Technicolor on the Lattice

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1 Introduction

The idea of Dynamical Electro–Weak Symmetry Breaking (DEWSB) due to the existence of a new strongly interacting sector at the TeV scale beyond the Standard Model of particle physics has been proposed many years ago [1, 2]. However the simplest models obtained by a naive rescaling of QCD are inadequate since they are in contradiction with the experimental evidence of precision Electro–Weak tests [3, 4]. Walking and conformal technicolor theories have been proposed [5, 6, 7, 8] whose dynamics should be sufficiently different from QCD so that they would not violate the experimental constraints (for recent reviews of techicolor models see [9, 10]). In particular the idea of using models with matter fields in representations other than the fundamental near the onset of the conformal window has been recently advocated [11, 12]. In this work we will focus on one of the possible candidate theories, the so-called Minimal Walking Technicolor theory, based on the gauge group SU(2) with two Dirac fermions in the adjoint representation. The attribute "minimal" in the name stems from having only 2 flavors and from being the theory with the estimated minimal value of the Peskin-Takeuchi S parameter among all theories with matter fields in one representation only.

Given the difficulty of dealing with strongly interacting theories, the analytical approaches which led to pinning down the promising candidates for realistic technicolor theories are all dependent on uncontrolled approximations or conjectures based on educated guesses. We approach the problem using Lattice Gauge Theory (LGT). This is a first principle framework in which quantitative predictions useful for phenomenology can be obtained, as it has been extensively demonstrated for QCD.

The lattice comunity has already shown renewed interest in this subject during the last two years [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30].

In this work we present a study of the spectrum of the gauge theory SU(2) with two Dirac adjoint fermions. Given the analytical uncertainties, it is not clear if this theory lies within the conformal window or not. Our first goal to understand the model is to determine if chiral symmetry is spontaneously broken or if the theory is IR conformal. We look for evidence of the existence of an IR fixed point by studying the dependence of the mesonic spectrum on the current quark mass. As explained in detail below, to obtain reliable results it is essential to control all the well-known sources of systematic errors introduced by the lattice approximation.

In Sect. 2 we give an overview on the treatment of systematic errors present in a lattice computation. In Sect. 3 we present our latest results and discuss the evidence produced. Our conclusions are given in Sect. 4.

2 Lattice simulations

Lattice Field Theory reduces the Euclidean path integral to an ordinary multidimensional integral by approximating the continuum space-time with a discrete four dimensional hypercubic lattice. To guarantee exact gauge invariance, gauge fields are replaced by parallel transporters $U_\mu(x)$ between neighbouring sites x and $x+\mu$, while fermion fields $\psi(x)$ live on the lattice sites. The action for matter fields in a representation R is given by[1]

$$S(U,\psi,\overline{\psi}) = \beta S_g(U) + \sum_{i=1}^{N_f} \overline{\psi_i^R} D_m(U^R) \psi_i^R \ , \tag{1}$$

where the exact form of the lattice gauge action S_g and the massive lattice Dirac operator D_m depend on the particular discretization used. For the simplest discretization, the action depends only on two bare parameters: the bare inverse coupling β and the bare quark mass am_0. While the link variables appearing in the gauge action are in the fundamental representation of the gauge group, the links in the lattice Dirac operator are in the same representation R as the fermions fields. The partition function, after integrating out the matter fields, takes the form:

$$Z = \int \exp[-\beta S_g(U)][\det\ D_m(U^R)]^{N_f} dU \ . \tag{2}$$

We use the Wilson action in our simulations: the gauge action is simply proportional to the real part of the trace of 1×1 plaquettes summed over the whole lattice; for the fermions the Wilson–Dirac operator is given by

$$D_m(U^R) = am_0 + \frac{1}{2}\sum_\mu \left[\gamma_\mu \left(\nabla_\mu + \nabla^*_\mu\right) - a\nabla^*_\mu \nabla_\mu\right] \ , \tag{3}$$

[1] We omit for the sake of simplicity all the position, color and spin indexes.

where ∇_μ is the discretized forward covariant derivative depending on the link U_μ^R and ∇_μ^* its adjoint. In a numerical lattice simulation the computation of the discretized path integrals is performed by Monte Carlo integration using importance sampling: an *ensemble* of gauge configurations is generated with probability proportional to $\exp[-\beta S_g(U)][\det\ D_m(U^R)]^{N_f}$. The expectation value of any observable can then be computed as a stochastic average over this ensemble of configurations.

2.1 Systematic Errors in Lattice Simulations

It is important to realize that the desired continuum results can only be recovered taking appropriate limits of numerical lattice simulations. Changing perspective, one can say that lattice simulations are always affected by systematic errors, which must be controlled to obtain reliable results. This is of the utmost importance for our calculations, since the theory we aim to studying, $SU(2)$ with Dirac adjoint fermions, is not understood theoretically and could be very different from the familiar theory of QCD. We need to ensure that we are observing genuine features of the continuum theory, and not artefacts of our lattice formulation.

Let us remind - and warn! - the reader about the sources of systematic errors. Besides statistical uncertainties, arising from the finite number of configurations in our statistical ensemble, there are:

finite-size, finite-temperature corrections

Numerical simulations are always performed on a lattice with finite extent in all four direction, usually in a geometry $T \times L^3$. On a 4-dimensional torus these expectation values can have large corrections, even if the infinite volume limit is reached at an exponential rate asymptotically . The absolute magnitude of finite size effects increases as the correlations in the lattice increase, i.e. they become big when the Compton lenghtwave of light modes propagating in the lattice is comparable with its size. Moreover, if the size of the lattice is not sufficiently large, the system may enter a phase that bears little resemblance to the large–volume theory we are intersted in. Some examples, based on analytical finite–volume results [31, 32, 33], have been recently discussed in Ref. [30].

explicit breaking of chiral symmetry

The lattice discretization breaks chiral symmetry explicitly. For Wilson–Dirac fermions chiral symmetry is recovered by tuning one parameter, the bare fermion mass, as one decreases the lattice spacing. Other formulations are known which possess better chiral properties, however all of them are much more computationally expensive than Wilson–Dirac fermions[2]. Even with better chiral formu-

[2]Kogut-Susskind staggered fermions also have better chiral properties than Wilson fermions for a comparable cost. However in this case one is limited to having a number of flavors which is a multiple of four, in order to avoid complications due to the so–called rooting of the fermion determinant.

lations, there is another technical problem: the zero modes of the lattice Dirac operator, which appear as the chiral limit is approached, make it impossible to run simulations at very light masses with the current algorithms. When using Wilson fermions on a given finite lattice there is a lower limit to the masses which are numerically accessible, which depends on the volume of the system.

lattice artefacts

Computations are performed on a discrete four dimensional lattice. Since we are ultimately interested in the continuum physics, we must consider the limit in which the lattice spacing, conventionally called a, goes to zero. However the lattice spacing a is not a parameter of the simulation that one can directly change (all quantities in a simulation are dimensionless). The continuum limit is recovered in a more subtle way: approaching the UV fixed point, at bare coupling $g = 0$, i.e. $\beta = \infty$, the system undergoes a continuous phase transition and, by universality, the microscopic details of the system becomes immaterial. To put it in a different way, the physics at the scale of the lattice spacing must decouple from the long range (continuum) physics which we are interested to study on the lattice.

To give reliable predictions, lattice simulations must be performed in a regime in which a precise hierarchy of scales is realized. To illustrate this point let us consider first the case of QCD. Numerical simulations, as explained above, are always performed with a finite quark mass, which explicitly induces a mass gap in the theory. To avoid finite-size effects in the computation of the low-lying spectrum, the lattice size L must be much bigger than the inverse mass of the lightest hadron $m_{\rm PS}$ we are trying to measure. Since we are interested in the chiral regime, this light hadron mass must be much smaller than the characteristic hadronic scale, at which the theory becomes strongly interacting: we can take for example the Sommer scale r_0, as a convenient quantity to measure on the lattice. Finally to avoid discretization errors, the lattice spacing must be much smaller than the reference scale for strong interactions r_0, so that the physics at the scale of the UV cutoff is weakly interacting. In summary we must have the following hierarchy of scales:

$$\left(\frac{L}{a}\right)^{-1} \ll a m_{\rm PS} \ll \left(\frac{r_0}{a}\right)^{-1} \ll 1 \ , \tag{4}$$

for the computations to be reliable, and only in this regime we can safely extrapolate to the chiral and continuum limit.

Let us now assume that the continuum theory we are interested in studying on the lattice has an IR fixed point in the massless limit. Introducing a mass term, as it is always the case in numerical simulations, explicitely breaks conformal invariance and the above considerations made for QCD remain valid also in this case. In particular it is still possible to define a scale in analogy to r_0 in QCD (see Sect. 3.2 below) which remains finite also in the massless limit. This scale however has a different interpretation than in QCD. The difference with the familiar QCD-like case is in the behavior of the two theories approaching the chiral limit. We will discuss below in Section 3 the signatures of the IR conformal case.

2.2 Simulation Code

Our results presented in the next Section are obtained using our own simulation code, written from scratch for the specific purpose of studying gauge theories with fermions in arbitrary representations [18]. This code was designed to be flexible, fast, easy to use and extend. Our simulation code is suitable for the study of gauge theories with:

- gauge group $SU(N)$ for any N;
- generic fermion representations. At present the code implements the fundamental (fund), adjoint (ADJ), 2-index symmetric (S) and antisymmetric (AS). It is easy to extend the code to other representations, like 3-index symmetric for example, and even to have fermions in two or more different representations at the same time. In particular no modifications for the computation of the HMC force are needed, which is typically the most complicated part of the code;
- any number of flavors. We use Wilson fermions and the RHMC algorithm, which is an exact algorithm for any number of flavors.

Since this was a new code, and lattice simulations in the past were mainly devoted only to QCD, we made a large effort to validate the code and to study its behavior in the parameter region of interest for physics studies. As part of this effort, we made a number of cross-checks:

- for $SU(3)$, $N_f = 2$ with different, established codes (we used M. Luscher's DD-HMC algorithm [34]);
- consistency among different representations (e.g. $SU(3)$ AS vs fund, $SU(2)$ SYM vs ADJ);
- observables with different codes (e.g. meson spectrum for $SU(2)$ ADJ in Chroma);
- large quark mass limit compared to pure gauge (quenched) spectrum;
- correctness of integrator, reversibility, acceptance probability;
- independence on integrator step size.

To control the stability of the simulations, which could incur in well-known problems close to the chiral limit [35], we monitored the lowest eigenvalue of the (preconditioned) Wilson–Dirac matrix. We found no instabilities in our runs. A detailed report will appear elsewhere [36].

3 SU(2) with 2 Adjoint fermions

Before looking at the actual numerical results, we will discuss the signatures of an IR fixed point. In this work, we search for indications of IR conformal behavior in the spectrum. If there is no IR fixed point, the theory is expected to be confining and chiral symmetry to be spontaneously broken. This is the familiar scenario of QCD: using the PCAC mass from the axial Ward identity, m, to parametrize the explicit breaking of chiral symmetry, we expect the usual scaling $m_{\rm PS}^2 \propto m$, as $m \to 0$; $F_{\rm PS}$ remains finite; the Gell-Mann–Oakes–Renner relation is satisfied and can be used to extract the non-zero chiral condensate: $(m_{\rm PS} F_{\rm PS})^2/m \to -\langle\overline{\psi}\psi\rangle$.

On the other hand, if the theory has an IR fixed point, and as before, we parametrize the explicit breaking of chiral and conformal symmetry with m, as $m \to 0$ we expect that:

- all hadron masses vanish proportionally to the same power of m: $m_{\rm had} \sim m^\alpha$;
- in particular the ratio of the vector to pseudoscalar meson mass remains finite: $m_{\rm V}/m_{\rm PS} \to \text{const} < \infty$;
- $F_{\rm PS}$ and $\langle\overline{\psi}\psi\rangle \to 0$.

Furthermore, if one measures the string tension σ as a function of m, we expect that σ vanishes in the chiral limit.

The behavior just described assumes that the system is in the continuum limit and in an infinite volume. However this is not the case in a lattice simulation. In particular the finite size of the system can be seen as a relevant coupling which will drive the system away from criticality under the renormalization group (RG) flow. As m is decreased towards the chiral limit, great care must be taken to control finite size effects as explained in Section 2. We will show below that these effects are quite big in the range of masses and lattice volumes which are currently used in numerical simulations. In particular it is important to keep in mind that an asymptotically free theory will look almost conformal in a very small volume and practical limitations can make it very difficult to distinguish between the two cases.

3.1 Spectrum

We now present the results of our numerical work. All the simulations used in this work are made at a fixed lattice spacing, corresponding to a bare coupling $\beta = 2.25$. This value of the coupling was chosen based on previous studies of the same theory [13, 19, 21] to avoid a bulk phase transition present at about $\beta = 2.0$.

We use four different lattices: 16×8^3, 24×12^3, 32×16^3 and 64×24^3. For each of these four lattices a number of ensembles corresponding to different quark masses were generated focusing in the range corresponding to pseudoscalar masses of 0.6–0.2 a^{-1}.

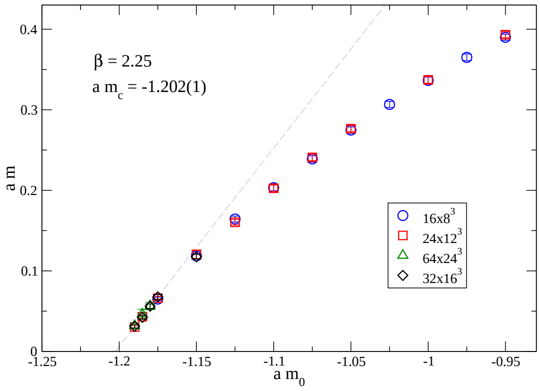


Figure 1: Extrapolation of the quark mass from the axial Ward identity to locate the chiral limit. As expected no significant finite size effects are present.

We explicitly control finite-size effects making simulations with different volumes. In this way we can reach the small mass region and study the finite-volume effects for the first time in this theory. All the previous studies cited above [13, 19, 21] have almost no information on this small quark mass region, having mostly pseudoscalar masses of order of the ultraviolet cutoff a^{-1} or heavier and no study of finite volume effects. For each lattice and quark mass we accumulated a statistical ensemble of about 5000 thermalized configurations, except at the largest volume for which we present only preliminary data based on approximately 400 thermalized configurations for each quark mass. The gauge configurations were generated using trajectories in the molecular dynamics integration of length 1 for the two smallest volumes and 1.5 for the two largest lattices, with integration parameters leading to an acceptance rate of about 85% in all cases, and to an integrated autocorrelation times for the lowest eigenvalue of the Wilson–Dirac operator of order 10 or less.

For each ensemble of configurations, we measured the quark mass from the axial Ward identity (PCAC mass) m, the pseudoscalar meson mass $m_{\rm PS}$, the vector mass $m_{\rm V}$ and the pseudoscalar decay constant $F_{\rm PS}$. We locate the chiral limit at the critical bare mass where the PCAC mass vanishes. This does not correspond to bare mass zero because the explicit breaking of chiral symmetry with Wilson fermions induces an additive renormalization of the quark mass. We show in Fig. 1 the extrapolation of m for different lattice sizes. Using a linear extrapolation, the chiral limit can be located at the critical bare mass $am_c = -1.202(1)$. As expected from the fact that m is essentially an UV quantity, no significant finite-size effects are visible and the measured values for this quantity agree within errors on all four lattices.

Our results for the mass of the pseudoscalar meson are presented in Fig. 2. The interesting region of small quark mass is enlarged in the right panel. The data presented for the 64×24^3 are still preliminary and have large statistical uncertainties.

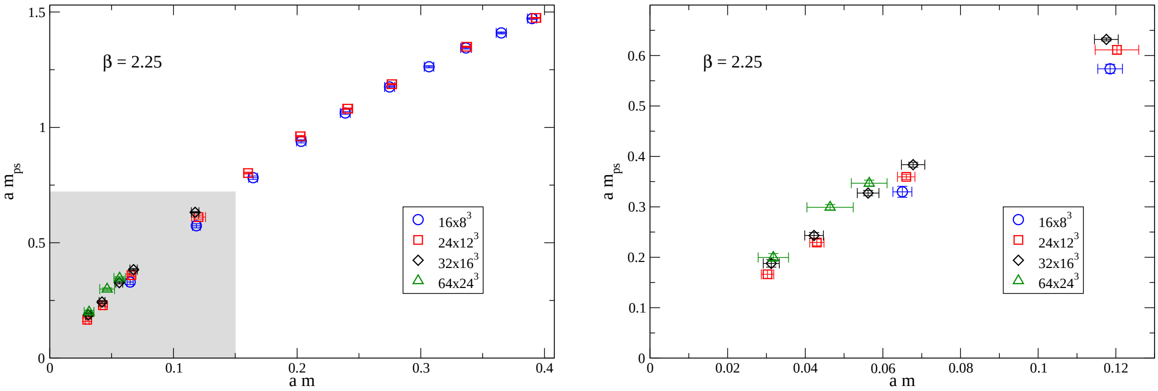


Figure 2: Pseudoscalar meson mass as a function of the PCAC mass. The interesting small mass region shaded in the left panel is enlarged on the right. Finite volume effects are evident and grow approaching the chiral limit.

Finite size effects on $m_{\rm PS}$ are evident and, as the PCAC mass is decreased, they increase as expected from the discussion of Sect. 2. To quantify this systematic effect and to keep it under control, we use larger lattices as the chiral limit is approached.

This large finite size effects are clearly visible in the range of volumes and quark masses explored. Due to this effect it is difficult to draw definite conclusions about the functional behavior of the pseudoscalar mass in the chiral limit. At the current level of accuracy, it is not possible to exclude a QCD-like behavior for the pseudoscalar mass, as well as for all the other quantities we measure, as it will be clear in the following. Data at a finite volume tend to suggest an IR conformal behavior, but it is not clear whether or not this behavior will persist once the infinite volume limit is taken before the chiral limit.

To exemplify this behavior which, as mentioned, is common to all the quantities we measure, consider the ratio $am_{\rm PS}^2/m$ shown in Fig. 3 (left). If the theory has an IR fixed point the ratio should vanish in the chiral limit, while it remains finite in the other case. The data appear to be well described by a linear function of the PCAC mass for $am \leq 0.15$. If we consider our two smallest volumes, the extrapolated value is quite small and compatible with zero within errors. As we move to larger volumes however, the meson masses increase and the extrapolated value starts to deviate from zero, while remaining very small.

We consider next another physically interesting quantity, the pseudoscalar decay constant $F_{\rm PS}$, shown in Fig. 3 (right). The chiral extrapolation is problematic and not conclusive also in this case due to large finite volume effects. This quantity however behaves in the opposite way than the pseudoscalar mass $m_{\rm PS}$. At finite volume the chiral extrapolation yields a finite result: $F_{\rm PS}$ reaches a plateau for masses smaller than some volume–dependent threshold value of the PCAC mass. However as the

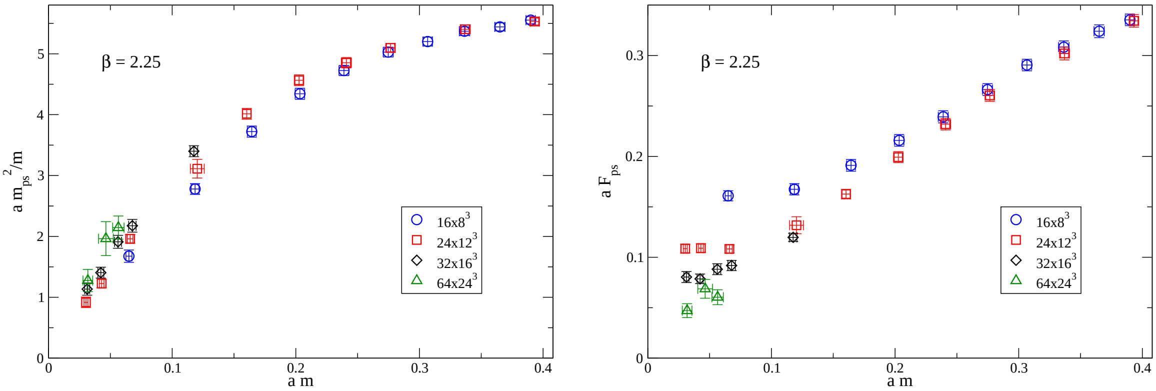


Figure 3: (Left) Ratio of the pseudoscalar mass squared to the PCAC mass. The extrapolation to the chiral limit suffers from large finite-volume effects. See the text for a discussion. (Right) Pseudoscalar decay constant near the chiral limit. Very large finite volume effects are present also in this case which cause the chiral extrapolation to be have large uncertainties.

volume is increased, the plateau value of $F_{\rm PS}$ decreases and a clear envelope for the curves at different volumes is visible in Fig. 3 (right). An extrapolation of this envelope to the chiral limit yields small numerical values, with large systematics due to choice of the range used in the fit. It is also interesting to note that the plateau values decrease approximatively like $1/L$, as one should expect in the conformal case.

The ratio of the vector to pseudoscalar mass is shown in Fig. 4 (left). This quantity is bounded to be greater than one [37] and in the heavy quark limit will tend to one. At large m finite volume effects are small, the ratio is bigger than 1, although deviating only by 4%, and decreasing as m increases, as expected in the heavy quark approximation. It should be clear that the region $am > 0.2$, which corresponds to mesons with masses of order of the inverse lattice spacing or more, is not suitable for studying the physics of the continuum, and that in the same region the theory is effectively quenched. Nevertheless what is remarkable in Fig. 4 (left) is the fact that in the whole mass range we were able to explore, no large separation between the pseudoscalar and vector mass is observed. Looking at our data in more detail, we observe that at fixed volume the ratio $m_{\rm V}/m_{\rm PS}$ presents a maximum for some value of m below which it becomes an increasing function of m. Absolute deviations from the maximum value ≈ 1.04 are small, and one may be tempted to conclude that this ratio remains finite even in the chiral limit. However, as for the quantities discussed above, finite volume corrections go in the direction of invalidating this conclusion. Larger lattices are needed to draw robust conclusions.

The chiral condensate would be a prime candidate to study chiral symmetry breaking. However due to the use of Wilson fermions, the direct measure of $\langle\overline{\psi}\psi\rangle$ is plagued

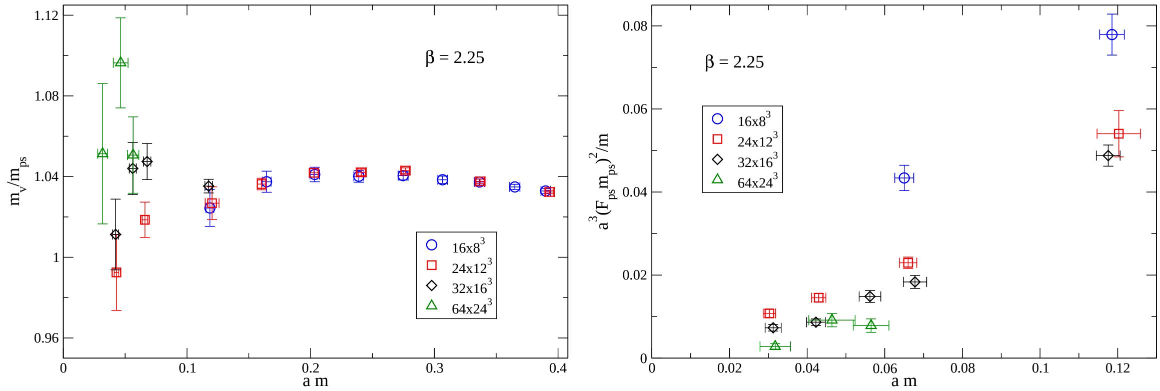


Figure 4: (Left) Comparison between the vector and pseudoscalar meson masses. At large PCAC mass, due to quenching the ratio is very near to one. Near the chiral limit large finite size effects show up. (Right) The GMOR relation can be used to extract information on the chiral condesate. The measure results however quite difficult in practice and we cannot distinguish any signal of chiral symmetry breaking.

with UV divergences which are notoriously difficult to tame. Using the GMOR relation an estimate for the chiral condensate can be obtained[3]. The method has been applied with success in the case of QCD, see e.g. Ref. [38]. We present our result for this quantity in Fig. 4 (right). Although there is a partial cancellation of the finite size effects coming from the pseudoscalar mass and the decay constant, the larger volume dependence of the latter dominates yielding large systematic errors. As a consequence an extrapolation is unfortunately not possible from our current set of the data. We observe that finite volume effects tend to make the condensate smaller, however the small numerical value of the bare condesate by itself is not meaningful: for example in a typical QCD simulation the value for this quantity is an order of magnitude smaller than the one presented here.

3.2 Discussion

Lattice computations are a key ingredient for understanding strongly interacting gauge theories which may possess an IR fixed point. To obtain reliable results however, systematic errors must be kept under control. It is only in recent years that the combination of increased computer power and more efficient algorithms has allowed reliable predictions in the chiral limit. A lot of effort is required in order to reach the parameter region of physical relevance for gauge theories with light quarks. When

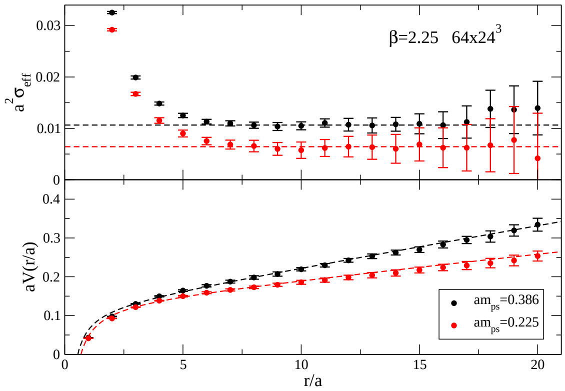
[3]We do not attempt here to compute the necessary multiplicative renormalization constant, since we are not interested to the actual physical value. Perturbative results for the renormalization of fermions bilinears can be found in Ref. [15].

considering theories other than QCD, lattice practitioners are challenged by the lack of information available to guide them. In this situations more than ever, sources of error should be kept under control with great care.

In this work we looked for evidence of an IR fixed point in the spectrum of the theory. Several other studies [13, 19, 18, 21, 20, 24] also investigate the light meson spectrum. However so far these investigations have been limited to heavy mesons with masses of the order of the UV cutoff, small volumes[4] and, in some cases, at finite temperature. All these studies suffer from being in a parameter region in which the systematic errors discussed in Sect. 2 mask the continuum physics. Such investigations are useful to understand the phase structure of the lattice theory and guide future investigations, for example to avoid bulk transitions which are present in the lattice formulation. However much more refined analyses are needed to extract the interesting physics. Work in this direction is in progress in all the groups involved in these studies.

A first step in this direction was presented in this paper. We made simulations at a fixed value of the lattice spacing, and aimed at reaching the chiral limit in a controlled way. Despite the use of relatively large volumes up to 64×24^3, finite size corrections make the extrapolation to the chiral limit difficult. As discussed in the previous section, the finite volume of the system can mimic the existence of an IR fixed point in the chiral limit. This makes the interpretation of our results quite difficult. A possible way to understand if the behavior of the data presented in the last section is already in the asymptotic region of small quark masses would be to identify a meaningful reference quantity to fix the scale in the theory. In QCD it is customary to use quantities such as the string tension σ or the Sommer scale r_0 in the chiral limit. For QCD, r_0 was chosen so that $r_0 \approx 1/\sqrt{\sigma}$, and thus the two quantities stand on equal footing. If the theory if conformal in the IR however, the string tension will vanish in the chiral limit, and it is therefore not a good candidate for setting the scale. One can nonetheless use r_0 as in QCD. In fact, one can always consider the static potential between two fundamental charges in the theory $V(r)$ and the force $F(r) \equiv dV(r)/dr$ associated with it. Then the quantity $H(r) \equiv r^2 F(r)$ can be used to define the coupling since it is proportional to $\alpha_s(\mu = 1/r)$ in perturbation theory. In a QCD–like theory $H(r)$ will run from zero to infinity and any value in this range can be used to set a scale r_0. For QCD conventionally the Sommer scale is defined by $H(r_0) = 1.65$. If on the other hand, the massless theory is conformal in the IR, at exactly zero quark mass $H(r)$ will run from zero to some finite value $h^\star$. In this case any value in the range $]0, h^\star[$ can be used to set a scale in the theory. The presence of a finite quark mass will spoil the IR conformal behavior. One would imagine that at non-zero quark mass m the coupling $H(x, m)$ will follow the massless $H(x)$ up to some length scale $\tilde{x}(m)$; the latter scale diverges as $m \to 0$, while at distances larger than

[4] There is only one work in the literature that uses lattices comparable to the ones used in this work [21], which however is limited to very heavy quark masses.



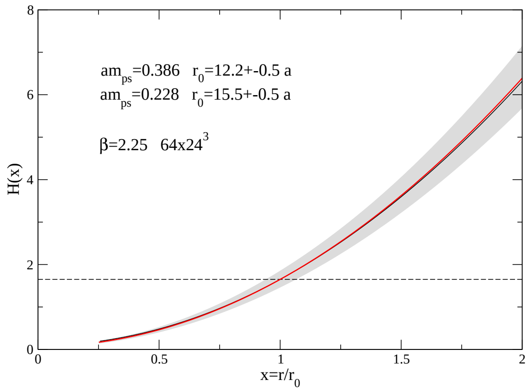


Figure 5: (Left figure) upper panel: string tension as extracted from the Creutz ratios of Wilson loops for two different values of the quark mass. Lower panel: the corresponding static potential between fundamental charges extracted from Wilson loops. (Right figure) Evolution of the coupling $H(r) = r^2 F(r)$ in the $\overline{q}q$ scheme in units of r_0. The red curve corresponds to the lighter quark mass, while the virtually identical black line to the heavier mass.

$\tilde{x}(m)$ the coupling will start to run again similar to what happens in QCD. However if we choose a value $h \in]0, h^\star[$ to define the scale r_0, such that $H(r_0, m) = h$, this quantity will remain finite in the chiral limit, and could be used as a reference scale. We show in Figs. 5 our preliminary results for the static potential, the string tension and the coupling in the $\overline{q}q$ scheme $H(r)$. These results are obtained on a 64×24^3 lattice at two of the smallest mass studied in this work, and only a limited number of configurations, and as such should be taken with a grain a salt. One step of HYP smearing was used to improve the signal. We extract the string tension from Creutz ratios of Wilson loops, Fig. 5 (left) upper panel, which can then be used as an input to fit the static potential $V(r)$ extracted from the (rectangular) Wilson loops. The parametrization used in the fit is the usual Cornell potential: $V(r) = \alpha + \sigma r + \gamma/r$. Using the fitted values for the string tension σ and γ, we can plot in Fig. 5 (right) the coupling $H(r)$, from which the Sommer scale can be defined: $H(r_0) = 1.65$. Here in the definition we used the same value as in QCD, as the measured coupling does not show a plateau in the limited range of distances explored in our data.

We measure a small string tension in units of the lattice spacing, $a\sqrt{\sigma} \simeq 0.1$, and a quite large $r_0 \simeq 15a$. Similar values for the string tension are also obtained when using a much more refined variational method based on expectation values of Polyakov loops (see Ref. [29] for a summary of our results; a more detailed report is in preparation). Moreover, the dependence of the string tension on the quark mass seems to be pronounced, while r_0 appears to be more stable. The coupling $H(r)$, when plotted in units of r_0 as in Fig. 5 (right), shows no residual dependence on the mass within errors. While the physical meaning of r_0 defined as above if the theory is conformal is not clear, such large values of r_0 may suggest that the pseudoscalar masses of order $\simeq 0.2a^{-1}$ or bigger, are still "heavy" and not in the chiral regime of the theory. For example in QCD the values we observed, i.e. $m_\pi r_0 \approx 3.4$, corresponds to $m_\pi \approx 1.4$ GeV in a box of $L \approx 0.8$ fm, and features similar to the ones we show in this report are expected.

4 Conclusions

We have presented a careful investigation of the mesonic spectrum of one of the prime candidate theories for a realistic technicolor model. To study the low-lying spectrum of the SU(2) gauge theory with two Dirac adjoint fermions we used numerical lattice simulations. Such numerical simulations are an extremely powerful tool to explore the non-perturbative dynamics of gauge theories which is otherwise inaccessible to theoretical speculations, but great care must be taken to control systematic errors. We presented here the first study of the low-lying spectrum aiming at the chiral limit in a controlled way. Using a series of four different lattice sizes we showed how finite-size effects could be kept under control in the range of quark masses explored. The resulting behavior of the different quantities analyzed, namely the pseudoscalar and vector meson mass and the pseudoscalar decay constant, show significant deviations from what is expected in the presence of spontaneous chiral symmetry breaking. In particular we showed that the pseudoscalar mass does not seem to scale with the square root of the quark mass, the ratio of the vector to the pseudoscalar mass differs from one only by a few percents, and F_{PS} seems to extrapolate to zero in the chiral limit. While our data alone is still not enough for a proof, if the behavior we observed persists down to arbitrary small quark masses this will be a clear evidence for the existence of an IR fixed point.

One of the major difficulties at present is to set a significant reference scale for comparing other measured quantities. We showed that a scale analogous to r_0 in QCD can always be defined even if the theory is conformal, but in this case its physical meaning is not clear. With the same definition as in QCD, the numerical value for r_0 is around $15a$ at the smallest quark mass used in this study. We also measured the string tension, obtaining quite small values in units of the lattice spacing. This is certainly consistent with an IR fixed point, but again not enough to draw any definitive conclusion.

To find solid evidence for the existence of an IR fixed point will certainly require a large numerical effort. The use of different and complementary approaches to the one presented here, like the study of gluonic observables, or the measure of the running couplings to study the renormalization group flow, will be beneficial.

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