Pomeron and Odderon: Saturation and Confinement and Gauge/String Duality

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At high energies, elastic hadronic cross sections are believed to be dominated by vacuum exchange. In leading order of the leading $1/N_c$ expansion this exchange process has been identified as the BFKL *Pomeron* or its strong AdS dual the closed string graviton [1]. However difference of particle anti-particle cross sections are given by a so-called *Odderon* carrying C = -1 quantum numbers identified in weak coupling with odd numbers of exchanged gluons. Here we show that the dual description associates this with the Neveu-Schwartz ($B_{\mu\nu}$) sector of closed string theory. Analysis of its strong coupling properties generalizes the AdS Pomeron of Ref [1] to this sector. We also consider eikonalization in AdS and discuss the effect due to confinement. We also discuss the extension of strong coupling treatment to central diffractive Higgs production at LHC.

1 Introduction

The subject of near-forward high energy scattering for hadrons has a long history. The traditional description of high-energy small-angle scattering in QCD has two components — a soft Pomeron Regge pole associated with exchanging tensor glueballs, and a hard BFKL Pomeron at weak coupling. On the basis of gauge/string duality, a coherent treatment of the Pomeron was provided [1]. These results agree with expectations for the BFKL Pomeron at negative t, and with the expected glueball spectrum at positive t, but provide a framework in which they are unified [2]. Therefore, a firm theoretical foundation for Pomeron in QCD has been established. It is now possible to identify a *dual Pomeron* as a well-defined feature of the curved-space string theory [1].

We focus here on the recent developments based on Maldacena's weak/strong duality, (AdS/CFT), relating Yang-Mills theories to string theories in (deformed) Anti-de Sitter space [1, 3, 4, 5, 6]. The application of this duality to Diffractive Scattering and the Pomeron physics represent an important area where a connection with the string-theory-based techniques can be made. Furthermore, it is now possible to extend this treatment to central diffractive production of Higgs at LHC.

In the large 't Hooft coupling, Pomeron can be considered as a *Reggeized Massive Graviton*, propagating in a 5-dimensional curved space, the so-called AdS_5, where both the IR (soft) Pomeron and the

UV (BFKL) Pomeron are dealt in a unified single step. The connection with the stringy aspects in a five-dimensional description is indeed very direct. In gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously. Indeed, Pomeron is directly related to the graviton and its higher spin partners on the leading (five-dimensional) Regge trajectory. In AdS/CFT, confinement is associated with a deformed AdS_5 geometry having an effective horizon, e.g., that for a blackhole. The solution to this is unknown and represents the major theoretical challenge in model-building. Each model leads to certain unique signature. LHC data can provide guide and direction in this endeavor.

2 Pomeron and QCD Parameter-Space

It is useful to take a step back in examining high energy scattering in QCD. From a theoretical stand point it is useful to consider a 3-parameter space varying the number of colors (N_c), the 't Hooft coupling ($\lambda = g^2 N_c$), and the virtuality of an external probe $1/Q$, e.g., that of a virtual photon.

Figure 1: The Pomeron in QCD viewed as a function of colors (N_c), the 'tHooft coupling ($g^2 N_c \simeq 0$ and the resolution (virtuality: $Q \sim 1/z$ of the probe.

Thus diffractive scattering (or Pomeron exchange) in QCD can now be considered in two steps. First one may consider the leading contribution in the $1/N_c$ expansion holding fixed the 't Hooft coupling $\lambda = g^2 N_c$. For example, instead of $N_c = 3$, the leading term for the Regge limit of 2-to-2 scattering in the this limit is the exchange of a network of gluons with the topology of a cylinder in the 't Hooft topological expansion or, in string language, the exchange of a closed string. This gives rise to what we call the "bare Pomeron" exchange. Taking into account high order terms in the $1/N_c$ leads to two effects: (1) The cylinder diagrams includes closed quark loops, leading to $q\bar{q}$ pairs or multi-hadron production via the optical theorem dominated by low mass pions, kaon etc. (2) The multiple exchange of the bare Pomeron which includes the eikonal corrections (or survival probability) and triple-Pomeron and higher order corrections in a Reggeon calculus. We will focus primarily on the "bare Pomeron" sector and will discuss only briefly higher order effect due to eikonalization.

In weak coupling coupling summations where $\lambda << 1$ and N_c large, the leading singularity (prior to full unitarization) is at

$$j_0 = 1 + (\ln 2/\pi^2)\ \lambda \tag{1}$$

where $\lambda = g^2 N_c$ is the 't Hooft coupling. Indeed, in the limit $\lambda \to 0$, this reduces to the so-called Low-Nussinov Pomeron, i.e., two-gluon exchange, as depicted in Fig. 1b. When this description is adequate, hadronic cross sections are expected to rise as a small power s^{j_0-1} until unitarity forces compliance with the Froissart bound. However there is an additional probe of the "Pomeron" as a function of virtuality Q^2 in off-shell photon scattering.

It is generally acknowledge that diffractive scattering is intrinsically a non-perturbative phenomena. In the limit where the 't Hooft coupling is large, weak coupling calculations become unreliable. In Ref. [1], it has been shown that the leading singularity in strong coupling in the comformal limit approached $j = 2$. In the language of the AdS/CFT, Pomeron is the graviton pole in the 5-dim AdS space where the AdS radius r serves the 5th dimension. (In what follows, this will be referred to as the strong coupling BPST Pomeron, or simply the BPST Pomeron.)

One of the more interesting developments of the BPST Pomeron is the recognition that the virtuality of an external probe, $1/Q$, can be identified with the AdS radius, $z = 1/r$. Conformal invariance, which allows a simultaneous scale transformations in the transverse size and the probe scale, can now be encoded as the isometry of the transverse Euclidean AdS_3.

3 Forward Scattering, Gauge/String Duality, and Confinement

For conformally invariant gauge theories, the metric of the dual string theory is a product, $AdS_5 \times W$, $ds^2 = \left(\frac{r^2}{R^2}\right)\eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{R^2}{r^2}\right)dr^2 + ds^2_W$, where $0 < r < \infty$. (Equivalently, one can introduce $z = R^2/r$ and express the metric as $ds^2 = e^{2A}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$, with $e^A = \frac{R}{z}$ in the conformal limit.) For the dual to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory the AdS radius R is $R^2 \equiv \sqrt{\lambda}\alpha' = (g^2_{\rm YM}N)^{1/2}\alpha'$, and W is a 5-sphere of this same radius. We will ignore fluctuations over W and also assume that $\lambda \gg 1$, so that the spacetime curvature is small on the string scale, and $g^2_{YM} \ll 1$ so that we can use string perturbation theory. (See [4, 18] for more references.)

The fact that 5-dim description enters in high energy collision can be understood as follows. In addition to the usual LC momenta, $p_\pm = p^0 \pm p^z$ (2d), and transverse impact variables, $\vec{b}$ (2d), there is one more "dimension": a "resolution" scale specified by a probe, e.g., $1/Q^2$ of virtual photon in DIS, (see Fig. 2a.) Because of conformal symmetry, these 5 coordinates transform into each others, leaving the system invariant. In the strong coupling limit, conformal symmetry is realized as the $SL(2,C)$ isometries of Euclidean AdS_3 subspace of AdS_5, where r can be identified with Q^2.

One important step in formulating the dual Pomeron involves the demonstration [7] that in exclusive hadron scattering, the dual string theory amplitudes at wide angle, due to the red-shifted local-momenta, $s \to \tilde{s} = (R/r)^2 s$ and $t \to \tilde{t} = (R/r)^2 t$, give the power laws that are expected in a gauge theory. It was also noted that at large s and small t that the classic Regge form of the scattering amplitude should be present in certain kinematic regimes [7]. Equally important is the fact that, with confinement, transverse fluctuations of the metric tensor G_{MN} in AdS acquire a mass and can be identified with a tensor glueball [8, 9]. It was suggested in [9] that, at finite λ, this will lead to a Pomeron with an intercept below 2. That is, Pomeron can be considered as a *Reggeized Massive Graviton.*

Figure 2: (a) Intuitive picture for AdS^5 kinematics. (b) Schematic representation of J-plane singularity structure. (c) Schematic form of Δ-j relation for $\lambda << 1$ and $\lambda >> 1$.

For a conformal theory in the large N_c limit, a dual Pomeron can always be identified with the leading eigenvalue of a Lorentz boost generator M_{+-} of the conformal group [4]. The problem reduces to finding the spectrum of a single J-plane Schrödinger operator. One finds that, in the strong coupling limit, conformal symmetry requires that the leading $C = +1$ Regge singularity is a fixed J-plane cut,

$$j_0^{(+)} = 2 - 2/\sqrt{\lambda} \tag{2}$$

For ultraviolet-conformal theories with confinement deformation, the spectrum exhibits a set of Regge trajectories at positive t, and a leading J-plane cut for negative t, the cross-over point being model-dependent. (See Fig. 2b.) For theories with logarithmically-running couplings, one instead finds a discrete spectrum of poles at all t, with a set of slowly-varying and closely-spaced poles at negative t.

3.1 Conformal Pomeron, Odderon and Analyticity

At high-energy, analyticity and crossing lead to $C = \pm 1$ vacuum exchanges, the *Pomeron* and the *Odderon*. The qualitative picture for Pomeron exchange in weak coupling [10] has been understood for a long time, in leading order expansion in g^2_{YM} and all order sum in $g^2_{YM} log(s/s_0)$. In the conformal limit, both the weak-coupling BFKL Pomeron and Odderons correspond to J-plane branch points, e.g., the BFKL Pomeoron is a cut at $j_0^{(+)}$, above $j = 1$. Two leading Odderons have been identified. (See [5, 11] for more references.) Both are branch cuts in the J-plane. One has an intercept slightly below 1 [12], and the second has an intercept precisely at 1 [13]. These are summarized in Table 1.

In the strong coupling limit, as we have already mentioned above, conformal symmetry dictates that the leading $C = +1$ Regge singularity is a fixed J-plane cut at $j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$. As λ increases, the "conformal Pomeron" moves to $j = 2$ from below, approaching the AdS graviton. We have recently shown [18] that the strong coupling *conformal odderons* are again fixed cuts in the J-plane, with intercepts specified by the AdS mass squared, m^2_{AdS}, for Kalb-Ramond fields [14],

$$j_0^{(-)} = 1 - m^2_{AdS}/2\sqrt{\lambda} + O(1/\lambda)\ . \tag{3}$$

Interestingly, two leading *dual odderons* can be identified, parallel the weak-coupling situation. One solution has $m^2_{AdS,(1)} = 16$. There is also a second solution where $m^2_{AdS,(2)} = 0$. We outline below how these features emerge in *Gauge/String duality.*

	Weak Coupling	Strong Coupling
$C = +1$: Pomeron	$j_0^{(+)} = 1 + (\ln 2)\,\lambda/\pi^2 + O(\lambda^2)$	$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$
$C = -1$: Odderon	$j_{0,(1)}^{(-)} \simeq 1 - 0.24717\,\lambda/\pi + O(\lambda^2)$ $j_{0,(2)}^{(-)} = 1 + O(\lambda^3)$	$j_{0,(1)}^{(-)} = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$ $j_{0,(2)}^{(-)} = 1 + O(1/\lambda)$

Table 1: Pomeron and Odderon intercepts at weak and strong coupling, with $\lambda = g_{YM}^2 N_c$ the 't Hooft coupling.

3.2 Flat-Space Expectation for $C = \pm 1$ Sectors

String scattering in 10-d flat-space at high energy leads to a crossing-even and crossing-odd amplitudes,

$$\mathcal{T}_{10}^{(\pm)}(s,t) \to f^{(\pm)}(\alpha' t)(\alpha' s)^{\alpha_\pm(t)} \,, \tag{4}$$

where $\alpha_+(t) = 2 + \alpha' t/2$ and $\alpha_-(t) = 1 + \alpha' t/2$ respectively. That is, at $t = 0$, a massless state with integral spin is being exchanged, e.g., for $C = +1$, one is exchanging a massless spin-2 particle, the ubiquitous graviton. Of course, the coefficient functions, $f^{(\pm)}(\alpha' t)$, are process-dependent.

Massless modes of a closed string theory can be identified with transverse fluctuations coming from a left-moving and a right-moving level-one oscillators, e.g., states created by applying $a_{1,I}^\dagger \tilde{a}_{1,J}^\dagger$ to the vacuum, i.e.,

$$a_{1,I}^\dagger \tilde{a}_{1,J}^\dagger |0; k^+, k_\perp\rangle, \tag{5}$$

with $k^2 = 0$. Since a 10-dim closed string theory in the low-energy limit becomes 10-dim gravity; these modes can be identified with fluctuations of the metric G_{MN}, the anti-symmetric Kalb-Ramond background B_{MN} [14], and the dilaton, ϕ, respectively. It is important to note that we will soon focus on AdS^5, i.e., one is effectively working at $D = 5$. With $D = 5$, the independent components for G_{MN} and B_{MN} are 5 and 3 respectively, precisely that necessary for having (massive) states with spin 2 and 1 [9]. For oriented strings, it can be shown that the symmetric tensor contributes to $C = +1$ and the anti-symmetric tensor contributes to $C = -1$.

3.3 Pomeron in Super Gravity Limit

Recall that, in flat space, exchanging a graviton leads to a near-forward amplitude behaving asymptotically as

$$A \sim \frac{s^2}{t} \tag{6}$$

The factor in the numerator, s^2, is due to spin-2 of a graviton with its coupling to the enregy-momentum tnesor, and the factor, $\frac{1}{t}$, is the propagator for a massless exchange. In moving to AdS, it has been shown in Ref. [4] that the scattering takes on a "local form",

$$A(s,t) \simeq g_0^2 \iint dz dz' P_{13}(z) \left(\widehat{s}^{j_0} G(\widehat{s}, t, z, z')\right) P_{24}(z') \tag{7}$$

with $j_0 = 2 - 2/\sqrt{\lambda}$ replacing 2 as the effective spin and G replacing $1/t$ as a new propagator, (here we have introduced a "local squared energy" $\widehat{s} = (zz's/R^2)$.) We will provide in this Appendix additional discussion on steps required in leading to these equations. There are two new aspects to (7): the local factorizable form, with $P_{ij}(z)$ proportional to the product of wave functions, $\phi_i(z)\phi_j(z)$, and a more complex new propagator, $G(\widehat{s}, t, z, z')$. We will begin by first discuss a simpler limit where $\lambda \to \infty$ before considering the case with λ finite. In the later case, the new features are $j_0 < 2$ and "diffusion" in AdS space. For the easy of presentation, we will first carry out the analysis in 4-d momentum representation before moving to an impact representation.

In the limit where the 't Hooft coupling $\lambda \to \infty$, consider elastic dilaton-dilaton scattering. To lowest order in string coupling, the amplitude is given by one-graviton exchange, which has been shown in Ref. [?] that the amplitude can be expressed in the Regge limit as

$$\begin{aligned} A(s,t) &= g_0^2 \int dz \int dz' \left(\sqrt{-g(z)} e^{-2A(z)} \phi_1(z)\phi_3(z)\right) \\ &\times \left((\alpha' s)^2 G^{(0)}(z,z';t)\right) \left(\sqrt{-g(z')} e^{-2A(z')} \phi_2(z')\phi_4(z')\right) \end{aligned} \tag{8}$$

Eq. (8) corresponds to an elementary spin-2 exchange, leading to a fixed s^2 behavior. The structure is fixed by coupling graviton in the bulk to a conserved energy-momentum tensor, e.g., $T^{mn} G_{mn;m'n'} T^{m'n'}$. The leading s behavior is obtained by keeping the $(++--)$-component of the graviton propagator. We have collected factors so that the first and the last terms on the right-hand side of (8) corrspond to $P_{13}(z)$ and $P_{24}(z')$ respectively. Eq, (8) can be interpretated as scattering between "partons" at z and z', via a graviton exchange, with the P_{13} and P_{24} serving as parton densities respectively. In this limit, the amplitude is purely real.

To have a better understanding on how the wave functions enter, let us introduce an IR cutoff, z_{IR} so that the spectrum for dilaton fluctuations becomes discrete, e.g., each state corresponds to a normalized solution of AdS_5 scalar wave equation in momentum space, $-\nabla_0 \phi_n(z) = 0$, or, more explicitly,

$$(1/\sqrt{-g})(\sqrt{-g} e^{-2A} \phi_n'(z))' = m_n^2 e^{-2A} \phi_n(z) \tag{9}$$

with masses m_n determined by appropriate boundary conditions. These wave functions can then be chosen to be normalized

$$\int_0^{z_{IR}} dz \sqrt{-g} e^{-2A} \phi_n(z)\phi_n(z) = \delta_{mn} \tag{10}$$

In the conformal limit, $P_{ij}(z) \equiv \sqrt{-g} e^{-2A} \phi_i(z)\phi_j(z) = (z/R)^{-3} \phi_i(z)\phi_j(z)$, and the normalization condition (10) can be written more compactly as $\int_0^{z_{IR}} dz P_{ij}(z) = \delta_{ij}$, thus justifying our treating $P_{ij}(z)$ as the parton density.

In Eq. (8), $G^{(0)}(z,z';t)$ is the AdS_5 scalar propagator in momentum space, i.e., it is the solution to the Laplacian equation, $-\nabla_0 G^{(0)}(z,z';t) = (z/R)^{-5} \delta(z-z')$, or, more explicitly,

$$\left(-z^5 \partial_z z^{-3} \partial_z - z^2 t\right) G^{(0)}(z,z';t) = R^{-4} z^5 \delta(z-z') \tag{11}$$

For scattering in the s-channel, the momentum transfer vector is spacelike, $t = -q_\perp^2 < 0$, and this can be reduced to the standard Bessel equation. In the conformal limit, i.e., with the IR removed,

$$G^{(0)}(z,z';t) = \left(\frac{zz'}{R^2}\right)^2 \int_0^\infty kdk \frac{J_2(kz)J_2(kz')}{k^2-t} \ . \tag{12}$$

Eq. (12) also serves as a spectral representation. In the conformal limit, the spectrum is continuous, for $t > 0$. However, if an IR cutoff is introduced, the spectrum becomes discrete. It is then clear that, in the case of hard-wall model, wave-functions $\phi_n(z)$ is given by

$$\phi_n(z) = c_n \left(\frac{z}{R}\right)^2 J_2(m_n z) \tag{13}$$

with m_n fixed by a Neumann condition at z_{IR} and c_n is a normalization constant.

It is often convenient to introduce another propagator

$$G^{(2)}(z,z';t) \equiv \left(\frac{zz'}{R^2}\right)^{-2} G^{(0)}(z,z';t) \tag{14}$$

so that (8) can be expressed as

$$A(s,t) \quad = \quad g_0^2 \int dz \int dz' P_{13}(z) \left((\alpha'\widehat{s})^2\, G^{(2)}(z,z';t)\right) P_{24}(z') \tag{15}$$

Since tensor Laplacian ∇_j can be expressed in terms of scalar Laplacian by $\nabla_j = (z/R)^{-j}\nabla_0(z/R)^j$, it follows that $G^{(2)}$ is the propagator for ∇_2, i.e.,

$$(-z\partial_z z\partial_z + 4 - z^2 t)G^{(2)}(z,z';t) = z\delta(z-z') \tag{16}$$

and one has

$$G^{(2)}(z,z';t) = \int_0^\infty kdk \frac{J_2(kz)J_2(kz')}{k^2-t} \ . \tag{17}$$

3.4 Analyticity and Crossing

Let us turn next to the fact that, due to curvature of AdS, at finite λ, the effective spin of the exchange is lowered from 2 to $j_0 = 2 - 2/\sqrt{\lambda}$ [1]. Focus on the amplitude for $C = +1$, (a similar analysis can also be carried out for $C = -1$.) Note that, in the limit $\lambda \to \infty$, both amplitudes, (6) and (15) are real. However, if the effective spin is shifted, e.g., $2 \to 2 - \epsilon$, with ϵ small, real-analyticity and crossing evenness of the amplitude requires, in flat spacetime,

$$A \sim (-\frac{1}{\epsilon} + i)s^{2-\epsilon} \tag{18}$$

which necessarily leads to the presence of an imaginary part. To regain the graviton pole, a physical particle with spin 2, its location must now be shifted. This can be accomplished by making the power t-dependent, e.g., $2 + (\alpha' t/2 - \epsilon)$, so that the power is $2 - \epsilon$ at $t = 0$, i.e.,

$$A \sim (\frac{1}{\alpha' t/2 - \epsilon} + i)s^{2+(\alpha' t/2-\epsilon)} \tag{19}$$

Note that, in the flat spacetime, this is precisely the expected Regge form for a graviton becoming slightly "massive". Conversely, at $t = 0$, if the the imaginary part of the amplitude is power-behaved, $\text{Im}A(s, 0) \sim s^{2-\epsilon}$, the real part will have a pole at $t = 0(\epsilon/\alpha')$. All these point to (i) we need to adopt a formalism which builds in automatically the analyticity requirement and at the same time facilitates in dealing with amplitudes which is power-behaved at high energy. The formalism which doe precisely that is use of the complex J-plane. In this formalism, one makes use of the fact that the full amplitude can be reconstructed once the ImA is given. To be precise, this is done by defining J-plane partial-wave amplitude A_j from ImA via a Mellin transform.

4 Pomeron in AdS

Recall that, in flat space string theory, the J-plane has a leading pole (in fact is meromorphic) so the high energy limit of the 2 to 2 scattering amplitude has the classic Regge from

$$A(s,t) \simeq \beta_{13}(t)\frac{1+e^{-i\alpha(t)}}{\sin \pi\alpha(t)}(\alpha' s)^{\alpha(t)}\beta_{24}(t) \tag{20}$$

where for the close string $\alpha(t) = 2 + \alpha' t/2$ for the exchange of a Reggized graviton.

$$A(s, -q_\perp^2) = \int_{-i\infty}^{i\infty} \frac{dj}{2\pi i}(\alpha' s)^j \frac{1+e^{-i\pi j}}{\sin \pi j} A_j(-q_\perp^2) \,, \tag{21}$$

$$A_j(-q_\perp^2) = \beta_{13}(-q_\perp^2) G_j(-q_\perp^2)\beta_{24}(-q_\perp^2) \,, \tag{22}$$

where the Reggeon "propagator" takes the form

$$G_j(-q_\perp^2) = \frac{1}{j - j_0 + \alpha' q_\perp^2} \,. \tag{23}$$

Here, again, for flat-space closed string, $j_0 = 2$ and $\alpha(t) = 2 - \alpha' q_\perp^2$.

Let us turn next to AdS. In Ref. [4], it has been shown that one obtains

$$\begin{aligned} A(s,t) &= g_0^2 \int dz dz' P_{13}(z) P_{24}(z') \left(- \int \frac{dj}{2\pi i}(\alpha' \widehat{s})^j \left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right) G_j(z, z'; t) \right) \\ &\equiv - \int \frac{dj}{2\pi i}(\alpha' s)^j \left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right) A_j(t) \end{aligned} \tag{24}$$

It follows that $A_j(t)$ is related to the imaginary part of $A(s, t)$ by a Mellin transform,

$$\begin{aligned} A_j(t) &= \int_0^\infty ds s^{-j-1} \text{Im}A(s,t) \\ &= g_0^2 \int dz dz' P_{13}(z) P_{24}(z') G_j(z, z'; t) \end{aligned} \tag{25}$$

4.1 Diffusion in AdS for Pomeron and Odderon

Let us next introduce diffusion in AdS. We will restrict ourselves to the conformal limit. Regge behavior is intrinsically non-local in the transverse space. For flat-space scattering in 4-dimension, the transverse space is the 2-dimensional impact parameter space, $\vec{b}$. In the Regge limit of s large and $t < 0$, the momentum transfer is transverse. Going to the $\vec{b}$-space, $t \to \nabla_b^2$, and the flat-space Regge propagator, for both $C = \pm 1$ sectors, is nothing but a diffusion kernel,

$$\langle \, \vec{b} \mid (\alpha' s)^{\alpha_\pm(0)+\alpha' t \nabla_b^2/2} \mid \vec{b}' \, \rangle, \tag{26}$$

with $\alpha_+(0) = 2$ and $\alpha_-(0) = 1$ respectively. In moving to a ten-dimensional momentum transfer $\tilde{t}$, we must keep a term coming from the momentum transfer in the six transverse directions. This extra term leads to diffusion in extra-directions, i.e., for $C = +1$,

$$\alpha' \tilde{t} \to \alpha' \Delta_P \equiv \frac{\alpha' R^2}{r^2} \nabla_b^2 + \alpha' \Delta_{\perp P}. \tag{27}$$

The transverse Laplacian is proportional to R^{-2}, so that the added term is indeed of order $\alpha'/R^2 = 1/\sqrt{\lambda}$. To obtain the $C = +1$ Regge exponents we will have to diagonalize the differential operator Δ_P. Using a Mellin transform, $\int_0^\infty d\tilde{s}\; \tilde{s}^{-j-1}$, the Regge propagator can be expressed as

$$\widehat{s}^{2+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i}\, \widehat{s}^j\, G^{(+)}(j) = \int \frac{dj}{2\pi i}\, \frac{\widehat{s}^j}{j - 2 - \alpha' \Delta_P/2} \tag{28}$$

where $\Delta_P \simeq \Delta_2$, the tensorial Laplacian. Using a spectral analysis, (see next sub-section), it leads to a J-plane cut at $j_0^{(+)}$.

A similar analysis can next be carried out for the $C = -1$ sector. We simply replace the Regge kernel by

$$\widehat{s}^{1+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i}\, \widehat{s}^j\, G^{(-)}(j) = \int \frac{dj}{2\pi i}\, \widehat{s}^j (j - 1 - \alpha' \Delta_O/2)^{-1} \; . \tag{29}$$

The operator $\Delta_O(j)$ can be fixed by examining the EOM at $j = 1$ for the associated super-gravity fluctuations responsible for this exchange, i.e., the anti-symmetric Kalb-Ramond fields, B_{MN}. One finds two solutions,

$$G^{(-)}(j) = \frac{1}{[j - 1 - (\alpha'/2R^2)(\Box_{Maxwell} - m^2_{AdS,i})]} \; , \tag{30}$$

$i = 1, 2$, where $\Box_{Maxwell}$ stands for the Maxwell operator. Two allowed values are $m^2_{AdS,1} = 16$ and $m^2_{AdS,2} = 0$. A standard spectral analysis then lead to a branch-cut at $j_0^{(-)}$, given by Eq. (3).

4.2 Regge and DGLAP Connection

With the spin of the exchange shifted from 2 to $2 - (1/2\sqrt{\lambda})\nabla_2$, in place of $G^{(2)}(z, z'; t)$, which is the propagator for ∇_2, we find that it is more appropriate to work with the J-dependent propagator, $G_j(z, z'; t)$, where

$$(2\sqrt{\lambda})(j - 2) - \nabla_2) G_j(z, z'; t) \quad = \quad z\, \delta(z - z') \; . \tag{31}$$

(We have shifted the normalization by $G_j \to 2\sqrt{\lambda} G_j$). When $\lambda \to \infty$, a finite limit can be reached only if $j \to 2$, where $G_2(z, z'; t) = G^{(2)}(z, z'; t)$, as expected. Writing out Eq. (31) explicitly, one has

$$(-z\partial_z z\partial_z + (2\sqrt{\lambda}))(j - j_0) - z^2 t)G_j(z, z'; t) \quad = \quad z\,\delta(z - z') \tag{32}$$

where $j_0 = 2 - 2/\sqrt{\lambda}$. This generalized propagator can now be found by a standard method of spectral analysis. One can continue to study the spectrum in t, with J fixed. There is now another option, spectrum in j, with t fixed.

In the conformal limit, and holding $j > j_0$ and real, (32) is Hermitian. The spectrum in t is again continuous, along its positive real axis. In analogy to Eq. (12), one finds

$$G_j(z, z'; t) \quad = \quad \int_0^\infty kdk \frac{J_{(\Delta(j)-2)}(kz) J_{(\Delta(j)-2)}(kz')}{k^2 - t} \ . \tag{33}$$

where

$$\Delta(j) = 2 + \sqrt{2\sqrt{\lambda}(j - j_0)} \tag{34}$$

It is also possible to obtain $\tilde{G}_j(z, z'; t)$ by considering its spectrum in j, with t fixed. It reduces to a Schrodinger problem, with the term $-z^2 t$ as potential. For $t < 0$, the potential is repulsive, and there is no bound state and the spectrum is continuous. One then finds that

$$G_j(z, z'; t) = \int_{-\infty}^{\infty} \frac{d\nu}{\pi^2} (\nu \sinh \pi\nu) \frac{K_{i\nu}(qz) K_{i\nu}(qz')}{2\sqrt{\lambda}(j - j_0) + \nu^2} \tag{35}$$

It is also useful to explore the conformal invariance as the isometry of transverse AdS_3. Upon taking a two-dimensional Fourier transform with respect to $q_\perp$, where $t = -q_\perp^2$, one finds that $G_j^{(\pm)}$ can be expressed simply as

$$G_j^{(\pm)}(z, x^\perp, z', x'^\perp) = \frac{1}{4\pi z z'} \frac{e^{(2-\Delta^{(\pm)}(j))\xi}}{\sinh \xi} \ , \tag{36}$$

where $\cosh \xi = 1 + v$,

$$v = \frac{(x^\perp - x'^\perp)^2 + (z - z')^2}{2zz'} \tag{37}$$

the AdS_3 chordal distance, and

$$\Delta^{(\pm)}(j) = 2 + \sqrt{2}\, \lambda^{1/4} \sqrt{(j - j_0^{(\pm)})} \tag{38}$$

is a J-dependent effective AdS_5 conformal dimension [1, 4, 18]. The $\Delta - j$ curve for $\Delta^{(\pm)}$ is shown in Fig. 2c. A related discussion on $\Delta(j)$ can be found in [15].

5 Unitarity, Absorption, Saturation and the Eikonal Sum

For simplicity, we will focus here on the $C = +1$ sector, assuming all crossing odd amplitudes vanish. It has been shown in Refs. [3, 4, 6] that, in the strong coupling limit, a 2-to-2 amplitude, $A(s, t)$, in the near-forward limit can be expressed in terms of a "generalized" eikonal representation,

$$A_{2\to 2}(s, t) = \int dz dz' P_{13}(z) P_{24}(z') \int d^2 b\, e^{-ib^\perp q_\perp} \widetilde{A}(s, b^\perp, z, z') \ , \tag{39}$$

where

$$\widetilde{A}(s, b^\perp, z, z') = 2is \left[1 - e^{i\chi(s,b^\perp,z,z')}\right] , \tag{40}$$

and $b^\perp = x^\perp - x'^\perp$ due to translational invariance. The probability distributions for left-moving, $P_{13}(z)$, and right moving, $P_{24}(z)$ particles are products of initial (in) and final (out) particle wave functions. The eikonal, χ, can be related to the strong coupling Pomeron kernel [1, 4], and can be expressed as the inverse Mellin transform of $G_j^{(+)}(x^\perp - x'^\perp, z, z')$.

We note the salient feature of eikonal scattering locally in transverse AdS_3, and the near-forward field-theoretic amplitude is obtained from a bulk eikonal amplitude after convolution. It is useful to focus our attention on the properties of the bulk eikonal formula $\widetilde{A}(s, b^\perp, z, z')$ itself. For χ real, it is elastic unitary. On the other hand, when χ is complex, (with $\text{Im}\chi > 0$), one has inelastic production. Absorption and saturation can now be addressed in this context. It is also important to note that, for Froissart bound, confinement is crucial. Discussion on these and related issues can be found in Ref. [4].

We end by pointing out one unique feature of strong coupling – the eikonal is predominantly real. To simplify the discussion, let us consider the second order contributions to the imaginary part of the elastic amplitude. The AGK cutting rule for the imaginary part of the elastic amplitude generalizes to

$$\cos(j_0\pi)|\chi|^2 = \left[1 - 2\sin^2(j_0\pi/2) - 2\sin^2(j_0\pi/2) + 2\sin^2(j_0\pi/2)\right] |\chi|^2 \tag{41}$$

where the first term on the right is due to the elastic scattering, the last term is due to two-cut-Pomeron contribution, and the second and the third are due to one-cut-Pomeron contributions. The tradition weak coupling approach to diffraction scattering has $j_0 \simeq 1$, leading to a net negative contribution: $-1 = 1 - 2 - 2 + 2$. This leads to absorption, already dominant at second order. However, for extreme strong coupling, one has $j_0 \simeq 2$, leading to a positive cut contribution: $1 = 1-0-0+0$. This is consistent with scattering being predominantly elastic. However, the real world is neither strictly weak coupling nor strong coupling. For $j_0 \simeq 1.5$, one finds the two-Pomeron contribution vanishes: $0 = 1 - 1 - 1 + 1$. That is, what used to be the dominant correction to elastic scattering now vanishes. Clearly, these issues deserve further examination.

For applications of [1, 3, 4, 5, 6] for DIS, see [16]. For a more proper treatment while taking into account of confinement effects, see [1].

6 Diffractive Production of Higgs at LHC

A promising production mechanism for Higgs meson at the LHC involves the forward proton-proton scattering $pp \to pHp$. Because of the exceptional signal to background discrimination, this may even be a discovery channel depending of course on the production cross section. The theoretical estimates generally involve the assumption of perturbative contribution of gluon fusion in the central rapidity region [**?**]. In these estimates the Pomeron is effectively replaced by two-gluon exchange referred to in the early literature as the Low-Nussinov Pomeron. In spite of the plausibility of this approach, there are considerable uncontrolled uncertainties. The Regge description for diffractive production is well known to be intrinsically non-perturbative.

Figure 3: (a) kinematics for single-Regge limit for 2-to-2 amplitudes, (b) Double-Regge kinematics for 2-to-3 amplitudes. (c) Cylinder Diagram for large N_c Higgs Production.

We have begun the analysis in strong coupling based on the AdS/CFT correspondence and conformal strong coupling BPST Pomeron [1] This amounts to a generalization of our previous AdS for 2-to-2 amplitudes to one for 2-to-3 amplitudes, e.g., from Fig. 3a to Fig. 3b. A more refined analysis for Higgs production involves a careful treatment for that depicted in Fig. 3c. While this also will have its uncertainties, a careful comparison between weak and strong coupling Pomeron should give better bounds on these uncertainties. Ultimately the strong coupling approach calibrated by comparison with experimental numbers for double diffraction heavy quark production, can provide increasing reliable estimates for Higgs production. The first required step is to clarify the Pomeron-Pomeron Higgs production vertex for the conformal Pomeron in both strong BPST and weak BFKL regimes. This is not a full calculation but a first critical ingredient.

Focusing only on contributions from Pomeron exchange, a flat-space 2-to-2 amplitude in the Regge limit can be expressed as $A(s,t) \simeq \beta_{13}(t)\frac{1+e^{-i\alpha(t)}}{\sin \pi\alpha(t)}(\alpha' s)^{\alpha(t)}\beta_{24}(t)$, as depicted in Fig. 3a. For a 5-point amplitude, there are five independent invariants, as indicated in Fig. 3b. In the kinematic region for diffractive scattering where transverse momenta of all produced particles are limited, $\kappa \equiv s_1 s_2/s$ is fixed, with $\kappa \simeq m_H^2 + q_\perp^2$, in the frame where incoming particles are longitudinal. Using a double J-plane representation, In the double-Regge region, a 2-to-3 amplitude can be expressed can be represented using a doulbe-J-plane representation, as

$$\begin{aligned} T(s, s_1, s_2, t_1, t_2) \quad &\simeq \quad \int_{-i\infty}^{i\infty} \frac{dj_1}{2\pi i}\, \xi(j_1)\; (\alpha' \widehat{s}_1)^{j_1} \int_{-i\infty}^{i\infty} \frac{dj_2}{2\pi i}\, \xi(j_2)\; (\alpha' \widehat{s}_2)^{j_2} \\ &\times \quad \beta_{13}(t_1) G_{j_1}(t_1) \mathcal{V}(t_1, t_2, \kappa) G_{j_2}(t_2) \beta_{24}(t_2) \end{aligned} \tag{42}$$

where $\xi(j) = \frac{1+e^{-i\pi j}}{\sin \pi j}$ is the signature factor and $\mathcal{V}$ is a new Pomeron-Particle-Pomeoern coupling.

To go to AdS, we simply need to replace G_j and $\mathcal{V}(t_1, t_2, \kappa)$ by corresponding generalizations. For instance, we replace $G_j(t)$ by $G_j^{(+)}(z, x^\perp, z', x'^\perp)$. The essential new feature is a new vertex, $\mathcal{V}(x_{1\perp} - x_{H\perp}, z_1', x_{2\perp} - x_{H\perp}, z_2')$, depicted in Fig. 3c. appropriate for a diffractive central Higgs production [18]. Here we apply the approximation implicity in scale separations. First we assume that $m_H \ll m_t$ so that the Higgs vertex is repaced by a source for $F^a_{\mu\nu}(x) F^a_{\mu\nu}$ at the boundary of AdS ($z \to 0$). The standard AdS/CFT dictionary leads to a bulk to boudary propagator $\Delta(x - x, z)$ for the interior of AdS to this point so that this vertex is a factorized product $\int d^2x_\perp dz V(x_{1\perp} - x_\perp, z_1', x_{2\perp} - x_\perp, z_2', z) \Delta_\perp(x_\perp - x_{H\perp}, z)$. In a subsequent analysis we will build on this result, adding corrections due to (1) conformal symmetry breaking, (2) the proton impact factor and (3) eikonal "survival" probability to obtain phenomenological results for double Pomeron Higgs production at the LHC. These will be reported in a series of future publications.

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