Ghost Magic: MGI, MLC & Qluonless QCD under Eikonal Approximations

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1 Introduction

This talk is based on the recent work with H. M. Fried (Brown), Y. Gabellini (INLN) and T. Grandou (INLN) on non-perturbative summation of all QCD virtual gluon exchanges[1]. A novel, non-perturbative approach to the theory of strong coupling is presented with functional methods; starting from well-known Schwingerian QED formulation[2], and then extending to QCD in a manifestly gauge-invariant (MGI) and manifestly Lorentz-covariant (MLC) fashion. The ghost magic is presented with quark scattering processes and leads to a new Quonless QCD formalism of no virtual gluons under Eikonal approximations. Contrary to the conventional ghost fields, it turns out that virtual gluons themselves can also be viewed as ghosts in the new formalism.

The talk is arranged as the following: first, a brief introduction of function method from QED to QCD is briefly discussed in Section 2; followed by the MGI and MLC QCD formulation in Section 3. Virtual gluon exchanges in fermion scattering under Eikonal approximations will be used as an example of MGI and MLC QCD in Section 4. The ghost magic with aids of Helpern's reformation leading to Qluonless QCD formalism in the quark scattering processes is in Section 5; and a brief summary is given in Section 6.

2 Schwingerian Formalism in Functional Method

The approach is based on the Schwingerian QED formalism by starting from the generating functional with a given Lagrangian and its extension to QCD. The QED Lagrangian (density) is given by

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu} - \overline{\psi}\left[m + \gamma\cdot(\partial - igA)\right]\psi = \mathcal{L}^{(0)} + \mathcal{L}'\{A, \psi, \overline{\psi}\}. \quad (1)$$

After adding the source terms, $\mathcal{L}_{\rm QED} \to \mathcal{L}_{\rm QED} + j\cdot A + \overline{\eta}\cdot\psi + \overline{\psi}\cdot\eta$, the generating functional derived from Schwinger's Action principle can be expressed as[2]

$$\begin{aligned}
\mathfrak{Z}_{\rm c}\{j,\overline{\eta},\eta\} &= \langle 0|\left(\exp\left\{i\int\left[j\cdot A + \overline{\eta}\cdot\psi + \overline{\psi}\cdot\eta\right]\right\}\right)_+|0\rangle \\
&= \langle\mathbf{S}\rangle^{-1}\exp\left[i\int\mathcal{L}'\left\{\frac{1}{i}\frac{\delta}{\delta j},\frac{1}{i}\frac{\delta}{\delta\overline{\eta}},\frac{-1}{i}\frac{\delta}{\delta\eta}\right\}\right]\cdot\mathfrak{Z}_{\rm c}^{(0)}\{j,\overline{\eta},\eta\},
\end{aligned} \tag{2}$$

where $\mathfrak{Z}_{\rm c}^{(0)}\{j,\overline{\eta},\eta\} = \exp\left\{\frac{i}{2}\int j\cdot\mathbf{D}_{\rm c}\cdot j + i\int\overline{\eta}\cdot\mathbf{S}_{\rm c}\cdot\eta\right\}$ represents an interaction-free system, and $\mathbf{S}_{\rm c}$ and $\mathbf{D}_{\rm c}$ are the free fermion propagator and photon propagator, respectively. The generating functional can be further rearranged to a more convenient form originated by H. M. Fried as[2]

$$\mathfrak{Z}_{\rm QED}[j,\eta,\overline{\eta}] = \langle\mathbf{S}\rangle^{-1}\, e^{\frac{i}{2}\int j\cdot\mathbf{D}_c\cdot j}\cdot e^{\mathfrak{D}_A}\cdot e^{i\int\overline{\eta}\cdot\mathbf{G}_c[A]\cdot\eta + \mathbf{L}[A]}\Big|_{A=\int\mathbf{D}_c\cdot j} \tag{3}$$

in terms of potential-theory-like Green's function $\mathbf{G}_c[A] = [m+\gamma\cdot(\partial - igA)]^{-1} = \mathbf{S}_{\rm c}\cdot[1-g\,(\gamma\cdot A)\,\mathbf{S}_{\rm c}]^{-1}$, closed-fermion-loop functional $\mathbf{L}[A] = \mathbf{Tr}\ln[1-ig(\gamma\cdot A)\mathbf{S}_{\rm c}]$, and linkage operator $e^{\mathfrak{D}_A}$ with $\mathfrak{D}_A = -\frac{i}{2}\int\frac{\delta}{\delta A}\cdot\mathbf{D}_c\cdot\frac{\delta}{\delta A}$. Dressed propagators and n-point functions can then be constructed by differentiating the generating functional with respect to corresponding sources and setting them to zeros at the end. The linkage operator inserts photon propagators among fermion propagators and closed-fermion-loops for every pair of A_μ-fields; and inherently it sums over all orders of Feynman graphs systematically.

The extension to QCD is similar with the QCD lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \overline{\psi}\,[m+\gamma\cdot(\partial - igA)]\,\psi, \tag{4}$$

where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ is the gluon field strength. To separate gluons' self-coupling, one can define the QED-like field strength as $\mathbf{F}^a_{\mu\nu} = G^a_{\mu\nu}\big|_{g=0} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ without self-coupling, then the QCD Lagrangian can be split into two parts as

$$\begin{aligned}
\mathcal{L}^{(0)}_{\rm QCD} &= -\overline{\psi}\,[m+\gamma_\mu\,\partial_\mu]\,\psi - \frac{1}{4}\mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu}, \\
\mathcal{L}'_{\rm QCD} &= +ig\,\overline{\psi}\,(\gamma_\mu\,A^a_\mu\lambda^a)\,\psi - \frac{1}{4}(G^a_{\mu\nu}G^a_{\mu\nu} - \mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu}),
\end{aligned} \tag{5}$$

where $\mathcal{L}^{(0)}_{\rm QCD}$ is quadratic in A^a_μ, and $\mathcal{L}'_{\rm QCD}$ contains both cubic and quartic A^a_μ-terms from self-coupling of gluons. The QCD generating functional becomes

$$\mathfrak{Z}_{\rm c}\{j,\overline{\eta},\eta\} = \langle\mathbf{S}\rangle^{-1}\exp\left[i\int\mathcal{L}'_{\rm QCD}\left\{\frac{1}{i}\frac{\delta}{\delta j},\frac{1}{i}\frac{\delta}{\delta\overline{\eta}},\frac{-1}{i}\frac{\delta}{\delta\eta}\right\}\right]\cdot\mathfrak{Z}_{\rm c}^{(0)}\{j,\overline{\eta},\eta\}, \tag{6}$$

where $\mathfrak{Z}_c^{(0)}\{j,\overline{\eta},\eta\} = \exp\left\{\frac{i}{2}\int j\cdot\mathbf{D}_c\cdot j + i\int\overline{\eta}\cdot\mathbf{S}_c\cdot\eta\right\}$ is the free generating functional with the free quark propagator $\mathbf{S}_c$ and free gluon propagator $\mathbf{D}_c$. Similar to QED, there is a catch on free gluon propagators due to non-physical degrees of freedom from gauge fields in the functional intgral.

3 MGI & MLC Formalism

To remove non-physical degrees of freedom of A_μ-fields in QED, one introduces a gauge fixing condition, $\mathcal{F}(A) = 0$, into the generating functional as

$$\mathfrak{Z}[j,\eta,\overline{\eta}] = \mathcal{N}\int \mathfrak{D}A\,\mathfrak{D}\psi\,\mathfrak{D}\psi^\dagger \boxed{\delta[\mathcal{F}(A)]\cdot\det\left[\delta\mathcal{F}/\delta\omega\right]}\cdot e^{i\int\mathcal{L}_{\rm QED}+j\cdot A+\overline{\eta}\cdot\psi+\overline{\psi}\cdot\eta}, \qquad (7)$$

where the gauge fixing constraint, $\delta[\mathcal{F}(A)]$, is to define a specific gauge and the Jacobian determinant, $\det\left[\delta\mathcal{F}/\delta\omega\right]$, is to guarantee the gauge invariance. However, the Jacobian, $\det\left[\delta\mathcal{F}/\delta\omega\right]$, itself might be independent of gauge fields under certain choice of gauge conditions, *eg.*, the axial gauge $\hat{n}_\mu A^a_\mu = 0$; such that

$$\begin{aligned}\mathfrak{Z}[j,\eta,\overline{\eta}] &= \mathcal{N}'\int \mathfrak{D}A\,\mathfrak{D}\psi\,\mathfrak{D}\psi^\dagger\,\delta[\mathcal{F}(A)]\cdot e^{i\int\mathcal{L}_{\rm QED}+j\cdot A+\overline{\eta}\cdot\psi+\overline{\psi}\cdot\eta} \\ &= \mathcal{N}\text{"}\int \mathfrak{D}[A]\,\delta[\mathcal{F}(A)]\,e^{-\frac{i}{4}\int\mathbf{F}^2}\cdot e^{i\int\overline{\eta}\cdot\mathbf{G}_c[A]\cdot\eta+\mathbf{L}[A]+i\int j\cdot A}.\end{aligned} \qquad (8)$$

The remaining $\delta[\mathcal{F}(A)]$ in the generating functional imposes a gauge fixing term, $-\frac{1}{2\zeta}(\hat{n}_\mu A^a_\mu)^2$, with the gauge parameter $\zeta \neq 0$ and $\hat{n}^2 = -1$ into the effective Lagrangian which is neither explicitly gauge independent nor Lorentz covariant. Furthermore, each Feynman graph in the perturbation approach might not be gauge invariant individually. To keep gauge invariant in QED, one needs to observe charge conservation when calculating any physical quantities.

The free gluon sector of the QCD action can be written as

$$i\int\mathcal{L}^{(0)}_{\rm gloun} = -\frac{i}{4}\int\mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu} = +\frac{i}{2}\int A^a_\mu\delta^{ab}\left[g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right]A^b_\nu, \qquad (9)$$

but there is no inverse of operator $[g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu]$. One then proceeds with Faddeev-Popov ansatz by inserting $\Delta_g[A]$ in the generating functional as

$$\mathfrak{Z}[j,\eta,\overline{\eta}] = \int \mathfrak{D}A\,\mathfrak{D}\psi\,\mathfrak{D}\psi^\dagger\,\delta[\mathcal{F}(A)]\cdot\Delta_g[A]\cdot e^{i\int\mathcal{L}_{\rm QCD}+j\cdot A+\overline{\eta}\cdot\psi+\overline{\psi}\cdot\eta}, \qquad (10)$$

such that $\Delta_g^{-1}[A] = \int\mathfrak{D}\omega\,\delta[\mathcal{F}(A)]$ is a functional integral over gauge parameter ω, elements of the gauge group, and $\Delta_g[A]$ itself is gauge invariant. Thus, the Faddeev-Popov ghosts are introduced into the theory in order to keep proper counting of physical degrees of freedom.

Instead, one can add and subtract a gauge fixing term, $-\frac{1}{2\zeta}(\partial_\mu A^a_\mu)^2$, with $\zeta \neq 0$ to the gluon sector of Lagrangian as

$$
\begin{aligned}
\mathcal{L}^{(0)}_{\text{gluon}} &= -\frac{1}{4}\mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu} - \frac{1}{2\zeta}(\partial_\mu A^a_\mu)^2, \\
\mathcal{L}'_{\text{gluon}} &= -\frac{1}{4}(G^a_{\mu\nu}G^a_{\mu\nu} - \mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu}) + \frac{1}{2\zeta}(\partial_\mu A^a_\mu)^2,
\end{aligned}
\tag{11}
$$

such that

$$
i\int \mathcal{L}^{(0)}_{\text{gloun}} = -\frac{i}{4}\int \mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu} - \frac{i}{2\zeta}\int (\partial_\mu A^a_\mu)^2 = -\frac{i}{2}\int A^a_\mu \left(\mathbf{D}^{\zeta\,-1}_{\text{c}}\right)^{ab}_{\mu\nu} A^b_\nu \tag{12}
$$

with $\left(\mathbf{D}^{\zeta\,-1}_{\text{c}}\right)^{ab}_{\mu\nu} = -\delta^{ab}\left[\delta_{\mu\nu}\partial^2 + \left(\frac{1}{\zeta} - 1\right)\partial_\mu\partial_\nu\right]$. Meanwhile, the overall Lagrangian $\mathcal{L}_{\text{QCD}} = \mathcal{L}^{(0)}_{\text{QCD}} + \mathcal{L}'_{\text{QCD}}$ is kept the same with

$$
\begin{aligned}
\mathcal{L}^{(0)}_{\text{QCD}} &= -\overline{\psi}\left[m + \gamma_\mu\,\partial_\mu\right]\psi - \frac{1}{4}\mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu} - \frac{1}{2\zeta}(\partial_\mu A^a_\mu)^2, \\
\mathcal{L}'_{\text{QCD}} &= +ig\,\overline{\psi}\left(\gamma_\mu\,A^a_\mu\lambda^a\right)\psi - \frac{1}{4}(G^a_{\mu\nu}G^a_{\mu\nu} - \mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu}) + \frac{1}{2\zeta}(\partial_\mu A^a_\mu)^2.
\end{aligned}
\tag{13}
$$

The Lagrangian and thus generating functional are kept both manifestly gauge invariant (MGI) and manifestly Lorentz covariant (MLC). In contrast, Faddeev-Popov's Δ_g is gauge invariant, but $e^{+\frac{i}{2\zeta}\int(\partial_\mu A^a_\mu)^2}$ is not. Hence, the QCD generating functional becomes

$$
\begin{aligned}
\mathfrak{Z}_{\text{c}}\{j,\overline{\eta},\eta\} &= \langle\mathbf{S}\rangle^{-1}\, e^{\frac{i}{2}\int j^a_\mu\cdot\mathbf{D}^{\zeta\,ab}_{\text{c}\,\mu\nu}\cdot j^b_\nu}\, e^{-\frac{i}{2}\int \frac{\delta}{\delta A^a_\mu}\mathbf{D}^{\zeta\,ab}_{\text{c}\,\mu\nu}\frac{\delta}{\delta A^b_\nu}}\, e^{i\int \mathcal{L}'_{\text{QCD}}\left[A,\frac{1}{i}\frac{\delta}{\delta\overline{\eta}},\frac{-1}{i}\frac{\delta}{\delta\eta}\right]} \cdot e^{i\int\overline{\eta}\cdot\mathbf{S}_{\text{c}}\cdot\eta} \\
&= e^{\frac{i}{2}\int j^a_\mu\cdot\mathbf{D}^{\zeta\,ab}_{\text{c}\,\mu\nu}\cdot j^b_\nu}\, e^{-\frac{i}{2}\int \frac{\delta}{\delta A^a_\mu}\mathbf{D}^{\zeta\,ab}_{\text{c}\,\mu\nu}\frac{\delta}{\delta A^b_\nu}}\, e^{i\int\mathcal{L}'_{\text{gluon}}[A]}\, e^{i\int\overline{\eta}\cdot\mathbf{G}_{\text{c}}[A]\cdot\eta}\,\frac{e^{\mathbf{L}[A]}}{\langle\mathbf{S}\rangle}
\end{aligned}
\tag{14}
$$

This is similar to QED except that the linkage operator $e^{\mathfrak{D}_A}$ contains an explicitly-gauge-dependent free gluon propagator $\mathbf{D}^\zeta_c$ in $\mathfrak{D}_A = -\frac{i}{2}\int\frac{\delta}{\delta A}\cdot\mathbf{D}^\zeta_c\cdot\frac{\delta}{\delta A}$, and the generating functional has an extra factor of $e^{i\int\mathcal{L}'_{\text{gluon}}[A]}$ from the non-linear, self-interaction of non-Abelian gauge fields. In comparison, $e^{i\int\mathcal{L}'_{photon}} = 1$ in QED and the effective QED action (or Lagrangian) might not be gauge invariant, and one needs to keep in mind that charge conservation for all calculations.

The contribution from the self-interaction of gluons is

$$
e^{i\int\mathcal{L}'_{\text{gluon}}[A]} = e^{-\frac{i}{4}\int G^a_{\mu\nu}G^a_{\mu\nu} + \left[\frac{i}{4}\int\mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu} + \frac{i}{2\zeta}\int(\partial_\mu A^a_\mu)^2\right]}, \tag{15}
$$

and the factor, $e^{+\frac{i}{4}\int\mathbf{F}^a_{\mu\nu}\mathbf{F}^a_{\mu\nu} + \frac{i}{2\zeta}\int(\partial_\mu A^a_\mu)^2} = e^{+\frac{i}{2}\int A\cdot\left(\mathbf{D}^\zeta_{\text{c}}\right)^{-1}\cdot A}$, will be shown to play an important role in the presentation of "Ghost Magic" in QCD. To deal with the remaining cubic and quartic terms in $-\frac{i}{4}\int G^a_{\mu\nu}G^a_{\mu\nu}$, one can introduce the Halpern's field strength reformulation with variable $\chi^a_{\mu\nu}$ as

$$e^{-\frac{i}{4}\int G^a_{\mu\nu}G^a_{\mu\nu}} = \mathcal{N}' \int \mathfrak{D}\chi \, e^{\frac{i}{4}\int \chi^a_{\mu\nu}\chi^a_{\mu\nu} + \frac{i}{2}\int \chi^a_{\mu\nu}G^a_{\mu\nu}}, \tag{16}$$

such that the generating functional with this representation is at most quadratic in gauge field A^a_μ, and the linkage operation can be easily carried out.

To further simplify the formulation, one can rewrite the gluon sector of QCD Lagrangian as[4]

$$G^a_{\mu\nu}G^a_{\mu\nu} = \frac{1}{N}\,\mathrm{tr}(G^a_{\mu\nu}\tilde{\lambda}^a\, G^b_{\mu\nu}\tilde{\lambda}^b) = \frac{1}{N}\,\mathrm{tr}\Big(G^a_{\mu\nu}\tilde{\lambda}^a\Big)^2 \tag{17}$$

in terms of the adjoint representation of $SU(N)$ with $(\tilde{\lambda}^a)_{bc} = -if^{abc}$ and $\mathrm{tr}(\tilde{\lambda}^a\tilde{\lambda}^b) = N\delta^{ab}$. If one defines $\hat{\chi}_{\mu\nu} \equiv \chi^a_{\mu\nu}\tilde{\lambda}^a$ and

$$\int \mathfrak{D}[\chi^a_{\mu\nu}\tilde{\lambda}^a]\, e^{\frac{i}{4N}\int \mathrm{tr}\left(\chi^a_{\mu\nu}\tilde{\lambda}^a\right)^2} = \int \mathfrak{D}\hat{\chi}\, e^{\frac{i}{4N}\int \mathrm{tr}(\hat{\chi}_{\mu\nu})^2} = \tilde{\mathcal{N}}^{-1}, \tag{18}$$

Halpern's reformulation can be changed to

$$e^{-\frac{i}{4N}\int \mathrm{tr}\left(G^a_{\mu\nu}\tilde{\lambda}^a\, G^b_{\mu\nu}\tilde{\lambda}^b\right)} = \tilde{\mathcal{N}} \int \mathfrak{D}\hat{\chi}\, e^{\frac{i}{4N}\int \mathrm{tr}\left(\chi^a_{\mu\nu}\tilde{\lambda}^a\, \chi^b_{\mu\nu}\tilde{\lambda}^b\right) + \frac{i}{2N}\int \mathrm{tr}\left(\chi^a_{\mu\nu}\tilde{\lambda}^a\, G^b_{\mu\nu}\tilde{\lambda}^b\right)}, \tag{19}$$

such that

$$e^{-\frac{i}{4}\int G^a_{\mu\nu}G^a_{\mu\nu}} = \tilde{\mathcal{N}} \int \mathfrak{D}\hat{\chi}\, e^{\frac{i}{4}\int \chi^a_{\mu\nu}\chi^a_{\mu\nu} + \frac{i}{2}\int \chi^a_{\mu\nu}G^a_{\mu\nu}}. \tag{20}$$

The adjoint Halpern's field $\hat{\chi}^{ab}_{\mu\nu}$ is symmetric under exchanges of combined indices (a,μ) and (b,ν), so its eigenvalues are non-negative. It will be convenient to set up calculations by using schemes similar to Random Matrix Theory with eigenvalues of $\hat{\chi}^{ab}_{\mu\nu}$, and the configurations of zero eigenvalues will not contribute to the generating functional as shown in the appendix of Ref. [1].

The generating functional is now only quadratic in gauge fields. To calculate n-point functions, one needs to simplify Green's function $\mathbf{G}_c[A]$ and fermion closed-loop function $\mathbf{L}[A]$. One finds that the Schwinger-Fradkin representations[2] of $\mathbf{G}_c[A]$ and $\mathbf{L}[A]$ are convenient for the task; and one of modified Fradkin representations is given by[3]

$$\begin{aligned}\langle p|\mathbf{G}_c[A]|y\rangle \;&=\; e^{-ip\cdot y}\cdot i\int_0^\infty ds\; e^{-ism^2}\cdot e^{-\frac{1}{2}\mathbf{Tr}\ln(2h)} \\ &\times \int d[u]\{m - i\gamma\cdot[p - gA(y-u(s))]\}\cdot e^{\frac{i}{4}\int_0^s ds'\,[u'(s')]^2}\cdot e^{ip\cdot u(s)} \\ &\times \left(e^{g\int_0^s ds'\sigma\cdot G(y-u(s'))}\cdot e^{-ig\int_0^s ds'\,u'(s')\cdot A(y-u(s'))}\right)_+,\end{aligned} \tag{21}$$

where $h(s_1, s_2) = \frac{1}{2}\left[(s_1+s_2)+|s_1-s_2|\right]$. Under Eikonal, Bloch-Nordsieck approximations, the Green's function can be further reduced to

$$\begin{aligned}\langle p|\mathbf{G}_c^{\rm BN}[A]|y\rangle &= e^{-ip\cdot y}\cdot i\int_0^\infty ds\; e^{-is(m^2+p^2)}\,\left\{m - i\gamma_\mu\,\left[p_\mu - g\,A_\mu^a(y+2sp)\lambda^a\right]\right\} \quad (22)\\ &\times\left(e^{g\int_0^s ds'\sigma_{\mu\nu}\,G^a_{\mu\nu}(y+2s'p)\lambda^a}\cdot e^{+2ig\int_0^s ds'\,\left[p_\mu\,A_\mu^a(y+2s'p)\lambda^a\right]}\right)_+,\end{aligned}$$

where the ordered exponential (OE) comes from non-commutative matrices of both color and spin, λ^a and $\sigma^{\mu\nu}$, respectively.

4 Virtual Gluon Exchanges in Fermion Scattering

Here, we will use quark-(anti)quark scattering with $p_1 + p_2 \to p_1' + p_2'$ as an example, especially, in the regime of small momentum exchanges, *i.e.*, $p' \simeq p$ under Eikonal approximations. The transition matrix element of the S-matrix, $\mathbf{S} = 1 + i\mathbf{T}$, for the quark-quark or quark-antiquark scattering in reduction formula can be written as

$$\begin{aligned}\langle p_1'p_2'|\mathbf{T}|p_1p_2\rangle &= -iZ_2^{-2}(2\pi)^{-6}\left[\frac{m^2}{E_1E_2E_1'E_2'}\right]^{1/2}\int d^4x_1\,e^{-ip_1\cdot x_1}\int d^4x_2\,e^{-ip_2\cdot x_2} \quad (23)\\ &\cdot\int d^4y_1\,e^{-ip_1'\cdot y_1}\int d^4y_2\,e^{-ip_2'\cdot y_2}\cdot\overline{u}_{s_1'}^{\alpha_1'}(p_1')\,\overline{u}_{s_2'}^{\alpha_2'}(p_2')\cdot u_{s_1}^{\beta_1}(p_1)\,u_{s_2}^{\beta_2}(p_2)\\ &\cdot(\mathfrak{D}_{y_1})^{\alpha_1'\beta_1'}\,(\mathfrak{D}_{y_2})^{\alpha_2'\beta_2'}\,(\overline{\mathfrak{D}}_{x_1})^{\alpha_1\beta_1}\,(\overline{\mathfrak{D}}_{x_2})^{\alpha_2\beta_2}\\ &\cdot\frac{\delta}{\delta\overline{\eta}_{\beta_1'}(y_1)}\cdot\frac{\delta}{\delta\overline{\eta}_{\beta_2'}(y_2)}\cdot\frac{\delta}{\delta\eta_{\alpha_1}(x_1)}\cdot\frac{\delta}{\delta\eta_{\alpha_1}(x_2)}\cdot\langle\mathbf{S}\rangle\mathfrak{Z}_c\left\{j,\overline{\eta},\eta\right\}\Big|_{\eta=\overline{\eta}=0;j=0}.\end{aligned}$$

The last line is related to the 4-point function

$$\mathbf{M}(x_1,y_1;x_2,y_2) = i^2\;e^{\mathfrak{D}_A}\,e^{i\int\mathcal{L}'_{\rm gluon}[A]}\,\mathbf{G}_{\rm c}(y_1,x_1|A)\,\mathbf{G}_{\rm c}(y_2,x_2|A)\,e^{\mathbf{L}_{\rm c}[A]}\Big|_{A=0}, \quad (24)$$

which will be subsequently operated with the mass-shell amputation for each external leg in the transition matrix.

To avoid vanishing of the transition matrix during doubly mass-shell amputation on each Green's function, especially under the Eikonal approximation, one can first calculate $\frac{\partial^2\mathbf{T}}{\partial g_1\partial g_2}$ instead of $\mathbf{T}$ directly, then integrate over both g_1 and g_2 to get back to $\mathbf{T}$ as

$$\mathbf{T} = \int_0^g dg_1\int_0^g dg_2\,\frac{\partial^2\mathbf{T}}{\partial g_1\partial g_2}. \quad (25)$$

The same scheme will also apply to the 4-point function $\mathbf{M}$. First, one assigns g_1 and g_2 as the gluon-quark coupling constants for the particle set $\mathbf{I}$ and $\mathbf{II}$, respectively, and

$$\frac{\partial}{\partial g}\,\mathbf{G}_{\rm c}(y,x|gA) = i\int d^4z\,\mathbf{G}_{\rm c}(y,z|gA)\,\left(\gamma_\mu A^a_\mu(z)\lambda^a\right)\,\mathbf{G}_{\rm c}(z,x|gA), \tag{26}$$

such that the differentiated 4-point function becomes

$$\begin{aligned}\frac{\partial^2\mathbf{M}}{\partial g_1\partial g_2} &= i^2\,e^{\mathfrak{D}_A}\,e^{i\int\mathcal{L}'_{\rm gluon}[A]}\,e^{\mathbf{L}[A]} \\ &\quad\times i\int d^4z_1\,\mathbf{G}^{\rm I}_{\rm c}(y_1,z_1|g_1A)\,\gamma^{\rm I}_\mu A^a_\mu(z_1)\lambda^a_{\rm I}\,\mathbf{G}^{\rm I}_{\rm c}(z_1,x_1|g_1A) \\ &\quad\times i\int d^4z_2\,\mathbf{G}^{\rm II}_{\rm c}(y_2,z_2|g_2A)\,\gamma^{\rm II}_\mu A^a_\mu(z_2)\lambda^a_{\rm II}\,\mathbf{G}^{\rm II}_{\rm c}(z_2,x_2|g_2A)\Big|_{A=0}.\end{aligned} \tag{27}$$

The formulation so far is still exact and the Linkage operation with $e^{\mathfrak{D}_A}$ will sum over all virtual gluon exchanges. Approximations can then be made for $\mathbf{G}^{\rm I,II}_{\rm c}[A]$ and $\mathbf{L}[A]$ for ease of calculations.

In high energy scattering, quarks and anti-quarks carry large energy, but 4-momenta of virtual gluon exchanges are relatively small compared to those of incident fermions under the Eikonal model. The calculation can be treated in large energy $s_{qq}=-(p_1+p_2)^2$ and fixed momentum transfer $t_{qq}=-(p_1-p_2)^2$ limit, *i.e.*, $s_{qq}\to\infty$ and $t_{qq}/s_{qq}\to 0$. Hence, we can invoke the Bloch-Nordsieck approximation with $p'_{1,2}\simeq p_{1,2}$, $\overline{u}(p'_{1,2})\gamma_\mu u(p_{1,2})\simeq -i\frac{p_{1,2\mu}}{m}$, and neglect any self-energy structures. After mass-shell amputation, Green's functions of outgoing and incoming fermions become

$$\mathbf{G}^{\rm BN}_{\rm msa}(p',z|A) = e^{ip'\cdot z}\,\left(e^{+g\int_0^\infty ds\left[\sigma_{\mu\nu}\,G^a_{\mu\nu}(z+2sp')\lambda^a\right]}\cdot e^{+2ig\int_0^\infty ds\left[p'_\mu\,A^a_\mu(z+2sp')\lambda^a\right]}\right)_+ \tag{28}$$

and

$$\mathbf{G}^{\rm BN}_{\rm msa}(z,p|A) = e^{-ip\cdot z}\,\left(e^{+g\int_{-\infty}^0 ds\left[\sigma_{\mu\nu}\,G^a_{\mu\nu}(z+2sp)\lambda^a\right]}\cdot e^{+2ig\int_{-\infty}^0 ds\left[p_\mu\,A^a_\mu(z+2sp)\lambda^a\right]}\right)_+, \tag{29}$$

respectively. Spin-related contribution can also be dropped under the Eikonal approximation as $\overline{u}(p')\,\sigma_{\mu\nu}\,G_{\mu\nu}\,u(p)\simeq\frac{(-i)^2}{2m^2}p_\mu p_\nu G_{\mu\nu}=0$; and the (differentiated) transition matrix element reduces to

$$\begin{aligned}\frac{\partial^2\mathbf{T}}{\partial g_1\partial g_2} &\simeq -iZ_2^{-2}(2\pi)^{-6}\left[\frac{m^2}{E_1E_2E'_1E'_2}\right]^{1/2}\frac{1}{m^2}\,e^{\mathfrak{D}_A}\,e^{i\int\mathcal{L}'_{\rm gluon}[A]}\,e^{\mathbf{L}_{\rm c}[A]} \\ &\quad\times\int d^4z_1\,e^{iq_1\cdot z_1}\,\left(e^{+2ig_1\int_0^\infty ds\left[p'_{1,\mu}\,A^a_\mu(z_1+2sp'_1)\lambda^a_{\rm I}\right]}\right)_+\,(p_1\cdot A^a_{\rm I}(z_1)\lambda^a_{\rm I}) \\ &\qquad\cdot\left(e^{+2ig_1\int_{-\infty}^0 ds\left[p_{1,\mu}\,A^a_\mu(z_1+2sp_1)\lambda^a_{\rm I}\right]}\right)_+ \\ &\quad\times\int d^4z_2\,e^{iq_2\cdot z_2}\,\left(e^{+2ig_2\int_0^\infty ds\left[p'_{2,\mu}\,A^a_\mu(z_2+2sp'_2)\lambda^a_{\rm II}\right]}\right)_+\,\left(p_2\cdot\,A^b_{\rm II}(z_2)\,\lambda^b_{\rm II}\right) \\ &\qquad\cdot\left(e^{+2ig_2\int_{-\infty}^0 ds\left[p_{2,\mu}\,A^a_\mu(z_2+2sp_2)\lambda^a_{\rm II}\right]}\right)_+\Big|_{A=0}.\end{aligned} \tag{30}$$

One then needs to separate gauge fields A^a_μ from the ordered exponential (OE) of color matrices λ^a as

$$
\begin{aligned}
& \left(e^{2ig\int_{-\infty}^{+\infty} ds\, p_\mu A^a_\mu(y+2sp)\lambda^a}\right)_+ \\
= & \int \mathfrak{D}\alpha\, \delta\big[\alpha^a(s) - 2gp_\mu A^a_\mu(y+2sp)\big] \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha^a(s)\lambda^a}\right)_+ \\
= & \ N' \int \mathfrak{D}\alpha \int \mathfrak{D}\Omega\, e^{i\int_{-\infty}^{+\infty} ds\, \Omega^a(s)\left[\alpha^a(s) - 2gp_\mu A^a_\mu(y+2sp)\right]} \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha^a(s)\lambda^a}\right)_+ ,
\end{aligned} \tag{31}
$$

and these OE's can be further represented by functional integrals as

$$
\begin{aligned}
& \left(e^{+2ig\int_0^{\infty} ds\left[p'_\mu A^b_\mu(z+2sp')\lambda^b\right]}\right)_+ \lambda^a \left(e^{+2ig\int_{-\infty}^{0} ds\,[p_\nu A^c_\nu(z+2sp)\lambda^c]}\right)_+ \\
= & \ N' \int \mathfrak{D}\alpha \int \mathfrak{D}\Omega\, e^{i\int_{-\infty}^{+\infty} ds\, \Omega(s)\cdot\alpha(s)}\, e^{ig\int d^4w\, \mathcal{R}(w)\cdot A(w)}\, e^{ig\int d^4w\, \mathcal{R}(w)\cdot A(w)} \\
& \times \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha(s)\cdot\lambda}\right)_+ ,
\end{aligned} \tag{32}
$$

where

$$
\mathcal{R}^a_\mu(w) = -p'_\mu \int_0^{+\infty} ds\, \Omega^a(s)\, \delta^{(4)}(w - z - sp') - p_\mu \int_{-\infty}^{0} ds\, \Omega^a(s)\, \delta^{(4)}(w - z - sp) \tag{33}
$$

after re-scaling $s \longrightarrow 2s$ in the integral.

5 Qluonless QCD

The transition matrix in the Eikonal representation can be written as[2]

$$
\mathbf{T} = \frac{is_{qq}}{2m^2} \int d^2\vec{b}\, e^{i\vec{q}\cdot\vec{b}} \left[1 - e^{i\mathbf{X}}\right], \tag{34}
$$

where $q = p' - p$ and the Eikonal exponential $e^{i\mathbf{X}}$ is given by

$$
\begin{aligned}
e^{i\mathbf{X}} \simeq & \ \mathcal{N}\, e^{\mathfrak{D}_A}\, e^{i\int \mathcal{L}'_{\text{gluon}}[A]}\, e^{\mathbf{L}[A]}\, N'_{\mathrm{I}} \int \mathfrak{D}\Omega_{\mathrm{I}} \int \mathfrak{D}\alpha_{\mathrm{I}}\, N'_{\mathrm{II}} \int \mathfrak{D}\Omega_{\mathrm{II}} \int \mathfrak{D}\alpha_{\mathrm{II}} \\
& \times e^{i\int_{-\infty}^{+\infty} ds\, \Omega_{\mathrm{I}}(s)\cdot\alpha_{\mathrm{I}}(s)}\, e^{ig\int d^4w\, \mathcal{R}_{\mathrm{I}}(w)\cdot A(w)} \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha_{\mathrm{I}}(s)\cdot\lambda_{\mathrm{I}}}\right)_+ \\
& \times e^{i\int_{-\infty}^{+\infty} ds\, \Omega_{\mathrm{II}}(s)\cdot\alpha_{\mathrm{II}}(s)}\, e^{ig\int d^4w\, \mathcal{R}_{\mathrm{II}}(w)\cdot A(w)} \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha_{\mathrm{II}}(s)\cdot\lambda_{\mathrm{II}}}\right)_+ \Bigg|_{A=0} .
\end{aligned} \tag{35}
$$

Hence, the interaction term related to the self-coupling of gluons becomes

$$
e^{i\int \mathcal{L}'_{\text{gluon}}[A]} = \tilde{\mathcal{N}} \int \mathfrak{D}\hat{\chi}\, e^{\frac{i}{4}\int \chi^a_{\mu\nu}\chi^a_{\mu\nu} + \frac{i}{2}\int \chi^a_{\mu\nu}[\mathbf{F}^a_{\mu\nu} + gf^{abc}A^b_\mu A^c_\nu] + \frac{i}{2}\int A^a_\mu \left(\mathbf{D}_{\mathrm{c}}^{\zeta^{-1}}\right)^{ab}_{\mu\nu} A^b_\nu} . \tag{36}
$$

Under the quenched approximation without particle generations, $e^{\mathbf{L}[A]}$ can be dropped to further simplify the calculation, and the Eikonal exponential becomes

$$
\begin{aligned}
e^{i\mathbf{X}} \;\simeq\;& \mathcal{N} \int \mathfrak{D}\Omega_{\mathrm{I}} \int \mathfrak{D}\alpha_{\mathrm{I}}\, e^{i\int_{-\infty}^{+\infty} ds\, \Omega_{\mathrm{I}}(s)\cdot\alpha_{\mathrm{I}}(s)} \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha_{\mathrm{I}}(s)\cdot\lambda_{\mathrm{I}}}\right)_{+} \\
&\times \int \mathfrak{D}\Omega_{\mathrm{II}} \int \mathfrak{D}\alpha_{\mathrm{II}}\, e^{i\int_{-\infty}^{+\infty} ds\, \Omega_{\mathrm{II}}(s)\cdot\alpha_{\mathrm{II}}(s)} \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha_{\mathrm{II}}(s)\cdot\lambda_{\mathrm{II}}}\right)_{+} \\
&\times \int \mathfrak{D}\hat{\chi}\, e^{\frac{i}{4}\int \chi^{a}_{\mu\nu}\chi^{a}_{\mu\nu}} \left\{ e^{\mathfrak{D}_{A}} \cdot e^{-i\int (\partial_\nu \chi^{a}_{\nu\mu})\, A^{a}_{\mu} + \frac{i}{2}\int g f^{abc}\chi^{a}_{\mu\nu}\, A^{b}_{\mu} A^{c}_{\nu} + \frac{i}{2}\int A\cdot\left(\mathbf{D}_{\mathrm{c}}^{\zeta\,-1}\right)\cdot A} \right. \\
&\quad \left. \times\; e^{ig\int d^4 w\, [\mathcal{R}_{\mathrm{I}}(w)\cdot A(w) + \mathcal{R}_{\mathrm{II}}(w)\cdot A(w)]} \Big|_{A=0} \right\},
\end{aligned} \tag{37}
$$

where $\hat{\mathcal{N}} \equiv \mathcal{N}\cdot N'_{\mathrm{I}}\cdot N'_{\mathrm{II}}\cdot\tilde{\mathcal{N}}$ is the collection of normalization constants, and $+\frac{i}{2}\int \chi^{a}_{\mu\nu}\,\mathbf{F}^{a}_{\mu\nu} = +\frac{i}{2}\int \chi^{a}_{\mu\nu}\,[\partial_\mu A^{a}_{\nu} - \partial_\mu A^{a}_{\nu}] = -i\int (\partial_\mu \chi^{a}_{\mu\nu})\, A^{a}_{\nu}$ has been applied.

One then collects all (gauge) field-dependent factors in the brackets of the Eikonal exponential with $\mathcal{Q}^{a}_{\mu} = g\mathcal{R}^{a}_{\mathrm{I}\mu} + g\mathcal{R}^{a}_{\mathrm{II}\mu} + \partial_\nu \chi^{a}_{\mu\nu}$ linear in A^{a}_{μ}, and $\mathcal{K}^{ab}_{\mu\nu} = g f^{cab}\chi^{c}_{\mu\nu} + \left(\mathbf{D}_{\mathrm{c}}^{\zeta\,-1}\right)^{ab}_{\mu\nu}$ quadratic in A^{a}_{μ}, respectively. After applying the linkage operator, the factor inside the brackets of Eq.(37) becomes

$$
\begin{aligned}
& e^{-\frac{i}{2}\int \frac{\delta}{\delta A^{a}_{\mu}}\, \mathbf{D}_{\mathrm{c}\,\mu\nu}^{\zeta\,ab}\, \frac{\delta}{\delta A^{b}_{\nu}}} \cdot e^{+\frac{i}{2}\int A^{a}_{\mu}\,\mathcal{K}^{ab}_{\mu\nu}\, A^{b}_{\nu} + i\int A^{a}_{\mu}\,\mathcal{Q}^{a}_{\mu}} \Big|_{A\to 0} \\
=\;& e^{-\frac{1}{2}\mathbf{Tr}\ln\left(1-\mathbf{D}_{\mathrm{c}}^{\zeta}\cdot\mathcal{K}\right)} \cdot e^{\frac{i}{2}\int \mathcal{Q}\cdot\left[\mathbf{D}_{\mathrm{c}}^{\zeta}\cdot\left(1-\mathcal{K}\cdot\mathbf{D}_{\mathrm{c}}^{\zeta}\right)^{-1}\right]\cdot\mathcal{Q}}.
\end{aligned} \tag{38}
$$

The kernel of quadratic $\mathcal{Q}^{a}_{\mu}$'s can be written as

$$
\mathbf{D}_{\mathrm{c}}^{\zeta}\cdot\left(1-\mathcal{K}\cdot\mathbf{D}_{\mathrm{c}}^{\zeta}\right)^{-1} \;=\; \mathbf{D}_{\mathrm{c}}^{\zeta}\cdot\left(1-\left[gf\cdot\chi + \mathbf{D}_{\mathrm{c}}^{\zeta\,-1}\right]\cdot\mathbf{D}_{\mathrm{c}}^{\zeta}\right)^{-1} = -\left(gf\cdot\chi\right)^{-1}, \tag{39}
$$

where internal, virtual gluon propagators are all canceled out; and this is the 'Ghost Magic' touted in the introduction. This feature is unique to QCD from the self-coupling part of the action with $+\frac{i}{2}\int A\cdot\left(\mathbf{D}_{\mathrm{c}}^{\zeta\,-1}\right)\cdot A$, which removes all gluon propagators added by the linkage operator. Eq.(38) can be further reduced as

$$
\begin{aligned}
& e^{-\frac{i}{2}\int \frac{\delta}{\delta A^{a}_{\mu}}\, \mathbf{D}_{\mathrm{c}\,\mu\nu}^{\zeta\,ab}\, \frac{\delta}{\delta A^{b}_{\nu}}} \cdot e^{+\frac{i}{2}\int A^{a}_{\mu}\,\mathcal{K}^{ab}_{\mu\nu}\, A^{b}_{\nu} + i\int A^{a}_{\mu}\,\mathcal{Q}^{a}_{\mu}} \Big|_{A\to 0} \\
=\;& \mathcal{N}_{g}\cdot e^{-\frac{1}{2}\mathbf{Tr}\ln\left(gf\cdot\chi\right)} \cdot e^{-\frac{i}{2}\int \mathcal{Q}\cdot\left[\left(gf\cdot\chi\right)^{-1}\right]\cdot\mathcal{Q}},
\end{aligned} \tag{40}
$$

where $e^{-\frac{1}{2}\mathbf{Tr}\ln\left(-\mathbf{D}_{\mathrm{c}}^{\zeta}\right)}$ from $e^{-\frac{1}{2}\mathbf{Tr}\ln\left(1-\mathbf{D}_{\mathrm{c}}^{\zeta}\cdot\mathcal{K}\right)}$ has been treated as an unimportant (divergent) constant and absorbed into the normalization constant $\mathcal{N}_{g}$. Hence, the Eikonal exponential becomes

$$
\begin{aligned}
e^{i\mathbf{X}} \;\simeq\; & \mathcal{N}' \int \mathfrak{D}\Omega_{\mathrm{I}} \int \mathfrak{D}\alpha_{\mathrm{I}}\, e^{i\int_{-\infty}^{+\infty} ds\, \Omega_{\mathrm{I}}(s)\cdot\alpha_{\mathrm{I}}(s)} \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha_{\mathrm{I}}(s)\cdot\lambda_{\mathrm{I}}}\right)_{+} \\
& \times \int \mathfrak{D}\Omega_{\mathrm{II}} \int \mathfrak{D}\alpha_{\mathrm{II}}\, e^{i\int_{-\infty}^{+\infty} ds\, \Omega_{\mathrm{II}}(s)\cdot\alpha_{\mathrm{II}}(s)} \left(e^{i\int_{-\infty}^{+\infty} ds\, \alpha_{\mathrm{II}}(s)\cdot\lambda_{\mathrm{II}}}\right)_{+} \\
& \times \int \mathfrak{D}\hat{\chi}\, e^{-\frac{1}{2}\mathbf{Tr}\ln(gf\cdot\chi)} \cdot e^{\frac{i}{4}\int \chi^2} \cdot e^{-\frac{i}{2}\int \mathcal{Q}\cdot\left[(gf\cdot\chi)^{-1}\right]\cdot\mathcal{Q}}.
\end{aligned}
\tag{41}
$$

In such functional method, both the Eikonal exponential and transition matrix sum over all virtual gluon exchanges for quark-quark or quark-antiquark scattering. The result shows that all internal, virtual gluon propagators are removed; and the interaction between quarks and antiquarks becomes effectively 'contact-like', and is now carried by $\chi^a_{\mu\nu}$-fields instead of 'free' gluons, *i.e.*, Qluonless QCD. The final formulation has no gluon propagators, thus it is good for any initial gauge fixings; and most importantly there is no gauge invariant problem in contrast to perturbative approaches.

How about lifting the quenched approximation? $\mathbf{L}[A]$ is gauge invariant[2], so adding closed-fermion-loop function will still keep the formulation both MGI and MLC. The closed-fermion-loop functional can be written as

$$
\mathbf{L}[A] = \mathbf{Tr}\ln\left[1 - g\left(\gamma_\mu\, A^a_\mu \lambda^a\right)\mathbf{S}_{\mathrm{c}}\right] = -i\int_0^g dg'\, \mathbf{Tr}\left\{(\gamma\cdot A)\,\mathbf{G}_{\mathrm{c}}[g'A]\right\}, \tag{42}
$$

such that $\mathbf{L}[A]$ can be expressed in a form similar to $\mathbf{G}_{\mathrm{c}}[A]$ as

$$
\mathbf{L}[A] = \cdots \left(e^{i\int \mathcal{R}'\cdot A}\right) \cdots . \tag{43}
$$

Hence, one can expand $e^{\mathbf{L}[A]}$ as

$$
e^{\mathbf{L}[A]} = 1 + \mathbf{L}[A] + \frac{1}{2!}\mathbf{L}^2[A] + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}\,\mathbf{L}^n[A], \tag{44}
$$

which leads to

$$
e^{\mathbf{L}[A]} = \sum_n \frac{1}{n!} \cdots \left(e^{i\int \sum_i \mathcal{R}'_i\cdot A}\right) \cdots . \tag{45}
$$

Then the calculation follows similar steps above with Linkage operation; and all internal virtual gluon propagators will be removed automatically. Thus, the magic ingredient for Qluonless QCD is still valid with and without the quenched approximation.

The Qluonless QCD presented here does not have any apparat ghosts. One then ask how those ghosts are introduced to the theory in other approaches. At first, a gauge fixing condition, $\mathcal{F}[A] = 0$, is introduced into the generating functional (integral) as

$$\mathfrak{Z}_{\rm c}\{j,\overline{\eta},\eta\} = \mathcal{N}\int \mathfrak{D}A\, e^{i\int \mathcal{L}_{\rm QCD}} \to \mathcal{N}'\int \mathfrak{D}A\,\delta(\mathcal{F}[A])\, e^{i\int \mathcal{L}_{\rm QCD}} \tag{46}$$

to remove un-physical degrees of freedom. But are both Lorentz covariance and Gauge invariance manifested with the insertion of gauge fixing conditions? Therefore, the Faddeev-Popov determinant $\det \mathcal{B}$ is added to compensate the gauge fixing factor $\delta(\mathcal{F}[A])$ in the functional integral as

$$\mathfrak{Z}_{\rm c}\{j,\overline{\eta},\eta\} = \mathcal{N}\int \mathfrak{D}A\,\delta(\mathcal{F}[A])\, e^{i\int \mathcal{L}_{\rm QCD}} \to \mathcal{N}'\int \mathfrak{D}A\,\delta(\mathcal{F}[A])\,\det \mathcal{B}\, e^{i\int \mathcal{L}_{\rm QCD}}, \tag{47}$$

where $\det \mathcal{B} = \exp\left[\mathbf{Tr}\ln \mathcal{B}\right] = \mathcal{N}''\int \mathfrak{D}\overline{\varsigma}\,\mathfrak{D}\varsigma\, e^{i\int \overline{\varsigma}\cdot\mathcal{B}\cdot\varsigma}$; and

$$\mathfrak{Z}_{\rm c}\{j,\overline{\eta},\eta\} \to \tilde{\mathcal{N}}\int \mathfrak{D}A\,\mathfrak{D}\overline{\eta}\,\mathfrak{D}\eta\,\delta(\mathcal{F}[A])\, e^{i\int \mathcal{L}_{\rm QCD}+i\int \overline{\varsigma}\cdot\mathcal{B}\cdot\varsigma}. \tag{48}$$

The effective Lagrangian becomes $\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \lambda\mathcal{F}^2[A] + \overline{\varsigma}\cdot\mathcal{B}\cdot\varsigma$, and Faddeev-Popov ghosts, $\overline{\varsigma}$ and ς, are introduced in the effective theory.

Similarly, one can also introduce ghosts into the Qluonless QCD formulation, especially, into the kernel of quadratic A^a_μ-dependence, $\mathcal{K}^{ab}_{\mu\nu} = gf^{cab}\chi^c_{\mu\nu} + \left(\mathbf{D}^{\zeta^{-1}}_{\rm c}\right)^{ab}_{\mu\nu}$. The relevant part of the linkage operation is

$$e^{-\frac{i}{2}\int \frac{\delta}{\delta A}\cdot\mathbf{D}^{\zeta}_{c}\cdot\frac{\delta}{\delta A}} \cdot e^{\frac{i}{2}\int A\cdot\mathcal{K}\cdot A}\Big|_{A=0} = e^{\frac{i}{2}\int A\cdot\left[\mathbf{D}^{\zeta}_{c}\frac{1}{1-\mathcal{K}\cdot\mathbf{D}^{\zeta}_{c}}\right]\cdot A} \cdot e^{-\frac{1}{2}\mathbf{Tr}\ln\left(1-\mathcal{K}\cdot\mathbf{D}^{\zeta}_{c}\right)}\Bigg|_{A=0}, \tag{49}$$

and one can add an arbitrary $\mathcal{B}^{ab}_{\mu\nu}$ into $\mathcal{K}^{ab}_{\mu\nu}$, such that $\mathcal{K} = gf\cdot\chi + (\mathbf{D}^{\zeta}_{\rm c})^{-1} \to \mathcal{K} = gf\cdot\chi + (\mathbf{D}_{\rm c\zeta})^{-1} + \mathcal{B}$, and $1-\mathcal{K}\cdot\mathbf{D}^{\zeta}_{c} = gf\cdot\chi\cdot\mathbf{D}^{\zeta}_{c} - \mathcal{B}\cdot\mathbf{D}^{\zeta}_{c}$. One then gets

$$e^{-\frac{i}{2}\int A\cdot\mathcal{B}^{-1}\cdot A} \cdot e^{-\frac{1}{2}\mathbf{Tr}\ln\mathcal{B}} \cdot e^{-\frac{1}{2}\mathbf{Tr}\ln(-\mathbf{D}^{\zeta}_{c})}\Big|_{A=0} = e^{-\frac{1}{2}\mathbf{Tr}\ln\mathcal{B}} \cdot e^{-\frac{1}{2}\mathbf{Tr}\ln(-\mathbf{D}^{\zeta}_{c})}. \tag{50}$$

Following similar steps above, $e^{-\frac{1}{2}\mathbf{Tr}\ln(-\mathbf{D}^{\zeta}_{c})}$ can be treated as just a (divergent) normalization constant, and the determinantal factor can be expressed in terms of functional integral as $[\det \mathcal{B}]^{-\frac{1}{2}} = \exp\left[-\frac{1}{2}\,\mathbf{Tr}\ln\mathcal{B}\right] = \mathcal{N}\int \mathfrak{D}\phi\, e^{\frac{i}{2}\int \phi\cdot\mathcal{B}\cdot\phi}$. Again bosonic ghost fields ϕ are introduced into the theory similar to the conventional approach. However, the new ghost fields do not participate in the interaction of interest; the physics is not altered with or without introducing extra ghosts into the theory. All gluon propagators are vanished in the gluonless QCD formalism and the interaction now is carried by the Halpern's χ-fields. That is, virtual gluons themselves can be interpreted as ghosts after all.

6 Summary

We presented a MGI and MLC formulation of QCD in terms of Schwingerian formalism with functional method. Ghost magic from summing all virtual gluon exchanges in quark scattering processes leads to the local, gluonless interaction, *i.e.*, the gluonless QCD. Possible extensions to more general conditions, for example, non-quenched, non-eikonal approximations, are also briefly discussed.

Such formulation can be applied to many problems, for example, a qualitative picture of color dynamics can be described with the gluonless QCD formalism. The estimate of scattering consists of Quasi-Abelian Limit which is automatically enforced such that functional integrals become regular integrals in d-dimensional space-time and $n = N_c^2 - 1$ colors, and ordered exponentials are reduced to ordinary exponentials. The color dynamics between quarks and/or anti-quarks can be shown to be related to the impact parameter at high energy approximations. Due to the time constraint, more details on estimations of quark scattering can be found in Ref. [1].

Finally, I would like to thank the directors and section chair of the 10th Workshop on Non-perturbative QCD for inviting me to present our recent work.

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