Minimal Walking Technicolor: Phenomenology and Lattice Studies

Kimmo Tuominen
Department of Physics, University of Jyväskylä, Finland
Helsinki Institute of Physics, University of Helsinki, Finland

1 Introduction and a concrete model

During recent years there has been lot of theoretical effort dedicated to understand the dynamics of (quasi)conformal gauge theories, i.e. theories in which the evolution of the coupling towards the infrared is governed by a (quasi)stable fixed point. Along these lines, important recent progress towards the understanding of the phase diagram of strongly coupled gauge theories as a function of the number of flavors, colors and matter representations was made in [1, 2]. These results, then, allowed one to uncover new walking technicolor theories where the walking behavior arises with minimal matter content and also led to concrete models of the unparticle concept [3] in terms of the underlying strong dynamics [4].

Particularly simple candidates for walking technicolor were identified in [1] as the ones with $N_f = 2$ either in the adjoint representation of SU(2) or in the sextet representation of SU(3). In this presentation we concentrate in detail on the first of the above mentioned two cases, i.e. the minimal walking technicolor (MWTC). As a specific technicolor model, this model resolves the naturality, hierarchy and triviality problems of the Standard Model (SM) in the usual way. Due to the proposed walking dynamics, if embedded within an extended technicolor (ETC) framework [5], this model allows one to suppress flavor changing neutral currents (FCNC) down to the observed level yet generating phenomenologically feasible quark mass spectrum. Since walking is achieved with minimal particle content of just two techniflavors, this model is expected to be compatible with the existing constraints on the S-parameter.

Let us first briefly recall the details of this model. The usual Standard Model Higgs sector is replaced by an SU(2) gauge theory with two Dirac flavors of technifermions, U and D, transforming according to the adjoint representation of the technicolor gauge group. Under SM gauge groups these are taken to be singlets under QCD color, while under $\mathrm{SU_L}(2)$ the left-handed techniquarks transform as a doublet and right-handed techniquarks as singlets. Being in the triplet representation of SU(2) these techniquarks constitute three electroweak doublets and hence the resulting $\mathrm{SU_L}(2)$ gauge theory is ill defined, suffering from the Witten anomaly. However, this anomaly is easy to cure by adding one further left-handed doublet, singlet under technicolor in order not to spoil the walking behavior, and singlet under QCD color. Hence, up to the charge assignments these new particles resemble an ordinary lepton doublet of SM. For electrically charged particles in this doublet we also need to add the right-handed weak singlet; for neutral particle the right-handed singlet is optional. So, summarizing, the particle content is

$$\begin{aligned} Q_L^a &= \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L, \qquad U_R^a, D_R^a \qquad (a=1,2,3) && (1) \\ L_L &= \begin{pmatrix} N \\ E \end{pmatrix}_L, \qquad N_R, E_R. && (2) \end{aligned}$$

For which the anomaly free hypercharge assigement is

$$\begin{aligned} Y(Q_L) &= \frac{y}{2}, \qquad Y(U_R, D_R) = (\frac{y+1}{2}, \frac{y-1}{2}) && (3) \\ Y(L_L) &= -3\frac{y}{2}, \quad Y(N_R, E_R) = (\frac{-3y+1}{2}, \frac{-3y-1}{2}), && (4) \end{aligned}$$

where y is any number. Note that for the choice $y = 1/3$, which we will consider in here, the techniquarks will carry similar electric charges as ordinary QCD quarks and the new lepton doublet has a neutral (N) and a singly charged (E) member. Hence from the weak interaction viewpoint this new beyond the SM matter sector resembles simply a fourth generation of ordinary matter. However, the properties of this fourth generation are very different from the usual quarks and leptons: The quarks do not carry QCD color, but are confined at energies below the electroweak scale by a new technicolor force. The new leptons need to be very massive in comparison to their SM counterparts; this ratio being particularly striking for the new neutrino.

Finally, we can write the concrete Lagrangian for the theory replacing the Higgs sector of the Standard Model

$$\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_\ell + \mathcal{L}_{\text{mass}}, \tag{5}$$

where

$$\begin{aligned} \mathcal{L}_Q &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{Q}_L \not{D} Q_L + i\overline{U}_R \not{D} U_R + i\overline{D}_R \not{D} D_R && (6) \\ \mathcal{L}_\ell &= i\overline{L}\not{D} L + i\overline{N}_R \not{D} N_R + i\overline{E}_R \not{D} E_R. && (7) \end{aligned}$$

The term $\mathcal{L}_{\text{mass}}$ determines the elementary fermion masses and is due to ETC interactions; we will discuss the part relevant for the new leptons in next section. The technicolor field strength is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{\text{TC}}\epsilon^{abc}A^b_\mu A^c_\nu \tag{8}$$

while the covariant derivative is e.g. for left-handed techniquark

$$D_\mu Q_L^a = (\delta^{ac}\partial_\mu g_{\rm TC} A_\mu^b \epsilon^{abc} - i\frac{g}{2} W_\mu \cdot \tau \delta^{ac} - ig' Y(Q_L) B_\mu \delta^{ac}) Q_L^c. \tag{9}$$

For right-handed techniquarks the weak interaction is absent and the hypercharge appearing in the last term is adjusted appropriately. For leptons similarly, with the replacement of $Q \to L$ and without the technicolor interaction.

In the following sections we will first consider some specific phenomenological signatures of this model and use these to motivate the first principles lattice study of the properties of the underlying gauge theory. The goal of the lattice study, then, is to establish if this underlying theory has an infrared stable fixed point in accordance with the original proposal of [1].

2 Phenomenology for colliders and cosmology

Recall that the existence of the fourth lepton generation was required by the internal consistency of the model. In this section we concentrate mostly on the phenomenological signatures originating from these leptons and review the consequences both for colliders and cosmology. Note that the fourth generation of leptons, being treateble within perturbation theory, provides a nicely accessible handle on some of the properties of this theory.

We do not consider an explicit ETC model for the elementary fermion masses. However, effectively in terms of the composite scalar degree of freedom in the model, the generic terms responsible for the masses of the fourth generation leptons are

$$\begin{aligned} \mathcal{L}_{\rm mass} = & \ \ldots \ (y\overline{L}_L \Phi E_R + {\rm h.c.}) + C_D \overline{L}_L \tilde{\Phi} N_R \\ & + \frac{C_L}{\Lambda}(\overline{L}^c \tilde{\Phi})(\tilde{\Phi}^T L) + \frac{C_R}{\Lambda}(|Phi^\dagger \Phi)\overline{N}_R^c N_R + {\rm h.c.} \end{aligned} \tag{10}$$

where $\tilde{\Phi} = i\tau^2\Phi^*$ and Φ is an $\mathrm{SU_L}(2)$ doublet, analogous to SM Higgs [6], which, upon condensation $\Phi \to (0, v)^T$, leads to the general neutrino mass matrix

$$M = \begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix}. \tag{11}$$

Diagonalization of this mass matrix leads to two Majorana eigenstates, N_2 (light) and N_1 (heavy), with mixing angle $\sin\theta$. Charged and neutral currents in terms of the mass eigenstates become

$$\begin{aligned} W_\mu^+ \overline{N}_L \gamma^\mu E_L &= \sin\theta \overline{N}_{2L} W^+ \gamma^\mu E_L + \cdots \\ Z_\mu \overline{N}_L \gamma^\mu N_L &= \sin^2\theta Z_\mu \overline{N}_{2L} \gamma^\mu N_{2L} + \cdots \\ & \quad + \sin(2\theta) Z_\mu \overline{N}_{1L} \gamma^\mu N_{2L} \end{aligned}$$

Omitted terms contain contributions from the heavier eigenstate. Note that depending on the mixing the couplings are weakened making N_2 (or N_1 in other cases) less easily detectable at colliders.

We can now compute the precision parameters S and T and compare with existing data. This work has been carried out for Dirac neutrino ($M_L = M_R = 0$) in [2] and for pure left-handed Majorana neutrino ($m_D = M_R = 0$) in [7]. The study for the general case was recently carried out in [8]. These three cases are shown in Figure 1, and in each three panels of the figure the techniquarks contribute $S \sim 1/(2\pi)$ [1], while their contribution to T is zero. The shaded regions in the figure are obtained by varying the lepton masses within the values M_Z and $10M_Z$. The lepton masses appear as model parameters and as we see from the figure their contribution to both S and T is essential in obtaining compatibility with the data since split masses allow one to generate positive contribution to T.

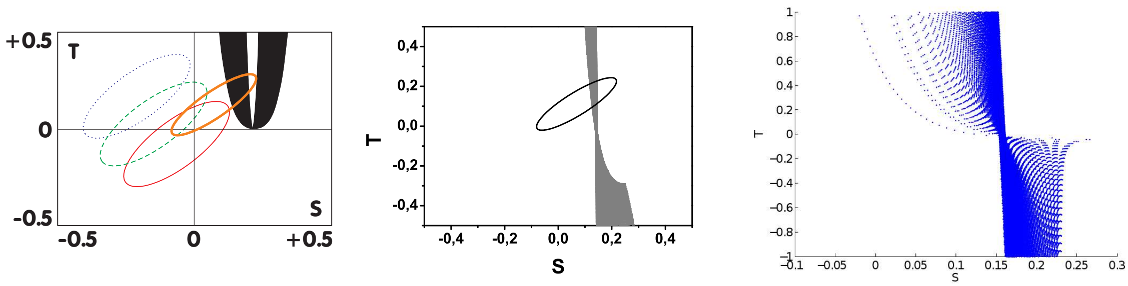

Figure 1: Contribution from the model to S and T parameters as the masses of the leptons are varied. The ellipsis in left and middle panels shows the 68% confidence contour from the LEPEWWG fit [9] assuming SM Higgs with a mass of 150 GeV (the staggered ellipses in the left panel correspond to the 90% contours of the PDG fit [10]). The figures correpond to Dirac neutrino (left), purely left-handed Majorana neutrino (middle) and mixed case with $\sin\theta = 0.3$ (right).

Since the model is compatible with the present data, it should be further tested at the LHC. If the lepton masses are within the accessible range as the constraints from precision data imply, they should provide clean signatures and direct further interest to the possible underlying strongly interacting sector which implies interesting collider signatures on its own [6]. Production of new neutrinos provides interesting signatures if the neutrino is not stable but mixes with the SM neutrinos. The most interesting production channels were identified and investigated in [8], see also [11]. For example, if we consider production of a pair of neutrinos, and subsequent decay $N_2 \to \ell W$, where $\ell = \mu, \tau$ or e, then the possible final states are

$$2\ell + 4\text{jets}, \qquad 3\ell + 2\text{jets} + \not{E}, \qquad 4\ell + \not{E}.$$

Apparently the third one is similar to what one would expect from the decay of a pair of neutralinos, but the other two should provide a way to distinguish neutrinos from neutralinos. The first one appears interesting since two same-sign leptons can appear in the final state due to the fact that the initial neutrinos are Majorana particles. Now focus on the $\ell^\pm\ell^\pm$ and $\ell^\pm\ell^\pm\ell^\mp$ final states arising from $Z^* \to N_2N_2$ and $W^* \to \ell^\pm N_2$ production channels. Following [12], we assume that N_2-neutrino couples to muons only and saturates the latest experimental bound on off-diagonal lepton mixing element $|V_{\mu\chi}|^2 < 0.0032$.

Table 1 illustrates numerically the scenario with two same-sign leptons in the final state. First two specific realizations of this scenario feature N_2-neutrino being the right-handed state ($\sin\theta = 0$) with N_1-neutrino mass equal to 90 GeV and 100 GeV. These masses were chosen to go in parallel with S and T analysis [8]. Third realization with M_N=135 GeV and mixing angle $\sin\theta = 0.5$ was selected to probe heavier neutrino masses. For event pre-selection we require the presence of two like-sign charged leptons with transverse momentum larger than 30 GeV, and an additional lepton of opposite charge. For the final event selection we also require: at least two jets in the final state with $p_T > 20$ GeV, and no b-tagged jets; missing energy smaller than 30 GeV; the transverse angle between the two leptons must be larger than $\pi/2$.

The signal and SM background cross sections for these selected events are also given in Table 1. The number of the selected events for an integrated luminosity of 10 fb^{-1} and the corresponding statistical significance are also presented there. The conclusion is that 100 GeV and 135 GeV neutrinos may be discovered early under these conditions (at full LHC center of mass energy at least).

Let us then briefly comment on cosmological consequences within MWTC model. We know that most of the energy content of the universe is obervable only through its gravitational interactions. If 75% is accounted for the dark energy as is implied by assumption of homogeneous cosmology with finely tuned cosmological constant, then we are left to explain 21 % of the energy content with some dark matter particle abundance. If inhomogeneities are important then this factor will be even larger, perhaps even as large as 96%.

The possibility that the observed dark matter abundance is explained by a weakly interacting massive particle (WIMP) remains an attractive hypothesis. This hypothesis has gained strong support due to recent observations of the colliding galaxy clusters, e.g. [13] and MWTC model provides several alternatives for a candidate WIMP.

First, the neutral technibaryon is an important candidate since it possesses an exactly conserved quantum number and also provides interesting connections to the ordinary matter-antimatter -asymmetry [14]. On the other hand, in the leptonic sector of the theory, if the fourth generation neutrino is stable it provides a natural WIMP candidate [7]. The earth based searches constrain both the Dirac neutrino [15, 16] and also a purely left handed Majorana neutrino [17] heavily. However, if the neutrino is a mixture of left and right handed gauge eigenstates, then with

Table 1: Signal cross-sections σ(in fb) with the corresponding leading SM background for three scenarios described in the text. Pre-selection and selection criteria are also described in the text.

$\mu^{\pm}\mu^{\pm}$ X (M_{N_1}=100 GeV)	Selected σ(fb)	Events/10 fb^{-1}	S/$\sqrt{B}$
$\mu^{\pm}N_1$: $\mu^{\pm}\mu^{\pm}$ + 2 jets	0.6	6	7.25
N_1N_1 : $\mu^{\pm}\mu^{\pm}$ + 4 jets	1.32	13.2	
SM background	0.7	7	
$\mu^{\pm}\mu^{\pm}$ X (M_{N_1}=90 GeV)	Selected σ(fb)	Events/10 fb^{-1}	S/$\sqrt{B}$
$\mu^{\pm}N_1$: $\mu^{\pm}\mu^{\pm}$ + 2 jets	0.113	1.1	1.89
N_1N_1 : $\mu^{\pm}\mu^{\pm}$ + 4 jets	0.39	3.9	
SM background	0.7	7	
$\mu^{\pm}\mu^{\pm}$ X (M_{N_1}=135 GeV)	Selected σ(fb)	Events/10 fb^{-1}	S/$\sqrt{B}$
$\mu^{\pm}N_1$: $\mu^{\pm}\mu^{\pm}$ + 2 jets	1.1	11	6.95
N_1N_1 : $\mu^{\pm}\mu^{\pm}$ + 4 jets	0.74	7.4	
SM background	0.7	7	

suitable mixing it can account for the dark matter abundance and have escaped direct detection so far [18].

Finally, since adjoint quarks can bind with technigluons, there exists interesting exotic states carrying baryon number and consisting of a quark and a gluon [19]. However, it is not clear if such states could be lighter than the baryonic Goldstone states constituting the usual technibaryonic candidate.

So far we have seen that a simple model can lead to surprisingly rich phenomenology. However, if MWTC is a viable model for electroweak symmetry breaking, the underlying theory must be near conformal, i.e. the β-function should have a (quasi)stable infrared fixed point at $g^* \neq 0$ for which $\beta(g^*) \simeq 0$. But does this happen? This question can be currently answered only by lattice simulations. Since the numerical simulations of gauge theories in four dimensions especially with fermions are expensive, the motivation should be strong. We hope to have provided enough phenomenological appeal for this model in order to motivate a detailed non-perturbative investigation of its foundational properties on the lattice. Therefore we now turn off all electroweak physics and simply concentrate on the underlying strong dynamics and aim to measure the running of the coupling constant in this theory.

3 The model on the lattice

Recall again the model which we study, SU(2) gauge theory with two adjoint Dirac flavors. The continuum theory is defined with the Lagrangian

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \overline{U}\,\not{D}U + \overline{D}\,\not{D}D$$

On the lattice this theory is defined by the action

$$S_{\rm lat} = S_G + S_F,$$

with the usual Wilson plaquette action S_G and the Wilson fermion action S_F which for our case of adjoint fermions is

$$S_{\rm F} = \sum_{f={\rm u,d}} \sum_{xy} \overline{\psi}_{f,x} M_{xy} \psi_{f,y}, \tag{12}$$

where

$$M_{xy} = \delta_{xy} - \kappa \sum_{\mu} \left[(1+\gamma_\mu)V_{x,\mu} + (1-\gamma_\mu)V^T_{x-\mu,\mu} \right]. \tag{13}$$

The only modification to the usual Wilson action for fundamental fermions is the replacement of the link variables $U_\mu(x)$ with the variables

$$V^{ab}_\mu(x) = 2{\rm tr}(S^a U_\mu(x) S^b U^\dagger_\mu(x)), \tag{14}$$

where S^a $a = 1,2,3$ are the generators of the fundamental representation, normalized as $\rm{Tr}S^aS^b = \frac{1}{2}\delta^{ab}$. The usual lattice parameters are the bare lattice coupling $\beta_L = 4/g^2_{\rm bare}$ and the hopping parameter $\kappa = 1/[8 + 2(am_{q,\rm bare})]$.

Because the Wilson fermion action breaks the continuum chiral symmetry, the quark mass is additively renormalised and we determine the quark mass through the PCAC relation

$$2m_Q = \left[\partial_t \langle V_A(x)\rangle / \langle V_P(x)\rangle\right]_{|t=L/2} \tag{15}$$

where V_A and V_P are (non-improved) axial and pseudoscalar current correlators (in the Schrödinger functional formalism [20]). For each lattice coupling β_L we tune κ so that m_Q vanishes. This determines the critical line $\kappa_c(\beta_L)$; from now on we assume that we always tune to the limit $m_Q = 0$.

In earlier studies [21, 22] it has been found that in this lattice theory there exists a critical coupling $\beta_{L,c} \approx 2$ so that when $\beta_L < \beta_{L,c}$ the theory shows signs of spontaneous chiral symmetry breaking, but reaching the limit $m_Q \to 0$ is prevented by the appearance of a first order transition where m_Q jumps to negative values. However, when $\beta_L > \beta_{L,c}$ the $m_Q \to 0$ limit can be reached without observing the spontaneous chiral symmetry breaking. This is the region where it is possible to find conformal behavior and on which which we shall concentrate here.

In the Schrödinger functional (SF) method [23, 24, 25] the coupling is measured utilizing a background field, and the scale at which the coupling is measured is determined by the finite size of the system; we consider lattices of volume $V = L^4 = (Na)^4$. At the classical level the gauge field boundary conditions, $\exp(-i\eta\tau_3 a/L)$ at $t = 0$ and $\exp(-i(\pi - \eta)\tau_3 a/L)$ at $t = L$, generate a constant chromoelectric field for which

$$\frac{\partial S^{\mathrm{cl.}}}{\partial \eta} = \frac{k}{g^2_{\mathrm{bare}}}, \tag{16}$$

where k is a function of $N = L/a$ and η. Using the quantum effective action Γ,

$$e^{-\Gamma} = \int \mathcal{D}[U, \psi, \overline{\psi}] e^{-S}, \tag{17}$$

the coupling constant is now defined through

$$\frac{\partial \Gamma}{\partial \eta} = \left\langle \frac{\partial S_{\mathrm{G}}}{\partial \eta} \right\rangle = \frac{k}{g^2(L)}. \tag{18}$$

The expectation value has explicit dependence only on the gauge action (for our non-improved fermion action) and we fix $\eta = \pi/4$ after taking the derivatives. This gives the coupling constant $g^2(L)$ as a function of the physical size of the lattice.

Our results[26] are shown in Figure 2, which displays the running towards a fixed point from below for large values of β_L and from above for small β_L. The points at small a/L are affected by small volume artifacts and should be discarded. Ideally one would like to perform a full step-scaling analysis. However, two factors make this procedure technically difficult for the case at hand. First, the evolution of $g^2(L)$ is very slow in the range of interest, much slower than in QCD. This means that a very large number of factor-2 steps would be necessary to cover the desired range. Because each step introduces statistical errors, the accuracy requirement for the simulations becomes very challenging. Second, as one can see from the figure, the evolution of the coupling is so slow that at small volumes finite lattice spacing artifacts mask it (at least with non-improved action). This means that the continuum limit in the step scaling function is difficult to obtain reliably.

Instead, we decided to use a simpler and more robust approach: we choose a set of values for β_L, and measure $g^2(N, \beta_L)$ on a range of volumes up to 20^4 giving us a set of discrete samples of the evolution of $g^2(L)$, to which we fit a following *β-function ansatz*:.

$$\beta(g) = -b_0 g^3 - b_1 g^5 + (b_0 g_*^{3-\delta} + b_1 g_*^{5-\delta})\, g^\delta \tag{19}$$

Here b_0 and b_1 are fixed by the perturbative two-loop β-function, and g_* and δ are fit parameters. Integrating $\beta(g) = dg(L)/d\ln L$ and constraining the resulting g^2 to the lattice measurements at $L/a = 12$ for each β_L gives the continuous lines shown in

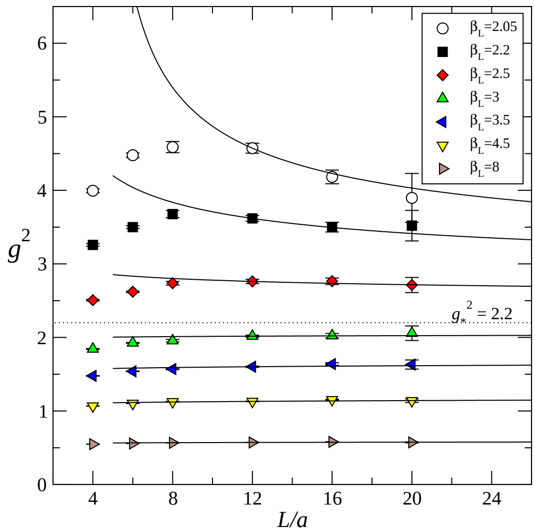

Figure 2: Lattice measurements of $g^2(L/a, \beta_L)$. Continuous lines show $g^2(L)$ integrated from (19), constrained to go through lattice points at $L/a = 16$.

Figure 2. The integrated $g^2(L/a, \beta_L)$ can be compared with the lattice measurements at $L/a = 16$ and 20, resulting in a χ^2 criterion for the fit. Good fits (χ^2/d.o.f~ 1) are obtained when parameters vary from $(g_*^2, \delta) \approx (2.0, 6)$ to $(3.2, 15)$ in a narrow region indicating that the parameters are strongly correlated. We take $g_*^2 = 2.2$, $\delta = 7$ to be our benchmark value. If we fix $\delta = 7$, g_*^2 can vary between 2.05 and 2.4. The resulting β-function is shown in Figure 3, together with the "error band" formed by the variation of the parameters in the region where acceptable fit is obtained.

Integrating the β-function gives the coupling g^2 which is shown in Figure 3 as a function of $\mu = 1/L$. The result splits into two branches: the asymptotically free one with $g^2 < g_*^2$, and the not asymptotically free with $g^2 > g_*^2$. These branches correspond to completely different theories and the scaling between the μ-values of the two branches is arbitrary in Figure 3.

Let us now discuss the interpretation of these results. When we combine the results of this analysis with the spectrum measurements of ref. [21], a consistent picture emerges. In the branch $g^2 < g_*^2$, the theory is asymptotically free, and in the infrared becomes conformally invariant, without chiral symmetry breaking. This is the physically interesting case. On the other hand, when $g^2 > g_*^2$, the theory is not asymptotically free, and probably does not have a continuum limit. For some other fermion discretisation this result may differ. Especially the approach to the continuum limit motivates a further study with fully $O(a)$ improved lattice formalism. There exists related studies for SU(3) gauge theory with either two Dirac flavors in the sextet representation [27] or twelve flavors in fundamental representation [28].

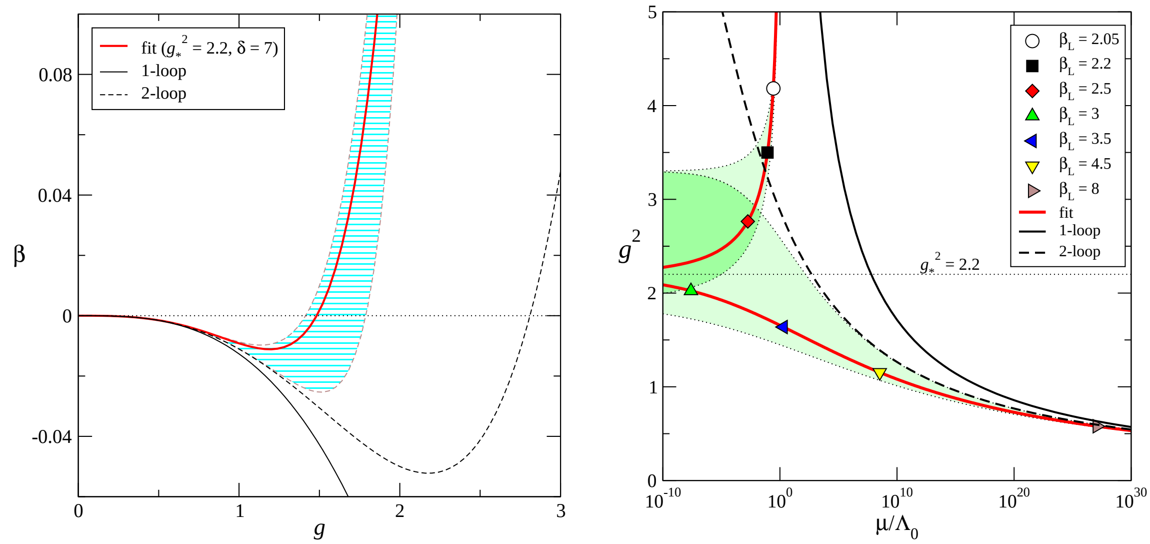

Figure 3: Left: The β-function obtained from (19), with $g_*^2 = 2.2$. The hatched area shows the estimated error range of the fit. Shown are also perturbative 1- and 2-loop β-functions. Right: $g^2(\mu = 1/L)$ determined from (19), together with perturbative g^2. Error bands are omitted for clarity. Scale Λ_0 is determined so that $\mu = \Lambda_0$ is at the IR Landau pole of the asymptotically free 1-loop coupling and at the UV Landau pole of the measured non-asymptotically free branch ($g^2 > g_*^2$). The $L/a = 16$ lattice points have been set on the curves at the measured g^2-values.

In conclusion, we have discussed a simple model for dynamical electroweak symmetry breaking which is viable in light of current data and provides phenomenologically interesting consequences for colliders and cosmology. Detailed, but still initial, lattice investigations of the underlying gauge theory have provided evidence in support on the existence of an infrared stable fixed point.

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