} J_m \omega_1^2$;

$E_{C2} = \frac{1}{2}(\delta_1\, l_1 + \delta_1\, l_2)\,(l_1\omega_1)^2$; $E_{C3} = \frac{1}{2}\mathrm{Q}\left(\frac{D}{d}\right)^4 (l_1\omega_1)^2$; that gives the average power:

$$E_{CT} = (E_{C1} + E_{Cm} + E_{C2})\frac{2}{T_C} + E_{C3}.$$

The potential energies of the rod, the stem and the debit of water are:

$\Delta E_{P2} = \delta_1\, l_1\; g\; \Delta h_2$, $\Delta E_{P3} = \delta_1\, l_2\; g\; \Delta h_3$, $\Delta E_{P4} = Q\; g\; H$,

that gives the power $\Delta E_P = (\Delta E_{P2} + \Delta E_{P3})\dfrac{2}{T_C} + \Delta E_{P4}$.

The energy of pressure of the debit [7] is: $E_{pr} = (p_{atm} + \rho_e\, g\, H)\;\pi\, 0{,}25 D^2 l_1\, \omega_1$;

the loss of energy is estimated to: $\Delta E_{pr} = \left[\left(0{,}63 + 0{,}37\left(\dfrac{S_1}{S_2}\right)^3\right)^{-1} - 1\right]^2 E_{pr}$;

it is minimal for $S_1 \to S_2$. The total pressure energy is: $E_T = E_{pr} + \Delta E_{pr}$.

The total mechanical power of pumping is: $E_m = E_{CT} + \Delta E_P + E_T$.

The moment of inertia J_1 of the rotor is equal to the sum of the moments of inertia of all pieces that compose it; these are: the blades, the stems of fixing, the hub, the stretchers, the hoop and the crank. It comes: $J_1 = n.J_{zz} + 4.J_I + J_M + n.J_T + J_C + J_m$, where successively:

$$J_{zz} = \sigma h\left[x_2^2(1+r_L) - \frac{2}{3}x_2 h(1+2r_L) + \frac{1}{6}(1+3r_L)h^2 \right]L + \frac{1}{6}\sigma h(1+r_L^2)(1+r_L)\cos^2\alpha L^3$$

$$J_{zz} = \sigma h\left[x_2^2(1+r_L) - \frac{2}{3}x_2 h(1+2r_L) + \frac{1}{6}(1+3r_L)h^2 \right]L + \frac{1}{6}\sigma h(1+r_L^2)(1+r_L)\cos^2\alpha L^3 \;;\; J_I = \frac{1}{3}\delta_1 r_C^3 \;;$$

$$J_M = \frac{1}{2}\lambda_M\, l_M\, \pi\left(r_2^4 - r_1^4\right) \;;\; J_T = \frac{1}{3}\delta_1\left(x_2^3 - r_C^3\right) \;; J_C = 2\pi\delta_C r_C{}^3 \;;\; J_m = \frac{1}{3}\delta_1 .l_1^3 .$$

While regrouping the coefficients of the same term in $P.\mu - E_m = 0$, we get the equation (c) expressing the constraint opposite the load that the pump must defeat. The expression (c) and the optimal conditions (a), (b) and (d) form the system (6) that represents the mathematical model of the energizing system formed by the wind machine and the mass of air in movement; it contains 3 equations with 4 unknowns (n,α,x_2,L):

$$\begin{cases}
\left(r_R;r_L\right) = \left(0{,}9;0{,}1\right) \quad (a); \quad \left[1-\dfrac{1{,}39}{n}\sin\alpha\right]^2\cos\alpha - \sin\alpha = 0 \quad (b);\\
0 = C_p\rho\left(1-\dfrac{1{,}39}{n}\sin\alpha\right)^2 r_R\left(2-r_R\right)\pi V_1^3\mu \quad x_2^2\\
\quad -\left[\left(2-3r_R+2r_R^2-\dfrac{1}{2}r_R^3\right)+\dfrac{1}{6}\left(1+r_L\right)\left(1+r_L^2\right)\left(\dfrac{\pi}{n}\right)^2\right]\dfrac{\sigma r_R}{\cos\alpha}\left(\lambda V_1\right)^3 x_2\\
\quad -\left[\left(1-r_R\right)^3\left(\dfrac{4}{3}\delta_C + 2\pi\delta_C - \dfrac{1}{3}n\delta_1\right)+\dfrac{1}{3}n\delta_1\right]\dfrac{1}{\pi}\left(\lambda V_1\right)^3 \qquad (6)\\
\quad -\left[\dfrac{1}{2}\lambda_M l_M\left(r_2^4-r_1^4\right)+2g\left(\delta_1 l_1+2\delta_1 l_2+2\delta_3 l_3\right)l_1\dfrac{1}{\pi}\right]\left(\lambda V_1\right) \quad \dfrac{1}{x_2}\\
\quad -\left[\rho_e gH + 2\left(p_{atm}+\rho_e gH\right)\pi\left(1+\left(\left(0{,}63+0{,}37\left(\dfrac{S_1}{S_2}\right)^3\right)^{-1}-1\right)^2\right)\right]\dfrac{D^2}{2}l_1\left(\lambda V_1\right)\dfrac{1}{x_2}\\
\quad -\left(\left(\dfrac{1}{3}\delta_1 l_1+\delta_1 l_2+\delta_3 l_3\right)\dfrac{1}{\pi}+\left(\dfrac{D}{d}\right)^4\dfrac{D^2}{4}\rho_e\right)l_1^2\left(\lambda V_1\right)^3 \quad \dfrac{1}{x_2^3}; \qquad (c)\\
L = \dfrac{\pi x_2}{n\cos\alpha}; \qquad (d)
\end{cases}$$

The equations (b) and (c) are solved by the dichotomy method [8]. The initial datas are the speed V_1 of the wind of starting, the mass of blades, the length l_1 of the crank and the depth *H* of the well. A Delphi program solves the system (6). While iterating n=3 to 40 with $\left(r_L\,;r_R\right) = (0{,}1;0{,}9)$, every value of the couple (n,α) of the equation (b) corresponds a value x_2 of (c) and a value *L* of (d). The *h* and *l* values are deducted of (a). The table 1 contains an extract of the acquired results. The optimal solutions are the values of (n,α,x_2,L) corresponding in the most elevated debit *Q*, either to the smallest radius x_2.

Other results are obtained making vary successively the speed V_1 of the wind for the starting, the density of the blade, the depth *H* of the well and the length l_1 of the crank (tables 2 to 5). These results serve numerical illustrations of the model and permit to find laws of interdependence (calculated by extrapolation) between the different parameters. These laws help to have an assessment of the influence of every element. The heaviest blades in plastic ($1,5 \leq \sigma \leq 2kg.m^{-2}$) are lighter than those made of sheet metal ($2,7 \geq \sigma \geq 2,1kg.m^{-2}$).

When the main parameters are fixed, their values are: H=10 m, l_1=0,075 m, $\sigma = 2$ kg.m^{-2}, $V_1 = 3$ m.s^{-1}. The following laws are gotten when one of these parameters varies:

for $3 \leq n \leq 23$, $x_2 = 1,9370 + \frac{2,1037}{n^{\frac{3}{2}}}$ and for $24 \leq n \leq 40$, we have : $x_2 = 0,0011.n + 1,9316$;

$x_2 = -0,0003\,H^2 + 0,0352\,H + 1,6299$; $x_2 = 0,0033\,\sigma^2 + 0,0726\,\sigma + 1,8020$;

$x_2 = -16,1957\,l_1^2 + 9,6567\,l_1 + 1,3210$;

$x_2 = 0,0295L^4 - 0,1946L^3 + 0,4484L^2 - 0,2649L + 2,0096$;

$L = \frac{8,4370}{n} - \frac{7,0536}{n^4}$; $Ap = \frac{0,5777}{n} + \frac{778,7039}{n^3}$;

$Snp = 15,9099 + \frac{33,7449}{n^2}$ when $3 \leq n \leq 15$;

$Snp = 0,0005n^2 + 0,0024n + 15,9327$ when $16 \leq n \leq 40$.

$Q = 0,24 - \frac{0,9141}{n^2} + \frac{1,7529}{n^3}$ when $3 \leq n \leq 23$,

$Q = (-0,0013.n + 2,4120).10^{-1}$ when $24 \leq n \leq 40$;

$Q = 0,0487x_2^2 - 0,3111x_2 + 0,6606$; $Q = -0,0003\,V_1^3 + 0,0118\,V_1^2 + 0,0641\,V_1 - 0,0535$;

$Q = -2,6677\,l_1^2 + 2,6345\,l_1 + 0,0546$; $Q = 0,0010H^2 - 0,0051H + 0,2834$;

$Q = -0,0104\,\sigma + 0,2588$.

Table 1: *Different values of parameters of the rotor for different numbers of blades (S_{np}= total surface of the n blades)* $\lambda = 0,8$; H = 10 m ; V_1 = 3 m.s^{-1} ; l_1 =0,075m ; σ =2kg.m^2

n	α, ^{0}	x_2, m	h, m	L, m	l, m	Snp m²	Q, m^3.h^{-1}
3	30,3810	2,33352966	2,100176	2,832641	0,283264	19,631852	0,199937
4	33,2382	2,20285446	1,982569	2,068532	0,206853	18,044436	0,211798
.....	.....	.....	......	.....	.....	.....	.....
16	41,6038	1,96821859	1,771396	0,516826	0,051682	16,112883	0,237047
17	41,7961	1,96583095	1,769247	0,487289	0,048729	16,121937	0,237335
18	41,9678	1,96398951	1,767590	0,461024	0,046102	16,135054	0,237557
19	42,1221	1,96260647	1,766345	0,437514	0,043751	16,151524	0,237725
20	42,2616	1,96161184	1,765450	0,416345	0,041634	16,170790	0,237845
21	42,3882	1,96094915	1,764854	0,397183	0,039718	16,192411	0,237926
22	42,5036	1,96057243	1,764515	0,379756	0,037975	16,216043	0,237971
23	**42,6093**	**1,96044373**	**1,764399**	**0,363836**	**0,036383**	**16,241397**	**0,237987**
24	42,7064	1,96053153	1,764478	0,349237	0,034923	16,268241	0,237976
25	42,7959	1,96080939	1,764548	0,335800	0,033580	16,296381	0,237943
26	42,8788	1,96125506	1,765130	0,323391	0,032339	16,325661	0,237888
.....	.....	.....	.....	.....	.....	.....	.....
39	43,5754	1,97636695	1,778730	0,236070	0,023607	16,768762	0,236070
40	43,6105	1,97797292	1,780175	0,214558	0,021455	16,805827	0,235878

Table 2: *Variation of the parameters according to the speed of wind to the starting, $H=10$ m, $\sigma=2$ kg.m^{-2}, $l_1=0{,}075$m.*

V_1	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8
n	23	21	19	18	17	16	15	15	14	14	13
x2	1,960	1,797	1,669	1,565	1,448	1,441	1,345	1,292	1,245	1,204	1,167
L	0,364	0,364	0,337	0,367	0,367	0,370	0,338	0,361	0,371	0,359	0,373
Q	0,237	0,302	0,372	0,447	0,525	0,607	0,693	0,782	0,874	0,968	1,065

Table 3: *Variation of the parameters according to the depth H*: *$V_1=3$ m.s^{-1}, $\sigma=2$ kg.m^{-2}, $l_1=0{,}075$m.*

H	5	7,5	10	12,5	15	17,5	20	22,5	25	27,5	30
n	21	22	23	24	25	26	26	27	28	29	29
x2	1,788	1,879	1,961	2,035	2,104	2,168	2,228	2,284	2,338	2,389	2,437
L	0,362	0,364	0,364	0,363	0,360	0,358	0,367	0,363	0,359	0,354	0,362
Q	0,260	0,248	0,237	0,229	0,221	0,215	0,209	0,204	0,199	0,195	0,191

Table 4: *Variation of the parameters according to the mass of the blades*: *$V_1=3$ m.s^{-1}, $H=10$ m, $l_1=0{,}075$m.*

σ	1,5	1,6	1,7	1,8	1,9	2	2,1	2,2	2,3	2,4	2,5	2,6	2 ,7
n	23	23	23	23	23	23	23	23	23	23	23	23	24
x2	1,918	1,927	1,935	1,943	1,952	1,960	1,969	1,978	1,987	1,995	2,004	2,013	2,022
L	0,356	0,358	0,3359	0,361	0,362	0,364	0,366	0,367	0,369	0,370	0,372	0,374	0,360
Q	0,243	0,242	0,241	0,240	0,239	0,237	0,236	0,235	0,234	0,233	0,232	0,231	0,230

Table 5: *Variation of the parameters according to the length of the crank* l_1: *$V_1=3$ m.s^{-1}, $\sigma=2$ kg.m^{-2}, $H=10$m.*

l_1	0,05	0,06	0,07	0,075	0,08	0,09	0,10	0,11	0,12	0,13	0,14	0,15
n	20	22	23	23	24	24	25	26	27	27	28	29
x_2	1,749	1,840	1,922	1,960	1,997	2,066	2,130	2,190	2,246	2,300	2,350	2,399
L	0,371	0,356	0,357	0,364	0,356	0,368	0,365	0,361	0,357	0,366	0,361	0,356
Q	0,178	0,203	0,226	0,238	0,249	0,271	0,292	0,312	0,332	0,352	0,370	0,389

3. Conclusion and perspectives

The kinematical analysis of the mechanism rod crank permitted to distinguish the domain of measurements where the field of speeds is strongly irregular from the domain where this one is regular. Thus, we could bring out a domain of viability of the system parameters and its limit. Every domain has its application. When the rod is, at least, seven times longer than the crank, the mechanism becomes practically symmetrical and the oscillation of the piston has the tendency to be a sinusoid. The invariant of the system is the cycle period. Only the length of the piston on the way up is slightly superior to the one of its coming down.

This periodic oscillating movement (or even cyclic) permits to make its mathematic form of controls by a hybrid system: discreet continuation of sequences (stop of the piston in top, stop of the piston below) of control on the continuous process (out-flow of wind and catchment's energy). It is the same for the discreet states of the pumping system (replenishment of water, expulsion of water) that alternate with states of continuous movement. It permits to have possibilities of efficient control already while focusing the survey on the sequence of transition of such a hybrid dynamic system.

The strong irregularity of speeds is solved and the risk of rupture of the rod limited. The mathematical model of the mechanism and the mathematical model of the pump permit to determine, a priori and in a convenient manner, the optimal measurements of a wind pump to horizontal axis. The radius of the rotor is an increasing function (nearly linear) of the depth of the well and the density of the blade, but it decreases with the presupposed speed of wind for the starting. The basis of the blades becomes smaller when their number increases.

The wind motor can be armed by a special mechanism to obtain the working speed of an alternator to produce the electric energy.

This work permits to manage, with optimal manner, from energetic point of view, the oscillating movement of the wind pump. It helps to conceive an air pump for the air-conditioning of dwelling or storeroom, the increasing of the efficience of the solar driers while increasing the retiring and incoming air flux. From theoretical point of view, this modelling has manipulability, is falsifiable and can be therefore the starting point of simulation for other improvements and developments according to the needs and the various creativeness; it can act as basis help tool to improve the control, the monitoring, the assessment, the normalization, the piloting of initiatives, or even simply for training of potentialities of the wind pump. The perspectives are immense; to each its development and its improvement.

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