

# Scattering amplitudes and AdS/CFT

*Cristian Vergu*  
*Brown University*  
*BOX 1843*  
*Providence, RI 02912*

## 1 Introduction and notation

Scattering amplitudes in the maximally supersymmetric  $\mathcal{N} = 4$  gauge theory are known to possess some remarkable features. The reasons for this are not completely understood, but some possible candidates are: the integrability properties of the theory, the existence of a gravitational dual and the existence of a twistor space description.

I will not try to give a detailed description of the  $\mathcal{N} = 4$  supersymmetric gauge theory, but only mention some of its important features. The particle spectrum of the theory can be described by a gauge field, four Weyl fermions and six real scalar fields, all transforming in the adjoint representation of a gauge group which I take to be  $SU(N_c)$ . The theory has a  $SU(4)_R$  symmetry group under which the four Weyl fermions transform in a **4** representation and the six scalars transform in a **6** representation.

In fact, this theory has an even larger symmetry, described by the supergroup  $PSU(2, 2|4)$ , which contains the conformal group  $SU(2, 2)$  as a subgroup. However, the implications of this symmetry for the scattering amplitudes are not easy to work out. In part, this is due to the fact the asymptotic states used in the scattering theory do not transform in irreducible representations of the global symmetry supergroup  $PSU(2, 2|4)$ .

There is a natural action of the (super-)conformal group on the transform of the amplitudes to twistor space. At this time, however, the transform to twistor space is best understood for tree-level amplitudes.

In the following, I will restrict the discussion to planar, on-shell scattering amplitudes.

The data needed to describe the external states is given by the on-shell momenta, the helicities of the particles, the color indices and by the transformations under the  $R$ -symmetry group  $SU(4)$ . This data can be neatly encoded in a super-wavefunction  $\Psi(p, \eta)$ , where  $p$  is the momentum and  $\eta^I$ , with  $I = 1, \dots, 4$  are four Grassmann

variables. The expansion of the super-wavefunction in powers of  $\eta$  yields

$$\begin{aligned} \Psi(p, \eta) = & g_+(p) + \psi_I(p)\eta^I + \frac{1}{2}\phi_{IJ}(p)\eta^I\eta^J + \\ & + \frac{1}{3!}\epsilon_{IJKL}\psi^I(p)\eta^J\eta^K\eta^L + \frac{1}{4!}g_-(p)\epsilon_{IJKL}\eta^I\eta^J\eta^K\eta^L, \end{aligned}$$

where  $g_+(p)$  is the wavefunction for helicity plus gluon,  $\psi_I(p)$  are the four fermion states with helicity  $\frac{1}{2}$ ,  $\phi_{IJ}(p)$  are the six scalars,  $\psi^I(p)$  are the four helicity  $-\frac{1}{2}$  fermions and finally  $g_-(p)$  is the helicity minus gluon. If we attribute to  $\eta$  a helicity of  $\frac{1}{2}$ , then the super-wavefunction  $\Psi(p, \eta)$  carries a helicity of  $+1$ .

By using these super-wavefunctions, one can compute superamplitudes, which contain information about the scattering of all the particles in the  $\mathcal{N} = 4$  supermultiplet. Before describing the scattering amplitudes themselves, let me take a detour to describe the color structure.

By inspection of the Feynman rules of a gauge theory, it is obvious that they can be written as a product of a color (or group theory) factor and a factor depending on the kinematics. The same is true for a full Feynman diagram; a diagram can be written as a product of a color factor and a factor depending on the kinematics. In explicit computations, it has proven fruitful to group the diagrams with the same color factors together. One reason for doing so is that gauge invariance leads to cancellations between different Feynman diagrams, but these cancellations can only happen between diagrams with the same color structure.

At leading order in the limit of large number of colors, the basis of color factors is very simple. It is given by single traces of products of generators of the gauge algebra, ordered in all possible ways. Because of the cyclicity of the trace, for an  $n$ -point amplitude there are  $(n-1)!$  such single-trace color factors. The scattering amplitudes can then be decomposed on the basis of these color factors as follows

$$\mathcal{A}_n = g^{n-2} \sum_{\sigma \in \mathcal{S}_n/\mathbb{Z}_n} \text{tr}(T^{\sigma(1)} \dots T^{\sigma(n)}) A_n(\sigma(1), \dots, \sigma(n)), \quad (1)$$

where  $T^a$  are the generators of the  $SU(N_c)$  gauge group and the sum runs over the cyclically unrelated permutations of the external lines. The  $su(N_c)$  algebra generators are normalized by  $\text{tr}(T^a T^b) = \delta^{ab}$  (which is different from the usual normalization  $\text{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$ ).

Let me emphasize that this color decomposition holds to all loops, but only to leading order in the number of colors. The subleading contributions in the number of colors contain multiple-trace contributions. In 't Hooft's double line notation, these correspond to non-planar diagrams, while the single-trace contributions correspond to planar diagrams.

The color-ordered amplitude  $A_n(1, \dots, n)$  is symmetric under cyclic permutations of the labels 1 to  $n$ . In the following I will only discuss color-ordered amplitudes.

The color-ordered amplitudes are simplest when written in spinor form. Each momentum  $p^\mu$  can be put into correspondence with a rank two tensor by using the Pauli matrices  $(\sigma^\mu)_{\alpha\dot{\alpha}} \equiv (\mathbf{1}, \vec{\sigma})_{\alpha\dot{\alpha}}$  and  $(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \equiv (\mathbf{1}, -\vec{\sigma})^{\dot{\alpha}\alpha}$

$$p_{\alpha\dot{\alpha}} = p_\mu (\sigma^\mu)_{\alpha\dot{\alpha}}, \quad p^\mu = \frac{1}{2} p_{\alpha\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}, \quad (2)$$

where I have used the identity  $(\sigma^\mu)_{\alpha\dot{\alpha}} (\bar{\sigma}^\nu)^{\dot{\alpha}\alpha} = 2\eta^{\mu\nu}$ , with the metric signature being  $(+, -, -, -)$ . The dotted and undotted indices transform differently; the undotted indices transform in the representation  $\mathbf{2}$  of  $SL(2, \mathbb{C})$ , while the dotted indices transform in the complex conjugate representation  $\bar{\mathbf{2}}$  of the same group.

A massless vector  $p_\mu$  can be written as

$$p_\mu (\sigma^\mu)_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}. \quad (3)$$

Therefore, to a massless vector we can associate two spinors  $\lambda$  and  $\tilde{\lambda}$ . On this space of spinors there is an  $SL(2, \mathbb{C})$  invariant bilinear form defined by

$$\langle ab \rangle = (\lambda_a)^\beta (\lambda_b)_\beta = -\epsilon^{\alpha\beta} (\lambda_a)_\alpha (\lambda_b)_\beta, \quad [ab] = (\lambda_a)_{\dot{\alpha}} (\lambda_b)^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} (\lambda_a)_{\dot{\alpha}} (\lambda_b)_{\dot{\beta}}, \quad (4)$$

where  $\epsilon$  is antisymmetric with  $\epsilon^{12} = \epsilon^{\dot{1}\dot{2}} = -1$ . With these conventions the dot product of two null vectors  $p$  and  $q$  is given by  $2p \cdot q = \langle pq \rangle [qp]$ .

Now, instead of writing the scattering amplitudes in terms of the external momenta  $p_i$  and polarization vectors  $\varepsilon_i(p_i)$ , we'll use the spinors  $\lambda_i, \tilde{\lambda}_i$  and the helicity  $h_i$ .

Among the scattering amplitudes the MHV amplitudes play a special role. These are the  $n$ -point gluon scattering amplitudes with two gluons of helicity minus and  $n-2$  gluons of helicity plus. In supersymmetric language, the tree-level MHV amplitudes are represented by the following degree eight, supersymmetry-invariant quantity (see ref. [4])

$$A_n^{\text{MHV},0} = i(2\pi)^4 \frac{\delta^8(\sum_{i=1}^n \lambda_i^\alpha \eta_i^I)}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}. \quad (5)$$

These MHV amplitudes are the simplest non-vanishing amplitudes. In the following I will describe them at loop level. The full MHV amplitude is proportional to the tree-level amplitude

$$A_n^{\text{MHV}} = A_n^{\text{MHV},0} M_n, \quad (6)$$

where  $M_n$  is a scalar depending only on the kinematics and not on the helicities or type of scattered particles.

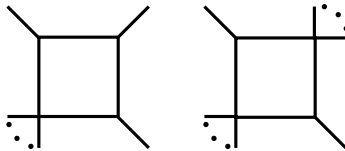


Figure 1: The one-loop integrals contributing to the MHV amplitude. The one on the left is called “one-mass box” and the one on the right is called “two-mass easy box.”

## 2 The unitarity method

Our computational tool will be the unitarity method (see refs. [5, 6, 7]). The unitarity method builds loop-level scattering amplitudes by using *on-shell* tree-level scattering amplitudes. It is profitable to use this method because the on-shell scattering amplitudes have significantly simpler forms than the off-shell scattering amplitudes.

The unitarity method aims to build the scattering amplitude at loop level from the knowledge of its singularities. When continued to the complex plane scattering amplitudes at loop level have complicated branch cut singularities in the kinematic invariants. These branch cuts have a physical interpretation in terms of exchange of real on-shell particles, and the discontinuities across them can be computed by integrating the product of the scattering amplitudes on both sides of the cut over the Lorentz invariant phase space of the exchanged particles.

Then, one aims to find a function which possesses the same discontinuities across the branch cuts. This function then gives the scattering amplitude, up to an additive ambiguity of an analytic function. I will describe later how to fix this remaining ambiguity.

To one loop, the search for a function with the right branch cuts is simplified by the knowledge of a basis of integrals. That is, every one-loop Feynman diagram can be reduced to a linear combination of box, triangle, bubble and tadpole integrals. Therefore, the problem of identifying the function possessing the right branch cut discontinuities reduces to the problem of finding the coefficients with which these integrals appear in the result. In practice this is done by computing the cuts in different channels and matching these cuts to an ansatz containing a linear combination of the integrals in the basis. Each cut computed yields a linear equation for these coefficients and by computing enough cuts one can fix all the coefficients.

The one loop results for  $\mathcal{N} = 4$  supersymmetric gauge theory are particularly simple. The answer is expressible as a linear combination of box integrals, with the external momenta distributed in different ways at the four corners of the box. For MHV, the integrals appearing are called “one-mass” and “two-mass easy” (see fig. 2).

At two loops the basis of integrals is not known, but one can try to put the product of tree amplitudes on both sides of the cut in a form resembling cuts of two-

loop integrals. Also, at higher loops generalized unitarity proves to be very useful. Generalized unitarity is a variant of the unitarity method where one computes the cut in several channels simultaneously.

Even though the basis of integrals is not known beyond one loop, it has been discovered (see ref. [8]) by examining the results of explicit computations (see refs. [9, 10]) that a class of integrals, called pseudo-conformal integrals, plays a special role. These integrals are pseudo-conformal in the following sense. One has to define a dual space with coordinates  $x_i$  such that the external momenta  $k_i$  are expressed in terms of dual coordinates by  $k_i = x_{i+1} - x_i$ . It is obvious that this parameterization solves the momentum conservation constraint. Then, to each loop of an integral one should associate another dual coordinate, such that the other momentum conservation constraints are satisfied.

The integrals are then invariant under inversion of the dual coordinates  $x^\mu \rightarrow \frac{x^\mu}{x^2}$ . Under this transformation, the kinematic invariants and the integration measure transform as

$$x_{ij}^2 \rightarrow \frac{x_{ij}^2}{x_i^2 x_j^2}, \quad d^4 x_p \rightarrow \frac{d^4 x_p}{(x_p^2)^4}, \quad (7)$$

where  $x_{ij} = x_i - x_j$  and  $x_p$  is a dual variable corresponding to a loop. The pseudo-conformal integrals are those which are invariant under inversion in this dual space. The reason for the name ‘‘pseudo’’ is that these integrals are invariant when taken in four dimensions. However, in explicit calculations one has to regularize the infrared divergences and for this a variant of dimensional regularization is used. This regularization explicitly breaks the invariance under inversion in dual space.

### 3 Scattering amplitudes and Wilson loops

Motivated by the ABDK iteration relations (see ref. [11]) and on an explicit four-point three-loop computation, Bern, Dixon and Smirnov proposed an all-order form for the MHV scattering amplitudes (see ref. [10]).

At four points, this proposal, called the BDS ansatz in the literature, has been confirmed at strong coupling in a paper [12] by Alday and Maldacena. Ref. [12] also established a connection with a Wilson loop whose sides are built from the light-like momenta of the scattered particles. However, this connection was initially restricted to strong 't Hooft coupling.

Soon after this, some weak-coupling computations (see refs. [14, 15]) confirmed that the connection between Wilson loops and scattering amplitudes survives at weak coupling as well. It also became clear that the connection is between MHV amplitudes and Wilson loops.

At weak coupling the Wilson loops live in the dual space defined above for the conformal integrals. In fact, one can see that the vertices of the polygonal contour

defining the Wilson loop have coordinates  $x_i$  such that the differences  $x_{i+1} - x_i = k_i$  are the null momenta of the scattered particles. This means that the conformal transformations in the dual space are just the usual conformal transformations for the Wilson loop.

However, the Wilson loops with cusps and light-like lines are UV-divergent, so the conformal transformations become anomalous. Ref. [16] derived a constraint on the anomaly. This constraint takes the form of a differential equation acting on the logarithm of the Wilson loop. Assuming the logarithm of the quantity  $M_n$  defined in eq. (6) satisfies the same anomaly equation as the logarithm of the Wilson loop, ref. [16] showed that the BDS ansatz is the unique solution at four and five points but starting at six points the most general solution is given by the BDS ansatz plus a function of the three conformal cross-ratios

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2}, \quad u_2 = \frac{x_{24}^2 x_{51}^2}{x_{25}^2 x_{41}^2}, \quad u_3 = \frac{u x_{35}^2 x_{62}^2}{x_{36}^2 x_{52}^2}. \quad (8)$$

Alday and Maldacena (see ref. [13]) also showed that, in the limit of a large number of particles, the BDS ansatz needs to be modified.

## 4 The six-point two-loop computation

Motivated by this, we computed (see ref. [17]) the two-loop six-point MHV scattering amplitude, and compared it to the BDS ansatz and to the Wilson loop computations.

The computation was done by using the unitarity method described in sec. 2. I will not detail the computation, but just give the final result. Starting at five points, the quantity  $M_n$  can be split in an even and an odd part. The (even) odd part is (even) odd under parity. We only computed the even part; the odd part was computed in ref. [18] and was shown to cancel in the logarithm of  $M_n$ .

The integrals contributing to the amplitude are shown in fig. 2. All the integrals in the even part of the amplitude are conformal, except the last two. These last two integrals can not be detected by four-dimensional cuts. In order to detect these integrals, one has to compute the cuts in an arbitrary dimension  $D$ . This is done by taking advantage of the fact that  $\mathcal{N} = 4$  supersymmetric gauge theory is the dimensional reduction of  $\mathcal{N} = 1$  supersymmetric gauge theory in ten dimensions.

However, it also turns out that the integrals which can't be detected by four-dimensional cuts cancel in the logarithm of  $M_n$ .

The integrals listed in fig. 2 are very hard to compute analytically. Their expansion in powers of  $\epsilon = \frac{4-D}{2}$ , the dimensional regularization parameter, starts with  $\frac{1}{\epsilon^4}$  and in ref. [17] the expansion is performed analytically through order  $\frac{1}{\epsilon^3}$ .

Because of the difficulty in evaluating the scattering amplitude analytically, we

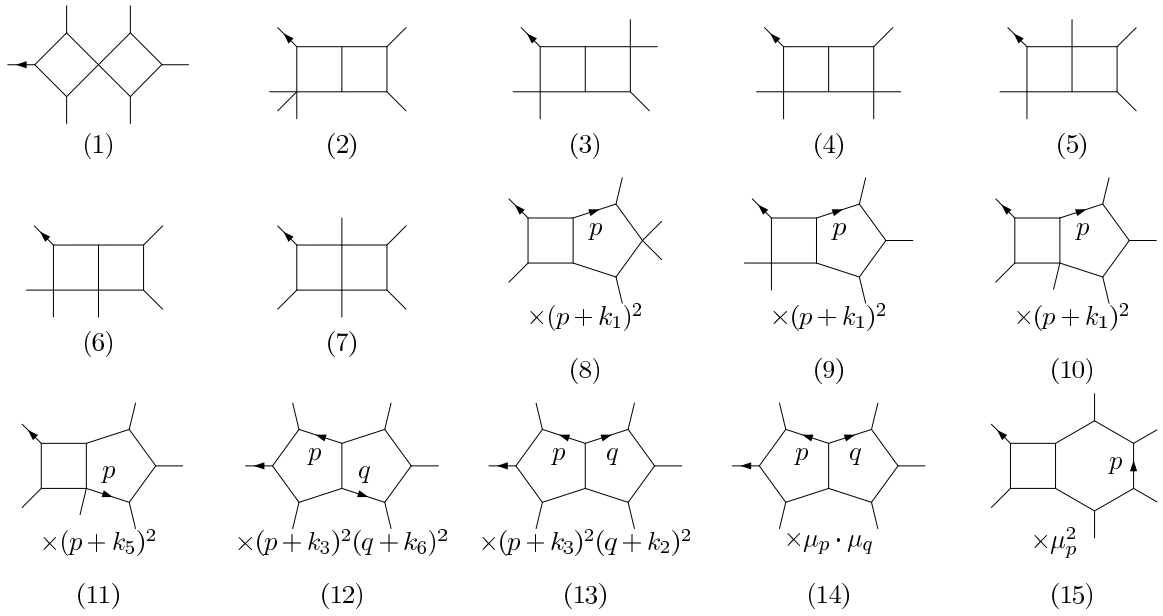


Figure 2: The integrals contributing to the even part of the six-point two-loop MHV amplitude. The labeling of the external starts with leg 1 marked by an arrow and proceeds clockwise. Under each integral is written a numerator factor which is necessary to make the integral conformal. This figure is taken from ref. [17].

Table 1: The numerical remainder compared with the ABDK/BDS ansatz for the kinematic points in eq. (9). The second column gives the conformal cross ratios defined in eq. (8). This table is taken from ref. [17].

kinematic point	$(u_1, u_2, u_3)$	$R_A$
$K^{(0)}$	$(1/4, 1/4, 1/4)$	$1.0937 \pm 0.0057$
$K^{(1)}$	$(1/4, 1/4, 1/4)$	$1.076 \pm 0.022$
$K^{(2)}$	$(0.547253, 0.203822, 0.881270)$	$-1.659 \pm 0.014$
$K^{(3)}$	$(28/17, 16/5, 112/85)$	$-3.6508 \pm 0.0032$
$K^{(4)}$	$(1/9, 1/9, 1/9)$	$5.21 \pm 0.10$
$K^{(5)}$	$(4/81, 4/81, 4/81)$	$11.09 \pm 0.50$

had to resort to numerical integration at several kinematic points, listed below

$$K^{(0)} : s_{i,i+1} = -1, \quad s_{i,i+1,i+2} = -2, \quad (9a)$$

$$K^{(1)} : s_{12} = -0.7236200, \quad s_{23} = -0.9213500, \quad s_{34} = -0.2723200, \quad (9b)$$

$$s_{45} = -0.3582300, \quad s_{56} = -0.4235500, \quad s_{61} = -0.3218573, \\ s_{123} = -2.1486192, \quad s_{234} = -0.7264904, \quad s_{345} = -0.4825841,$$

$$K^{(2)} : s_{12} = -0.3223100, \quad s_{23} = -0.2323220, \quad s_{34} = -0.5238300, \quad (9c)$$

$$s_{45} = -0.8237640, \quad s_{56} = -0.5323200, \quad s_{61} = -0.9237600, \\ s_{123} = -0.7322000, \quad s_{234} = -0.8286700, \quad s_{345} = -0.6626116,$$

$$K^{(3)} : s_{i,i+1} = -1, \quad s_{123} = -1/2, \quad s_{234} = -5/8, \quad s_{345} = -17/14, \quad (9d)$$

$$K^{(4)} : s_{i,i+1} = -1, \quad s_{i,i+1,i+2} = -3, \quad (9e)$$

$$K^{(5)} : s_{i,i+1} = -1, \quad s_{i,i+1,i+2} = -9/2. \quad (9f)$$

In table 1 we present the results for the remainder function  $R_A$ , defined to be the difference between the computed result and the BDS ansatz.

The kinematical points  $K^{(0)}$  and  $K^{(1)}$  have the same conformal cross-ratios and the corresponding values of the remainder function  $R_A$  are equal, within the numerical uncertainties. This provides some support for the conjecture that the remainder function only depends on conformal cross-ratios.

For the Wilson loop, one can use the analog of the BDS ansatz and define a remainder function  $R_W$ . The fact that  $R_W$  is non-zero was discovered in ref. [19]. The two remainder functions  $R_A$  and  $R_W$  differ by an inessential constant. This constant can be determined by taking a collinear limit in the Wilson loop result, but it turns out to be better from the point of view of numerical errors to compare the differences of  $R_A$  and  $R_W$  at two different kinematic points. The results are presented in table 2. The agreement between the third and fourth column provides strong



Table 2: The comparison between the remainder functions  $R_A$  and  $R_W$  for the MHV amplitude and the Wilson loop. To account for various inessential constants, we subtract from the remainders their values at the standard kinematic point  $K^{(0)}$ , denoted by  $R_A^0$  and  $R_W^0$ . The third column contains the difference of remainders for the amplitude, while the fourth column has the corresponding difference for the Wilson loop. The numerical agreement between the third and fourth columns provides strong evidence that the finite remainder for the Wilson loop is identical to that for the MHV amplitude. This table is taken from ref. [17].

kinematic point	$(u_1, u_2, u_3)$	$R_A - R_A^0$	$R_W - R_W^0$
$K^{(1)}$	$(1/4, 1/4, 1/4)$	$-0.018 \pm 0.023$	$< 10^{-5}$
$K^{(2)}$	$(0.547253, 0.203822, 0.881270)$	$-2.753 \pm 0.015$	$-2.7553$
$K^{(3)}$	$(28/17, 16/5, 112/85)$	$-4.7445 \pm 0.0075$	$-4.7446$
$K^{(4)}$	$(1/9, 1/9, 1/9)$	$4.12 \pm 0.10$	$4.0914$
$K^{(5)}$	$(4/81, 4/81, 4/81)$	$10.00 \pm 0.50$	$9.7255$

numerical evidence for the equality of the finite parts of the scattering amplitudes and Wilson loops.

## 5 Conclusion and further developments

In conclusion, the BDS ansatz was shown to break down at six points, but the MHV amplitudes—Wilson loops was shown to hold. It is important to further test this correspondence. One way to do so would be to find the remainder function analytically for the scattering amplitude and for the Wilson loop and show that they agree.

Recently, the two-loop Wilson loop results have become available for an arbitrary number of points (see ref. [21]). On the scattering amplitudes side, the contributing integrals have been identified and their coefficients have been computed (see refs. [22, 23]), but no numerical results are available beyond two-loop six-point.

Another big unknown is how to modify the BDS ansatz to make it hold at six points and beyond. Of course, before attempting to do this, a more detailed understanding of the remainder function needs to be obtained.

At strong coupling, the extension of the correspondence between scattering amplitudes and Wilson loops beyond the leading order in  $\sqrt{\lambda}$  was shown by using a new duality called fermionic  $T$ -duality (see ref. [24]).

Recently some new scattering amplitudes were computed at strong coupling in refs. [25, 26]. It would be desirable to extend these computations beyond what was done so far.

Also, at strong coupling the helicity dependence and the reason for the special role played by the MHV amplitudes remain obscure.

There is already a proposal about the structure of the next-to-MHV amplitudes (i.e. amplitudes with three helicity minus gluons). This proposal was put forward in ref. [27], where the dual conformal symmetry mentioned above was extended to a dual conformal supersymmetry.

I would like to thank Zvi Bern, Lance Dixon, David Kosower, Radu Roiban, Marcus Spradlin and Anastasia Volovich for collaboration on the topics presented here. I would also like to thank the organizers of the Tenth Workshop on QCD for the invitation to speak.

The author is supported in part by the US Department of Energy under contract DE-FG02-91ER40688 and by the US National Science Foundation under grant PHY-0643150.

## References

- [1] A. Ferber, Nucl. Phys. B **132**, 55 (1978).
- [2] E. Witten, Commun. Math. Phys. **252**, 189 (2004) [arXiv:hep-th/0312171].
- [3] S. J. Parke and T. R. Taylor, Phys. Rev. Lett. **56**, 2459 (1986).
- [4] V. P. Nair, Phys. Lett. B **214**, 215 (1988).
- [5] Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B **425**, 217 (1994) [arXiv:hep-ph/9403226].
- [6] Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B **435**, 59 (1995) [arXiv:hep-ph/9409265].
- [7] Z. Bern, L. J. Dixon and D. A. Kosower, Nucl. Phys. B **513**, 3 (1998) [arXiv:hep-ph/9708239].
- [8] J. M. Drummond, J. Henn, V. A. Smirnov and E. Sokatchev, JHEP **0701**, 064 (2007) [arXiv:hep-th/0607160].
- [9] Z. Bern, J. S. Rozowsky and B. Yan, Phys. Lett. B **401**, 273 (1997) [arXiv:hep-ph/9702424].
- [10] Z. Bern, L. J. Dixon and V. A. Smirnov, Phys. Rev. D **72**, 085001 (2005) [arXiv:hep-th/0505205].
- [11] C. Anastasiou, Z. Bern, L. J. Dixon and D. A. Kosower, Phys. Rev. Lett. **91**, 251602 (2003) [arXiv:hep-th/0309040].

- [12] L. F. Alday and J. M. Maldacena, JHEP **0706**, 064 (2007) [arXiv:0705.0303 [hep-th]].
- [13] L. F. Alday and J. Maldacena, JHEP **0711**, 068 (2007) [arXiv:0710.1060 [hep-th]].
- [14] J. M. Drummond, G. P. Korchemsky and E. Sokatchev, Nucl. Phys. B **795**, 385 (2008) [arXiv:0707.0243 [hep-th]].
- [15] A. Brandhuber, P. Heslop and G. Travaglini, Nucl. Phys. B **794**, 231 (2008) [arXiv:0707.1153 [hep-th]].
- [16] J. M. Drummond, J. Henn, G. P. Korchemsky and E. Sokatchev, arXiv:0712.1223 [hep-th].
- [17] Z. Bern, L. J. Dixon, D. A. Kosower, R. Roiban, M. Spradlin, C. Vergu and A. Volovich, Phys. Rev. D **78**, 045007 (2008) [arXiv:0803.1465 [hep-th]].
- [18] F. Cachazo, M. Spradlin and A. Volovich, Phys. Rev. D **78**, 105022 (2008) [arXiv:0805.4832 [hep-th]].
- [19] J. M. Drummond, J. Henn, G. P. Korchemsky and E. Sokatchev, Phys. Lett. B **662**, 456 (2008) [arXiv:0712.4138 [hep-th]].
- [20] J. M. Drummond, J. Henn, G. P. Korchemsky and E. Sokatchev, Nucl. Phys. B **815**, 142 (2009) [arXiv:0803.1466 [hep-th]].
- [21] C. Anastasiou, A. Brandhuber, P. Heslop, V. V. Khoze, B. Spence and G. Travaglini, JHEP **0905**, 115 (2009) [arXiv:0902.2245 [hep-th]].
- [22] C. Vergu, Phys. Rev. D **79**, 125005 (2009) [arXiv:0903.3526 [hep-th]].
- [23] C. Vergu, arXiv:0908.2394 [hep-th].
- [24] N. Berkovits and J. Maldacena, JHEP **0809**, 062 (2008) [arXiv:0807.3196 [hep-th]].
- [25] L. F. Alday and J. Maldacena, arXiv:0903.4707 [hep-th].
- [26] L. F. Alday and J. Maldacena, arXiv:0904.0663 [hep-th].
- [27] J. M. Drummond, J. Henn, G. P. Korchemsky and E. Sokatchev, arXiv:0807.1095 [hep-th].