The state of cold quark matter:
a model-independent view

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Abstract

From a model-independent point of view, we address the possibility that quark clustering could occur in cold quark matter at realistic baryon densities because of the likely strong coupling between quarks in compact stars.

1   Introduction

In this paper, I would like to present an idea about the state of cold quark matter, for your comments and suggestions. It is generally supposed that color superconductivity (CSC) will occur in dense quark matter. However, besides this CSC state, another one may be possible: strong coupling could cause quarks to *cluster* in realistic dense quark matter in which case a *solid* state of quark matter would exist at low temperatures.

This conjecture may alleviate many problems that challenge astronomers and astrophysicists. For instance, we still understand neither the supernova explosion mechanisms, nor the central engines of Gamma-ray bursts (GRBs). Also, what's the nature of pulsar-like stars, especially the so-called magnetar candidates (AXPs, anomalous X-ray pulsars, and SGRs, soft gamma-ray repeaters)? The conjecture also has bearing on some related problems in physics. For example, what are the properties of the strongly coupled quark-gluon plasma (sQGP)? How can one draw the QCD (quantum chromo-dynamics) phase diagram? All of these problems, I think, may be related to a *big* challenge: the physics of (cold) matter at supra-nuclear density. In principle, that equation of state could be derived from the first principles (i.e., QCD), however this is as yet not possible because of the strong noneperturbative effects at low energy scale. One requirement for solving the big problems is to have physicists and astronomers to exchange information during their work when trying to "dig a tunnel".

The outlines of this paper is as follows. In §2, I will start with an analogous but simple phase diagram, that of water (H_2O). Then, in §3, the conjectured QCD states of cold quark matter, color-superconducting *v.s.* quark-clustered, are discussed, and our ideas to understand various observations are presented in §4. Finally, I summarize in §5.

2   An analogy: the H_2O phase diagram

Let's begin with the phase diagram of water, rather than the QCD phase diagram, since the H_2O phases make it easier for us to understand the essential but similar physics.

Suppose I give you a large number of particles of electrons (e), protons (p), and oxygen nuclei (^{16}O), at room temperature of $T \sim 300$ K and density of $\rho \sim 1$ g/cm^3, with a number proportion of

$$n_e : n_p : n_O = 10 : 2 : 1.$$

Certainly attractive and repulsive forces exist between those particles. My question is: What's the state of such a particle system? You know the answer from experience: liquid water! However, as a physicist who may have never seen water, can you predict the state of this particle system by calculations, starting from the laws of physics?

We can first compute the number densities of the particles,

$$n_e = 3.35 \times 10^{23}/\text{cm}^3, \quad n_p = n_e/5, \quad n_O = n_e/10,$$

and also the distances between each kind of particle,

$$l_e = n_e^{-1/3} \sim 1 \times 10^{-8}\text{cm}, \quad l_p \sim 2 \times 10^{-8}\text{cm}, \quad l_O \sim 3 \times 10^{-8}\text{cm}.$$

Then we can also calculate their quantum wavelengths,

$$\lambda_e = \frac{h}{\sqrt{2m_e kT}} \sim 8 \times 10^{-7}\text{cm}, \quad \lambda_p \sim 2 \times 10^{-8}\text{cm}, \quad \lambda_O \sim 4 \times 10^{-9}\text{cm}.$$

By comparing the distances and the wavelengths, you my conclude: quantum Fermi-Dirac statistics applies to the electrons and protons, while classical Maxwell-Boltzmann statistics applies to oxygen-16.

Furthermore, one can also estimate the chemical potentials of the degenerate electrons and protons. Assuming, for simplicity, zero temperature and ignoring the electromagnetic interactions, one comes to

$$\begin{cases} \mu_e = \dfrac{\hbar^2}{2m_e}(3\pi^2)^{2/3} \cdot n_e^{2/3} = 18\text{eV} = 2 \times 10^5\text{K} \gg T, \\[2mm] \mu_p = \dfrac{\hbar^2}{2m_p}(3\pi^2)^{2/3} \cdot n_p^{2/3} = 0.003\text{eV} = 28\text{K} \lesssim T. \end{cases} \quad (1)$$

If interactions are neglected, the conclusions are then as follows:

$$
\begin{cases}
\textit{Electrons}: & \text{strongly degenerate Fermi gas at } T \sim 0; \\
\textit{Protons}: & \text{weakly degenerate, classical gas}; \\
O^{16} \textit{ nuclei}: & \text{classical gas (M} - \text{B statistics)}.
\end{cases}
$$

However, these conclusions would be wrong, because interactions do play an important role. Due to the electro-magnetic interaction, particle clusters have to form in the system: 13 particles (10 electrons, two protons, and one oxygen nucleus) will clustered in a group as one water molecular, H_2O.

Let's see how much further we could go using fundamental physics. Considering simply a cluster composed of an electron (e) and a proton (p), with a length scale l_{ep}, from Heisenberg's uncertainty relation, one infers a kinetic energy of $\sim p^2/m_e \sim \hbar^2/(m_e l_{ep}^2)$, which has to be comparable to the interaction energy of $E_{ep} \sim e^2/l_{ep}$ in order to have a bound state. One then finds,

$$
l_{ep} \sim \hbar^2/(m_e e^2) = \frac{1}{\alpha_{em}} \frac{\hbar c}{m_e c^2}, \quad E_{ep} \sim \alpha_{em} \frac{\hbar c}{l_{ep}} = \alpha_{em}^2 m_e c^2, \tag{2}
$$

where $\alpha_{em} = e^2/(\hbar c) = 1/137$ is the coupling constant of the electro-magnetic interaction. Such a simple estimate is very effective because we have $l_{ep} = 0.53$ Å, which is very close to the Bohr radius $a_B = 0.52$ Å, and interaction energy $E_{ep} = 27$ eV, approximately equals to the Rydberg constant $R_H = 13.6$ eV. Therefore, one has to modify the previous conclusion: there will be clusters of Hydrogen atoms ($e+p$) [as well as Oxygen atoms ($8e+^{16}O$)] in the particle system because $E_{ep} \gtrsim \text{Max}[\mu_e, \mu_p]$.

What's next? What's the real phase diagram of the system? This is still a challenging problem to solve from first principles only, and we have to use experiential knowledge. Certainly, we know that larger clusters (H_2O, i.e., the water molecules) will form if the residual electromagnetic interaction between H- and O-atoms is considered. Note that we are still *far away* from our final goal of reproducing the global phase diagram calculated from first principles, even though we know the elementary composition of the system, H_2O. Fortunately, we have this diagram although it is a long and hard work for experimentalists to obtain.

From above, we see that it is very difficult to infer the properties even for the very simple case of water. Actually, for any electromagnetic particle system, a general calculation of matter properties, including the phase diagram, is too complex to be possible because of the interaction, even though it has coupling $\alpha_{em} \ll 1$. Indeed, fascinating new quantum phenomena are continually emerging, which are related to the degenerate quantum gas, and a recent conference was focused on these topics (http://coldatom.castu.tsinghua.edu.cn/).

And then, what about the QCD phase diagram for quarks, for which the coupling parameters may well be close to and possibly even greater than 1?

3 Cold quark matter: color-superconducting or quark-clustering?

In this section, I would like to present a few points about the states of QCD matter, which I hope will lead to comments and suggestions.

Certainly, due to the nature of asymptotic freedom, when temperature T or baryon chemical potential μ_B are high in the QCD phase diagram, there is a phase in which quarks are deconfined. At the high density, low temperature regime on which we are focused, cold dense quark matter could be of Fermi gas or liquid *if* the interaction between quarks is negligible. However, the questions is: can the density in realistic compact stars be so high that we can neglect the interaction?

The average density of a pulsar-like star with a mass of 1.4 $M_\odot$ and a radius of 10 km is $2.4\rho_0$, where ρ_0 is the nuclear density. Even at this density, the quark degree of freedom could appear since the critical density n_c to break nucleons,

$$n_c = \frac{3}{4\pi r_n^3} \simeq 1.5 r_1^{-3} \rho_0,$$

is only about $2\rho_0$ if $r_1 \equiv r_n/(1 \text{ fm}) = 0.9$, where r_n denotes the nucleon radius. It could then be reasonable to consider realistic dense matter in compact stars already as quark matter.

For three-flavor quark matter with density of, e.g., $\rho = 3\rho_0$, one may make estimates similar to those done above for water. We have number densities for each flavor of quark, u, d, and s, of $n_u \simeq n_d \simeq n_s \sim (3 \times 0.16 = 0.48) \text{ fm}^{-3} \sim 5 \times 10^{38}/\text{cm}^3$, and typical separations,

$$l_u \simeq l_d \simeq l_s \sim 1.3 \text{ fm}.$$

The typical distance between quarks is then $l_Q \simeq l_u/(3^{1/3}) \sim 0.9$ fm. The quantum wavelength, however, is order of

$$\lambda_q \simeq \frac{h}{\sqrt{2m_q kT}} = 5 \times 10^3 m_{300}^{-1/2} T_6^{-1/2} \text{ fm} \gg l_Q$$

where quarks are assumed to be dressed, with mass of $m_q = m_{300} \times 300$ MeV, and $T_6 = T/(10^6 \text{ K})$. So quantum Fermi-Dirac statistics seems to apply to the case of cold quark matter. By turning off the interaction, a further calculation of the quark chemical potential at zero temperature shows

$$\mu_u^{NR} \simeq \mu_d^{NR} \sim \mu_s^{NR} \approx \frac{\hbar^2}{2m_q}(3\pi^2)^{2/3} \cdot n_u^{2/3} = 380\text{MeV} \gg T \tag{3}$$

if quarks are considered moving non-relativistically, or

$$\mu_u^{ER} \simeq \hbar c (3\pi^2)^{1/3} \cdot n_u^{1/3} = 480\text{MeV} \gg T \tag{4}$$

if quarks are considered moving extremely relativistically. Thus, with this estimate of a quark chemical potential order of 0.4 GeV, much higher than thermal kinetic energy, we would infer that realistic dense quark matter is a strongly degenerate Fermi gas.

However, as was shown in the last section for the case of water, interactions between quarks near the Fermi surface may play an important role. Similar to the condensed matter physics of low-temperature superconductivity, quark pairs may form due to attraction mediated by gluons, and cold quark matter may in a BCS-like color superconductivity (CSC) state (see [1] for a recent review). Such a CSC solution was found in perturbative quantum chromo-dynamics (pQCD) at extremely high density, and is supposed to exist in realistic quark matter of compact stars according QCD-based effective models.

But, what if the color-interaction is strong rather than weak? Similar to the case of clustered water molecular discussed previously, would the strong interaction result in the formation of quark clusters? There are repulsive and attractive interactions between two quarks, which is very analogous to the $\{e, p, O\}$-system presented in §2. A similar estimate to Eq.(2) for the length scale (l_q) and interaction energy (E_q) in a quark cluster gives if quarks are dressed,

$$l_q \sim \frac{1}{\alpha_s} \frac{\hbar c}{m_q c^2} \simeq \frac{1}{\alpha_s} \text{ fm}, \quad E_q \sim \alpha_s^2 m_q c^2 \simeq 300 \alpha_s^2 \text{ MeV}. \tag{5}$$

This is dangerous for the CSC state since E_q is approaching and even greater than ~ 400 MeV if $\alpha_s > 1$. So, what do we know about α_s for realistic dense quark matter from recent work on perturbative and non-perturbative QCD?

At extremely high density, the QCD coupling decreases as density increases, approximately as [2]

$$\alpha_s(\mu) \equiv \frac{g_s^2}{4\pi} \approx \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)} \tag{6}$$

in pQCD, where $\beta_0 = (11 - 2n_f/3)/(4\pi)$, n_f is the number of quark flavors, the renormalization parameter $\Lambda = (200 \sim 300)$ MeV, and the energy scale μ likely of order the chemical potential estimated in Eq.(3) and Eq.(4). One sees from Eq.(6) that $\alpha_s \sim 0.1 \gg \alpha_{\text{em}} = 1/137$ even if the density is unreasonably high, $\sim 10^6 \rho_0$. However, Eq.(6) cannot applicable when $\rho < 10\rho_0$, since there non-perturbative effects dominate.

How about α_s in non-perturbative QCD? This is a challenging problem. Indeed, non-perturbative QCD is related to one of the seven Millennium Prize Problems named by the Clay Mathematical Institute. Certainly, the coupling is strong at low energies, and the perturbative formulation is not applicable. Nevertheless, there are some approaches to the non-perturbative effects of QCD, one of which uses the Dyson-Schwinger equations (DSE). This method was tried by Fischer et al. [3, 4], who formed,

$$\alpha_s(x) = \frac{\alpha_s(0)}{\ln(e + a_1 x^{a_2} + b_1 x^{b_2})}, \tag{7}$$

where $a_1 = 5.292 \text{ GeV}^{-2a_2}$, $a_2 = 2.324$, $b_1 = 0.034 \text{ GeV}^{-2b_2}$, $b_2 = 3.169$, $x = p^2$ with p the typical momentum in GeV, and that α_s freezes at $\alpha_s(0) = 2.972$. For our case of dense quark matter at $\sim 3\rho_0$, the chemical potential is ~ 0.4 GeV [Eqs.(3) and (4)], and thus $p^2 \simeq 0.16 \text{ GeV}^2$. Thus, it appears that the coupling in realistic dense quark matter should be greater than 2, being close to 3 in the Fischer's estimate presented in Eq.(7). Therefore, $E_q > \mu_u$, and quarks should be clustered in realistic dense quark matter, as was found in the case for water in §2.

What kind of quark clusters could exist in cold quark matter? Generally speaking, as quarks are Fermions, the exchange of two quarks in position and inner spaces should change the sign of the wave function of the quantum system of the particles. For a quark system, the interaction via attraction channel may result in the formation of quark clusters if the wave function $\Psi(q_1, q_2, ...)$ is symmetric when exchanging two quarks in position space, but $\Psi(q_1, q_2, ...)$ should be asymmetric in inner (e.g., spin, flavor, and color) spaces. The Q_α-cluster [5] has such a wave function, in which the spin states $(\{|1/2\rangle, |-1/2\rangle\})$ are 2, the flavor states $(\{u, d, s\})$ are 3, and the color states $(\{r, g, b\})$ are 3 in one ground state of position space. The total number of quarks in a Q_α-cluster is then $2 \times 3 \times 3 = 18$. Certainly di-Q_α-clusters ($Q_{2\alpha}$), tri-Q_α-clusters ($Q_{3\alpha}$), and even more massive clusters could also be possible due to the attraction between quarks or Q_α-clusters.

In dense quark matter at $\rho \sim 3\rho_0$, the distance between Q_α-clusters is about

$$l_{Q_\alpha} \sim (\frac{3 \times 0.48}{18})^{-1/3} \simeq 2 \text{ fm},$$

while the cluster length would be $l_q \sim 1\text{fm}/\alpha_s$ [from Eq.(5)], which could be smaller than l_{Q_α}. However, now the question arises about what state these clusters would be in: could the quark-cluster's quantum wave-length be much longer than l_{Q_α}? The answer is "yes" if the interaction between the clusters is negligible. But there will be residual interaction between the clusters, and we need such an interaction to have associated energies of

$$E_{\text{cluster}} \sim \frac{\hbar^2}{2m_{Q_\alpha} l_{Q_\alpha}^2} \sim 1 \text{ MeV}$$

to localize the clusters (where the mass of non-relativistic clusters $m_{Q_\alpha} \simeq 300 \times 18$ MeV). Thus, we infer that the potential drop between clusters has to be deeper than ~ 1 MeV in order to have classical quark-clusters, rather than a cluster quantum gas. This could be reliable since the energy scale of strong interaction is generally higher than 1 MeV (e.g., in nuclear physics).

How and where can we experimentally test and determine the QCD phases of cold quark matter? Observations of pulsar-like stars are certainly useful, and in §4 one will see that we could need a solid state of quark stars to help us understand different observed manifestations.
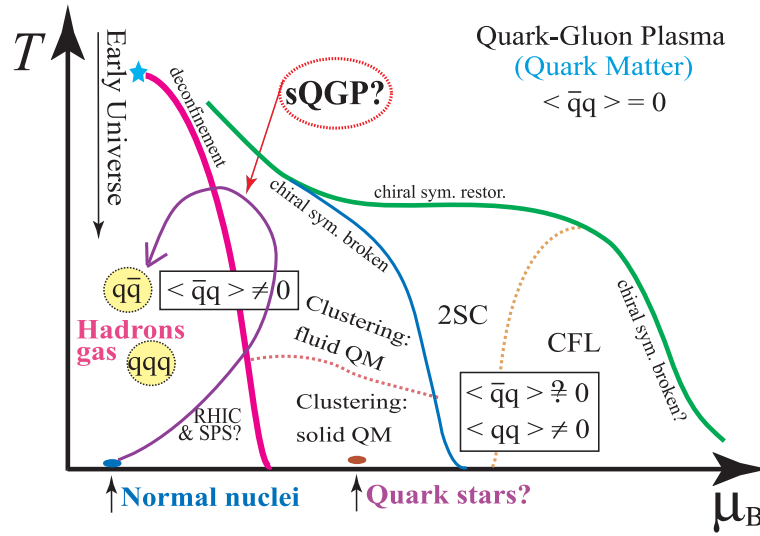
Figure 1: Schematic illustration of the QCD phase diagram speculated from different manifestations of astrophysical compact stars. A quark cluster state is conjectured when chiral symmetry is broken though quarks are still unconfined. The cold quark matter with quark clusters should be in a solid state at low temperature.

4 To understand different manifestations of pulsar-like stars in quark star models

What can quark-clustering do for us? I think there are at least four advantages, which I will discussed below.

A stiffer equation of state. It is conventionally though that the maximum mass of quark stars should be lower than that of neutron stars, because the quarks in quark matter (e.g., in the MIT bag model) are relativistic and then the equation of state of quark matter is soft. However, quark clusters move non-relativistically in our case, and hence the equation of state of clustering quark matter is stiff, likely even stiffer than that of hadronic matter. It may well be possible to obtain a maximum mass of $\geq 2M_\odot$. Normal neutron star models could be ruled out but a quark star model would be preferred if astronomers detect a pulsar-like star with mass higher than the maximum mass of reliable neutron star models.

Solidified quark matter. Similar to normal matter, dense quark matter could also be in a solid state if the interaction energy between quark clusters is much higher than the kinetic energy, kT. Two kinds of solid would be possible, dependent on the penetration probability of quark clusters through barriers between them. A *classical* solid may form if the barrier penetration is negligible in case of strong interaction, while a *quantum* solid could exist if the penetration is significant in case of weak interaction. An astrophysically conjectured QCD phase diagram, with the inclusion of such a solid state of quark matter, is shown in

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Fig. 1. As the baryon density decreases, the states of cold quark matter may change from BCS to BEC and even quark-clustering phases. A solid state of dense quark matter could help understand pulsar glitch, precession, and even a Planckian thermal spectrum. The idea of quark clustering (quark-molecular) could also be tested using the strongly coupled quark gluon plasmas (sQGP) created in relativistic heavy ion colliders. Certainly quark clusters are in a liquid state because of high temperature of collider experiments.

Energy release during a starquake. Elastic and gravitational energies could develop when solid stars evolve, and would be released since a solid stellar object would inevitably have starquakes when the strain energy increases to a critical value. Assuming the two kinds of energies are of a same order, one could have a huge energy release, of

$$\Delta E \simeq \frac{GM^2}{R} \cdot \frac{\Delta R}{R} \sim 10^{53}\frac{\Delta R}{R} \quad \text{erg,}$$

during star-quakes if the stellar radius changes from R to $\sim (R - \Delta R)$ (and momentum of inertia changes accordingly from I to $\sim I(1 - 2\Delta R/R)$). Such kind of starquakes may be responsible for the bursts and glitches observed in soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs).

Ferro-magnetism? What is the origin of the strong magnetic fields of pulsars? This is still a matter of debate more than 40 years after the discovery. In solid quark stars, ferro-magnetism might be the origin of the magnetic field, and such a strong field would not decay significantly (as observed in radio pulsars). It would seem very worthwhile to study the spontaneously breaking mechanism of magnetic symmetry in solid quark matter.

We have already done some modeling for different manifestations of pulsar-like stars in solid quark star regime, and compared neutron and quark star models. We find that the quark star model is attractive. For instance, from the drifting subpulse phenomenon of radio pulsars, one infers that the particles on a pulsar's surface are strongly bound in the standard Ruderman-Sutherland model, with binding energy $E_b \gtrsim 10$ keV, but the expected energy is $E_b \lesssim 1$ keV for normal neutron stars or crusted strange quark stars. However, the binding energy would be effectively infinite for both positively (*u*-quarks) and negatively (*d*- and *s*-quarks, and electrons) charged particles on the quark surface, and we [6] thus suggested that radio pulsars should be bare strange quark stars in order to solve completely the "binding energy problem" first posed ten years ago.

5   Conclusions

We suggest that realistic cold quark matter in compact stars would well be in a solid state, where quark clustering occurs because of strong coupling between quarks of quark matter at only a few times of nuclear density. At the same time, a solid quark-star model could help us understand different manifestations observed in pulsar-like stars. In the future, in order to fully and globally know the real QCD phases, it is essential to combine three

approaches: lattice QCD, QCD-based effective models (e.g. DSE, NJL), and phenomeno-logical models (e.g. in astrophysics). A solid state of cold quark matter may well have physical implications in the research of fundamental strong interaction.

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Discussion

J. Wambach: I am not quite sure if necessarily strong interactions mean that you have clustering. When you think of nuclear matter, where the interaction between two nucleons is very strong, so that from perturbation theory we cannot get anything. But nuclear matter, as far as we know, is a liquid, and we do not need clusters when the density becomes very low. So somehow it is not clear to me that it is necessary that if you have very strong interactions, you automatically get clusters.

R. Xu: Nuclear matter at the nuclear saturation density should be in a liquid state because nuclei can be well understood in the nuclear liquid model. Nevertheless, the degree of free-dom of nuclear matter at low density is nucleon, while the degree of quark matter at higher densities is quark and gluon. The interaction of the quark matter might then be stronger than that of nuclear matter. Even in some heavy nuclei, α-clusters are supposed to exist, and an α-decay would occur if an α-cluster penetrates the Coulomb barrier. Additionally,

a quark molecular model is suggested to explain the features of sQGP detected in RHIC experiment. My point is: if quarks could be clustered in dense and cold quark matter, it should be very natural for us to expect to form quark molecular in the experimental hot quark matter at higher temperature.

T. Schaefer: I had sort of a similar comment about making it a solid. In principle it should be much easier to make nuclear matter into a solid than quark matter, because the conditions are much more favourable, the particles are much heavier and you have repulsive short-distance interactions. But not even nuclear matter, under standard conditions, ever solidifies. So it seems to me that, the higher you go in density, the harder it should be to make it a solid. Because quarks are mass-less, it takes a lot of kinetic motion. So, firstly, it is not clear how you would solidify this.

R. Xu: So you believe the nuclear matter to be in a solid state?

T. Schaefer: No, we know that we cannot make nucleons in a solid, so for nucleons it already doesn't work, and for quarks it should be harder still.

J. Horvath: Let me add a comment on that. He doesn't have actually the quarks, themselves, but he has a cluster in a single object that has a mass of several times the mass of a nucleon. Therefore, what is clustered is the quarks and the basic object is now a cluster. So, the clusters are in a periodic structure, if I understood it correctly. So the nucleons already don't show a solid, this does not argue that the quarks cannot. The quarks are inside the clusters, so he takes the clusters as a whole. Correct?

R. Xu: Yes. The degrees would change from nucleon to quark as the density increases. Only residual color interaction exists between nucleons, while color-charged ones between quarks. The very strong coupling between quarks may results in the formation of quark-clusters as a new degree of freedom at a few nuclear densities.

R. Ouyed: The density you showed is three times saturation density, which is very close to the average density of any compact star, so in your model then there is no such thing as a neutron star (or little room for neutron stars, if this phase exists).

R. Xu: Yes, our picture is very simple, only quark stars exist, and we are trying to understand different manifestations of pulsar-like stars in this regime these years, although it is still very difficult to calculate the critical density of de-confinement phase transition because of non-perturbative coupling there.

V. de la Incera: My question is what the role of the gluons is here.

R. Xu: Although the gluon degree can not be negligible in hot quark matter, it could be integrated as the interaction between quarks in our cold quark matter. So the gluon themselves may only play an important role at high temperature.

T. Schaefer: Let me show another sort of tiny comment. The cluster you wrote down, this eighteen quark state, in terms of quantum numbers is what people used to call H dibaryons, so in some sense what you suggest is H dibaryon fluid. And that indeed seems incredibly reasonable, a great idea, except that people could never really find any evidence for this particular cluster that seemed so incredibly favourable in terms of its quantum numbers.

R. Xu: Yes, the study of multi-quark particles has a long history, and it is recently a hot topic to search the experimental evidence of such new hadron states (e.g., the pentaquark). Thought multi-quark particles could be similar to the quark clusters discussed here, they are in different kinds of vacuum and the quark-clusters may be populated in quark matter but is very difficult to exist as hadrons because of decaying to other lighter hadrons in our vacuum. One point could be relevant to electron's participating in our vacuum, so that massive quark clusters are unstable in our daily life but would be stable in cold strange quark matter where electricity is negligible. Certainly, both H-dibaryon-like and Q_α-like quark clusters are candidate degrees of freedom in cold quark matter.

B. Zhang: A question from an astrophysical point of view. In terms of sub-pulse drifting pulsars you talked about, the main astrophysics evidence in favour of neutron stars these days is the small hot spot. The old problem of the quark stars was that it is hard to concentrate the heat in a small cap, so do you have a new idea about how to solve the problem?

R. Xu: Your question is related to the thermal conductivity of cold quark matter, which is still not certain up to now. The thermal conductivity of liquid-like or gas-like cold quark matter is high, but it could be low for solid quark matter because of quarks being clustered and possible electron-phonon interaction. Solid quark stars would have hot spots if the thermal conductivity is not as high as speculated in conventional literatures.

P. Zhuang: I think probably we can ask people to calculate the probability of a pentaquark at high density. If this probability increases with increasing density, then probably this lends support to your idea. We can ask, for instance, Boqiang to calculate the pentaquark at three nuclear density, and to see what happens.

Y. Liu: A group in Nankai University had performed such types of calculations. The results showed that the probability decreased quite rapidly.

P. Zhuang: The question is: if you try to consider the lattice QCD at high density, this is a big problem and is not easy to solve.