Derivation of the gluon condensate from holographic approach

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We investigate AdS/QCD duality for gluon condensate power correction by deriving glueball correlation functions. We point out the advantage of this dual-model approach in term of power correction. This holographic approach allows, for a perfect gauge invariant way to fix the quadratic correction which is assumed to encode the confinement at short distances. In gluon condensate particular case, this dual-model provides a new qualitative picture for the power correction.

1 Introduction

The Gauge/Gravity correspondence has recently provided exciting connexion between string and gauge field theories. The discovery of this duality has revived the hope of eventually understanding the classic strong coupling mysteries of QCD.
Starting with AdS/CFT correspondence in which Maldacena has proposed that, on the product of d+1 dimension AdS space with a compact manifold, the large N limit of certain conformal field theories in d dimension can be described in term of string theories. With various modifications, physicists have extended the dual-model and are describing QCD with the so-called AdS/QCD correspondence. In QCD, power corrections play a important role due to the fact that tested theory of strong interaction becomes form the consistency of fundamental parameters such as the strong coupling α_s. From experimental data and perturbative prediction confrontation, one usually extract α_s considering the power correction. Correctness of assuming QCD theory depends on the consistency of this parameter from different experiments and different observables.
In this work, we will briefly describe the ongoing developed model and we will consider a bulk critical action which derives the glueball correlator. Comparing with OPE QCD results, we will show the new qualitative picture for the gluon condensate power correction.

2 The model

Let us first briefly describe the model. Maldacena has proposed the duality but the original proposal was for conformal theories. Starting with AdS/CFT, various modification have been found to extend it into gauge/string duals and AdS/QCD such as mass gap, confinement, and supersymmetry breaking[1].
The dual string spacetime is the product of five-dimensional space with a Euclidean metric:

$$ds^2 = R^2 \frac{h(z^2)}{z^2}(dx^i dx^i \pm dz^2) \tag{1}$$

where $h(z^2)$ contains a metric field as for example $h(z^2) = e^{2A(z)}$ with $A(z)$ is the mentioned metric field. A(z)$\overset{z\rightarrow 0}{\longrightarrow}$ 0 is required so that an approximate conformal symmetry of the Yang-Mills theories is reproduced [2]. One can already introduce the coefficient λ characteristic of the model in this metric field or includes it in the dilaton field of the bulk action.
Since QCD is not conformal, AdS/QCD approach makes it very attractive phenomenology because it is not only nearly conformal theory at UV, but also results in linear Reegy-like trajectory for mesons[3]. Holographic calculation usually follows theses following steps:

- Bulk action consideration
- Equation of motion by variation of the bulk action
- Equation of motion resolution giving the critical field
- Bulk critical action by inserting critical field in the action
- Two functional derivatives giving correlator

This correlation function plays fundamental role in holographic formulation because many expected QCD results might be derived such as mass spectrum, gluon condensate,etc.

3 Bulk critical action

Let us to consider the following metric:

$$ds^2 = g_{MN}(x)dx^M dx^N = e^{2A(z)}\frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \tag{2}$$

where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric.
Considering a massless bulk field φ, one can find its minimal action with the following form:

$$S[\varphi, g, \phi] = \frac{1}{2k^2}\int d^5x\sqrt{|g|}e^{-\phi(z)}g^{MN}\partial_M\varphi\partial_N\varphi \tag{3}$$

By variation of this bulk action, one ends with the field equation:

$$\left[\partial_z^2 + (\partial_z\phi(z) + \frac{3}{z})\partial_z + q^2\right]\widehat{\varphi}(q, z) = 0 \tag{4}$$

where $\widehat{\varphi}(q, z)$ the four-dimensional Fourrier transform.
The bulk critical action is obtained by inserting the critical field (which is the solution of the equation of motion (4)) in the previous action (3).
Through the bulk-to-boundary propagator $\widehat{K}(q, z)$, one can establish the relation [4] between the critic field $\varphi_{crit}$and its boundary source value $\varphi^{(s)}$ as:

$$\varphi_{crit}(x, z) = \int d^4x' K(x - x'; z, 0)\varphi_0^{(s)}(x')$$

$$\widehat{\varphi}_{crit}(x, z) = \int \frac{d^4q}{(2\pi)^4}e^{-iqx}\widehat{K}(x - x'; z, 0)\int d^4x' e^{iqx'}\widehat{\varphi}_0^{(s)}(x') \tag{5}$$

The bulk critical action takes the form:

$$S[\widehat{\varphi}_{crit}] = \frac{1}{2k^2}\int d^5x\sqrt{|g|}e^{-\phi(z)}g^{MN}\partial_M\widehat{\varphi}_{crit}\partial_N\widehat{\varphi}_{crit} \tag{6}$$

4 Holographic Glueball correlator

We take $\widehat{K}(q;\epsilon\to 0)=1$ and $\widehat{K}(0,z)=1$ as boundaries conditions. Inserting the solution (5) into equation (4) and deriving twice with respect to $\varphi^{(s)}$:

$$\frac{\delta^2 S[\varphi_{crit}]}{\delta\varphi_0(x_1)\delta\varphi_0(x_2)}\bigg|_{\varphi_0(x_1)=\varphi_0(x_2)=0} = Corr(x_1,x_2) = \int d^4q e^{iq(x_1-x_2)}\widehat{Corr}(q) \tag{7}$$

One needs to decompose the critical action in order to show the bulk and boundary contributions as: $S_{crit}=S_{int}+S_{bound}$. Since the action in the interior space does not depend on the source field, it remains only to take twice functional derivatives of the following action at the boundary space:

$$S_{bound}[\widehat{\varphi}_{crit}] = -\frac{1}{2k^2}\int d^4x[h^3(z)e^{-\phi(z)}\widehat{\varphi}_{crit}\partial_z\widehat{\varphi}_{crit}] \tag{8}$$

where $h(z)=\frac{R}{z}e^{A(z)}$.
Then, the correlator evaluated at x_1 and x_2 takes the forme:

$$Corr(x_1,x_2) = -\frac{R^3}{k^2}\frac{e^{-\phi(z)}}{z^3}e^{3A(z)}\left[\int d^4 e^{iq(x_1-x_2)}\widehat{K}(q,z)\partial_z\widehat{K}(q,z)\right] \tag{9}$$

By comparison, we obtain the soft-wall scalar glueball correlator:

$$\widehat{Corr}(-q^2) = -\frac{R^3}{k^2}\left[\frac{e^{-\phi(z)}}{z^3}\widehat{K}(q,z)\partial_z\widehat{K}(q,z)\right]_{z=\epsilon\to 0} \tag{10}$$

In the soft-wall background, the analytical solution [5] for $\widehat{K}(q,z)$ is:

$$\widehat{K}(q,z) = \frac{\pi}{4}(qz)^2\left[\frac{Y_1(qz_m)}{J_1(qz_m)}J_2(qz) - Y_2(qz)\right] \tag{11}$$

where J_ν, Y_ν are Bessel functions and $\widehat{K}(0,z)=1$.
In soft-wall background, one introduces the dilaton field in which the coefficientλ of holographic model appears in the first time:

$$\phi^{(sw)}(z)=\lambda^2z^2, \quad A^{(sw)}(z)\equiv 0 \tag{12}$$

After plugging this analytical solution (11) into the general expression (10), we obtain the soft-wall correlator at spacelike momanta $Q^2=-q^2$:

$$\widehat{Corr}(Q^2) = -\frac{2R^3}{k^2}\lambda^4\left[1+\frac{Q^2}{4\lambda^2}(1+\frac{Q^2}{4\lambda^2})\psi(\frac{Q^2}{4\lambda^2})\right] \tag{13}$$

Using the asymptotic expansion of the digamma function, equation (13) can be rewritten for $Q^2 >> 4\lambda^2 > \Lambda^2_{QCD}$ as:

$$\widehat{Corr}(Q^2) = -\frac{2}{\pi^2}Q^4\left[ln\frac{Q^2}{\mu^2}+\frac{4\lambda^2}{Q^2}ln\frac{Q^2}{\mu^2}+\frac{2^25}{3}\frac{\lambda^4}{Q^4}-\frac{2^4}{3}\frac{\lambda^6}{Q^6}+\frac{2^5}{15}\frac{\lambda^8}{Q^8}+...\right] \tag{14}$$

5 Gluon condensate power corrections

Now, the QCD OPE says [5]:

$$\widehat{\Pi}^{(OPE)} = \left[A_0 + A_1 ln(\frac{Q^2}{\mu^2}) + A_2 ln^2(\frac{Q^2}{\mu^2})\right] Q64ln(\frac{Q^2}{\mu^2}) + \left[B_0 + B_1 ln(\frac{Q^2}{\mu^2})\right]\left\langle G^2\right\rangle \tag{15}$$

$$+\left[C_0 + C_1 ln(\frac{Q^2}{\mu^2})\right]\frac{\left\langle gG^3\right\rangle}{Q^2} + D_0\frac{G^4}{Q^4}$$

where:

$$\begin{aligned}
&A_0 = -(N_c^2-1)/(4\pi^2)\\
&N_c(Nf) = 3\\
&B_0 = 4 + 49\alpha_s/3\pi\\
&C_0 = 8\\
&D_0 = 8\pi\alpha_s
\end{aligned}$$

The gluon condensates are defined at the OPE scale μ as:

$$\begin{aligned}
&\left\langle G^2\right\rangle := \left\langle G^a_\mu \nu G^{a\mu\nu}\right\rangle\\
&\left\langle gG^3\right\rangle := \left\langle g f_{abc} G^a_{\mu\nu} G^{b\nu}_\rho G^{a\mu\nu}\right\rangle\\
&\left\langle G^4\right\rangle := 14\left\langle (f_{abc}G^b_{\nu\rho}G^{\rho c}_\nu)^2\right\rangle - \left\langle (f_{abc}G^b_{\mu\nu}G^c_{\rho\lambda})^2\right\rangle
\end{aligned}$$

Comparing with these two equations (14) and (15), one ends up with relations for the three lowest-dimensional gluon condensates (defined at the OPE scale $\mu \sim 1GeV$):

$$\left\langle G^2\right\rangle \simeq -\frac{10}{3\pi^2}\lambda^4, \quad \left\langle gG^3\right\rangle \simeq \frac{4}{3\pi^2}\lambda^6, \quad \left\langle G^4\right\rangle \simeq -\frac{8}{15\pi^3\alpha_s}\lambda^8. \tag{16}$$

This dual formulation makes appear a new qualitative picture for the power correction:

$$\left\langle G^2\right\rangle \sim \lambda^2 \sim \Lambda^4_{QCD} \tag{17}$$

The value of λ is fixed from the scope of Regge trajectory of mesons, $\lambda \approx 0.9GeV^2$[3].

6 Conclusion

In this paper, we gave a brief description of AdS/CFT which is extended into AdS/QCD after various modifications. Starting with a given metric and a standard action in holographic formulation, we obtained the bulk critical action containing the bulk-to-boundary propagator. Taking two functional derivatives with respect of source field, we had scalar glueball correlation function in the soft-wall background. Comparing with QCD OPE results, we found the gluon condensate.
The coefficient λ, introduced and characteristic of the developed model plays the role of quadratic correction.This coefficient is associated with short distance and then assumed to encode the confinement at such distance.
The dual model provides also means to describe the quadratic correction which has never been found in terms of the direct formulation. One suggests that a dual formulation might be very good for describing physics at strong coupling.

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