Combustion gasdynamics of the neutron → strange matter conversion: towards an assessment of realistic scenarios

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1   Abstract

A variety of descriptions of the conversion of a neutron into a strange star have appeared in the literature over the years. Generally speaking, these works treat the process as a mere phase transition or ignore everything but microscopic kinetics, attempting to pin down the speed of the conversion and its consequences. We revisit in this work the propagation of the hypothetical "combustion" $n \to SQM$ in a dense stellar environment. We address in detail the instabilities affecting the flame and present new results of application to the turbulent regime. The acceleration of the flame, the possible transition to the distributed regime and further deflagration-to-detonation mechanism are addressed. As a general result, we conclude that the burning happens in (at least) either the turbulent Rayleigh-Taylor or the distributed regime. In both cases the velocity of the conversion of the star is several orders of magnitude larger than v_{lam}, making the latter irrelevant in practice for this problem. A transition to a detonation is by no means excluded, actually it seems to be favored by the physical setting, but a definitive answer would need a full numerical simulation.

2   Introduction

The interest in hypothetical absolutely stable phases of QCD at high density ([1]) has been going on for more than two decades. In addition to the celebrated and widely studied SQM, renewed work on pairing in dense matter ([2]) opened the possibility of an extra gain of energy, possibly enhancing the prospects for a "ΔSQM" stability ([3]).

There are many approaches to this problem dealing with evolved configurations of compact stars made from SQM or ΔSQM (refs) and their observable features. Another important question is how exactly the "normal" (proto) neutron stars become exotic objects. This could happen early in their lives (on $\sim$ *seconds via* classical nucleation or later if the conversion is quenched and depends ultimately on accretion, driven by quantum effects (bombaci). Even though the compression of the central regions of the star to $\rho \geq 3\rho_0$ and temperatures $T \sim 10 MeV$ favor the conversion $n \to SQM$,it is still possible that the nucleation could be postponed [4]. In the remaining of this work we will assume a "hot" conversion on $\sim s$ timescale, focusing on the precise form of propagation and related energetic issues.

The fate of a just-nucleated stable drop of SQM (or Δ SQM) matter has been addressed in several works (for example, [5], [6]). Usually,it was *assumed* that the propagation of the conversion front proceeds by diffusion of the s-quarks ahead, carrying the seed of flavor conversion that releases energy after relaxation. This "chemical" equilibration depends on weak interactions and occurs in a timescale $\tau_w \sim 10^{-9}s$,much faster than dynamical times of the order of a ms. In some works not even the formation of a front is acknowledged, and a macroscopic synchronization of the conversion is assumed, that is, deconfinement followed by a decay of d into s quarks, a process which is unlikely given that the conditions for that should be extreme in a region of the order of $\sim 10 km$ ([7], [8]).

In general, kinetic descriptions postulating diffusion lead to a well-known slow combustion mode termed *deflagration*, which is familiar to anyone. Moreover, in these works only the *laminar* regime is studied,leading to front velocities $\ll c_{sound} \sim c$,neglecting therefore all complications related to the behavior actually observed in flames. Specifically,it has been known for years ([9]) that at least two instabilities are important for the flame propagation: the Landau-Darrehius and Rayleigh-Taylor modes. While the first desestabilizes all wavelengths at the liner level (but is controlled by non-linear terms),the latter acts on large scales and is always active and important. Both instabilities develop on timescales $\sim \tau_{dynamical} \sim 10^{-3}s$, and therefore their effects should be considered from the scratch, right after the beginning of the propagation.

3 The cellular stage of the propagation

After a few ms, the action of the LD instability leads to a wrinkling of the flame and the development of a so-called *cellular structure*. As a result of this, the fuel consumption rate rises (because it is controlled by the total area, which has become bigger) and the flame accelerates. However, the non-linear effects neglected by Landau play an important role stabilizing the flame, which looks nested Several approaches have been attempted One of these employed concepts of fractal geometry to argue

that the velocity of the flame (by itself a fractal) can be described by the expression [10]

$$u_{cell} = u_{lam} \left(\frac{l_{max}}{l_{min}} \right)^{0.6(1-\rho_2/\rho_1)^2} \tag{1}$$

Whereas the maximum length l_{max} is not difficult to find, and is ultimately bounded by the largest scale in the flame, the minimum length is more tricky: if l_{th} represents the (microscopic) scale of the dissipation,then a related scale $\sim 100 \times l_{th}$ exists. This is known as the *Markstein* length. An inspection of eq.(1) shows that the value of the exponent,however, does not allow a vary large increase of the velocity. Actually the flame speeds up in numerical simulations ([11]) but not dramatically. Independently of this, the cellular stage is short and more dramatic effects happen along the propagation as described below.

4 Disruption of the flame and turbulent cascade

The action of gravity directed against the propagation speed has been observed to disrupt the cellular flame generating a *turbulent cascade* also on short timescale. Above a length L at which $u_{cell} = u'(L)$ (where u' is the velocity fluctuating part), disruption of bubbles occur. This is the so-called *Gibson* scale. The estimation of this length requires the specification of the turbulent spectrum. Assuming a Kolmogorov form, one can check that

$$l_{Gibson} \propto \left(u_{cell}/u'(L) \right)^3 \leq 10^{-4} cm \tag{2}$$

since this is a small length, but still $\gg l_{th}$ because the thermal scale is truly microscopic, we can classify the propagation as belonging to the *flamelet regime*. In this situation, the flame propagation is still controlled by diffusion, but the total burning rate is determined by turbulence, and termed the *flame brush*. When the flame brush is developed, turbulent eddies turnover control the transport and fuel consumption rate, thus u_{turb} is now unrelated to diffusion. A rough depiction of the evolution of the flame is given in Fig 1.

The determination of u_{turb} is possible by resorting to the basic expressions of the RT instability, although more elaborated models can be found in the literature, for example the *Sharp-Wheeler* [12] model, and also a fractal model quite similar to the one employed for the cellular regime [13]. In the latter the velocity of the flame is

$$u_{RT} = u_{lam} \left(\frac{L}{l_{min}} \right)^{D-2} \tag{3}$$

with D the fractal dimension of the front. For any reasonable value of D, and given that L is bounded from above by the largest turbulent eddies and l_{min} can not
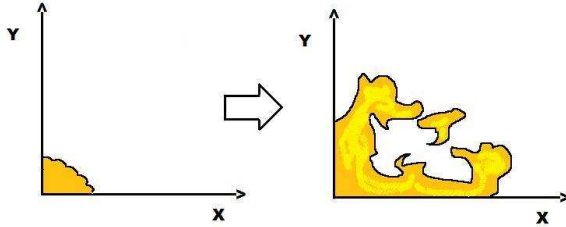
Figure 1: Scheme of the deformation of flame from the initial laminar stage to the turbulent cascade (short cellular stage not shown).

be smaller than the Gibson scale, it is concluded that $u_{RT} \equiv u_{turb} \gg u_{lam}$ deep inside the star, for distances $\leq 1\,km$.

5   Distributed regime and detonations

The flame now accelerated to $u_{turb} \gg 10^4 cms^{-1}$ may be still subject to changes according to the actual physical conditions. One intriguing possibility is the disruption of the flame by turbulence, a condition reached when $l_{Gibson} < l_{th}$. In principle, this can be reached early along the propagation, but it should be remembered that l_{th} is microscopic and therefore not easy to beat. However, the so-called *distributed* regime, in which no definite flame front occurs, is sometimes described by the mixing of fuel and ashes interacting strongly with turbulent eddies.

Even though it is not clear that the distributed regime ensures, it is important to consider its consequences, because observations show that it is one of the preconditions for a jump to the *detonation* branch. Alternatively, the burning may proceed outwards with a high (but subsonic) velocity $u_{turb} \leq 10^9 cms^{-1}$, much faster than suggested by the simplest laminar analysis.

One of the popular proposed mechanisms for this jump to the detonation branch is the so-called Zel'dovich gradient. It is often described as a synchronic burning of a mixed (fuel+ashes) region. As a necessary condition, the region should be small ($l_c \leq 10cm$) [14], and also the mixing time smaller than the burning time, a condition written as

$$\tau_{mix} \leq \frac{l_c}{u'(l_c)} \tag{4}$$

translated into a conservative bound $l_c < 10^{-5}cm$. This is smaller, than the Gibson scale and makes the necessary condition irrelevant. A thorough examination

of this problem would require the solution of the reactive Euler equations (see, for instance, [15]) and has not been attempted until now.

6 Conclusions

We have discussed semiqualitatively the realistic features of a $n \rightarrow SQM$ burning in (just born) proto-neutron stars. The conversion, which has been often treated as a simple phase transition, should feature a quite complex evolution driven by LD and RT instabilities, in a close analogy to thermonuclear explosions of white dwarfs [16]. These instabilities are always present and act very promptly, this is why they can not be ignored and make the laminar analysis just a "zero-time" academic exercise. After a few dynamical timescales the velocity is likely to be high, but subsonic (turbulent deflagration), or even supersonic (detonation) if the jump to that branch is achieved. In both cases the outcome of the conversion should affect the external layers, by blowing up and allow a bare quark surface with high photon luminosity [17] [18], by enhanced neutrino heating onto a stalled prompt shock [19], or by a direct piston action [20]. A small magnetic field is also enough to produce substantial asymmetry of the front [21], at least at a linear level. All these features suggest that a new round of calculations is guaranteed.

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