From confinement to adjoint zero-modes

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1 Introduction

Recently [1] we have been embarked in a program aiming at obtaining the analytic expression and properties for the zero-modes of the Dirac equation in the adjoint representation in the background of caloron solutions [2, 3]. Obviously, calorons are configurations that play an important role in the semiclassical study of Yang-Mills theory at finite temperature, and our zero-modes inherit this importance for the case of Supersymmetric Yang-Mills theory. However, our work fits into a much broader context which has guided part of the work done by our group in a period spanning more than a decade. It is precisely a short promenade over this intellectual itinerary, as the title reflects, what we are presenting in this talk and contribution of the conference. Although, a good deal of the ideas reviewed can be found in published papers, we find it useful to include them here, since the ultimate goal has not been accomplished and some of them are not widely known.

The starting point of our promenade is the problem of Confinement in non-abelian gauge theories in four space-time dimensions. After several general considerations about the problem itself, we review our proposal [4] for a microscopic mechanism explaining confinement, which is based upon the presence of fractional topological charge instantons in the Yang-Mills vacuum. Its merits and its problems are presented. In the following section we review some of the results obtained in the last decade which validate the term "instanton quarks" for these fractional instantons. In particular, its connection to calorons is explained. The next section, deals with adjoint zero-modes and how they fit into this program, to conclude reviewing some aspects of our results for calorons. We end with some closing remarks. Bon Voyage!

2 Confinement

Confinement is a fascinating non-perturbative property that we believe is shared by QCD, the subject matter of this conference. Many papers have been written to give and explanation of this property and a proof that it occurs in QCD. Very often, sterile conflict and debate originates by semantic problems about what is meant by Confinement, and what is there to be explained. Thus, we want to start this section by discussing some aspects:

What is Confinement?

Confinement in QCD is tied to the observation that there seems to be no asymptotic states having fractional baryon number, electric charge, and the rest of quantum numbers of quark fields. In seeking for an intuitive reason why this is so, one concludes that this must be due to the strength of the interaction among these fundamental fields. This idea led to the effective or macroscopic description of the phenomenon which was understood and explained in the 70's.

Effective theory of Confinement

The fundamental work of K. Wilson, G. 't Hooft, S. Mandelstam, and others enabled to reach an understanding of the origin of the forces leading to Confinement. It was first established that Confinement can be defined in Yang-Mills theories (without quarks) as a property leading to a linear rise (with separation) of the potential between two (non-dynamical) quark sources. Wilson's lattice formulation of gauge theories allows to analyze the problem from a classical statistical mechanics point of view. The area law serves to characterize the Confinement phase, which turns out to be a typical phase of a gauge theory. This contributed to lift what could be initially regarded as a strange proposal by Weinberg, into a natural and presumably plausible possibility.

In an abelian gauge theory, Gauss law and rotational invariance predicts how the potential energy among two charges depends upon distance. In two space-time dimensions linear confinement takes place. In 3+1 dimensions we have the standard Coulomb law and no confinement. However, if the electric flux is concentrated along 1-dimensional flux tubes instead linear confinement takes place. No matter how bizarre this possibility might look, 't Hooft and Mandelstam realized that all one needs is a dual version of what happens with magnetic flux in superconductors.

Microscopic mechanism of Confinement

The previous point explains the nature of the Confinement property. It is the equivalent of the description of Superconductivity as the result of the condensation of a charged field. However, this is not the end. One still needs to explain why a particular system exhibits this property and others don't. We dub this explanation, a *microscopic mechanism for confinement*. For ordinary superconductors this is provided by BCS theory. A similar explanation for high temperature superconductors is still under debate. Equivalently, it is simple to prove confinement in certain systems. For example, it is simple to compute the string tension in Z_N gauge theories in a 2-d lattice, in terms of the finite free energy of vortices. In 3-d vortices are one dimensional and they are ineffective in leading to confinement at zero temperature.

However, in compact abelian gauge theories Confinement in 3-d is obtained as a result of the finiteness of the free energy of monopoles. In 4-d abelian gauge theories neither vortices nor monopoles are effective in leading to Confinement at sufficiently low temperatures.

The previous considerations are firmly established and provide the setting for discussing the occurrence of Confinement in 4-d (3+1) non-abelian gauge theories. Many authors have proposed and attempted to employ vortices and monopoles to prove Confinement for those theories. Any explanation should show how it works specifically for non-abelian theories in the continuum limit.

Fractional topological charge mechanism for Confinement

In a series of papers [4], our group proposed in the 90's a microscopic mechanism based on the role played by structures present in the Yang-Mills vacuum and carrying fractional topological charge. These objects are specific of non-abelian theories and are point-like in 4-d, as are vortices in 2-d and monopoles in 3-d. Our proposal resembles strongly that made by Callan, Dashen and Gross many year earlier [5]. These authors, however, based their proposal upon the singular solutions introduced by Alfaro, Fubini and Furlan [6], called merons. Our fractional charge structures are non-singular.

In our opinion, our proposal has certain appealing features which we will now list:

- The existence of self-dual and smooth classical configurations with lumps of fractional topological charge $Q = 1/N$ is firmly established mathematically. These configurations have been analyzed numerically in great detail [7]. They have infinite action but finite action density and are local minima of the action.
- The counting of moduli parameters describing self-dual configurations is provided by the index theorem. The result is that there are four degrees of freedom for each $Q = 1/N$ lump, which we associate with its position. The space of self-dual configurations can be regarded as a 4-d gas of fractional instantons.
- One of the typical reasons why instantons are not considered good candidates for confinement is because their free energy diverges in the large N limit. Our configurations with fractional charge escape this problem.
- It is well-known that the QCD vacuum has a non-zero topological susceptibility. In our model the same objects are responsible for this phenomenon as for Confinement. This establishes a relation between the topological susceptibility and the string tension which is in rough agreement with the data.
- This mechanism is specific of 4-d non-abelian gauge theories.
- Since classical Yang-Mills is a conformal theory fractional instanton solutions exist in all sizes. However, quantum fluctuations break the conformal invariance

and end up determining the characteristic size of these configurations in physical units. Like for the case of instantons the small sizes are more improbable, while large sizes are limited by the presence of neighboring structure. This ends up producing a fairly dense media, best described as a liquid.

- It is known that instantons do not produce Confinement. We gave both heuristic and numerical evidence that these configurations do indeed give a non-vanishing string tension. We prepared non-thermal configurations containing an array of fractional instantons and measured the Wilson loop in them. The linear potential term is seen clearly in Fig. 1.

- Our group was led to the idea of fractional instantons by the study of the evolution of the Yang-Mills dynamics from small to large periodic spatial volumes. The minimum energy in each electric flux sector was measured from small sizes, where semiclassical methods are precise, to large volumes, where the infinite volume theory is recovered. For twisted boundary conditions in space (non-zero 't Hooft magnetic flux), the evolution is smooth as shown in Fig. 2 (Left) and energies fuse into the value dictated by the string tension. The Monte Carlo results match for small volumes with the computation from perturbation theory and the semiclassical approximation. For the latter, the role of fractional instantons is crucial. Fig. 2 (Right) shows a typical configuration (after smoothening) displaying the presence of fractional (and ordinary) instantons as a function of time.

- Early on the presence of fractional topological charge configurations in the Yang-Mills vacuum was advocated by Zhitnitsky [8] to explain a puzzle in the theta vacuum dependence with N.

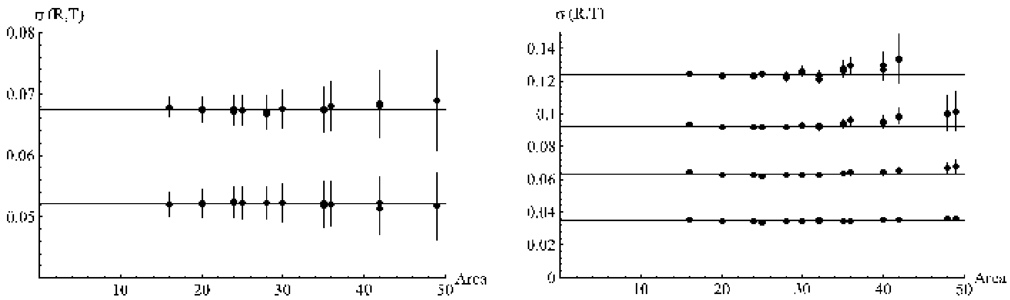

Figure 1: The string tension vs the area of the Wilson loop measured on non-thermal configurations containing an array of fractional instantons. Horizontal lines present results after different number of cooling steps (increasing from bottom to top).

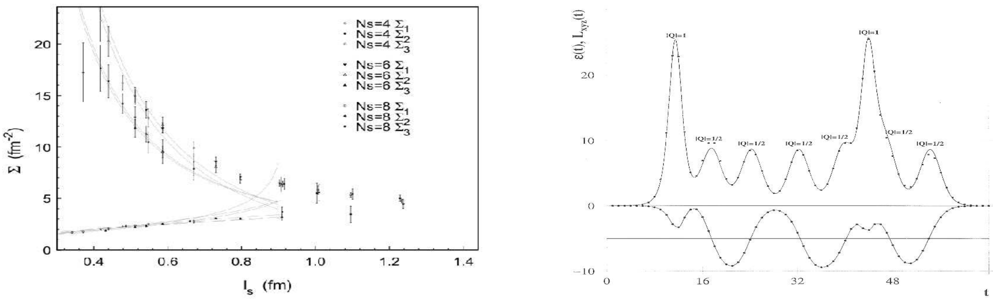

Figure 2: Left: Effective string tension extracted from the electric flux energies as a function of the size of the twisted box. Right: Energy profile (top) and Polyakov line (bottom) of a typical Monte-Carlo configuration for a small box.

Despite all its appealing features, our program did not succeed in producing conclusive evidence of our description of the Yang-Mills vacuum and its corresponding explanation of the Confinement phenomenon in non-abelian gauge theories in 4-d. The main reason is due to the following difficulties:

- Although we know that they exist and their shape and properties have been studied numerically, we lack an analytic formula for a fractional instanton configuration. Calorons (see later) have ameliorated the situation.

- As explained previously the dynamical role of these configurations, and the typical scales involved depend on its quantum weights. Lacking a precise analytic formula for the configurations we also lack an analytic handle over these weights.

- In any given analytic description of the Yang-Mills vacuum one would like to have an ansatz describing a typical vacuum configuration. In achieving this goal one faces enormous difficulties. First of all, if the picture we describe is correct the dilute gas approximation is non-valid. A dense environment of fractional instantons of equal sign could be described by complicated high topological charge formulas. In addition, one has to face the problem that fractional instantons of opposite signs should be present and the configuration is not even a classical solution of the equations of motion.

- Ultimately, in order to check any proposal of this kind one should try to resort to the analysis of Monte Carlo generated lattice configurations. However, raw configurations tend to be very rough as a result of ultraviolet divergences. A smoothening technique is required. Although, we presented evidence that the

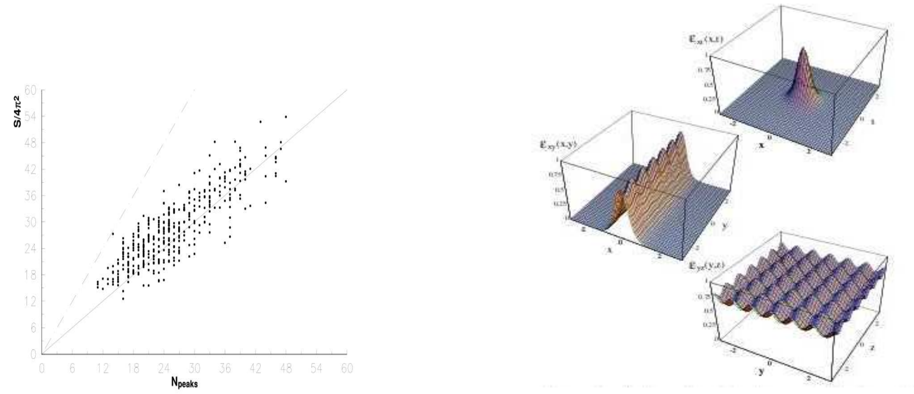

Figure 3: Left: Scatter plot showing the action in units of $4\pi^2$ vs the number of action-density peaks for several cooled Monte Carlo configurations. Right: Several 2-d sections of the action density of the self-dual vortex solution.

lumps present in smoothened configurations carry fractional charge (See Fig. 3 Left), this and other results have been questioned because they employed the cooling method as a smoothening technique.

- Our work had also to face certain misunderstanding on the role played by twisted boundary conditions employed in some of our papers. It should be realized that for large volumes the boundary conditions are inessential. However, this is not the case for small volumes. Twisted boundary conditions are more effective in showing fractional instantons in the low density regime. To understand the situation one should compare with the similar situation taking place in 1-d double-well scalar models. While kinks are crucial to understand the dynamics at large volumes, antiperiodic boundary conditions are more efficient than periodic ones in approaching this limit from small volumes.

3 Fractional instantons and related objects

Although no analytic expression has been obtained for the 4d fractional instantons, there has been considerable progress on other classical structures, **calorons**, which are intimately connected to them.

A caloron is a self-dual Yang-Mills configuration in $S_1 \times R^3$. In other words, it is an instanton-like configuration which is periodic in one direction. In a series of papers [2, 3], analytic expressions were obtained for $Q = 1$ calorons. These formulas show in glory detail how for a wide range of parameters of the solution, these objects decompose into N (for SU(N) gauge group) lumps carrying non-integer charges adding up to 1. This is precisely the dissociation that we advocate, and exhibit numerically, to take place in 4d. The individual lumps which make a caloron were called *constituent monopoles* in Ref. [2]. The way in which the topological charge is split into all its constituents is dictated by the holonomy of the configuration $P_\infty = \mathrm{diag}\{e^{i2\pi Z_i}\}$.

The approach followed by Kraan and van Baal to obtain the analytic formulas for calorons makes use of the **Nahm transform**. This is a transformation which maps SU(N) self-dual gauge fields with topological charge Q onto SU(Q) self-dual fields with topological charge N. The transformed field $\hat{A}_\mu(z)$ is obtained as follows:

$$\hat{A}_\mu(z) = i \int dx\, \psi^\dagger(x,z) \frac{\partial}{\partial z_\mu} \psi(x,z) \tag{1}$$

where ψ is the solution of the modified Weyl equation in the fundamental representation

$$\bar{\sigma}_\mu (D_\mu - 2\pi i z_\mu) \psi(x,z) = 0 \tag{2}$$

One important property is that the Nahm transform is an involution. Furthermore, periodic gauge fields map onto periodic gauge fields with inverse periods.

In the case of $Q = 1$ calorons the Nahm transform gives rise to 1-d periodic abelian gauge fields $\hat{A}_\mu(z)$, which are self-dual except at discontinuities located at the eigenvalues of the holonomy $z = Z_i$. After solving for $\hat{A}_\mu(z)$, applying a Nahm transformation, one obtains the caloron formulas, which depend upon $4N$ real parameters, in agreement with the index theorem. As mentioned previously the action density is generically arranged into N independent monopole lumps, whose mass (topological charge) is proportional to $Z_{i+1} - Z_i$. (The fermion zero-modes in the fundamental representation have also been studied [9]).

Instanton-quarks

What connection is there between the constituent monopoles and the fractional instantons of our model of Confinement? It was found [10] that the constituent monopoles of minimal holonomy ($Tr(P) = 0$) calorons can be obtained as 1-d arrays of our fractional instantons. This explains the periodicity in 1-d. In this fashion the latter appear as the building blocks from which calorons are built. This substantiates the idea of instanton quarks and identifies fractional instantons with them. Although, instanton quarks cannot exist isolated th ey can be arranged into self-dual (and hence classically stable) smooth structures. One-dimensional arrangements can be identified with the world line of monopoles, which as mentioned previously coincide with the constituent monopoles of calorons. Similarly two-dimensional arrangements describe the world sheet of non-abelian self-dual vortices which have been shown to exist (See Ref. [11] and Fig. 3 Right).

At zero-temperature we do not see any reason why there should be a hierarchy among distances to neighboring fractional instantons, which should rather resemble a 4-d liquid. Ultimately, the question of whether instantons dissociate into instanton quarks and their possible tendency to arrange into 1,2,3, or 4-dimensional structures is determined by free energy (or quantum weight) of each type of configuration. On general grounds entropy favors dissociation. Recent results on calorons point in this direction as well [12].

4 Adjoint quarks as probes

Recently the interest in the study of Yang-Mills theory with quarks in the adjoint representation has boosted. Traditionally, most of the results were focused in the case of gluino fields in Supersymmetric Yang-Mills theory. Lately other QCD-like theories have received a lot of attention, some of which possess quarks in the adjoint. The reasons are varied ranging from studies in Technicolor, large N reduction, and in general as testing grounds of several concepts of Yang-Mills dynamics.

There is another goal of using adjoint quarks, which we have been concerned with, and fits into the general program of elucidating the structure of the Yang-Mills vacuum and the origin of Confinement. As mentioned previously, any proposal should be contrasted and checked versus configurations obtained by Monte Carlo simulations on the lattice. The main difficulty is how to extract the information about the structures contained in these configurations filtering the higher momentum fluctuations. Traditionally cooling and smearing algorithms have been used to derive smooth enough configurations which can be analyzed for structures. Although the methodology is generally accepted in extracting global quantities, as the total topological charge, their usage to probe local structures has been criticized, since both mechanisms might modify the initial gauge configuration in an uncontrolled way.

In the last years, a new approach based on the Dirac operator has been employed [13]. The low-lying eigenmodes of the Dirac operator can be regarded as observables of a pure gauge theory which are less sensitive to the high momentum fluctuations of the field. Furthermore, they should serve to track the structures present in a given configuration and their topological nature. From this general idea to a particular proposal one has to determine which eigenmodes will be used and how. It is also possible to apply this idea for quarks in any representation, although most of the work has focused upon the fundamental representation. One difficulty seems to be that even if the configuration is smooth the fermionic densities of the zero-modes do not exactly reproduce the shape of the gauge action density. In particular, for instantons for example, they fall off with a different power. There is a notable exception to this for the case of the adjoint representation. This is precisely the idea that inspired a new proposal [14], which is explained in the next paragraph.

For any gauge field configuration, A_μ, which is a solution of the classical equations of motion, and any constant 4-spinor v, there exists a zero-mode of the Dirac operator in the adjoint representation $\psi^a_{ss} = \frac{1}{8} F^a_{\mu\nu} [\gamma_\mu, \gamma_\nu] v$, whose density is proportional to the gauge action density. This is the "supersymmetric zero-mode". Furthermore, each of its chiral components reproduces the self-dual or anti-self-dual part of the action density. This is not the only property that makes the supersymmetric zero-mode special. A look at the previous expression shows that, for an appropriate choice of v, $\mathrm{Im}(\psi^a_{ss})_1 = 0$ at all points of space. In particular, for the Weyl representation of the Dirac matrices and positive chirality, $v = (1, 0, 0, 0)$ does the job.

The previous facts form the basis of our filtering method [14]. One selects the quasi-zero mode of the Dirac operator in each chirality that satisfies the reality condition explained in the previous paragraph. The corresponding density provides a filtered version of the action density and topological charge density. Practical implementations applied to configurations on the lattice give promising results. As an example, suppose that one starts from a smooth SU(3) Yang-Mills $Q = 1$ configuration, an instanton. If we now apply even a small number of heat-bath sweeps at $\beta = 6.4$, the configurations becomes very rough and neither the action density nor the topological charge density reveal the underlying instanton structure. This is displayed in the left and middle graphs of Fig. 4. On the right, the filtered action density is displayed, showing the instanton with the right size and location. The latter is obtained as the density of the ground state of the operator O_+ obtained by projecting the square of Neuberger's Dirac operator onto the space of positive chirality vectors satisfying the reality condition.

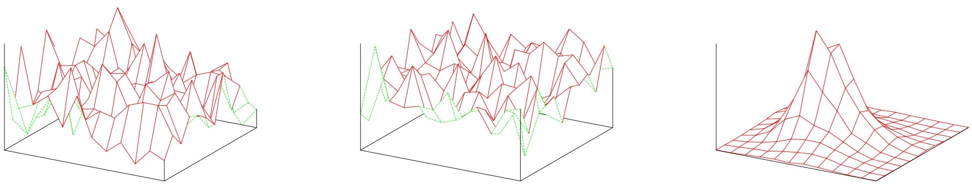

Figure 4: From left to right: 2-d slice of the action, topological charge and susy zero-mode densities after applying eight, $\beta{=}6.4$, heat-bath sweeps to an instanton.

5 Adjoint zero-modes for calorons

Motivated by the ideas explained in previous sections, our group has recently derived analytic formulae for the adjoint (gluino) zero-modes in the background of $Q = 1$, SU(N) calorons with non-trivial holonomy [1]. We have studied both periodic and antiperiodic solutions relevant for supersymmetric and finite temperature compactifications respectively. Here we will present a brief summary of our results and refer the interested reader to the original papers for further details.

We note that Weyl adjoint-zero modes appear in CP-pairs which can be arranged into a 2×2 quaternionic matrix: $\Psi = (\psi, \psi_C)$. Our construction relies on the relation between solutions of the left-handed Weyl equation and self-dual deformations of the gauge field δA_μ. It can be shown that $\Psi \equiv \delta A_\mu \sigma_\mu$ is a solution of the adjoint Weyl equation, $\bar{\sigma}_\mu D_\mu \Psi = 0$, provided that δA_μ satisfies the background gauge condition ($D_\mu \delta A_\mu = 0$). This connection allows to obtain adjoint zero-modes by differentiating the gauge potential with respect to the parameters of the moduli space of solutions. As an example, the supersymmetric zero-mode discussed in the previous section can be derived from the deformations of the gauge field associated to translations, i.e. $\delta A_\mu = \partial_\rho A_\mu + D_\mu(-A_\rho) = F_{\rho\mu}$. The construction provides zero-modes that are periodic in the thermal cycle with the same period as the gauge field A_μ. In order to obtain antiperiodic solutions, relevant for finite temperature, one has to resort to what we have called the *replica* trick. It is based on the observation that antiperiodic solutions become periodic in the double period and can thus be extracted from deformations of the $Q = 2$ solution. This *replica* procedure can be easily generalized to higher number of replicas and allows to obtain zero-modes with arbitrary periodicity.

Although the general caloron solution for arbitrary charge Q and its moduli, is not known, the self-dual deformations of the replicated caloron gauge field can be derived by making use of the Nahm-ADHM formalism introduced in section 3. The Nahm-dual gauge field associated to the L-times replicated caloron is a one-dimensional U(L) gauge field given by $\hat{A}^R_\mu(z) = \mathrm{diag}(\hat{\mathrm{A}}_\mu(\mathrm{z}+(\mathrm{n}-1)/\mathrm{L}))$, $n = 1, \cdots, L$, with $\hat{A}_\mu(z)$ the Nahm-dual of the $Q=1$ caloron gauge field. Without entering into details, it can be shown that the problem of finding the adjoint zero-modes of the Dirac equation can be mapped into the simpler one of solving the Nahm-dual adjoint Dirac equation, with gauge field $\hat{A}^R_\mu(z)$ and delta function sources at $z = Z_a + (n-1)/L$. The general solution of the latter for arbitrary number of replicas and the formulas linking them to adjoint zero-modes of arbitrary periodicity can be found in [1] and will not be detailed here. Here we will focus on discussing how the spatial structure of the zero-modes relates to the location of the constituent monopoles. We will show how the results fit nicely into the predictions of the index theorem [15, 16].

In the limit of well separated constituent monopoles the structure of periodic zero-modes of the caloron is rather simple. There is a CP-pair of zero-modes centered at each monopole, in accordance with the prediction of the Callias index theorem [15]. On the contrary, anti-periodic zero-modes exhibit a more complicated pattern which we will illustrate for gauge group SU(3). The number of adjoint zero-modes predicted

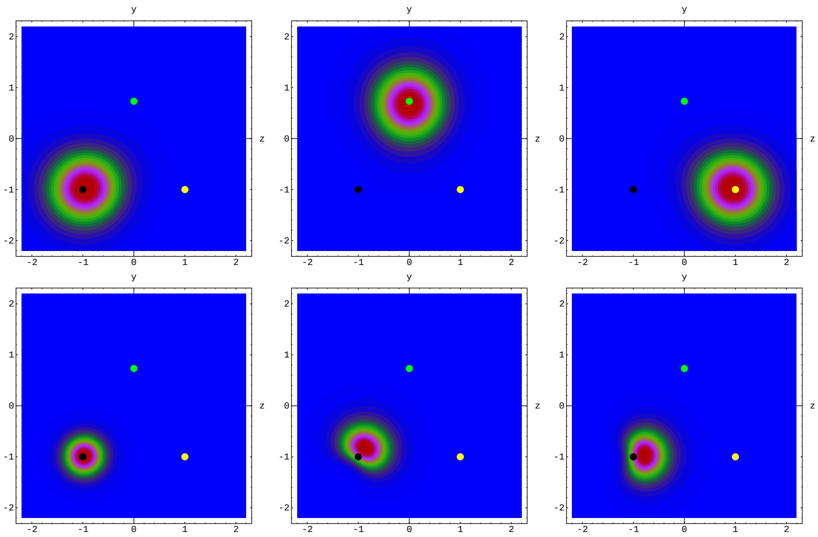

Figure 5: Contour plots of the SU(3) antiperiodic zero-mode densities at $x = t = 0$. The top (bottom) row corresponds to monopole masses: $m_1 = m_2 = m_3 = 2\pi/3$ ($m_1 = 4\pi/3$, $m_2 = m_3 = \pi/3$). The small filled circles indicate the monopole positions.

by the index theorem for the SU(3) L-times replicated caloron is given by [16]:

$$I_{\rm adj}(L, n_1, n_2, n_3) = \sum_{a=1}^{3} 2n_a(3q_a + \epsilon)\,, \tag{3}$$

where the index a parameterizes each of the constituent monopoles, $\epsilon \equiv L - \sum_a q_a$, and $q_a \equiv [Lm_a/2\pi]$, with $[v] \equiv \max\{n \in \mathcal{Z} \,|\, n \leq v\}$, and m_a the masses of the constituent monopoles. For the L-times replicated caloron $n_a = 1$, $\forall a$, and the total index correctly gives $6L$ adjoint zero-modes. According to the formula, out of those there are $2(3q_a + \epsilon)$ associated to the a-th constituent monopole. Periodic zero-modes correspond to $L = 1$ for which $I_{\rm adj} = \sum_a 2n_a$, i.e. each monopole carries two periodic zero modes as expected. Antiperiodic zero-modes can be analyzed by looking at the $L = 2$ case. Their distribution among the constituent monopoles depends on the values of the q_a which can be 0 or 1 for m_a smaller or larger than π. We can distinguish three different cases:

- $\vec{q} = (0, 0, 0)$ for which $I_{\rm adj} = \sum_a 4n_a$ and two periodic plus two anti-periodic zero modes are attached to each monopole.
- $\vec{q} = (1, 0, 0)$ for which $I_{\rm adj} = 8n_1 + 2n_2 + 2n_3$, implying that a single monopole supports all 6 anti-periodic zero-modes.
- $\vec{q} = (1, 1, 0)$ for which $I_{\rm adj} = 6n_1 + 6n_2$. This is a limiting case corresponding to one massless constituent monopole with no zero-modes, while each of the other two,

with $m = \pi$, support 4 antiperiodic zero-modes.
The previous expectation matches with our results as shown in Fig. 5. This example displays contour plots for the antiperiodic zero-mode densities for a caloron having monopoles located in the vertices of an equilateral triangle with masses $m_1 = m_2 = m_3 = 2\pi/3$ (Top), and $m_1 = 4\pi/3$, $m_2 = m_3 = \pi/3$ (Bottom).

6 Closing remarks

We want to close this manuscript by thanking the organizers for the invitation to contribute to this conference. Special thanks go to Mithat Unsal who convinced us to participate in this exciting session. In this talk we have attempted to review a series of results obtained by the authors, putting them in perspective within a much broader research program. We are aware that the goal was too ambitious for such a short space, but we hope it has triggered the interest to consult the original papers and induce discussion.

References

[1] M. García Perez and A. González-Arroyo, JHEP **0611** (2006) 091. M. García Perez, A. González-Arroyo and A. Sastre, Phys. Lett. B **668** (2008) 340; JHEP **0906** (2009) 065.

[2] T. C. Kraan and P. van Baal, Phys. Lett. B **428** (1998) 268; Nucl. Phys. B **533** (1998) 627; Phys. Lett. B **435** (1998) 389.

[3] K. M. Lee, Phys. Lett. B **426** (1998) 323. K. M. Lee and C. h. Lu, Phys. Rev. D **58**, 025011 (1998).

[4] M. García Perez, A. González-Arroyo and P. Martínez, Nucl. Phys. Proc. Suppl. **34** (1994) 228. A. González-Arroyo and P. Martínez, Nucl. Phys. B **459** (1996) 337. A. González-Arroyo, P. Martínez and A. Montero, Phys. Lett. B **359** (1995) 159. A. González-Arroyo and A. Montero, Phys. Lett. B **387** (1996) 823.

[5] C. G. . Callan, R. F. Dashen and D. J. Gross, Phys. Rev. D **17** (1978) 2717.

[6] V. de Alfaro, S. Fubini and G. Furlan, Phys. Lett. B **65** (1976) 163.

[7] M. García Pérez, A. González-Arroyo and B. Soderberg, Phys. Lett. B **235** (1990) 117. M. García Pérez and A. González-Arroyo J. Phys. A **26** (1993) 2667 [arXiv:hep-lat/9206016].

[8] A. R. Zhitnitsky, Phys. Lett. B **291** (1992) 465. Nucl. Phys. B **340** (1990) 56.

[9] M. García Perez, A. González-Arroyo, C. Pena and P. van Baal, Phys. Rev. D **60** (1999) 031901. M. N. Chernodub, T. C. Kraan and P. van Baal, Nucl. Phys. Proc. Suppl. **83** (2000) 556. F. Bruckmann, D. Nogradi and P. van Baal, Nucl. Phys. B **666** (2003) 197.

[10] M. García Perez, A. González-Arroyo, A. Montero and P. van Baal, JHEP **9906** (1999) 001. M. García Perez, A. González-Arroyo, C. Pena and P. van Baal, Nucl. Phys. B **564** (2000) 159.

[11] A. Gonzalez-Arroyo and A. Montero, Phys. Lett. B **442** (1998) 273. A. Montero, Phys. Lett. B **483** (2000) 309.

[12] D. Diakonov and V. Petrov, Phys. Rev. D **76** (2007) 056001, and references therein.

[13] See for example F. Bruckmann, PoS **LAT2007** (2007) 006 and references therein.

[14] A. González-Arroyo and R. Kirchner, JHEP **0601** (2006) 029. M. García Perez, A. González-Arroyo and A. Sastre, PoS **LAT2007** (2007) 328.

[15] C. Callias, Commun. Math. Phys. **62** (1978) 213.

[16] E. Poppitz and M. Unsal, JHEP **0903** (2009) 027.