

Total Cross-sections at very high energies: from protons to photons

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1 Introduction

A good knowledge of total cross-sections of high energy photon-proton and photon-photon interactions, in an energy domain where no data are available, is important from the point of view of understanding high energy cosmic ray data and planning of the TeV energy e^+e^- colliders respectively. This requires developing a model which can explain the current energy dependence of these cross-sections observed in the laboratory experiments and then using it to predict the cross-sections in the required higher energy regime. Apart from this very prosaic reason for studying the subject, high energy behaviour of total hadronic cross-sections is an issue of great theoretical importance. Very general arguments based on unitarity, analyticity and factorisation in fact imply a bound on the high energy behaviour of total hadronic cross-sections [1]. This bound predicts, independent of the details of the strong interaction dynamics, that asymptotically $\sigma_{tot} \leq C(\log s)^2$. All the experimentally measured hadronic cross-sections seem to rise with energy [2], although it is not clear whether the rate is the same for all the hadronic processes; nor is it clear whether the asymptotic behaviour is already reached at the current energies. In view of the important clues to the strong interaction dynamics that this energy dependence holds and the equally strong need of its precise knowledge in the high energy regime for the planning of future experiments, or the understanding of the high energy cosmic ray data, it is not surprising that this has been a subject of intense theoretical investigations [3, 4].

Since QCD has now been established as *the* theory of strong interactions, it is of course of import to seek an understanding of this interesting bound in a QCD based picture and try to see clearly as to which piece of the QCD dynamics is it most closely

related to. A description of one effort [5] in this direction, viz. a QCD based model to describe the energy dependence of the total hadronic cross-section and its extension [6] to the case of the high energy behaviour of the photon induced processes is the subject of this note. We will first summarise very briefly the current experimental situation on the observed energy dependence for all the hadronic cross-sections including the photon induced ones. Then we will describe the original minijet model [7] which tries to calculate this dependence in a QCD based picture. After pointing out the problem of 'too fast' an energy rise predicted in these models, we will then show how the resummation of soft gluons can tame the rise [8] and how it is possible to obtain a satisfactory description of the current data with stable predictions at the LHC [9] within the framework. Then we turn to how the model can be extended to describe photon-induced processes in terms of the measured photon structure function and present our predictions for the high energy photon-proton cross-sections obtained in this model [6].

2 The data for pp to $\gamma\gamma$

With the $\bar{p}p/pp$, γp and $\gamma\gamma$ cross-sections in the milibarn, microbarn and nanobarn range[10], it is possible to accommodate them all in the same figure by multiplying the γp and $\gamma\gamma$ cross-sections by 330 and $(330)^2$ respectively. This is an *ad hoc* factor, approximately given by Vector Meson Dominance (VMD) and the quark parton model [6]. Such compilation of all the proton and photon data for the hadronic cross-sections is shown in Figure 1, from [6]. While γp and $\bar{p}p/pp$ data could be interpreted as indicating the same rise, at least at presently reached accelerator energies, the figure seems to indicate that the $\sigma^{\gamma\gamma}$ data from LEP rise somewhat faster than the others. In the left panel of Fig. 1 the band corresponds to the predictions for the total $\bar{p}p/pp$ cross-sections according to the Block-Nordsieck (BN) model which we shall describe in the coming section. The same band is also shown in the right panel, together with predictions of other models [11] for purely proton processes. The figure makes clear that any model has to address three issues: 1) what makes the cross-sections rise, 2) what makes them obey the Froissart bound and 3) whether they indicate any breakdown of factorisation between the proton and photon processes.

3 Minijet models

Minijet models were one of the early QCD based models which made an effort to understand the energy rise in terms of the rising gluon content of the hadrons at small values of x and the basic QCD cross-section [12]. The cross-section for the jet

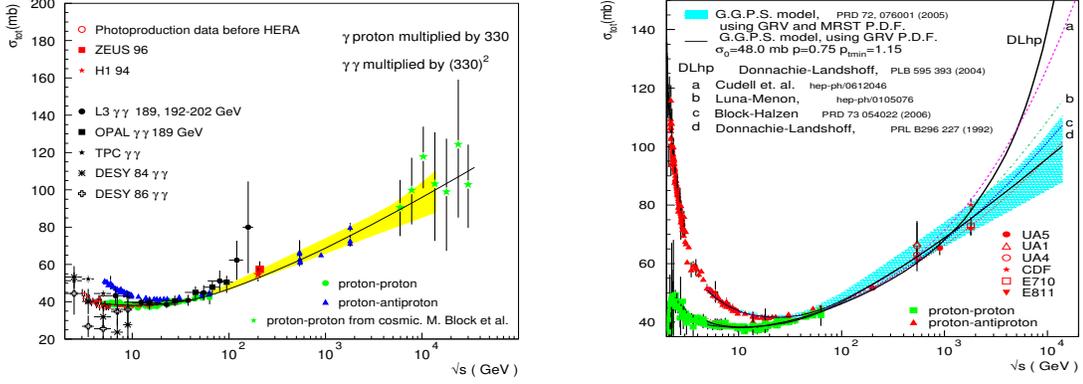


Figure 1: Proton and photon normalised cross-sections [6] with the band of predictions for proton-proton from [9], also shown on the right, together with estimates from various models.

production in collisions of two hadrons A, B in the process:

$$A + B \rightarrow X + jet \quad (1)$$

is obtained by convoluting the parton-parton subprocess cross-section with the given parton densities and integrating over all values of incoming parton momenta and outgoing parton transverse momentum p_t , according to the expression

$$\sigma_{jet}^{AB}(s, p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^1 dx_1 \int_{4p_t^2/(x_1 s)}^1 dx_2 \times \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t} \quad (2)$$

where A and B are the colliding hadrons or photons. By construction, this cross-section depends on the particular parametrization of the parton densities evaluated at scale p_t^2 . This cross-section strongly depends on the lowest p_t value on which one integrates, viz. p_{tmin} . The term *mini-jet* was introduced long ago [13] to indicate all those low p_t processes which are amenable to a perturbative QCD calculation but are actually not observed as hard jets; p_t indicating the scale at which α_s is evaluated in the mini-jet cross-section calculation. One can have $p_{tmin} \approx 1 \div 2 \text{ GeV}$. This minijet cross-section rises very fast with \sqrt{s} , the rate of rise is controlled by the dependence of the parton densities at low- x values and p_{tmin} used. In the simplest version of the model [7] σ^{tot} was assumed to be given by

$$\sigma^{tot} = \sigma^0 + \sigma^{jet}(s, p_{tmin}).$$

Eventhough these early calculations caught the essence of QCD dynamics that can cause the rise of total and inelastic cross-section with energy, they predicted a rise with energy which depended on a parameter p_{tmin} arbitrarily varying with energy. This was made necessary by the known low-x behaviour of the parton densities, which predicted a very fast rise with energy, leading to unitarity violation. The unitarity is achieved by embedding the minijet cross-section in an eikonal picture [14]. In fact many features of not just the cross-section but also quantities such as multiplicity distributions etc. can be successfully described in the eikonalsed minijet picture [15].

In the eikonal formulation, σ^{tot} is obtained from an eikonal function $\chi(b, s)$ which describes the impact parameter distribution during the collision, namely

$$\sigma_{elastic} = \int d^2\mathbf{b} |1 - e^{i\chi(b,s)}|^2 \quad (3)$$

$$\sigma_{total} = 2 \int d^2\mathbf{b} [1 - e^{-\mathcal{I}m\chi(b,s)} \cos\Re\chi(b,s)] \quad (4)$$

$$\sigma_{total\ inelastic} \equiv \sigma_{inel} = \int d^2\mathbf{b} [1 - e^{-2\mathcal{I}m\chi(b,s)}] \quad (5)$$

Neglecting the real part of the eikonal for the hadronic processes, one gets a very simple expression for the total cross-section

$$\sigma_{total} = 2 \int d^2\mathbf{b} [1 - e^{-\bar{n}(b,s)/2}] \quad (6)$$

with $\bar{n}(b, s)$ the average number of inelastic collisions.

Minijet models with the well motivated QCD input, embedded into the eikonal representation, are able to describe the early rise correctly. However, without any further energy dependent input they often fail to obtain the presently exhibited leveling off at high energy, a behaviour already consistent with the Froissart bound [16]. It is clear that one needs additional input: within QCD there is one important effect which can modify the energy dependence, and this is soft gluon emission from the scattering partons. We shall discuss this effect in the next section.

4 Block-Nordsiek Model

The BN model differs from and improves on the usual mini-jet models, including the eikonalsed ones, in three significant ways, namely 1) by implementing perturbative QCD input through currently used PDFs for the mini-jet cross-sections, 2) introducing soft gluon k_t -resummation to control the rise as the basic mechanism which describes the impact parameter distribution of the collision, with the upper scale in soft gluon resummation linked by kinematics to the mini-jet cross-section and finally 3) pushing the soft gluon integral into the InfraRed (IR) region. We have discussed the mini-jet calculation in the previous section. Here we shall illustrate the basic features of resummation and our approach to it.

4.1 Resummation

We shall start by recalling some properties of soft photon resummation. In QED, the general expression for soft photon resummation in the energy-momentum variable K_μ can be obtained order by order in perturbation theory [17, 18, 19] as

$$d^4P(K) = d^4K \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x - h(x,E)} \quad (7)$$

where $d^4P(K)$ is the probability for an overall 4-momentum K_μ escaping detection,

$$h(x, E) = \int d^3\bar{n}(k) [1 - e^{-ik \cdot x}] \quad (8)$$

and

$$d^3\bar{n}(k) = \frac{d^3k}{2k_0} |j_\mu(k, \{p_i\})|^2 \quad (9)$$

The electromagnetic current j_μ depends on the momenta of the emitting matter fields $\{p_i\}$ and to leading order in α_{QED} is given by

$$j_\mu(k, \{p_i\}) = -\frac{ie}{(2\pi)^{3/2}} \sum_i \epsilon_i \frac{p_{i\mu}}{p_i \cdot k} \quad (10)$$

with the sum running on all the entering particles ($\epsilon_i = +1$) and antiparticles (-1). For outgoing fields, the signs are reversed.

The expression in Eq.(7) can also be obtained employing the methods of statistical mechanics [20]. One starts with a discrete variable representation

$$d^4P(K) = \sum_{n_k} P(\{n_k\}) \delta^4(K - \sum_k n_k k) d^4K \quad (11)$$

and sums on all the possible Poisson distributed soft photon configurations, i.e.

$$P(\{n_k\}) = \prod_k \frac{\bar{n}_k^{n_k}}{n_k!} \exp[-\bar{n}_k] \quad (12)$$

where n_k is the number of photons emitted with momentum \mathbf{k} . From Eqs. (11,12) one can determine the probability of observing a 4-momentum loss K accompanying a charged particle reaction by using the integral representation for the 4-dimensional δ -function and exchanging the order between forming the product in $P(\{n_k\})$ and the summation over all the distributions. Then, going from the sum over discrete values to an integral over soft photon momenta, one can obtain Eqs. (7, 8). The derivation being semi-classical, it cannot give any information on $d^3\bar{n}(k)$. For this, one needs to use the perturbative expression for the electromagnetic current as in Eqs. (9,10). Notice that in this derivation, it is energy momentum conservation which ensures the

cancellation of the infrared divergence between soft and real quanta emission. This follows from the semi-classical approach, since in the infrared region the uncertainty principle does not allow to distinguish between real and virtual quanta.

From Eq. (11), the integration over the 3-momentum variable gives

$$dP(\omega) = \frac{\beta}{\gamma^\beta \Gamma(1 + \beta)} \frac{d\omega}{\omega} \left(\frac{\omega}{E}\right)^\beta \quad (13)$$

with γ the Euler's constant, $\beta \equiv \beta(\{p_i\}, m_e)$ defined as the integration over the soft photon angular distribution, i.e. β such that

$$\int_{\Omega_k} d^3\bar{n}(k) = \beta \frac{dk}{k} \quad (14)$$

and E is the maximum energy allowed to soft photon emission. With E being the energy scale of the reaction, the integration of Eq.(13) up to the energy resolution ΔE , gives the well know behaviour $(\Delta E/E)^{\simeq \alpha \log E/m_e}$ proposed in the early days of QED [21].

While it is easy to obtain a closed form for the energy distribution, a closed form for the momentum distribution is not available. It is also not necessary, since the first order expression in α_{QED} is adequate. More interesting, for applications to strong interactions, is the transverse momentum distribution, namely

$$d^2P(\mathbf{K}_\perp) = d^2\mathbf{K}_\perp \frac{1}{(2\pi)^2} \int d^2\mathbf{b} e^{-i\mathbf{K}_\perp \cdot \mathbf{b} - h(b, E)} \quad (15)$$

with

$$h(b, E) = \int d^3\bar{n}(k) [1 - e^{i\mathbf{k}_\perp \cdot \mathbf{b}}] \quad (16)$$

For large transverse momentum values, by neglecting the second term and using a cut-off term as lower limit of integration, the above expression coincides with the Sudakov form factor [22]. In QCD, Eq. (16) is used to discuss low- p_t transverse momentum distributions through the expression [23, 24]

$$h(b, M) = \frac{4C_F}{\pi} \int_{1/b}^M \alpha_s(k_t^2) \frac{dk_t}{k_t} \log \frac{2M}{k_t} \quad (17)$$

where $C_F = 4/3$.

We use resummation to approach the very large impact parameter region, which plays a role in such quantities as the total cross-section. For this, we need to explore the IR region and use the full range of integration of Eq. (16). In the region $k_t < \Lambda$ we propose to use an expression for the soft gluon spectrum which takes into account the effect of a rising confining potential. The expression to use will then be singular as $k_t \rightarrow 0$, but must be integrable. As discussed in many of our publications [5, 8],

we shall interpolate between the asymptotic freedom region (AF) and the IR through the following expression for the strong coupling constant

$$\alpha_s(k_t^2) = \frac{12\pi}{(33 - 2N_f)} \frac{p}{\ln[1 + p(\frac{k_t^2}{\Lambda^2})^p]} \quad (18)$$

which coincides with the usual one-loop formula for values of $k_t \gg \Lambda$, while going to a singular limit for small k_t , and generalizes Richardson's ansatz for the one gluon exchange potential to values of $p \leq 1$ [25]. However, notice that, in order to make the integral finite, we need $p < 1$. Also analyticity requires $p > 1/2$ [8].

Having thus extended the region of interest in Eq. (17) to the large impact parameter values, we use the Fourier transform of Eq. (15) to describe the impact parameter distribution, and input it into the average number of hadronic collisions, $\bar{n}(b, s)$, generated by mini-jets, i.e. we write

$$n_{hard}(b, s) = A_{BN}(b, s)\sigma_{jet} \quad (19)$$

with

$$A_{BN}(b, s) \equiv A_{BN}(b, M) = A_0 e^{-h(b, M)} \quad (20)$$

with the normalization constant A_0

$$A_0 = \frac{1}{2\pi \int b db e^{-h(b, M)}} \quad (21)$$

and with the integral in Eq. (17) extended down to $k_t = 0$, to be used in Eq. (6). The subscript BN indicates that this b -distribution is obtained from soft gluon k_t -resummation into the IR region.

Another important quantity in soft gluon resummation, is the upper limit of integration, the scale E in QED calculation, and which we have indicated with M in the QCD integral. M is in general a function of the incoming and outgoing parton momenta, and can be determined by the kinematics, as discussed in [6] and following [26]. Upon integration over all the PDFs, it is seen to be of the order of p_{tmin} and slowly varying with the c.m. energy.

4.2 BN model for protons

In order to apply our QCD description, inclusive of mini-jets and soft k_t -resummation, to scattering processes, we split the average number of collision $\bar{n}(b, s)$ as

$$\bar{n}(b, s) = n_{NP}(b, s) + n_{hard}(b, s) \quad (22)$$

and parametrize the first term, which is meant to include all those processes which cannot be described through parton-parton scattering discussed in the previous section. At low energies, i.e. \sqrt{s} approximately up to $20 \div 30 GeV$, $\bar{n}(b, s) \approx n_{NP}(b, s)$,

whereas as the c.m. energy increases, the second term will asymptotically dominate. To see how, in the BN model, soft gluon resummation helps to tame the rise of the mini-jet cross-section and bring satisfaction of the Froissart bound, we shall thus look at the expression for σ_{total} at extremely high energies, namely

$$\sigma_T(s) = 2 \int d^2\mathbf{b} [1 - e^{-n_{hard}/2}] \quad (23)$$

In the previous edition of this Conference, we have presented a preliminary version of the argument for satisfaction of the Froissart bound in our model [27]. A more accurate argument has now been given in [8]. Basically, we find that extending the integration in Eq. (17) into the IR region with our IR singular, but integrable, α_s introduces a cut-off in b -space, which behaves at least like an exponential ($1 < 2p < 2$). Such cut-off allows to evaluate the integral of Eq. (23) at the value where the integrand suddenly decreases. Inserting the asymptotic expression for σ_{jet} at high energies, which grows like a power of s , and $A_{BN}(b, s)$ from Eq. (20), in such large- b , large- s limit, we obtain

$$n_{hard} = 2C(s)e^{-(b\bar{\Lambda})^{2p}} \quad (24)$$

where $2C(s) = A_0(s)\sigma_1(s/s_0)^\varepsilon$. The resulting expression for σ_T

$$\sigma_T(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}] \quad (25)$$

leads to

$$\bar{\Lambda}^2 \sigma_T(s) \approx \left(\frac{2\pi}{p}\right) \int_0^{u_0} du u^{\frac{1-p}{p}} = 2\pi u_0^{1/p} \quad (26)$$

with

$$u_0 = \ln\left[\frac{C(s)}{\ln 2}\right] \approx \varepsilon \ln s \quad (27)$$

To leading terms in $\ln s$, we therefore derive the asymptotic energy dependence

$$\sigma_T \rightarrow [\varepsilon \ln(s)]^{(1/p)} \quad (28)$$

Remembering that $1/2 < p < 1$ [1], the above result shows that, with soft gluon momenta integrated into the IR region, $k_t < \Lambda$, and a singular but integrable coupling to the quark current, our model leads to a behaviour consistent with the Froissart-Martin bound [1].

4.3 The BN model for photons

To apply the BN model to photon processes, we follow refs. [28, 29], and estimate the total cross-section as

$$\sigma_{tot}^{\gamma p} = 2P_{had} \int d^2\mathbf{b} [1 - e^{-n^{\gamma p}(b,s)/2}] , \quad n^{\gamma p}(b, s) = n_{NP}^{\gamma p} + A_{BN}^{\gamma p}(b, s) \frac{\sigma_{jet}^{\gamma p}}{P_{had}} \quad (29)$$

with $P_{had} = 1/240 \approx \mathcal{O}(\alpha)$ to represent the probability that a photon behaves like a hadron and with photon PDF's used to calculate the mini-jet cross-sections and the scale energy parameter M . A full discussion of how to evaluate this parameter, called q_{max} when it is averaged over the densities, can be found in [6]. q_{max} depends on the energy of the subprocesses and, being evaluated using the PDFs of the processes under consideration, depends on the specific choice of the parametrisation used for the parton densities in the photon and the proton. As q_{max} increases with energy, the growth of the total cross-section due to mini jets is tempered by soft gluon emission. The calculated values of q_{max} , for all the available parton densities reach some sort of saturation at high energies, which in turn reflects in the total cross-sections reaching a stable slope.

The application of our model to photon total cross-sections shows some interesting features. While in the present accelerator energy range, γp data could be described also through factorization models, at very high energies in the TeV range, predictions differ. We show this in Fig. (2). The left panel plots the data up to the highest accel-

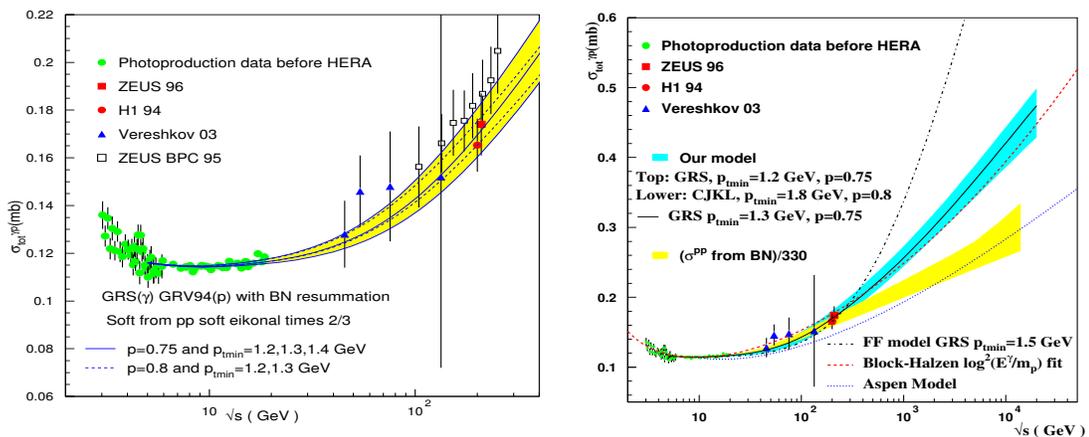


Figure 2: γp total cross section from [6] in different energy ranges.

erator energies. Data in this region come from cosmic rays [30], from extrapolation of virtual photon data taken with the BPC [31] and from H1 and Zeus experiments [32]. In this energy range data can be accommodated by many models, including factorization models, in which models for proton cross-sections are extended to photons, by just multiplying the proton curve by a constant factor [33] or by extrapolating eikonal models with scaling factors in the impact parameter description of photons [34] or by assuming for photons the same rise with energy of proton cross-section [35]. However, as one can see from the right panel, the situation changes as the c.m. energy of the γp system reaches into the TeV range. Here the discrepancy with factorization models,

exemplified by the lower band, is clearly indicated both by the upper band obtained through our BN model, as well as by the dashed curve within it. This curve was independently obtained in [16] from a fit to accelerator data and confirms the validity of our model into an energy range so far inaccessible through particle accelerators.

5 Conclusions

We have discussed the results from a mini-jet model which incorporates soft gluon k_t -resummation as a taming effect on the rapid rise with energy of low- x initiated mini-jet cross-sections. This has been applied to both $pp/\bar{p}p$ and γp processes. We find that soft k_t -resummation, inclusive of IR gluons with $k_t < \Lambda$, plays a crucial role in transforming the power like rise of the jet cross-sections into a more subdued logarithmic behaviour. We accomplish this through the use of a phenomenological ansatz for the coupling between soft gluons and the quark current which gives an expression singular but integrable.

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References

- [1] M. Froissart, Phys. Rev. **123** (1961) 1053, A. Martin, Phys. Rev. **129** (1963) 1432, A. Martin, Nuovo Cim. A **42** (1965) 930,
- [2] C. Amsler *et al.*, The Review of Particle Physics, Phys. Lett.**B667** (2008) 1.
- [3] For a recent review, see M. Block, Phys.Rept.**436** (2006) 71-215. e-Print: hep-ph/0606215, and references therein;
- [4] Richard C. Brower, Marko Djuric, Chung-I Tan, published in the Proceedings of the 38th International Symposium on Multiparticle Dynamics (ISMD08). J. Bartels et al. DESY-PROC-2009-01. e-Print: arXiv:0812.1299 [hep-ph]
- [5] R.M. Godbole, A. Grau, , G. Pancheri, Y.N. Srivastava, *Phys.Rev.***D72** 076001 (2005) e-Print: hep-ph/0408355; A. Grau, G. Pancheri and Y. N. Srivastava, Phys. Rev. D **60** (1999) 114020 [arXiv:hep-ph/9905228]; A. Corsetti, A. Grau,

- G. Pancheri and Y. N. Srivastava, Phys. Lett. B **382** (1996) 282 [arXiv:hep-ph/9605314].
- [6] R.M. Godbole, A. Grau, , G. Pancheri, Y.N. Srivastava, Eur. Phys. J. C **63** (2009) 69, e-Print: arXiv:0812.1065 [hep-ph].
- [7] D. Cline, F. Halzen and J. Luthe, Phys. Rev. Lett. **31** (1973) 491; T. K. Gaisser, F. Halzen, Phys. Rev. Lett. **54** (1985) 1754; G. Pancheri, Y. Srivastava, Phys.Lett. **B159** (1985) 69.
- [8] A. Grau, R.M. Godbole, G. Pancheri and Y.N. Srivastava, *Soft Gluon k_t -Resummation and the Froissart bound*, e-Print: arXiv:0908.1426 [hep-ph].
- [9] A. Achilli, R. M. Godbole, A. Grau, R. Hegde, G. Pancheri and Y. Srivastava, Phys. Lett. B **659** (2008) 137 [arXiv:0708.3626 (hep-ph)].
- [10] Our Refs. [6, 9] contains the references to all the data plotted in Figure 1.
- [11] M. M. Block and F. Halzen, Phys. Rev. **D73** (2006) 054022 [arXiv:hep-ph/0510238]; E. G. S. Luna and M. J. Menon, arXiv:hep-ph/0105076; J. R. Cudell and O. V. Selyugin, arXiv:hep-ph/0612046; A. Donnachie and P. V. Landshoff, Phys. Lett. **B595** (2004) 393 [arXiv:hep-ph/0402081]
- [12] L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys.Rept. **100** (1983)1;
- [13] R. Horgan and M. Jacob, Nucl. Phys. **B179** (1981) 441; G. Pancheri and C. Rubbia, Nucl. Phys. **A** (1984)
- [14] A. Capella, J. Kaplan, A. Krzywicki, D. Schiff, Nuovo Cim. **A63** (1969) 141. L. Durand, H. Pi, Phys.Rev.**D40** (1989) 1436.
- [15] T. Sjöstrand and M. van Zijl, Phys. Rev. **D36**, 2019 (1987).
- [16] M. Block and F. Halzen, Phys.Rev. **D70** (2004) 091901. e-Print: hep-ph/0405174; ibidem Phys. Rev. **D72** (2005) 036006, Erratum-ibid.**D72** (2005)039902. e-Print: hep-ph/0506031.
- [17] J.M. Jauch and F. Rohrlich, Hel. Phys. Acta **27** (1954) 613; *The Theory of Photon and Electrons*, Cambridge 1955.
- [18] E. L. Lomon, Nucl Phys. **1** (1956) 101; Phys. Rev. **113** (1959) 726.
- [19] D.R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. **13** (1961) 379.
- [20] E. Etim, G. Pancheri and B. Touschek, Nuovo Cimento **51B** (1967) 276.

- [21] J. Schwinger, Phys.Rev. **75** (1949) 898-899.
- [22] V.V. Sudakov Soviet Physics JETP **3** (1956) 65.
- [23] Y. I. Dokshitzer, D.I. Dyakonov and S.I. Troyan, Phys. Lett. **79B** (1978) 269.
- [24] G. Parisi and R. Petronzio, Nucl.Phys. **B154** (1979) 427.
- [25] J. L. Richardson, Phys. Lett. **B82** (1979) 272.
- [26] M. Greco and P. Chiappetta, Nucl. Phys. **B221** (1983) 269.
- [27] A. Achilli, R. Godbole, A. Grau, A G. Pancheri and Y.N. Srivastava, SLAC eConf C0706044, Eds. B. Mueller, M. A. Rotondo and C-I Tan.
- [28] Raj Gandhi, Ina Sarcevic, Phys. Rev. **D44** (1991) 10-14.
- [29] R.S. Fletcher, T.K. Gaisser and F. Halzen, Phys. Lett. **B298** (1993) 442; Phys. Rev. **D45** (1992) 377-381, Erratum-ibid.**D45** (1992) 3279.
- [30] G.M. Vereshkov, O.D. Lalakulich, Yu.F. Novoseltsev, R.V. Novoseltseva, Phys.Atom.Nucl.**66** (2003) 565-574, Yad.Fiz.**66** (2003) 591-600.
- [31] ZEUS Collaboration, J. Breitweg et al., EPJC **7** (1999) 609, hep-ex/9809005v1; D. Haidt , *The transition from $\sigma(\gamma^*p)$ to $\sigma(\gamma p)$, Bologna 2001, Deep inelastic scattering* 287-290, and refs. therein; ZEUS Collaboration, J. Breitweg et al., Phys. Lett. **B407** (1997) 432, hep-ex/9707025v3; ZEUS Collaboration, J. Breitweg et al., Phys. Lett. **B487** (2000) 53, hep-ex/0005018v2; B. Surov, DESY-THESIS-1998-004; A. Bornheim, in the *Proceedings of the LISHEP International School on High Energy Physics, Brazil, 1998*, hep-ph/9806021.
- [32] H1 Collaboration, S. Aid et al., Zeit. Phys. **C69** (1995) 27, hep-ex/9509001; ZEUS collaboration, S. Chekanov et al., Nucl. Phys. **B627** (2002) 3, hep-ex/0202034.
- [33] C. Bourrely, J. Soffer and T.T. Wu, Mod. Phys. Lett. **A15** (2000), 9-13. [arXiv: hep-ph/9903438]
- [34] M. Block, E. Gregores, F. Halzen and G. Pancheri, Phys. Rev. **D60** (1999) 054024, e-Print: hep-ph/9809403.
- [35] A. Donnachie and P. V. Landshoff, Phys. Lett. B **296** (1992) 227 [arXiv:hep-ph/9209205]; A. Donnachie and P. V. Landshoff, Phys. Lett. B **595**(2004) 393 [arXiv:hep-ph/0402081].