Breakdown of the operator product expansion of deep inelastic scattering in the 't Hooft model

Antonio Pineda
(i) Grup de Física Teòrica and IFAE, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona, Spain
(ii) Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China

We consider deep inelastic scattering in the 't Hooft model. Being solvable, this model allows us to directly compute the moments associated to the cross section at next-to-leading order in the $1/Q^2$ expansion. We perform the same computation using the operator product expansion. We find that all the terms match in both computations except for one in the hadronic side, which is proportional to a non-local operator. The basics of the result suggest that a similar phenomenon may occur in four dimensions in the large N_c limit. These proceedings are based on results obtained in Refs. [1, 2].

1 Introduction

It has been more than 35 years since QCD was vindicated as the theory of the strong interactions [3]. Unfortunately, analytic solutions describing hadrons and their interactions in terms of the degrees of freedom (quarks and gluons) and parameters (coupling constant and quark masses) that appear in the QCD Lagrangian have remained elusive. Leaving aside symmetry considerations (or how they are realized), the only quantitative and analytic scheme to check the dynamics of QCD from first principles is through (perturbative) weak-coupling computations. The problem then is to relate those computations to experiments, which are naturally described in terms of hadrons. In order to do so two key ingredients are used:

1) Asymptotic freedom [3, 4], which states that the renormalized coupling goes to zero at large Euclidean momentum.

2) The operator product expansion (OPE) [5] and, more generally, factorization. So far, the OPE has only been proven within perturbation theory [6].

In practice, the combination of these two points is stated as the fact that the multiplication of two operators (sandwiched between physical states) enjoys the following expansion at short distances

$$\hat{A}(x)\hat{B}(0) = \sum_n C_n(x)\hat{O}_n(0)\,, \tag{1}$$

where $\hat{O}_n$ are local operators with increasing dimensionality in n and the coefficients C_n can be computed in perturbation theory (or at least at weak coupling).

The (Fourier transform of the) OPE probes these correlators at large Euclidean momentum. Therefore, the OPE is not directly accessible to experiment and one has to resort to dispersion relations to check it [These may have potential problems of their own; basically the lack of knowledge of the asymptotic behavior of the correlators to ensure that one can neglect the contributions at infinity when using the Cauchy theorem. This will not affect the main conclusions of this paper, since this ambiguity should have an impact on, at most, a finite number of moments]. This practical version of the OPE is at the basis of computations at large Euclidean momentum of (the moments in) deep inelastic scattering (DIS) and the vacuum polarization tensor, which so far have been thought to be among the more solid predictions of QCD, since they are not affected by quark-hadron duality problems [We do not enter in this paper on the use of the OPE and factorization methods for quantities living in, or affected by, the Minkowski cut. These quantities are usually regarded as less fundamental, and are the ground on which the discussion about quark-hadron duality takes place, see for instance [7] and references therein. For those observables one can easily find examples where perturbation theory fails in the large N_c limit. For instance, the imaginary part of the vacuum polarization tensor becomes a sum of infinitely narrow resonances in the large N_c limit, as opposed to the smooth result obtained from perturbation theory]. Therefore, the importance of setting the OPE and the factorization methods used in quantum field theories, specially in QCD, on solid theoretical ground can hardly be overemphasized. So far, it was thought that the use of the OPE (in its non-perturbative formulation [8]) was secure, even tough it has not been proven in QCD. It has been only partially checked in models, for instance in two dimensional QCD in the large N_c limit: the 't Hooft model [9]. This theory is superrenormalizable and asymptotically free, so it is a nice ground on which to test the OPE [In the 't Hooft model there are no marginal operators. Therefore, the coupling constant does not run and has dimensions; no renormalons should then arise]. This was done at the lowest order in the OPE in Refs. [10, 11] for the vacuum polarization and for DIS off a meson with nice agreement between the results of the model and the OPE expectations. In Ref. [14] the OPE was semi-analytically checked at next-to-leading order (NLO) in the $1/Q^2$ expansion, with logarithmic accuracy, for the vacuum polarization. In Refs. [1, 2] DIS was considered at NLO, where further details of the computation can be found. Here We will only give the main results and sketch the derivation.

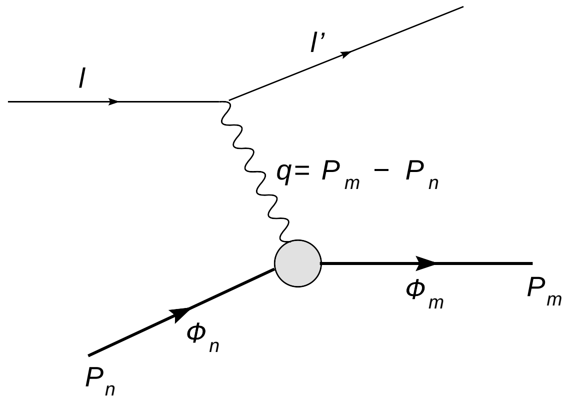

Figure 1: Deep-inelastic scattering off a light meson. The momentum of the photon q is incoming.

2 Deep Inelastic scattering

The cross section describing this process is proportional to the imaginary part of the following correlator on which we will focus in the following

$$\begin{aligned} T^{\mu\nu}(q) &= i\int d^2x e^{iq\cdot x}\langle ij;n|Tj^\mu(x)j^\nu(0)|ij;n\rangle \\ &\equiv \left(P_n^\mu - \frac{q^\mu q\cdot P_n}{q^2}\right)\left(P_n^\mu - \frac{q^\mu q\cdot P_n}{q^2}\right)T(Q^2,x_B)\,, \end{aligned} \tag{2}$$

where $x_B = Q^2/(2P_n\cdot q)$, $Q^2 = -q^2$, P_n is the momentum of the meson and i, j stand for the flavor of the quark and antiquark, respectively, which form the bound state. $j^\mu(x) = \sum_h j_h^\mu(x)$, where $j_h^\mu(x) = e_h\overline{\psi}_h\gamma^\mu\psi_h(x)$.

The imaginary part of $T^{\mu\nu}$ is proportional to the differential cross section when $x_B \geq 0$, and in the light-cone frame reads[1]

$$\begin{aligned} \mathrm{Im}T^{++} &= \frac{1}{2}\sum_m\int\frac{dP_m^+}{2(2\pi)P_m^+}\left|\langle ij;m|j^+(0)|ij;n\rangle\right|^2(2\pi)^2\delta^2(q+P_n-P_m) \\ &= \pi\sum_m\left|\langle ij;m|\sum_{h=i,j}e_h j_h^+(0)|ij;n\rangle\right|^2\delta\left(M_m^2-M_n^2-Q^2\frac{(1-x_B)}{x_B}\right)\,. \end{aligned}$$

Note that in four dimensions one would have a sum over infinitely narrow resonances as well.

[1] We will work the light-cone quantization frame [12]. This quantization frame may be convenient when dealing with nearly massless particles. In four dimensions this line of research has been pursued by many groups, see [13] for a review. In two dimensions it can be seen that it is a natural framework on which to solve QCD_{1+1} in the large N_c limit.

Using dispersions relations, up to a finite number of subtractions, one should have (see also the discussion in Ref. [15])

$$T(Q^2, x_B) = \frac{2}{\pi}\int_0^{x_B^{max}} dy_B \frac{1}{y_B}\frac{\mathrm{Im}T(Q^2, y_B)}{1-\left(\frac{y_B}{x_B}\right)^2 - i\epsilon}\,, \tag{3}$$

where $x_B^{max} = 1/(1+(P_0^2 - P_n^2)/Q^2)$. Therefore, $T(Q^2, x_B)$ admits an analytic expansion in $1/x_B$ for $x_B > x_B^{max}$,

$$T(Q^2, x_B) = 4\sum_{N=0,2,4,\ldots} M_N(Q^2)\frac{1}{x_B^N}\,, \tag{4}$$

where

$$M_N(Q^2) \equiv \frac{1}{2\pi}\int_0^{x_B^{max}} dy_B y_B^{N-1}\mathrm{Im}T(Q^2, y_B)\,. \tag{5}$$

3 Matrix elements and final computation

For the explicit computation it is convenient to work with the kinematical variable $x = -q^+/P_n^+$ (we take $q^+ < 0$ and $q^- \to +\infty$), which satisfies the equality $x_B = x/(1-\frac{M_n^2}{Q^2}x^2)$.

General expressions for the matrix elements in terms of the 't Hooft wave functions, $\phi_n^{ij}(x)$, are presented in Ref. [2]. They are obtained using similar techniques to those in Ref. [16], where the Lagrangian, the 't Hooft equation, as well as the definitions of the bound states are given. Approximated expressions, valid when $1 - x_B \gg \beta^2/Q^2$ ($\beta^2 = g^2N_c/(2\pi)$ is the 't Hooft coupling), were also obtained in Ref. [2] using Eqs. (51) and (52) from [14], as well as similar techniques to those used in Ref. [16]:

$$\begin{aligned}\langle ij; m|\overline{\psi}_i\gamma^-\psi_i|ij;n\rangle &= 2\pi\beta\frac{m_i}{-q^+}\\ &\times\left[\left(1+\frac{m_{i,R}^2+m_{j,R}^2}{2m\pi^2\beta^2}+\frac{m_im_j}{m\pi^2\beta^2}(-1)^m - \frac{m_i^2+(-1)^m m_im_j}{Q^2}\right)\phi_n^{ij}(x_m)\right.\\ &\left.+(m_{i,R}^2+(-1)^m m_im_j)\frac{x_m}{Q^2}\phi_n'^{ij}(x_m)\right] + o\left(\frac{1}{Q^2}\right)\,,\end{aligned} \tag{6}$$

where $m_{i,R}^2 = m_i^2 - \beta^2$. Using

$$\langle ij; m|\overline{\psi}_i\gamma^+\psi_i|ij;n\rangle = x^2\frac{(P_n^+)^2}{Q^2}\langle ij; m|\overline{\psi}_i\gamma^-\psi_i|ij;n\rangle\,, \tag{7}$$

which holds both for particle and antiparticle, one can obtain the plus component of the current.

One then obtains ImT with $\mathcal{O}(1/Q^2)$ precision when $1-x \gg \beta^2/Q^2$

$$\mathrm{Im}T \simeq 4\pi\left(\frac{2\pi\beta x}{Q^2}\right)^2 \frac{1}{\left(1+x^2\frac{M_n^2}{Q^2}\right)^2}\sum_{m=0}^{\infty}\delta\left(M_m^2 - M_n^2(1-x) - Q^2\frac{(1-x)}{x}\right) \quad (8)$$
$$\times\left[e_i m_i\left\{\phi_n^{ij}(x)\left(1-\frac{m_i^2+m_im_j(-1)^m}{Q^2}\right.\right.\right.$$
$$+\left(\frac{m_{i,R}^2+m_{j,R}^2}{2Q^2}+\frac{m_im_j}{Q^2}(-1)^m\right)\frac{x}{1-x}\Bigg)$$
$$+\frac{x}{Q^2}(m_{i,R}^2+m_im_j(-1)^m)\frac{d\phi_n^{ij}(x)}{dx}\Bigg)\Bigg\}$$
$$-(-1)^m e_j m_j\left\{\phi_n^{ij}(1-x)\left(1-\frac{m_j^2+m_jm_i(-1)^m}{Q^2}\right.\right.$$
$$+\left(\frac{m_{j,R}^2+m_{i,R}^2}{2Q^2}+\frac{m_jm_i}{Q^2}(-1)^m\right)\frac{x}{1-x}\Bigg)$$
$$\left.\left.\left.+\frac{x}{Q^2}(m_{j,R}^2+m_jm_i(-1)^m)\frac{d\phi_n^{ij}(1-x)}{dx}\right)\right\} + o\left(\frac{1}{Q^2}\right)\right]^2 .$$

Up to a prefactor, the terms $e_{i/j}m_{i/j}\{\cdots\}$ represent the contribution from the matrix element of the current of the quark i and the antiquark j, respectively. Note the relative sign $-(-1)^m$ between both contributions, which can be obtained from symmetry arguments.

Eq. (8) is one key result. By inserting this expression in the moments and using the Euler-MacLaurin expansion we obtain ($x_{max} = 1 - M_0^2/Q^2 + \mathcal{O}(1/Q^4)$)

$$M_N(Q^2) = \frac{8}{Q^4}\int_0^{x_{max}} dx\left(\frac{x}{1-\frac{M_n^2}{Q^2}x^2}\right)^N \frac{x}{1-\frac{M_n^4}{Q^4}x^4}$$
$$\times\left[e_i^2m_i^2\left(\phi_n^{ij}(x)\right)^2 + e_j^2m_j^2\left(\phi_n^{ij}(1-x)\right)^2\right.$$
$$+2e_i^2m_i^2\phi_n^{ij}(x)\left[-\frac{m_i^2}{Q^2}\phi_n^{ij}(x) + x\frac{m_{i,R}^2}{Q^2}\frac{d\phi_n^{ij}(x)}{dx}\right]$$
$$+2e_j^2m_j^2\phi_n^{ij}(1-x)\left[-\frac{m_j^2}{Q^2}\phi_n^{ij}(1-x) + x\frac{m_{j,R}^2}{Q^2}\frac{d\phi_n^{ij}(1-x)}{dx}\right]$$
$$-4e_ie_jm_i^2m_j^2\frac{2x-1}{Q^2(1-x)}\phi_n^{ij}(x)\phi_n^{ij}(1-x)$$
$$\left.-2e_ie_jm_i^2m_j^2\frac{x}{Q^2}\frac{d}{dx}\left(\phi_n^{ij}(x)\phi_n^{ij}(1-x)\right)\right], \quad (9)$$

which is correct with $\mathcal{O}(1/Q^2)$ precision (for finite N). Note that with this precision we can replace $x_{max} = 1$. Note as well that the last two terms in Eq. (9) are $\mathcal{O}(m^2/Q^2)$

suppressed with respect to the leading term. One comment is in order here. When using the Euler-MacLaurin expansion we had to deal with terms proportional to $(-1)^m$. Such terms give a potential contribution of relative order $o\,(1/Q^2)$, beyond the precision of our computation. Nevertheless, we also have terms proportional to $(-1)^{2m}=1$. They come from the interference of the particle and antiparticle currents and are no longer suppressed by the sign-alternating behavior, only by the prefactor, which is $\mathcal{O}(1/Q^2)$. Therefore, these terms contribute at $\mathcal{O}(1/Q^2)$. We anticipate that they will be the ones responsible for the violation of the OPE.

It must also be noted that Eq. (9) is not valid for all values of N. The factor x^{N-1} in the definition of M_N effectively selects the region of x that contributes the most to the integral. This is easily seen if we express x^N as

$$x^N = e^{N\ln(1-(1-x))} = e^{-N(1-x)+O((1-x)^2)} \,. \tag{10}$$

As $N\to\infty$, only the region $1-x\lesssim 1/N$ will give a sizable contribution. As Eq. (9) assumes that the region $\beta/Q\lesssim 1-x$ dominates the integral, it is only valid for $N\lesssim Q/\beta$. To be more precise, for N finite (though otherwise it could be large) the precision of our calculation is $1/Q^2$, if N scales with Q, the precision of our computation deteriorates, in particular for $N\sim Q/\beta$, the precision of our computation would be $1/Q$, since there are (in principle) terms of $\mathcal{O}(N\beta^3/Q^3)\sim\beta^2/Q^2$, which we have not considered.

Using Eq. (4) and the approximated expressions we have obtained for the moments in Eq. (9), we can obtain an approximated expression for T, which can actually be written as a dispersion relation formula:

$$T^{Eul.}(Q^2,x_B) = \frac{2}{\pi}\int_0^1 \frac{dy_B}{y_B}\frac{\mathrm{Im}T^{Eul.}(Q^2,y_B)}{1-\left(\frac{y_B}{x_B}\right)^2 - i\epsilon} \,, \tag{11}$$

where $\mathrm{Im}T^{Eul.}$ is given by Eq. (8), performing the substitution $\sum_m \to \int dm$. $T^{Eul.}$ is not a good approximation to T when we approach the physical cut. This has to do with the fact that our expressions for the moments are not valid when $N\to\infty$ ($x\to 1$ limit). Nevertheless, it can be considered a generating functional for the moments with not very large N. Therefore, it is the expression we would expect to be equal to the OPE result. In order to ease the comparison, we write $T^{Eul.}$ in a factorized way:

$$\begin{aligned} T^{Eul.}(Q^2,x_B) &= -2\left(\frac{4}{Q^2}\right)^2 \\ &\times \int_{-\infty}^{\infty} dy \left\{ e_i^2 J_i(x,y) f_i(y) + e_j^2 J_j(x,y) f_j(y) + e_i e_j J_{int.}(x,y) f_{int.}(y) \right\} \,, \end{aligned} \tag{12}$$

where

$$f_i(y) \equiv \left[\phi_n^{ij}(y)\right]^2 = \int_{-\infty}^{\infty} \frac{dx^-}{2(2\pi)} e^{-iyP_n^+\frac{x^-}{2}} \langle ij;n|\psi_{i,+}^\dagger(x^-)\psi_{i,+}(0)|ij;n\rangle \,,$$

$$f_j(y) \equiv \left[\phi_n^{ij}(1-y)\right]^2 = -\int_{-\infty}^{\infty}\frac{dx^-}{2(2\pi)}e^{-iyP_n^+\frac{x^-}{2}}\langle ij;n|\psi_{j,+}^\dagger(0)\psi_{j,+}(x^-)|ij;n\rangle\,,$$

$$f_{int.}(y) \equiv \frac{m_i m_j}{y(1-y)}\phi_n^{ij}(y)\phi_n^{ij}(1-y) = \frac{(P_n^+)^2}{N_c}\int_{-\infty}^{\infty}\frac{dx^-}{2(2\pi)}e^{-iyP_n^+\frac{x^-}{2}} \times\int_{-\infty}^{\infty}dz^-\langle ij;n|\psi_{i,-}^\dagger(x^-)\psi_{j,-}(z^-)\psi_{j,+}^\dagger(0)\psi_{i,+}(z^-)|ij;n\rangle\,. \tag{13}$$

In the above expressions we have not inserted the Wilson line:

$$\Phi(x^-,y^-) = P[e^{(ig\int_{y^-}^{x^-}dz^-A^+(z^-))}]\,, \tag{14}$$

between the quark fields ψ to make gauge invariance explicit, since we are working in the light-cone gauge, $A^+=0$.

The functions J are defined as[2]

$$J_{i(j)}(x,y) \equiv \frac{m_{i(j)}^2x^2y}{y^2-x^2+i\epsilon}\left[1-2\frac{m_{i(j)}^2}{Q^2}-2\frac{M_n^2}{Q^2}y^2+\frac{m_{i(j),R}^2}{Q^2}y\frac{d}{dy}\right]\,, \tag{15}$$

$$J_{int.}(x,y) \equiv 2\frac{m_i m_j}{Q^2}\left[x^2\frac{2y^2(1-2y)}{y^2-x^2+i\epsilon}-x^3\frac{d}{dx}\frac{y^2(1-y)}{y^2-x^2+i\epsilon}\right]\,. \tag{16}$$

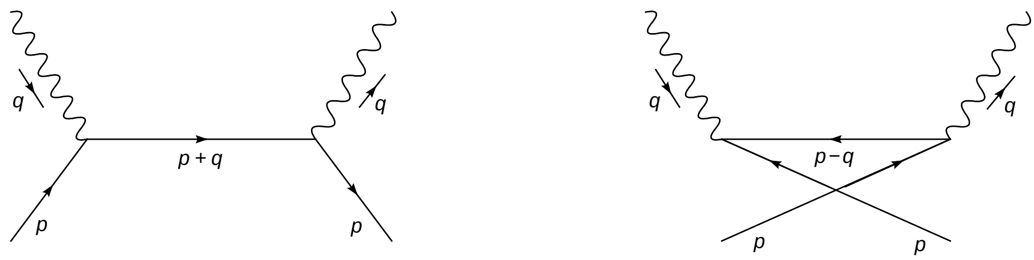

Figure 2: *Diagrams contributing to the perturbative computation at leading order. The momentum p is the momentum of the quark inside the meson n.*

We can now perform the same computation using the OPE (we need to do the computation at one loop). In order to avoid spurious differences with the hadronic result, we compute $\mathrm{Im}T^{OPE}$ and use dispersion relations afterwards (actually, one can

[2]It is possible to redefine J so that it has the functionality $J(y/x)$. This implies redefining $f(y)$ by some powers of y, equivalent to introducing some extra ∂^+ derivatives. We do not do so in this paper because it would increase the length of the formulae. Note as well that the derivatives in J_i tend to change the variable of $f_i(y)$ to $f_i(y(1+m_{i,R}^2/Q^2))$, consistent with the interpretation from the perturbative computation in the OPE.

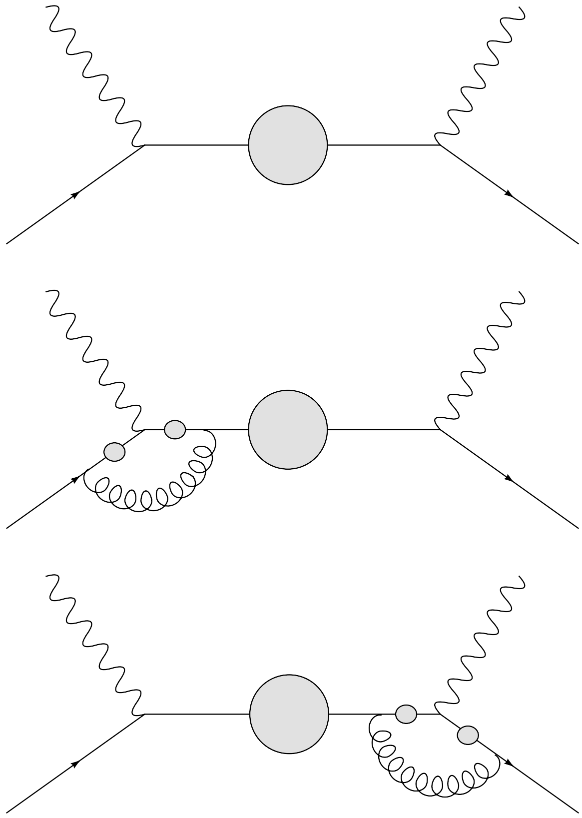

Figure 3: *Diagrams contributing to the perturbative computation at $O(\beta^2/Q^2)$.*

already see the failure of the OPE calculation from the comparison of $\mathrm{Im}T^{OPE}$ and $\mathrm{Im}T^{Eul}$). From the diagrams in Figs. 2 and 3 we then obtain

$$T^{OPE}(Q^2, x_B) = -2\left(\frac{4}{Q^2}\right)^2 \int_{-\infty}^{\infty} dy \left\{ e_i^2 J_i(x,y) f_i(y) + e_j^2 J_j(x,y) f_j(y) \right\} .$$

We can easily see that $T^{OPE} \neq T^{Eul}$. If we consider the moments generated by T^{OPE}, M_N^{OPE}, they can be expressed in terms of non-perturbative local operators (for the explicit expressions see [2]) as expected. Nevertheless, M_N has extra terms. If we consider the difference, we obtain

$$\begin{aligned} M_N(Q^2) - M_N^{OPE}(Q^2) &= \frac{16 e_i e_j}{Q^4} \frac{1}{N_c} \frac{m_i m_j}{Q^2} \\ &\times \int dz^- \langle ij; n | \psi_{i,-}^\dagger(0) \frac{(-i\overleftarrow{D}^+)^{N+2}}{(P_n^+)^{N+1}} \left[(N+4)\left(1 - \frac{-i\overleftarrow{D}^+}{P_n^+}\right) - 2\frac{-i\overleftarrow{D}^+}{P_n^+} \right] \\ &\times \psi_{j,-}(z^-) \psi_{j,+}^\dagger(0) \psi_{i,+}(z^-) | ij; n \rangle \ , \end{aligned} \tag{17}$$

which is expressed in terms of non-local operators. We take this result as evidence of the existence of OPE-violating terms. Let us stress that this is the first time, that we are aware of, that an analytic calculation in a quantum field theory exhibits OPE-violating contributions.

4 Conclusions

The possible existence of OPE-breaking effects in QCD has already been discussed in the past. As early as in Ref. [17] numerical evidence for the existence of OPE-breaking effects in the gluon condensate was claimed. Nevertheless, it is still unclear whether those effects can be associated to ultraviolet renormalons and/or higher orders in perturbation theory (for a recent discussion see [18]). Over the years there has also been some discussion on the possible existence of a $\langle A^2\rangle_{min.}$ condensate. This object should actually correspond to a non-local gauge-invariant condensate, though its explicit form is unknown for QCD [19]. Finally, there are some models that may produce effects that break the OPE, see for instance [20]. Nevertheless, those OPE-breaking effects would affect the static potential and the vacuum polarization. Regarding this we would like to emphasize that we do not find any OPE-breaking effect in the static potential or the vacuum polarization in the 't Hooft model. The static potential can be computed exactly in the 't Hooft model within perturbation theory. Therefore, there is no room there for effects associated to a sort of $\langle A^2\rangle_{min.}$ condensate. With the present precision of our computation, we also do not see OPE-breaking effects in the vacuum polarization.

The existence of this OPE-breaking effect is much associated to the large N_c analysis we have done, and the existence of sign-alternating effects. It should not be difficult to devise similar large N_c-inspired models in 4 dimensions, which would produce OPE-violating terms. In those models, however, it might be difficult to disentangle the OPE-violating terms from standard OPE contributions, since they both would scale in the same (or a very similar) way and, at the end of the day, one fits higher twist effects to data.

The new term is analytic in $1/Q^2$ and can actually be written in a factorized form. Therefore, one may think whether one could extend the standard formulation of the OPE in order to include this sort of terms. It would also be interesting to get a better understanding of this term from a diagrammatic analysis, if possible.

In conclusion, we have shown that in the 't Hooft model, at NLO in the $1/Q^2$ expansion, the moments associated to DIS receive a contribution that:
1) cannot be written in terms of local operators,
2) cannot be matched with any OPE-like contribution we are aware of.
We take these two facts as "smoking-gun" signals for the breakdown of the OPE in DIS in the 't Hooft model at NLO in the $1/Q^2$ expansion. Note that it appears

as a subleading (NLO) effect in DIS. Moreover, it is important that we considered the interference between two currents, otherwise this effect would be suppressed by a factor of, at least, order $1/Q^4$. For the vacuum polarization we do not see this sort of effects with the present precision [14].

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