$SU(3)$ mass-splittings of heavy-baryons in QCD

R.M. Albuquerque [*] [a], S. Narison [†] [b], M. Nielsen [a],

[a] Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil.

[b] Laboratoire de Physique Théorique et Astroparticules, CNRS-IN2P3 & Université de Montpellier II, Case 070, Place Eugène Bataillon, 34095 - Montpellier Cedex 05, France.

We extract directly (for the first time) the heavy-baryons (spin 1/2 and 3/2) mass-splittings due to $SU(3)$ breaking using double ratios of QCD spectral sum rules (QSSR) in full QCD, which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios commonly used for determining the heavy baryon masses. As a result, we provide (relatively) robust predictions of the $\Omega_Q^{(*)}(Qss)$ and $\Xi_Q^{(*)}(Qsq)$ masses (testable at Tevatron and LHCb) once the masses of the $\Lambda_Q(Qdd)$ and $\Sigma_Q^{(*)}(Qdd)$ are given. Like in the case of the light baryons, the leading term controlling the mass-splittings is the ratio $\kappa \equiv \langle\bar{s}s\rangle/\langle\bar{d}d\rangle$ of the condensate, where they decrease when κ increases. The optimal value of the mixing $b \simeq (-1/5 \sim 0)$ of the interpolating currents for the spin 1/2 baryons, confirms the one for light baryons and the previous range for the non-strange heavy baryons. We also predict the hyperfine splittings $\Omega_Q^* - \Omega_Q$ and $\Xi_Q^* - \Xi_Q$.

1. Introduction

QSSR [1,2] à la SVZ [3] has been used earlier in full QCD [4–6] and in HQET [7] for understanding charming and beautiful baryons masses. Recent observations at Tevatron of families of b-baryons [8,9] and of the Ω_c^* baryon by Babar and Belle [10] have stimulated different recent theoretical activities for understanding their nature [11–16,18]. QSSR results are in quite good agreement with recent experimental findings but with relatively large uncertainties. The inaccuracy of these results is mainly due to the value of the heavy quark mass and of its ambiguous definition when working to LO in the radiative α_s corrections in full QCD and HQET [1], where the heavy quark mass is the main driving term in the QCD expression of the baryon two-point correlator used in the QSSR analysis. Another source of uncertainty is the effect of the QCD continuum which parametrizes the higher baryon masses contributions to the spectral function and the *ad hoc* choices of interpolating baryon currents used in different literatures. In this paper, we shall concentrate on the analysis of the heavy baryons mass-splittings due to $SU(3)$ breaking using double ratios (DR) of QCD spectral sum rules (QSSR), which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios used in the literature to determine the absolute value of heavy baryon masses.

• **For the spin 1/2 baryons**, and following Ref. [4], we work with the lowest dimension currents:

$$\begin{aligned}
\eta_{\Xi_Q} &= \epsilon_{abc}\left[(q_a^T C\gamma_5 s_b) + b(q_a^T C s_b)\gamma_5\right] Q_c, \\
\eta_{\Lambda_Q} &= \eta_{\Xi_Q} \quad (s\to q) \\
\eta_{\Omega_Q} &= \epsilon_{abc}\left[(s_a^T C\gamma_5 Q_b) + b(s_a^T C Q_b)\gamma_5\right] s_c, \\
\eta_{\Sigma_Q} &= \eta_{\Omega_Q} \quad (s\to q)\ , \qquad (1)
\end{aligned}$$

where b is *a priori* an arbitrary mixing parameter. Its value has been found to be:

$$b = -1/5\ , \qquad (2)$$

in the case of light baryons [29] and in the range [4–6]:

$$-0.5 \leq b \leq 0.5\ , \qquad (3)$$

for non-strange heavy baryons . The corresponding two-point correlator reads:

$$\begin{aligned}
S(q) &= i\int d^4x\ e^{iqx}\ \langle 0|\mathcal{T}\overline{\eta}_Q(x)\eta_Q(0)|0\rangle \\
&\equiv \hat{q}F_1 + F_2\ , \qquad (4)
\end{aligned}$$

where F_1 and F_2 are two invariant functions.

• **For the spin 3/2 baryons**, we follow Ref. [5] and work with the interpolating currents:

$$\begin{aligned}
\eta^\mu_{\Xi_Q^*} &= \sqrt{\frac{2}{3}}\left[(q^T C\gamma_\mu Q)s + (s^T C\gamma_\mu Q)q + (q^T C\gamma_\mu s)Q\right] \\
\eta^\mu_{\Omega_Q^*} &= \frac{1}{\sqrt{2}}\eta^\mu_{\Xi_Q^*} \quad (q\to s) \\
\eta^\mu_{\Sigma_Q^*} &= \frac{1}{\sqrt{2}}\eta^\mu_{\Xi_Q^*} \quad (s\to q)\ , \qquad (5)
\end{aligned}$$

where an anti-symmetrization over colour indices is understood. The normalization in Eq. (5) is chosen in such a way that in all cases one gets the same perturbative contribution. The corresponding two-point correlator reads:

$$\begin{aligned}
S^{\mu\nu}(q) &= i\int d^4x\ e^{iqx}\ \langle 0|\mathcal{T}\overline{\eta}^\mu_Q(x)\eta^\nu_Q(0)|0\rangle \\
&\equiv g^{\mu\nu}\left(\hat{q}F_1 + F_2\right) + \ldots \qquad (6)
\end{aligned}$$

2. The spin 1/2 two-point correlator in QCD

In this letter, we extend the previous analysis in [4,5] by including the new $SU(3)$ breaking m_s correction terms.

• **The $\Lambda_Q(Qqq)$ and $\Xi_Q(Qsq)$ baryons**

The expression for Λ_Q has been (first) obtained in the chiral limit $m_q = 0$ in [5], and the one of Ξ_Q including $SU(3)$ breaking in [14]. One can notice that due to the expression of the current the m_s corrections vanish to leading order in α_s for the perturbative term, while the $D = 6$ condensates for the $SU(2)$ case of [5] needs the following replacement in the $SU(3)$ case:

$$\rho\langle\bar{q}q\rangle^2 \to \rho\langle\bar{q}q\rangle\langle\bar{s}s\rangle\ , \qquad (7)$$

[*] *E-mail addresses:*, rma@if.usp.br (R.M. Albuquerque), snarison@yahoo.fr (S. Narison), mnielsen@if.usp.br (M. Nielsen).
[†] Corresponding author.

[1] However, radiative corrections have been evaluated in [16] but (unfortunately) for a particular choice of the interpolating currents.

where $\rho = 2 \sim 3$ indicates the violation of the four-quark vacuum saturation [31,1,27]. The additional $SU(3)$ breaking corrections for the Ξ_Q are [14]:

- F_1 :

$$
\begin{aligned}
\mathrm{Im}F_1^{m_s}|_{\bar{s}s} &= -\frac{m_s}{2^4\pi}(1-x^2)\Bigg[(1-b^2)\langle\bar{q}q\rangle - \\
&\quad \frac{(1+b^2)}{2}\langle\bar{s}s\rangle\Bigg], \\
F_1^{m_s}|_{mix} &= \frac{m_s}{2^5\pi^2}\frac{1}{m_Q^2-q^2}\Bigg\{\langle\bar{s}Gs\rangle\ \frac{(1+b^2)}{6} + \\
&\quad \langle\bar{q}Gq\rangle(1-b^2)\Bigg\},
\end{aligned} \tag{8}
$$

- F_2 :

$$
\begin{aligned}
\mathrm{Im}F_2^{m_s}|_{\bar{s}s} &= -\frac{m_s m_Q}{2^3\pi}(1-x)\Bigg[(1+b^2)\langle\bar{q}q\rangle - \\
&\quad \frac{(1-b^2)}{2}\langle\bar{s}s\rangle\Bigg], \\
F_2^{m_s}|_{mix} &= \frac{m_s m_Q}{2^5\pi^2}\frac{1}{m_Q^2-q^2}\Bigg\{\langle\bar{s}Gs\rangle\ \frac{(1-b^2)}{6} + \\
&\quad \langle\bar{q}Gq\rangle(1+b^2)\Bigg\},
\end{aligned} \tag{9}
$$

where $x \equiv m_Q^2/s$ and $\langle\bar{s}Gs\rangle \equiv g\langle\bar{s}\sigma_{\mu\nu}\lambda_a/2G_a^{\mu\nu}s\rangle$.

• The $\Sigma_Q(Qqq)$ and $\Omega_Q(Qss)$ baryons

The expression for Σ_Q has been (first) obtained in [4]. The additionnal $SU(3)$ breaking terms for the Ω_Q are:

- F_1 :

$$
\begin{aligned}
\mathrm{Im}F_1^{m_s}|_{pert} &= \frac{3m_s m_Q^3}{2^8\pi^3}(1-b^2)\times \\
&\quad \left[\frac{2}{x}+3-6x+x^2+6\ln x\right], \\
\mathrm{Im}F_1^{m_s}|_{\bar{s}s} &= \frac{3m_s\langle\bar{s}s\rangle}{2^6\pi}(1+b)^2\left(1-x^2\right), \\
F_1^{m_s}|_{mix} &= -\frac{m_s\langle\bar{s}Gs\rangle}{2^7 3\pi^2}\Bigg[\frac{1}{m_Q^2-q^2}(7+22b+7b^2) \\
&\quad -6(1+b)^2\int_0^1\frac{d\alpha(1-\alpha)}{m_Q^2-(1-\alpha)q^2}\Bigg], \\
F_1^{m_s}|_{D=6} &= -\frac{m_s m_Q\rho\langle\bar{s}s\rangle^2(1-b^2)}{8(m_Q^2-q^2)^2}\ .
\end{aligned} \tag{10}
$$

- F_2 :

$$
\begin{aligned}
\mathrm{Im}F_2^{m_s}|_{pert} &= \frac{3m_s m_Q^4}{2^8\pi^3}(1-b^2)\times \\
&\quad \left(\frac{1}{x^2}-\frac{6}{x}+3+2x-6\ln x\right), \\
\mathrm{Im}F_2^{m_s}|_{\bar{s}s} &= -\frac{3m_s m_Q\langle\bar{s}s\rangle}{2^5\pi}(3+2b+3b^2)(1-x), \\
F_2^{m_s}|_{mix} &= \frac{m_s m_Q\langle\bar{s}Gs\rangle}{2^7 3\pi^2}\times \\
&\quad \Bigg[\frac{1}{m_Q^2-q^2}(25+22b+25b^2) \\
&\quad -3(5+6b+5b^2)\times \\
&\quad \int_0^1\frac{d\alpha}{m_Q^2-(1-\alpha)q^2}\Bigg], \\
F_2^{m_s}|_{D=6} &= -\frac{m_s\rho\langle\bar{s}s\rangle^2(1-b^2)}{8(m_Q^2-q^2)}\left[1+\frac{m_Q^2}{m_Q^2-q^2}\right],
\end{aligned} \tag{11}
$$

We have checked the existing results in [4] obtained in the chiral limit and all our previous results agree with these ones.

3. The spin 3/2 two-point correlator in QCD

The QCD expression of the two-point correlator for the $\Sigma_Q^*(Qqq)$ has been (first) obtained in the chiral limit $m_{u,d}=0$, to LO in α_s and up to the contributions of the $D=6$ condensates in [4]. In this letter, we extend the previous analysis by including the new $SU(3)$ breaking m_s correction terms and consider the $SU(3)$ breaking of the ratio of quark condensates $\langle\bar{s}s\rangle \neq \langle\bar{q}q\rangle$.

• The $\Sigma_Q^*(Qqq)$ and $\Xi_Q^*(Qsq)$ baryons

The additionnal terms and replacement due to $SU(3)$ breaking for the Ξ_Q^* compared with the one of the $\Sigma_Q^*(Qqq)$ in [5] are:

- F_1 :

$$
\begin{aligned}
\mathrm{Im}F_1^{m_s}|_{pert} &= \frac{m_s m_Q^3}{48\pi^3}\left[\frac{2}{x}+3-6x+x^2+6\ln x\right], \\
\mathrm{Im}F_1|_{\bar{s}s} &= -\frac{m_Q}{6\pi}\Big[\langle\bar{q}q\rangle+\langle\bar{s}s\rangle\Big](1-x)^2\ , \\
\mathrm{Im}F_1^{m_s}|_{\bar{s}s} &= -\frac{m_s}{12\pi}\Big[2(1-x^2)\langle\bar{q}q\rangle-(1-x^3)\langle\bar{s}s\rangle\Big], \\
\mathrm{Im}F_1|_{mix} &= \frac{7M_0^2}{3^2 2^3\pi}\Big[\langle\bar{q}q\rangle+\langle\bar{s}s\rangle\Big]\frac{x^2}{m_Q}\ , \\
F_1^{m_s}|_{mix} &= \frac{m_s M_0^2}{144\pi^2}\Bigg[\frac{12\langle\bar{q}q\rangle-9\langle\bar{s}s\rangle}{m_Q^2-q^2} + \\
&\quad 2\int_0^1\frac{d\alpha(1-\alpha)}{m_Q^2-(1-\alpha)q^2}\times \\
&\quad \Big[(1-3\alpha)\langle\bar{s}s\rangle+\langle\bar{q}q\rangle\Big]\Bigg], \\
F_1|_{D=6} &= \frac{4}{9}\frac{\rho\langle\bar{s}s\rangle\langle\bar{q}q\rangle}{m_Q^2-q^2}\ , \\
F_1^{m_s}|_{D=6} &= -\frac{2}{9}m_Q m_s\frac{\rho\langle\bar{s}s\rangle\langle\bar{q}q\rangle}{\left(m_Q^2-q^2\right)^2}\ .
\end{aligned} \tag{12}
$$

- F_2 :

$$
\begin{aligned}
\mathrm{Im}F_2^{m_s}|_{pert} &= \frac{m_s m_Q^4}{192\pi^3}\left[\frac{3}{x^2}-\frac{16}{x}+12+x^2-12\ln x\right], \\
\mathrm{Im}F_2|_{\bar{s}s} &= -\frac{m_Q^2}{18\pi}\Big[\langle\bar{q}q\rangle+\langle\bar{s}s\rangle\Big]\left(\frac{2}{x}-3+x^2\right)\ , \\
\mathrm{Im}F_2^{m_s}|_{\bar{s}s} &= -\frac{m_s m_Q}{12\pi}(1-x)\Big[6\langle\bar{q}q\rangle-(1+x)\langle\bar{s}s\rangle\Big], \\
\mathrm{Im}F_2|_{mix} &= \frac{M_0^2}{18\pi}\Big[\langle\bar{q}q\rangle+\langle\bar{s}s\rangle\Big]\left(1+\frac{3}{4}x^2\right)\ , \\
F_2^{m_s}|_{mix} &= \frac{m_s m_Q M_0^2}{72\pi^2}\Bigg[3\frac{(3\langle\bar{q}q\rangle-\langle\bar{s}s\rangle)}{m_Q^2-q^2} +
\end{aligned}
$$

$$\langle\bar{q}q\rangle\int_0^1 \frac{d\alpha}{m_Q^2-(1-\alpha)q^2}\Bigg] ,$$

$$F_2|_{D=6} = \frac{2}{3}\frac{m_Q\rho\langle\bar{s}s\rangle\langle\bar{q}q\rangle}{m_Q^2-q^2} ,$$

$$F_2^{m_s}|_{D=6} = -\frac{2}{9}\frac{m_s m_Q^2\rho\langle\bar{s}s\rangle\langle\bar{q}q\rangle}{(m_Q^2-q^2)^2} , \tag{13}$$

where $x \equiv m_Q^2/s$ and $\langle\bar{s}Gs\rangle \equiv g\langle\bar{s}\sigma_{\mu\nu}\lambda_a/2G_a^{\mu\nu}s\rangle \equiv M_0^2\langle\bar{s}s\rangle$.

• **The $\Omega_Q^*(Qss)$ baryons**

Compared with the expression of the $\Sigma_Q^*(Qqq)$ in [5], the additionnal $SU(3)$ breaking terms for the Ω_Q^* are:

- F_1 :

$$\mathrm{Im}F_1^{m_s}|_{pert} = \frac{m_s m_Q^3}{24\pi^3}\left[\frac{2}{x}+3-6x+x^2+6\ln x\right],$$

$$\mathrm{Im}F_1^{m_s}|_{\bar{s}s} = -\frac{m_s\langle\bar{s}s\rangle}{6\pi}\left(1-2x^2+x^3\right),$$

$$F_1^{m_s}|_{mix} = \frac{m_s M_0^2\langle\bar{s}s\rangle}{72\pi^2}\Bigg[\frac{3}{m_Q^2-q^2}+ 2\int_0^1\frac{d\alpha(1-\alpha)(2-3\alpha)}{m_Q^2-(1-\alpha)q^2}\Bigg] ,$$

$$F_1^{m_s}|_{D=6} = -\frac{4}{9}\frac{m_Q m_s\rho\langle\bar{s}s\rangle^2}{\left(m_Q^2-q^2\right)^2} . \tag{14}$$

- F_2 :

$$\mathrm{Im}F_2^{m_s}|_{pert} = \frac{m_s m_Q^4}{96\pi^3}\left[\frac{3}{x^2}-\frac{16}{x}+12+x^2-12\ln x\right],$$

$$\mathrm{Im}F_2^{m_s}|_{\bar{s}s} = -\frac{m_s m_Q\langle\bar{s}s\rangle}{6\pi}\left[5-6x+x^2\right],$$

$$F_2^{m_s}|_{mix} = \frac{m_s m_Q M_0^2\langle\bar{s}s\rangle}{36\pi^2}\Bigg[\frac{6}{m_Q^2-q^2}+ \int_0^1\frac{d\alpha}{m_Q^2-(1-\alpha)q^2}\Bigg] ,$$

$$F_2^{m_s}|_{D=6} = -\frac{4}{9}\frac{m_s m_Q^2\rho\langle\bar{s}s\rangle^2}{(m_Q^2-q^2)^2}. \tag{15}$$

We have checked the existing results in [5] obtained in the chiral and $SU(2)$ limits and agree with these ones.

4. Form of the sum rules and QCD inputs

We parametrize the spectral function using the standard duality ansatz: "one resonance"+ "QCD continuum". The QCD continuum starts from a threshold t_c and comes from the discontinuity of the QCD diagrams. Transferring its contribution to the QCD side of the sum rule, one obtains the finite energy Laplace/Borel sum rules:

$$|\lambda_{B_q^{(*)}}|^2 M_{B_q^{(*)}}\ e^{-M_{B_q^{(*)}}^2\tau} = \int_{t_q}^{t_c} ds\ e^{-s\tau}\ \frac{1}{\pi}\mathrm{Im}F_2(s) ,$$

$$|\lambda_{B_q^*}|^2\ e^{-M_{B_q^{(*)}}^2\tau} = \int_{t_q}^{t_c} ds\ e^{-s\tau}\ \frac{1}{\pi}\mathrm{Im}F_1(s) , \tag{16}$$

where $\lambda_{B_q^{(*)}}$ and $M_{B_q^{(*)}}$ are the heavy baryon residue and mass; $\tau \equiv 1/M^2$ is the sum rule variable. Consistently, we also take into account the $SU(3)$ breaking at the quark and continuum threshold:

$$\sqrt{t_q}|_{SU(3)} \simeq \left(\sqrt{t_q}|_{SU(2)} \equiv m_Q\right) + \bar{m}_{q_1} + \bar{m}_{q_2} ,$$

$$\sqrt{t_c}|_{SU(3)} \simeq \left(\sqrt{t_c}|_{SU(2)} \equiv \sqrt{t_c}\right) + \bar{m}_{q_1} + \bar{m}_{q_2} , \tag{17}$$

where $q_{1,2} \equiv q$ or/and s depending on the channel. $\bar{m}_{q_i}$ are the running light quark masses. m_Q is the heavy quark mass, which we shall take in the range covered by the running and on-shell mass (see Table 1) because of its ambiguous definition when working to LO. One can estimate the baryon masses from the following ratios:

$$\mathcal{R}_i^q = \frac{\int_{t_q}^{t_c} ds\ s\ e^{-s\tau}\ \mathrm{Im}F_i(s)}{\int_{t_q}^{t_c} ds\ e^{-s\tau}\ \mathrm{Im}F_i(s)} , \qquad i=1,2 ,$$

$$\mathcal{R}_{21}^q = \frac{\int_{t_q}^{t_c} ds\ e^{-s\tau}\ \mathrm{Im}F_2(s)}{\int_{t_q}^{t_c} ds\ e^{-s\tau}\ \mathrm{Im}F_1(s)} , \tag{18}$$

where at the τ-stability point :

$$M_{B_q^{(*)}} \simeq \sqrt{\mathcal{R}_i^q} \simeq \mathcal{R}_{21}^q . \tag{19}$$

These quantities have been used in the literature for getting the baryon masses and lead to a typical uncertainty of 15-20% [4–6] [3]. In order to circumvent these problems, we work with the double ratio of sum rules (DR)[20]:

$$r_i^{sd} \equiv \sqrt{\frac{\mathcal{R}_i^s}{\mathcal{R}_i^d}} , \qquad r_{21}^{sd} \equiv \frac{\mathcal{R}_{21}^s}{\mathcal{R}_{21}^d} . \tag{20}$$

which take directly into account the $SU(3)$ breaking effects. These quantities are obviously less sensitive to the choice of the heavy quark masses and to the value of the continuum threshold than the simple ratios $\mathcal{R}_i$ and $\mathcal{R}_{21}$ [2]. Analogous DR quantities have been used successfully (for the first time) in [20] for studying the mass ratio of the $0^{++}/0^{-+}$ and $1^{++}/1^{--}$ B-mesons, in [21] for extracting f_{B_s}/f_B, in [22] for estimating the $D \to K/D \to \pi$ semi-leptonic form factors and in [23] for extracting the strange quark mass from the $e^+e^- \to I=1,0$ data. For the numerical analysis whe shall introduce the RGI quantities $\hat{\mu}$ and $\hat{m}_q$ [24]:

$$\bar{m}_q(\tau) = \frac{\hat{m}_q}{\left(-\log\sqrt{\tau}\Lambda\right)^{2/-\beta_1}}$$

$$\langle\bar{q}q\rangle(\tau) = \hat{\mu}_q^3\left(-\log\sqrt{\tau}\Lambda\right)^{2/-\beta_1}$$

$$\langle\bar{q}Gq\rangle(\tau) = \hat{\mu}_q^3\left(-\log\sqrt{\tau}\Lambda\right)^{1/-3\beta_1} M_0^2 , \tag{21}$$

where $\beta_1 = -(1/2)(11-2n/3)$ is the first coefficient of the β function for n flavours. We have used the quark

[3]More accurate results quoted in the recent QSSR literature [14, 15] do not take into account the uncertainties due to the heavy quark mass definitions and to the arbitrary choice of the baryonic interpolating currents.

[2]One may also work with the double ratio of moments $\mathcal{M}_n$ based on different derivatives at $q^2=0$ [20]. However, in this case the OPE is expressed as an expansion in $1/m_Q$, which for a LO expression of the QCD correlator is more affected by the definition of the heavy quark mass to be used.

mass and condensate anomalous dimensions reviewed in [1]. We shall use the QCD parameters in Table 1. At the scale where we shall work, and using the paramaters in the table, we deduce:

$$\rho = 2.1 \pm 0.2 \ , \tag{22}$$

which controls the deviation from the factorization of the four-quark condensates. We shall not include the $1/q^2$ term discussed in [25,26],which is consistent with the LO approximation used here as the latter has been motivated for a phenomenological parametrization of the larger order terms of the QCD series.

Table 1
QCD input parameters. For the heavy quark masses, we use the range spanned by the running $\overline{MS}$ mass $\overline{m}_Q(M_Q)$ and the on-shell mass from QSSR compiled in page 602,603 of the book in [1].

Parameters	Values	Ref.
Λ	(353 ± 15) MeV	[27,9]
$\hat{m}_d$	(6.1 ± 0.5) MeV	[28,1,9]
$\hat{m}_s$	(114.5 ± 20.8) MeV	[28,1,9]
$\hat{\mu}_d$	(263 ± 7) MeV	[28,1]
$\kappa \equiv \langle \bar{s}s\rangle/\langle \bar{d}d\rangle$	(0.7 ± 0.1)	[28,1]
M_0^2	(0.8 ± 0.1) GeV^2	[29,30,20]
$\langle \alpha_s G^2\rangle$	$(6.8 \pm 1.3) \times 10^{-2}$ GeV^4	[27,31–35,2]
$\rho\alpha_s\langle \bar{d}d\rangle^2$	$(4.5 \pm 0.3) \times 10^{-4}$ GeV^6	[27,31]
m_c	$(1.18 \sim 1.47)$ GeV	[1,28,36,9]
m_b	$(4.18 \sim 4.72)$ GeV	[1,28,36,9]

5. The masses of the spin 1/2 octet baryons

As a preliminary step of the analysis, we check the different results obtained in full QCD and in the chiral limit [4,5]:

$$\begin{aligned} M_{\Sigma_c} &= (2.45 \sim 2.94) \text{ GeV} \ , \\ M_{\Sigma_b} &= (5.70 \sim 6.62) \text{ GeV} \ , \\ M_{\Sigma_c} - M_{\Lambda_c} &\leq 207 \text{ MeV} \ , \\ M_{\Sigma_b} - M_{\Lambda_b} &\leq 163 \text{ MeV} \ , \end{aligned} \tag{23}$$

which we confirm. However, we have not tried to improve these results due to the ambiquity in the definition of the heavy quark mass input mentioned earlier at LO.

• $\boldsymbol{\Xi_c(csq)/\Lambda_c(cqq)}$

– *Choice of the sum rules:* in so doing we choose (after iterations) $b = 0$ and study in Fig. 1a) and Fig. 1b), the τ-behaviour of the double ratio of sum rules (DR) for two extremal values of t_c ($M^2_{\Sigma_c}$: beginning of τ-stability and 12 GeV^2: beginning of t_c-stability). The appearance of the extremas for $\tau \simeq 0.3$ GeV^{-2} depends strongly on the value of t_c at which one cannot extract an optimal result. Therefore, among the three DR, we retain r_{21}^{sd} continuous (red) which is the most stable in τ and t_c .

– *Choice of the currents:* we show in Fig. 1c), by fixing τ=0.9 GeV^{-2} and t_c= 12 GeV^2 the b-behaviour of the result which is very stable like in the range given in Eq. (3). However, this generous range does not favour the *ad hoc* choice around 1 used in the existing literature [14,15]. The absolute extremum happens at $b = 0$, which is compatible with the one $b = -1/5$ in Eq. (3)

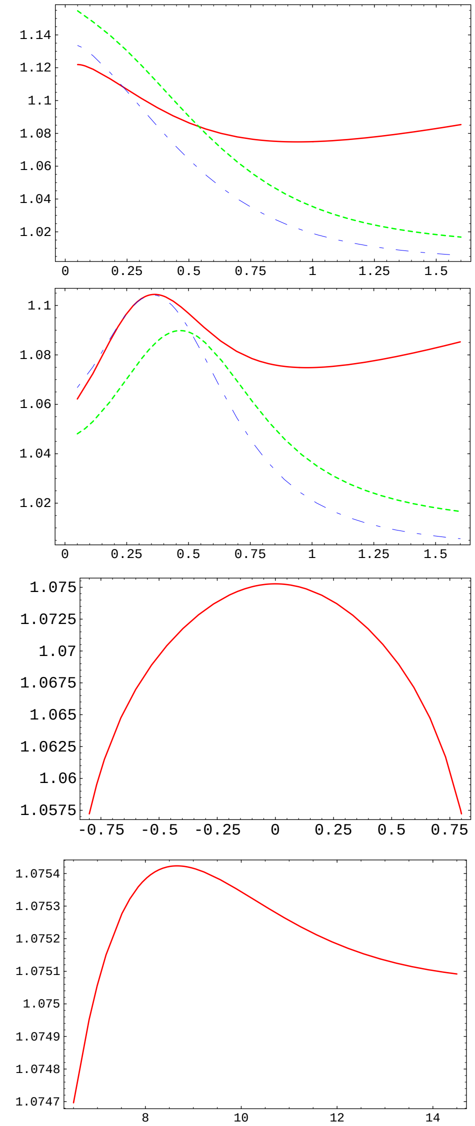

Figure 1. $\boldsymbol{\Xi_c/\Lambda_c}$: a) τ-behaviour of the double ratio of sum rules (DR) given $b = 0$ and $t_c = M^2_{\Sigma_c} = 6$ GeV^2: r_1^{ds} dashed-dotted (blue), r_2^{sd} dotted (green), r_{21}^{sd} continuous (red); b) the same as a) but for t_c =12 GeV^2; c) b-behaviour of r_{21}^{sd} for given values of $\tau = 0.9$ GeV^{-2} and $t_c = 12$ GeV^2; d) t_c-behaviour of the DR given $b = 0$ and $\tau = 0.9$ GeV^{-2}.

for the light baryon systems [29]. For definiteness, we shall work with:

$$b = -(1/5 \sim 0) \ . \tag{24}$$

– *t_c stabilities:* we show in Fig. 1d) the t_c behaviours of r_{21}^{sd} at fixed $\tau = 0.9$ GeV^{-2} and $b = 0$.

– *Results:* we can deduce the DR:

$$r_{\Xi_c}^{sd} = 1.075(0.1)(0.6)(0.4)(3.4)(11)(17)(6) \ , \tag{25}$$

where we have considered the mean value from $t_c = 6$ GeV^2 (beginning of τ-stability) to the beginning of t_c-stability (12 GeV^2). The errors are due respectively to the values of $\tau = (0.9 \pm 0.2)$ GeV^{-2}, b, t_c, m_c, m_s, the ratio $\kappa = \langle \bar{s}s\rangle/\langle \bar{d}d\rangle$ and the factorization of the four-quark condensate ρ. κ gives the most important error while the ratio of masses increases when the one of the quark condensate decreases. The other QCD parameters give negligible errors. Using as input the data [9]:

$$M_{\Lambda_c}^{exp} = (2286.46 \pm 0.14) \text{ MeV} \ , \tag{26}$$

and adding the different errors quadratically, one can deduce:

$$M_{\Xi_c} = (2449 \pm 49)\ \text{MeV}\ , \tag{27}$$

which agrees with the data [9]:

$$M^{exp}_{\Xi_c} = (2467.9 \pm 0.4)\ \text{MeV}\ . \tag{28}$$

This result is also given in Table 2.

• $\boldsymbol{\Xi_b(bsq)/\Lambda_b(bqq)}$

We repeat the previous analysis in the case of the b quark. The analysis of the ratio of sum rules shows similar curves than for the charm case except the obvious change of scale. It also indicates that r^{sd}_{21} has the best τ- and t_c-stabilities, from which we deduce:

$$r^{sd}_{\Xi_b} = 1.048(4)(2)(3)(1)(7)(9)(7)\ , \tag{29}$$

where we have considered the mean value from $t_c = 34$ GeV^2 (beginning of τ-stability) to the beginning of t_c-stability (60 GeV^2). We have used the optimal value $\tau = 0.35 \pm 0.05$ GeV^{-2}. The sources of the errors are the same as in the charm quark case. Using the data [9]:

$$M^{exp}_{\Lambda_b} = (5620.2 \pm 1.6)\ \text{MeV}\ , \tag{30}$$

and adding the different errors quadratically, one get:

$$M_{\Xi_b} = (5888 \pm 81)\ \text{MeV}\ , \tag{31}$$

which agrees with the data [9]:

$$M^{exp}_{\Xi_b} = (5792.4 \pm 3.0)\ \text{MeV}\ , \tag{32}$$

Although the errors look quite large in the two cases of charm and bottom compared with the data, the predictions are more robust than the recent estimates in [14,15], where we expect that the errors have been underestimated.

• $\boldsymbol{\Omega_c(css)/\Sigma_c(cqq)}$

We do an analysis similar to the one in the previous section. The result for the c-quark is shown in Fig 2 . One can notice that the optimal choice of the current is the same as in Eq. (2) which we fix to the value b=0. One can notice from in Fig 2a) and Fig 2b), that r^{sd}_{21} does not give a result consistent with the one from r^{sd}_i $(i = 1, 2)$ and it is also less stable in τ than the two others. The appearance of a (false) extremum at small τ-values is also strongly affected by t_c at which one cannot extract any optimal result. We shall not also retain r^{sd}_1 as it is not stable versus b. The final result from r^{sd}_2 is:

$$r^{sd}_{\Omega_c} = 1.141(12)(0.4)(17)(10)(13)(29)(1)\ , \tag{33}$$

and come from the mean of t_c values from 6 (beginning of τ-stablity) to 12 GeV^2 (beginning of t_c-stability). The sources of the errors are the same as before and come from $\tau = (0.8 \pm 0.2)$ GeV^{-2}, b, t_c, m_c, m_s, κ and ρ (deviation from factorization of the four-quark condensate). The other QCD parameters gives negligible errors. Using this previous result together with the experimental averaged value [9]:

$$M^{exp}_{\Sigma_c} = 2453.6\ \text{MeV}\ , \tag{34}$$

one can deduce the result in Table 2 in agreement with the data.

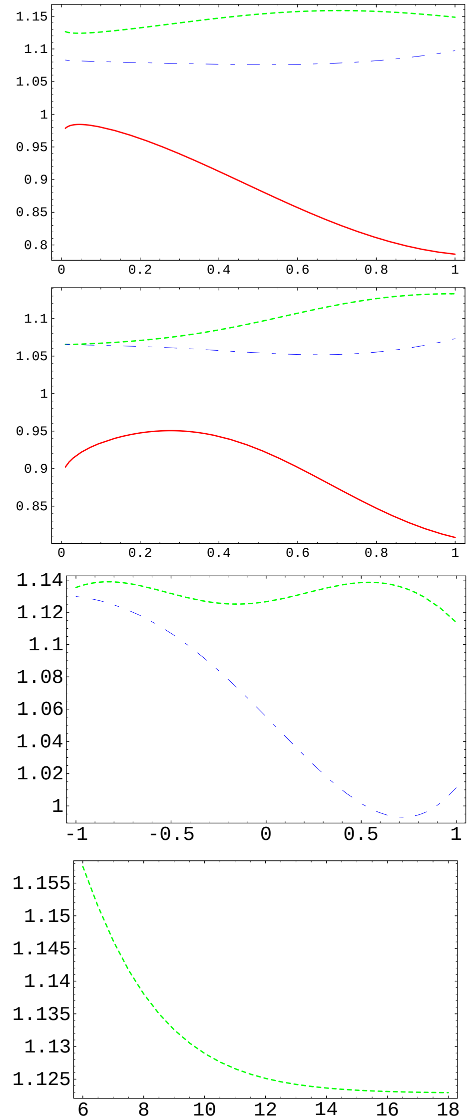

Figure 2. $\boldsymbol{\Omega_c/\Sigma_c}$: a) τ-behaviour of the DR in the charm case given $b = 0$ and $t_c = 6$ GeV^2: r^{ds}_1 dashed-dotted (blue), r^{ds}_2 dotted (green), r^{ds}_{21} continuous (red); b) the same as in a) but for $t_c = 11$ GeV^2; c) b-behaviour of the DR r^{ds}_1 and r^{ds}_2 given $\tau = 0.8$ GeV^{-2} and $t_c = 11$ GeV^2; d) t_c-behaviour of r^{ds}_2 given $b = 0$ and $\tau = 0.8$ GeV^{-2}.

• $\boldsymbol{\Omega_b(bss)/\Sigma_b(bqq)}$

We repeat the previous analysis in the case of the b-quark. The curves present the same qualitative behaviour as in the case of the charm, where, only r^{ds}_2 survives the different tests of stabilities. Here, the t_c-behaviour is almost flat from $t_c = 34$ GeV^2 (beginning of τ-stability). The optimal value is taken at the extremum $\tau = (0.25 \pm 0.05)$ GeV^{-2}. Then, we obtain:

$$r^{sd}_{\Omega_b} = 1.051(2)(0.5)(1)(4)(3)(11)(1)\ , \tag{35}$$

with the same sources of errors as before. Using this value together with the experimental averaged value [9]:

$$M^{exp}_{\Sigma_b} = 5811.2\ \text{MeV}, \tag{36}$$

one can deduce the result in Table 2 in agreement with the data [37], which, however, needs to be confirmed by some other experiments.

6. The masses of the spin 3/2 decuplet baryons

As a preliminary step of the analysis, we check the different results obtained in [5]:

$$M_{\Sigma_c^*} = (2.15 \sim 2.92)\ \text{GeV}\ ,$$
$$M_{\Sigma_b^*} - M_{\Sigma_c^*} = 3.3\ \text{GeV}\ , \qquad (37)$$

and confirm them. However, like in the octet case, we have not tried to improve these (old) results.

• $\Xi_c^*(csq)/\Sigma_c^*(cqq)$

We repeat the previous DR analysis for the case of the Ξ_c^*. We show in Fig. 3a) and Fig. 3b) the τ-behaviour of the mass predictions for two extremal values of t_c between $M^2_{\Sigma_c^*}$ (beginning of the τ-stability) and 12 GeV^2 (beginning of t_c-stability). We do not retain r_{21}^{sd} which differs completely from r_1^{ds} and r_2^{ds}, while we do not consider r_1^{ds} which becomes τ-instable when t_c increases. We show in Fig. 1c) the t_c-behaviour of r_2^{ds} given τ=0.7 GeV^{-2}.

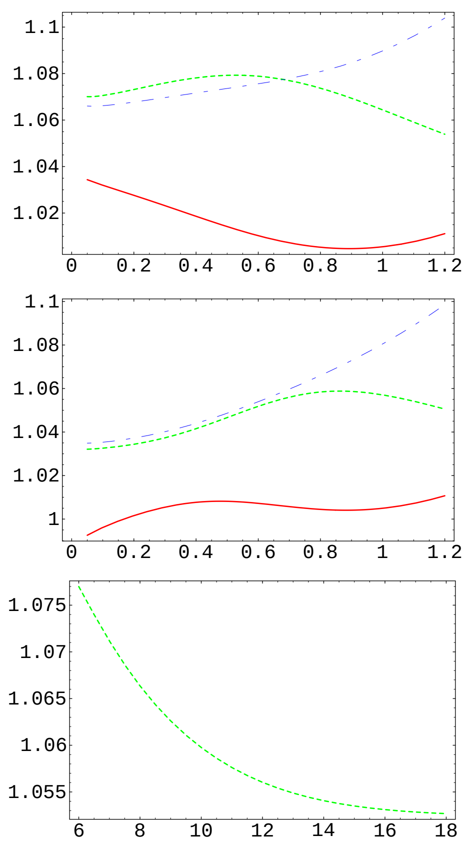

Figure 3. Ξ_c^*/Σ_c^*: a) τ-behaviour of the double ratio of sum rules (DR) by giving $t_c = M^2_{\Sigma_c^*} = 6$ GeV^2 : r_1^{ds} dashed-dotted (blue), r_2^{sd} dotted (green) and r_{21}^{sd} continuous (red); b) the same as in a) but for $t_c = 12$ GeV^2 c) t_c-behaviour of r_2^{ds} for a given optimal $\tau = 0.9$ GeV^{-2}.

We deduce the optimal value:

$$r^{sd}_{\Xi_c^*} = 1.065(1)(10)(4)(4)(17.5)(1)\ . \qquad (38)$$

The errors are due respectively to the values of $\tau = (0.7 \pm 0.2)$ GeV^{-2}, t_c, τ, m_c, m_s, $\kappa \equiv \langle\bar{s}s\rangle/\langle\bar{d}d\rangle$ and ρ (factorization of the four-quark condensate). The ones due to some other parameters are negligible. Using the data [9]:

$$M^{exp}_{\Sigma_c^*} = (2517.97 \pm 1.17)\ \text{MeV}\ , \qquad (39)$$

and adding the different errors quadratically, we deduce the results in Table 2.

• $\Xi_b^*(bsq)/\Sigma_b^*(bqq)$

We extend the analysis to the case of the bottom quark. The curves are qualitatively similar to the charm case. We deduce:

$$r^{sd}_{\Xi_b^*} = 1.024(0.4)(2.5)(1)(1)(7)(0.6)\ . \qquad (40)$$

The sources of the errors are the same as for the Ξ_c^*, where here $\tau = (0.22 \pm 0.04)$ GeV^{-2} and t_c between $M^2_{\Sigma_b^*}$ and 70 GeV^2 (beginning of t_c-stability).The ones due to some other parameters are negligible. Using the averaged data:

$$M^{exp}_{\Sigma_b^*} = (5832.7 \pm 6.5)\ \text{MeV}\ , \qquad (41)$$

and adding the different errors quadratically, we deduce the result in Table 2.

• $\Omega_c^*(css)/\Sigma_c^*(cqq)$

We pursue the analysis to the case of the $\Omega_c^*(css)$. We

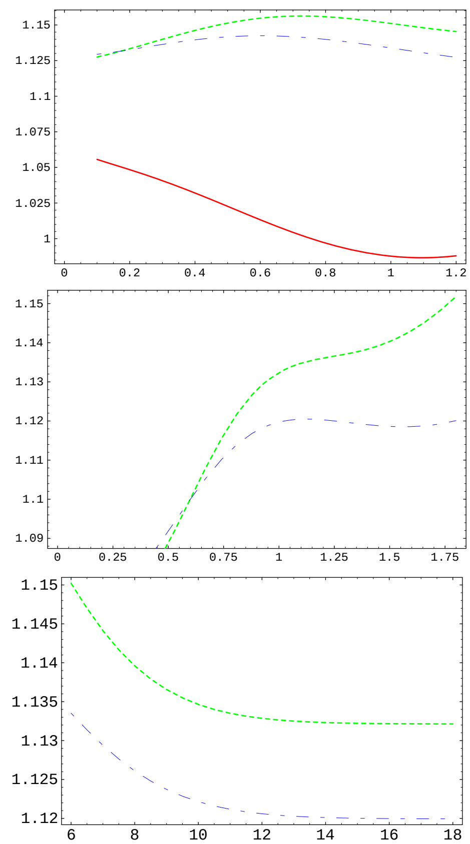

Figure 4. Ω_c^*/Σ_c^*: a) τ-behaviour of the double ratio of sum rules (DR) by giving $t_c = M^2_{\Sigma_c^*}$: r_1^{ds} dashed-dotted (blue), r_2^{sd} dotted (green) and r_{21}^{sd} continuous (red); b) the same as in a) but for $t_c = 12$ GeV^2 (beginning of the t_c-stability); c) t_c-behaviour of the DR r_2^{ds} giving $\tau = 1$ GeV^{-2}.

show the τ-behaviour of the different DR in Fig. 4a). From this figure, we shall not retain r_{21}^{ds} which differs completely from r_1^{ds} and r_2^{ds}. We show the same τ-behaviour in Fig. 4b) but for $t_c = 12$ GeV^2 (beginning of the t_c-stability). Given the optimal value of $\tau = 1$ GeV^{-2}, we show the t_c-behaviour of the DR r_1^{ds} and r_2^{ds} in Fig. 4c). The final result is the mean from r_1^{ds} and r_2^{ds}:

$$r^{sd}_{\Omega^*_c} = 1.135(6)(0.4)(7.5)(11.5)(14)(30.5)(0.5)\ . \tag{42}$$

The 1st error is due to the choice of r_i^{sd}. The other ones are due to $\tau = (1.0 \pm 0.2)$ GeV^{-2}, t_c, m_c, m_s, κ and ρ. The most important error comes from κ while the ratio of masses increases when the one of the quark condensate decreases. The other QCD parameters give negligible errors. Using the averaged data in Eq. (41), and adding the different errors quadratically, one can deduce the result in Table 2.

• $\boldsymbol{\Omega^*_b(bss)/\Sigma^*_b(bqq)}$

We repeat the previous analysis in the b-channel. The curves are qualitatively analogue to the ones of the charm. We shall not consider r_{21}^{sd} because of its incompatibility with the other ones. From the mean of r_1^{ds} and r_2^{ds}, we deduce:

$$r^{sd}_{\Omega^*_b} = 1.051(5.5)(0.3)(2)(1.5)(4)(15)(1.5)\ , \tag{43}$$

where the sources of the errors are the same as for Ω^*_b, where $\tau = (0.30\pm0.05)$ GeV^{-2}. Using the averaged data in Eq. (41), and adding the different errors quadratically, one can deduce the result in Table 2.

7. Summary and Conclusions

Table 2
QSSR predictions of the strange heavy baryon masses in units of MeV from the double ratio (DR) of sum rules with the QCD input parameters in Table 1 and using as input the observed masses of the associated non-strange heavy baryons.

Baryons	$r^{sd}_{B^*_Q}$	Mass	Data
Octet			
Ξ_c	1.075(21)	2458(50)	2467.9 ± 0.4
Ω_c	1.141(39)	2800(96)	2697.5 ± 2.6
Ξ_b	1.048(15)	5888(81)	5792.4 ± 3.0
Ω_b	1.051(12)	6108(71)	6165.0 ± 13
Decuplet			
Ξ^*_c	1.065(21)	2682(53)	2646.1 ± 1.3
Ω^*_c	1.135(37)	2858(92)	2768.3 ± 3.0
Ξ^*_b	1.024(8)	5973(44)	–
Ω^*_b	1.051(17)	6130(99)	–

• We have directly extracted (for the first time) the heavy baryons decuplet mass-splittings due to $SU(3)$ breaking using double ratios (DR) of QCD spectral sum rules(QSSR), which are less sensitive to the exact value and the definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios commonly used in the current literature for determining the heavy baryon masses. As a result, we have provided (relatively) robust predictions of the $\Xi^{(*)}_Q$ and $\Omega^{(*)}_Q$ masses once the ones of the associated non-strange heavy baryons are known from the data. The different results are summarized in Table 2.

Table 3
QSSR predictions of the strange heavy baryon hyperfine splittings in units of MeV from the double ratio (DR) of sum rules with the QCD input parameters in Table 1 and using as input the predicted values in Table 2

Hyperfine Splittings	Observed
$M_{\Xi^*_c} - M_{\Xi_c}$=224(52)	179(1)
$M_{\Omega^*_c} - M_{\Omega_c}$=58(94)	70(3)
$M_{\Xi^*_b} - M_{\Xi_b}$=85(63)	–
$M_{\Omega^*_b} - M_{\Omega_b}$=22(85)	$M_{\Sigma^*_b} - M_{\Sigma_b} = 22$

• Combining the previous predictions for the decuplet with the ones for the octet, we give in Table 3 predictions of the hyperfine mass-splittings. These results agree quite well with the data and with some expectations from quark models.

• Like in the case of the light baryons [29], it is remarkable to notice that the leading term controlling the mass-splittings is the ratio $\kappa \equiv \langle\bar{s}s\rangle/\langle\bar{d}d\rangle$ of the condensate rather than the running mass $\bar{m}_s$. This ratio gives, after the choice of the continuum threshold t_c, the largest errors in $r^{sd}_{B^{(*)}_Q}$.

• One can notice that for $SU(3)$ symmetric quark condensates $\langle\bar{s}s\rangle \simeq \langle\bar{d}d\rangle$, the predictions tend to be lower than the present predictions which deteriorate the agreement with the observed masses in different channels. This feature might explain the understimate of the Ω_b predictions observed in some quark models [11,?].

Acknowledgements

R.M.A and M.N. acknowledge the financial support from FAPESP and CNPq (Brazil). S.N. has been partially supported by the CNRS-IN2P3 within the project "Hadron Physics in QCD" and within the French-China Particle Physics Laboratory (FCPPL). We thank Jean-Marc Richard for some discussions.

REFERENCES

1. For reviews, see e.g.: S. Narison, *Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol.* **17** (2004) 1-778 [hep-ph/0205006]; S. Narison, *World Sci. Lect. Notes Phys.* **26** (1989) 1 ; S. Narison, *Acta Phys. Pol.* **B 26** (1995) 687; S. Narison, *Riv. Nuovo Cim.* **10 (N2)** (1987) 1; S. Narison, *Phys. Rept.* **84** (1982) 263 .
2. J.S. Bell and R.A. Bertlmann, *Nucl. Phys.* **B 177** (1981) 218. **B187** (1981) 285; R.A. Bertlmann, *Acta Phys. Austr.* **53** (1981) 305.
3. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B 147** 385, 448 (1979).
4. E. Bagan, M. Chabab, H.G. Dosch, S. Narison, *Phys. Lett.* **B 278** (1992) 367.
5. E. Bagan, M. Chabab, H.G. Dosch, S. Narison, *Phys. Lett.* **B 287** (1992) 176.
6. E. Bagan, M. Chabab, S. Narison, *Phys. Lett.* **B 306** (1993) 350.
7. E. Bagan, M. Chabab, H.G. Dosch, S. Narison, *Phys. Lett.* **B 301** (1993) 243.
8. The D0 collaboration: V. Abazov et al. *Phys. Rev. Lett.* **99** (2007) 052001; *arXiv: 0808.4142 [hep.ex]*; The CDF collaboration: D. Acosta et al., *Phys. Rev. Lett.* **96** (2006) 202001; T. Aaltonen et al. *Phys. Rev. Lett.* **99** (2007) 052002; ibid. *Phys. Rev. Lett.* **99** (2007) 202001.
9. PDG, C. Amsler et al., *Phys. Lett.* **B 667** (2008) 1.
10. The Babar collaboration: B. Aubert et al., *Phys. Rev. Lett.* **97** (2006) 232001; ibid *Phys. Rev. Lett.* **99** (2007) 202001; The Belle collaboration: R. Chistov et al., *Phys. Rev. Lett.* **97** (2006) 162001.
11. For recent reviews, see e.g.: M. Karliner, B. Keren-Zur, J.H. Lipkin, J.L. Rosner,*Annals Phys.* **324** (2009) 2; E. Klempt, J.M. Richard, *arXiv:0811.2711 [hep-ph]* and references therein.
12. R.S. Marques de Carvalho et al., *Phys.Rev.* **D 60** (1999) 034009.
13. F. Hussain, J.G. Korner, M. Kramer, G. Thompson, *Z. Phys.* **C 51** (1991) 321.
14. F.O. Durães, M. Nielsen, *Phys. Lett.* **B 658** (2007) 40.
15. X. Liu, H-X. Chen, Y-R. Liu, A. Hosaka, S-L. Zhu, *arXiv:0710.0123 [hep-ph]*; Z-G. Wang, *arXiv:0809.3038 [hep-ph]*; ibid, *arXiv:0704.1106 [hep-ph]*; J-R. Zhang, M-Q. Huang, *arXiv:0811.3266 [hep-ph]*, ibid, *arXiv:0810.5396 [hep-ph]*; ibid, *arXiv:0805.0479 [hep-ph]*.
16. S. Groote, J.G. Korner, O.I. Yakovlev, *Eur. Phys. J.* **C 58** (2008) 355.
17. D. Ebert, R.N. Faustov and V.O. Galkin, *Phys. Rev.* **D 72** (2005) 034026.
18. W. Detmold, C-J. D. Lin, M. Wingate, *arXiv:0812.2583 [hep-lat]*; R. Lewis, R.M. Woloshyn, *Phys. Rev.* **D 79** (2009) 014502; H. Na, S. Gottlieb, *arXiv:0812.1235 [hep-lat]*.
19. E. Jenkins, *Phys. Rev.* **D 77** (2008) 034012.
20. S. Narison, *Phys. Lett.* **B 210** (1988) 238; *Phys. Lett.* **B 605** (2005) 319.
21. S. Narison, *Phys. Lett.* **B 322** (1994) 327.
22. S. Narison, *Phys. Lett.* **B 337** (1994) 166; *Phys. Lett.* **B 668** (2008) 308; *Phys. Lett.* **B 327** (1994) 354.
23. S. Narison, *Phys. Lett.* **B 358** (1995) 113; *Phys. Lett.* **B 466** (1999) 345.
24. E.G. Floratos, S. Narison and E. de Rafael, *Nucl. Phys.* **B 155** (1979) 155.
25. K. Chetyrkin, S. Narison and V.I. Zakharov, *Nucl. Phys.* **B 550**, 353 (1999); For a review, see e.g. S. Narison, *Nucl. Phys. Proc. Suppl.* **164** (2007) 225.
26. For a review, see e.g. V.A. Zakharov, *Nucl. Phys. (Proc. Sup.)* **B 74** (1999) 392; V.I. Zakharov, *Nucl. Phys. Proc. Suppl.* **164** (2007) 240; V.I. Zakharov, contribution at QCD 08 (Montpellier); R. Akhoury and V.I. Zakharov, *Phys. Lett.* **B 438** (1998) 165; F.V. Gubarev, M.I. Polikarpov and V.I. Zakharov, hep-th/9812030.
27. S. Narison, *Phys. Lett.* **B 673** (2009) 30.
28. For reviews, see e.g.: S. Narison, *Phys.Rev.* **D 74** (2006) 034013; *arXiv:hep-ph 0202200*; *Phys. Lett.* **B 216** (1989) 191; *Phys. Lett.* **B 358** (1995) 113; *Phys. Lett.* **B 466** (1999) 345; S. Narison, H.G. Dosch, *Phys. Lett.* **B 417** (1998) 173; S. Narison, N. Paver, E. de Rafael and D. Treleani, *Nucl. Phys.* **B 212** (1983) 365; S. Narison, E. de Rafael, *Phys. Lett.* **B 103** (1981) 57; C. Becchi, S. Narison, E. de Rafael, F.J. Yndurain, *Z. Phys.* **C 8** (1981) 335.
29. Y. Chung et al.*Z. Phys.* **C 25** (1984) 151; H.G. Dosch, M. Jamin and S. Narison, *Phys. Lett.* **B 220** (1989) 251.
30. B.L. Ioffe, *Nucl. Phys.* **B 188** (1981) 317 , **B 191** (1981) 591; A.A.Ovchinnikov and A.A.Pivovarov, *Yad. Fiz.* **48** (1988) 1135.
31. G. Launer, S. Narison and R. Tarrach, *Z. Phys.* **C 26** (1984) 433.
32. S. Narison, *Phys. Lett.* **B 300** (1993) 293; ibid **B 361** (1995) 121.
33. R.A. Bertlmann, G. Launer and E. de Rafael, *Nucl. Phys.* **B 250** (1985) 61; R.A. Bertlmann et al., *Z. Phys.* **C 39** (1988) 231.
34. F.J. Yndurain, hep-ph/9903457.
35. S. Narison, *Phys. Lett.* **B 387** (1996) 162.
36. S. Narison, *Phys. Lett.* **B 197** (1987) 405; **Phys. Lett. B 341** (1994) 73; *Phys. Lett.* **B 520** (2001) 115.
37. The D0 collaboration: V.M. Abazov et al., *arXiv: 0808.4142 [hep.ex]*
38. The CDF collaboration: T. Aaltonen et al., *arXiv: 0905.3123 [hep.ex]*.