Phase Structure and Transport Properties
of Dense Quark Matter

Thomas Schäfer
Department of Physics
North Carolina State University
Raleigh, NC 27695

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1 Introduction

This goal of this meeting is the search for exotic states of matter in compact astrophysical objects. Over the years many possible signatures for phase transitions in compact stars have been suggested, for example unusual masses, radii or cooling histories, or sudden changes in the spin frequency. In this contribution we will take a very conservative approach and study the predictions of weak coupling QCD for the equilibrium and transport properties of the densest phases of QCD. This includes the color-flavor-locked (CFL) phase [1], and phases that arise from modifications of the basic CFL pairing pattern due to the effects of the non-zero strange quark mass. A flavor rotation of the CFL condensate leads to kaon condensation (CFL-K) [2], and a spatial modulation of the CFL state leads to the meson supercurrent state (curCFL) [3, 4].

We refer the reader to our recent review [5] for a detailed discussion of quark matter phases at lower density. Some of these phases are stable in weak coupling, and their properties are rigorously computable in perturbative QCD. This includes crystalline color superconductivity [6, 7], and single flavor spin-one color superconductivity [8, 9]. Other phases require strong coupling, for example the 2SC phase [10, 11], chiral density waves (and the quarkyonic phase) [12, 13, 14], or gapless color superconductivity [15, 16].

The goal of our research program is to compute the properties of all these phases, and to determine whether hybrid neutron/quark matter stars containing quark matter in the CFL phase or one of the lower density quark phases are consistent with observation. Current data on the masses and radii of compact stars are consistent with pure neutron stars as well as with hybrid quark matter stars [17, 18]. There are some results that indicate that other observational properties also do not clearly distinguish neutron stars from hybrid quark stars. In that case evidence for the existence of a high density phase in compact stars may have to come from careful studies of the mass-radius relation of compact stars [19], or from the observation of gravitational

waves from binary compact star mergers. Laboratory experiments involving heavy ions have established meaningful constraints on the equation of state of dense matter [20], and the next generation of these experiments may demonstrate the existence of a first order transition at densities that are achieved in compact stars.

2 Matter at the highest densities

2.1 The CFL Phase

Calculations based on weak-coupling QCD indicate that the ground state of three flavor baryonic matter at the highest densities is the color-flavor-locked (CFL) phase [1, 21, 22]. The CFL phase is characterized by a pair condensate

$$\langle \psi_i^a C \gamma_5 \psi_j^b \rangle = (\delta_i^a \delta_j^b - \delta_j^a \delta_i^b) \phi. \tag{1}$$

This condensate leads to a gap in the excitation spectrum of all fermions and completely screens the gluonic interaction. Both the chiral $SU(3)_L \times SU(3)_R$ and color $SU(3)$ symmetry are broken, but a vector-like $SU(3)$ flavor symmetry remains unbroken.

The gap in the fermion spectrum can be computed in perturbative QCD. The nine quarks species (three flavors and three colors) form an octet and a singlet under the unbroken $SU(3)$ flavor symmetry. The octet and singlet quarks have gaps Δ_0 and $2\Delta_0$, respectively, where [23, 24, 25, 26]

$$\Delta_0 \simeq 2^{-1/3} 512 \pi^4 \mu \left(\frac{2}{3g^2}\right)^{5/2} \exp\left(-\frac{3\pi^2}{\sqrt{2}g} - \frac{\pi^2 + 4}{8}\right). \tag{2}$$

Here, μ is the chemical potential and g is the strong coupling constant. At this order in g we can compute the coupling by evaluating the one-loop running coupling at the scale μ. At large μ the coupling constant is small and the gap is exponentially small compared to the Fermi energy $E_f = \mu$. For densities relevant to neutron stars $\mu < 500$ MeV and the coupling is not small. In this regime higher order correction to equ. (2) are not small and the gap is quite uncertain. It is remarkable, however, that both extrapolations of the weak coupling result to the regime of moderate coupling [27] as well as model calculations based on Nambu-Jona Lasinio (NJL) or similar interactions [28] give gaps in the range $\Delta_0 \simeq (50 - 100)$ MeV at a density $\rho \simeq 5\rho_0$, where ρ_0 is the nuclear matter saturation density.

Perturbation theory can also be used to compute the gluon screening mass. Screening arises from the Meissner effect, as it does in ordinary superconductors. We find [29, 30]

$$m^2 = \frac{21 - 8\log(2)}{54} \frac{g^2 \mu^2}{2\pi^2}, \tag{3}$$

which shows that the gluon screening length is short compared to the coherence length $\xi \sim \Delta_0^{-1}$. We conclude that color superconductivity is type I. We note that electromagnetism is unbroken, and the photon is not screened. The physical photon in the CFL phase is a mixture of the bare photon and the bare gluon, and interesting effects occur at the interface of normal nuclear matter and the CFL phase [31].

2.2   Effective theory of the CFL phase

For excitation energies smaller than the gap the only relevant degrees of freedom are the Goldstone modes associated with the breaking of $SU(3)_L \times SU(3)_R$ chiral symmetry, the $U(1)_B$ phase symmetry associated with baryon number, and the approximate $U(1)_A$ phase symmetry. The interaction of the low energy modes is described by an effective field theory. The structure of the effective lagrangian is determined by the symmetries of the CFL phase, and the coefficients that appear in the lagrangian can be computed in perturbative QCD. We will see that, with one notable exception, these coefficients agree with estimates based on naive dimensional analysis.

The effective lagrangian for the low energy modes has two important applications. First, it determines the spectrum and the interactions of quasi-particles. Based on this knowledge we can compute the specific heat and the transport properties of the CFL phase. Second, the effective theory determines the response of the CFL phase to non-zero quark masses and to external fields, such as lepton chemical potentials or magnetic fields. The effective lagrangian therefore determines the phase structure at non-asymptotic densities.

The Goldstone mode associated with superfluidity is related to the phase φ of the order parameter

$$\langle \psi_i^\alpha C\gamma_5 \psi_j^\beta \rangle = \epsilon^{\alpha\beta A}\epsilon_{ijB}e^{2i\varphi}\phi_A^B \,, \tag{4}$$

where ϕ_A^B parametrizes the color-flavor orientation of the order parameter. The field φ transforms as $\varphi \to \varphi + \alpha$ under $U(1)_B$ transformation of the quark fields $\psi \to \exp(i\alpha)\psi$. At leading order in the weak coupling limit the effective lagrangian is completely fixed by Lorentz invariance and $U(1)_B$ symmetry. One can show that [32]

$$\mathcal{L} = \frac{3}{4\pi^2}\left[(\partial_0\varphi - \mu)^2 - (\nabla\varphi)^2\right]^2 + \ldots, \tag{5}$$

where ... denotes terms that are higher order in g, or terms of the form $\partial^n\varphi^m$ with $n > m$. In the low energy limit we can expand the lagrangian in powers of $\partial\varphi$. We also rescale the field as $\phi = (3\mu/\pi)\varphi$ in order to make it canonically normalized. We will refer to ϕ as the phonon field. The phonon lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial_0\phi)^2 - \frac{1}{2}v^2(\partial_i\phi)^2 - \frac{\pi}{9\mu^2}\partial_0\phi(\partial_\mu\phi\partial^\mu\phi) + \frac{\pi^2}{108\mu^4}(\partial_\mu\phi\partial^\mu\phi)^2 + \ldots, \tag{6}$$

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where $v = 1/\sqrt{3}$ is the speed of sound. We observe that three and four phonon vertices are suppressed by powers of $|\partial\phi|/\mu^2$.

The effective lagrangian for the Goldstone modes associated with chiral symmetry breaking has the same structure as the chiral lagrangian at $T = \mu = 0$. The main difference is that Lorentz invariance is broken and only rotational invariance is a good symmetry. The effective lagrangian is given by [33]

$$
\begin{aligned}
\mathcal{L}_{eff} = \; & \frac{f_\pi^2}{4} \text{Tr} \left[\nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v_\pi^2 \partial_i \Sigma \partial_i \Sigma^\dagger \right] + \left[B \text{Tr}(M\Sigma^\dagger) + h.c. \right] \\
& + \left[A_1 \text{Tr}(M\Sigma^\dagger) \text{Tr}(M\Sigma^\dagger) + A_2 \text{Tr}(M\Sigma^\dagger M\Sigma^\dagger) \right. \\
& \left. + A_3 \text{Tr}(M\Sigma^\dagger) \text{Tr}(M^\dagger\Sigma) + h.c. \right] + \dots .
\end{aligned}
\tag{7}
$$

Here $\Sigma = \exp(i\phi^a \lambda^a / f_\pi)$ is the chiral field, f_π is the pion decay constant and M is the mass matrix. The chiral field and the mass matrix transform as $\Sigma \to L\Sigma R^\dagger$ and $M \to LMR^\dagger$ under chiral transformations $(L, R) \in SU(3)_L \times SU(3)_R$. In order to determine the structure of the effective theory we treat M as a field, but in practice we are interested in the case $M = \text{diag}(m_u, m_d, m_s)$.

The coefficients $f_\pi, v_\pi, B, A_i, \dots$ can be computed in weak coupling perturbation theory. In the case of f_π and v_π this is most easily done my matching the screening masses for flavored gauged fields. The coefficients B, A_i are related to the quark mass dependence of the condensation energy in the CFL phase.

At leading order in α_s the Goldstone boson decay constant and velocity are [29]

$$
f_\pi^2 = \frac{21 - 8\log(2)}{18} \left(\frac{p_F^2}{2\pi^2} \right), \quad v_\pi^2 = \frac{1}{3}.
\tag{8}
$$

The coefficient B is related to instanton effects. We find [34]

$$
B = c \left[\frac{3\sqrt{2}\pi}{g} \Delta \left(\frac{\mu^2}{2\pi^2} \right) \right]^2 \left(\frac{8\pi^2}{g^2} \right)^6 \frac{\Lambda_{QCD}^9}{\mu^{12}},
\tag{9}
$$

which shows that B is strongly suppressed at large chemical potential. The A_i terms receive perturbative contributions and are given by [29, 35]

$$
A_1 = -A_2 = \frac{3\Delta^2}{4\pi^2}, \qquad A_3 = 0.
\tag{10}
$$

Finally, one can show that $X_L = MM^\dagger/(2p_F)$ and $X_R = M^\dagger M/(2p_F)$ act as effective chemical potentials for left and right-handed fermion. These effective chemical potentials appear in the time derivative of the chiral field [2],

$$
\nabla_0 \Sigma = \partial_0 \Sigma + i \left(\frac{MM^\dagger}{2p_F} \right) \Sigma - i\Sigma \left(\frac{M^\dagger M}{2p_F} \right).
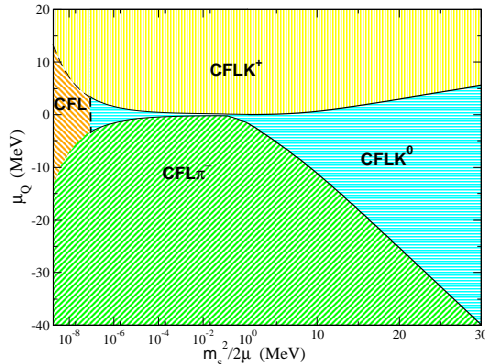\tag{11}
$$

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Figure 1: Phase structure of CFL matter as a function of the effective chemical potential $\mu_s = m_s^2/(2p_F)$ and the lepton chemical potential μ_Q, from Kaplan and Reddy [36]. A typical value of μ_s in a neutron star is 10 MeV.

3   Matter at non-asymptotic density

At non-asymptotic densities we can not rely on perturbative QCD calculations to determine the magnitude of the gap parameter, but the argument that pairing occurs and that the CFL phase is energetically favored is quite general, and does not depend on details of the interaction. The dominant stress on the CFL phase at non-asymptotic densities arises from flavor symmetry breaking due to the quark masses. We will focus on the physically relevant case $m_s \gg m_d \simeq m_u$. In this case the main expansion parameter is $m_s^2/(\mu\Delta)$, which is the ratio of the mass correction to the Fermi energy of the strange quark over the magnitude of the gap.

3.1   Kaon condensation

Using the chiral effective lagrangian we can determine the dependence of the order parameter on the quark masses. The effective potential for the order parameter is

$$V_{eff} = \frac{f_\pi^2}{4}\text{Tr}\left[2X_L\Sigma X_R\Sigma^\dagger - X_L^2 - X_R^2\right] - A_1\left[\left(\text{Tr}(M\Sigma^\dagger)\right)^2 - \text{Tr}\left((M\Sigma^\dagger)^2\right)\right]. \quad (12)$$

The first term contains the effective chemical potential $\mu_s = m_s^2/(2p_F)$ and favors states with a deficit of strange quarks (with strangeness $S = -1$). The second term favors the neutral ground state $\Sigma = 1$. The lightest excitation with positive strangeness is the K^0 meson. We therefore consider the ansatz $\Sigma = \exp(i\alpha\lambda_4)$ which

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allows the order parameter to rotate in the K^0 direction. The vacuum energy is

$$V(\alpha) = -f_\pi^2 \left(\frac{1}{2} \left(\frac{m_s^2 - m^2}{2p_F} \right)^2 \sin(\alpha)^2 + (m_K^0)^2 (\cos(\alpha) - 1) \right), \tag{13}$$

where $(m_K^0)^2 = (4A_1/f_\pi^2)m(m + m_s)$. Minimizing the vacuum energy we obtain

$$\cos(\alpha) = \begin{cases} 1 & \mu_s < m_K^0 \\ \frac{(m_K^0)^2}{\mu_s^2} & \mu_s > m_K^0 \end{cases} \tag{14}$$

Using the perturbative result for A_1 we can get an estimate of the critical strange quark mass. We find

$$m_s(crit) = 3.03 \cdot m_d^{1/3} \Delta^{2/3}, \tag{15}$$

from which we obtain $m_s(crit) \simeq 70$ MeV for $\Delta \simeq 50$ MeV. This result suggests that strange quark matter at densities that can be achieved in neutron stars is kaon condensed. The phase structure as a function of the strange quark mass and non-zero lepton chemical potentials was studied by Kaplan and Reddy [36], see Fig. 1. We observe that if the lepton chemical potential is non-zero charged kaon and pion condensates are also possible.

3.2   Fermions in the CFL phase

Kaon condensation occurs for $\mu_s/\Delta \sim \sqrt{mm_s}/\mu \ll 1$. For conditions relevant to neutron stars μ_s/Δ can get significantly larger, reaching $\mu_s/\Delta \sim 1$. In this case some of the fermion modes may become gapless or almost gapless [16]. In order to study this regime we have to include fermions in the effective field theory. The effective lagrangian for fermions in the CFL phase is [37, 38]

$$\begin{aligned} \mathcal{L} &= \text{Tr}\left(N^\dagger i v^\mu D_\mu N \right) - D\text{Tr}\left(N^\dagger v^\mu \gamma_5 \{\mathcal{A}_\mu, N\} \right) - F\text{Tr}\left(N^\dagger v^\mu \gamma_5 [\mathcal{A}_\mu, N] \right) \\ &+ \frac{\Delta}{2} \left\{ \left(\text{Tr}\left(N_L N_L \right) - [\text{Tr}\left(N_L \right)]^2 \right) - (L \leftrightarrow R) + h.c. \right\}. \end{aligned} \tag{16}$$

$N_{L,R}$ are left and right handed baryon fields in the adjoint representation of flavor $SU(3)$. The baryon fields originate from quark-hadron complementarity [39]. We can think of N as describing a quark which is surrounded by a diquark cloud, $N_L \sim q_L \langle q_L q_L \rangle$. The covariant derivative of the nucleon field is given by $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$. The vector and axial-vector currents are

$$\mathcal{V}_\mu = -\frac{i}{2} \left\{ \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right\}, \qquad \mathcal{A}_\mu = -\frac{i}{2} \xi \left(\nabla_\mu \Sigma^\dagger \right) \xi, \tag{17}$$

where ξ is defined by $\xi^2 = \Sigma$. F and D are low energy constants that determine the baryon axial coupling. In perturbative QCD we find $D = F = 1/2$. The effective
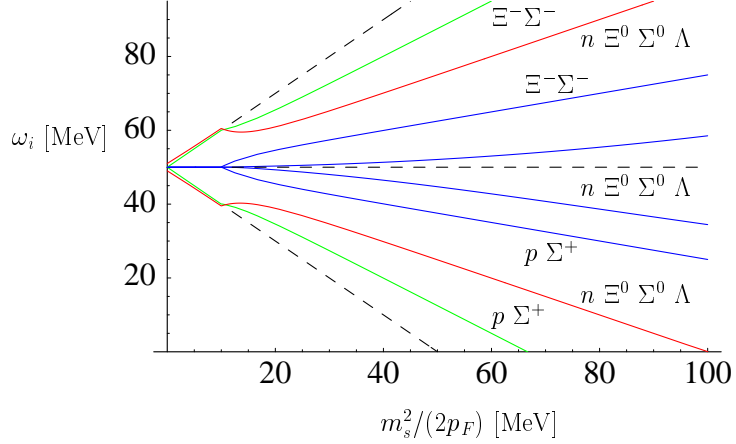
Figure 2: This figure shows the fermion spectrum in the CFL phase. For $m_s = 0$ there are eight fermions with gap Δ and one fermion with gap 2Δ (not shown). Without kaon condensation gapless fermion modes appear at $\mu_s = \Delta$ (dashed lines). With kaon condensation gapless modes appear at $\mu_s = 4\Delta/3$.

chemical potentials (X_L, X_R) appear in the covariant derivative of the nucleon field. We have

$$
\begin{aligned}
D_0 N &= \partial_0 N + i[\Gamma_0, N], \\
\Gamma_0 &= -\frac{i}{2}\left\{\xi\left(\partial_0 + iX_R\right)\xi^\dagger + \xi^\dagger\left(\partial_0 + iX_L\right)\xi\right\},
\end{aligned}
\tag{18}
$$

where $X_L = MM^\dagger/(2p_F)$ and $X_R = M^\dagger M/(2p_F)$ as before. (X_L, X_R) covariant derivatives also appears in the axial vector current given in equ. (17).

We can now study how the fermion spectrum depends on the quark mass. Since the field N has the quark numbers of the baryon octet and singlet we will use $(p, n, \Sigma, \Xi, \Lambda)$ to label the fields. In the CFL state we have $\xi = 1$. For $\mu_s = 0$ the octet has an energy gap Δ and the singlet has gap 2Δ. As a function of μ_s the excitation energy of the proton and neutron is lowered, $\omega_{p,n} = \Delta - \mu_s$, while the energy of the cascade states Ξ^-, Ξ^0 particles is raised, $\omega_\Xi = \Delta + \mu_s$. All other excitation energies are independent of μ_s. As a consequence we find gapless (p, n) and $(\Xi^-, \Xi^0)^{-1}$ excitations at $\mu_s = \Delta$. The situation is more complicated when kaon condensation is taken into account. In the kaon condensed phase there is mixing in the $(p, \Sigma^+, \Sigma^-, \Xi^-)$ and $(n, \Sigma^0, \Xi^0, \Lambda^8, \Lambda^0)$ sector. For $m_K^0 \ll \mu_s \ll \Delta$ the spectrum is given by

$$
\omega_{p\Sigma^\pm\Xi^-} = \left\{\begin{array}{l} \Delta \pm \frac{3}{4}\mu_s, \\ \Delta \pm \frac{1}{4}\mu_s, \end{array}\right. \qquad \omega_{n\Sigma^0\Xi^0\Lambda} = \left\{\begin{array}{l} \Delta \pm \frac{1}{2}\mu_s, \\ \Delta, \\ 2\Delta. \end{array}\right.
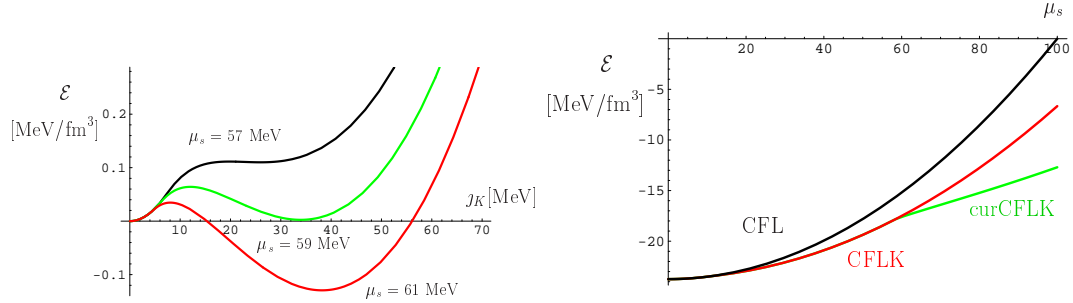\tag{19}
$$

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Figure 3: Left panel: Energy density as a function of the current $\jmath_K$ for several different values of $\mu_s = m_s^2/(2p_F)$ close to the phase transition. Right panel: Ground state energy density as a function of μ_s. We show the CFL phase, the kaon condensed CFL (CFLK) phase, and the supercurrent state (curCFLK).

Numerical results for the eigenvalues are shown in Fig. 2. We observe that mixing within the charged and neutral baryon sectors leads to level repulsion. There are two modes that become light in the CFL window $\mu_s \leq 2\Delta$. One mode is a linear combination of proton and Σ^+ particles, as well as Ξ^- and Σ^- holes, and the other mode is a linear combination of the neutral baryons $(n, \Sigma^0, \Xi^0, \Lambda^8, \Lambda^0)$.

3.3 Meson supercurrent state

What happens when gapless fermions appear in the spectrum? Several authors have shown that gapless fermion modes lead to instabilities in the current-current correlation function [40, 41]. Motivated by these results we have examined the stability of the kaon condensed phase against the formation of a non-zero current [3, 4]. Consider a spatially varying $U(1)_Y$ rotation of the kaon condensate

$$U(x)\xi_{K^0}U^\dagger(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & ie^{i\phi_K(x)}/\sqrt{2} \\ 0 & ie^{-i\phi_K(x)}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{20}$$

This state is characterized by non-zero currents $\vec{\mathcal{V}}$ and $\vec{\mathcal{A}}$. In order to determine the stability of the CFLK state we compute the vacuum energy as a function of the kaon current $\vec{\jmath}_K = \vec{\nabla}\phi_K$. The meson part of the effective lagrangian gives a positive contribution

$$\mathcal{E} = \frac{1}{2}v_\pi^2 f_\pi^2 \jmath_K^2. \tag{21}$$

A negative contribution can arise from gapless fermions. In order to determine this contribution we have to calculate the fermion spectrum in the presence of a non-zero

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current. The spectrum is determined by the effective lagrangian (16). The dispersion relation of the lowest mode is approximately given by

$$\omega_l = \Delta + \frac{(l - l_0)^2}{2\Delta} - \frac{3}{4}\mu_s - \frac{1}{4}\vec{v} \cdot \vec{\jmath}_K, \tag{22}$$

where $l = \vec{v} \cdot \vec{p} - p_F$ and we have expanded ω_l near its minimum $l_0 = (\mu_s + \vec{v} \cdot \vec{\jmath}_K)/4$. Equation (22) shows that there is a gapless mode if $\mu_s > 4\Delta/3 - \jmath_K/3$. The contribution of the gapless mode to the vacuum energy is

$$\mathcal{E} = \frac{\mu^2}{\pi^2} \int dl \int \frac{d\Omega}{4\pi} \, \omega_l \theta(-\omega_l), \tag{23}$$

where $d\Omega$ is an integral over the Fermi surface. In Fig. 3 we show the ground state energy as a function of the current. We observe that there is a first order transition to a state with a non-zero current. The ground state energy as a function of μ_s is shown in Fig. 3, see [42, 43] for more details. Once the current becomes large the effective theory ceases to be reliable and states with multiple currents may appear. These states can be thought of as continuously connected to the crystalline quark matter phase [6].

4 Transport Properties

Non-equilibrium properties, such as shear viscosity, bulk viscosity, thermal conductivity and neutrino emissivity, play an important role in constraining the structure of compact stars. Shear and bulk viscosity control r-mode instabilities which, if not suppressed by viscous damping, lead to a fast spin-down of rapidly rotating compact stars. Neutrino emissivity controls the cooling behavior of the star, and neutrino opacities determine the spectral shape of the initial neutrino burst. In addition to these specific constraints there has been significant progress, in both theory and observation, of tying together the rotation of the star, the magnetic field, and thermal properties. Exploiting these connections in order to constrain the phase structure of compact star matter will require a detailed understanding of transport properties.

4.1 Hydrodynamics of the CFL phase

The spontaneous breaking of $U(1)_B$ is related to superfluidity, and the $U(1)_B$ effective theory can be interpreted as superfluid hydrodynamics [32]. We can define the fluid velocity as

$$v_\alpha = -\frac{1}{\mu_0} D_\alpha \varphi, \tag{24}$$

where $D_\alpha \varphi \equiv \partial_\alpha \varphi + (\mu, 0)$ and $\mu_0 \equiv (D_\alpha \varphi D^\alpha \varphi)^{1/2}$. Note that this definition ensures that the flow is irrotational, $\vec{\nabla} \times \vec{v} = 0$. The identification (24) is motivated by the

fact that the equation of motion for the $U(1)$ field φ can be written as a continuity equation

$$\partial^\alpha(n_0 v_\alpha) = 0, \tag{25}$$

where $n_0 = 3\mu_0^3/\pi^2$ is the superfluid number density. At $T = 0$ the superfluid density is equal to the total density of the system, $n = dP/d\mu|_{\mu=\mu_0}$. The energy-momentum tensor has the ideal fluid form

$$T_{\alpha\beta} = (\epsilon + P)v_\alpha v_\beta - P g_{\alpha\beta}, \tag{26}$$

and the conservation law $\partial^\alpha T_{\alpha\beta} = 0$ corresponds to the relativistic Euler equation of ideal fluid dynamics. We conclude that the effective theory for the $U(1)_B$ Goldstone mode accounts for the defining characteristics of a superfluid: irrotational, non-dissipative hydrodynamic flow.

At non-zero temperature the hydrodynamic description of a superfluid contains dissipative terms. Similar to the two fluid model of liquid helium we can describe the CFL phase, or any other relativistic superfluid, as a mixture of an ideal superfluid and a dissipative normal component [44, 45]. We will denote the densities of the superfluid and normal components by ρ_s and ρ_n. We also define u_μ to be the velocity of the normal component, and w_μ to be the difference between the superfluid and normal velocities. The normal fluid provides both non-dissipative and dissipative contributions to the energy momentum tensor. In the rest frame of the normal fluid the dissipative terms are

$$\delta T_{ij} = -\eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right) - \delta_{ij}\zeta_1\partial_k\left(\rho_s w_k\right) - \delta_{ij}\zeta_2\partial_k u_k \tag{27}$$

and

$$\delta T_{0i} = -\kappa\partial_i T . \tag{28}$$

Here η is the shear viscosity, $\zeta_{1,2}$ are bulk viscosities, and κ is the thermal conductivity. Two additional bulk viscosities, $\zeta_{3,4}$, control dissipative corrections to the equation of motion for the superfluid velocity. There is a symmetry relation between the kinetic coefficients that requires that $\zeta_4 = \zeta_1$. Note that in the normal phase there is only one bulk viscosity, $\zeta \equiv \zeta_2$.

4.2 Transport Coefficients

The normal fluid is composed of quasi-particle excitations. In the CFL phase all quark modes are gapped and the relevant excitations are Goldstone bosons. At very low temperature, transport properties are dominated by the massless Goldstone boson φ associated with the breaking of the $U(1)_B$ symmetry. The effective lagrangian (6) determines the rates for the relevant scattering processes.
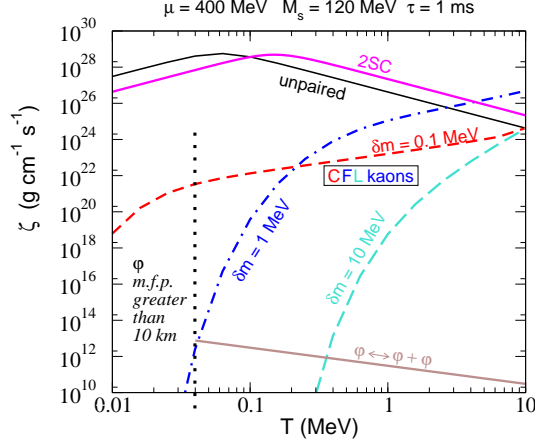
Figure 4: Bulk viscosity $\zeta \equiv \zeta_2$ as a function of temperature for an oscillation period $\tau = 2\pi/\omega = 1$ ms. CFL phase: contribution from the process $K^0 \leftrightarrow \varphi + \varphi$ for different values of $\delta m \equiv m_{K^0} - \mu_s$ and contribution from $\varphi \leftrightarrow \varphi + \varphi$. 2SC phase and unpaired quark matter: contribution from the process $u + d \leftrightarrow u + s$.

Shear viscosity is related to momentum transport. At low temperature the shear viscosity of the CFL phase is determined by $\varphi + \varphi \leftrightarrow \varphi + \varphi$ scattering. Manuel et al. find [46]

$$\eta \simeq 1.3 \times 10^{-4} \frac{\mu^8}{T^5} \,. \tag{29}$$

The bulk viscosity is sensitive to particle number changing processes. This includes purely strong decays like $\varphi \leftrightarrow \varphi + \varphi$, or electroweak processes like the strangeness changing reaction $K^0 \to \varphi + \varphi$. We first consider the pure QCD contribution. Bulk viscosity vanishes in an exactly scale invariant system. For realistic quark masses the dominant source of scale breaking is the strange quark mass. The contribution from the process $\varphi \leftrightarrow \varphi + \varphi$ is [47]

$$\zeta_2 \simeq 0.011 \frac{m_s^4}{T} \,. \tag{30}$$

We show this contribution in Fig. 4. The electroweak process $K^0 \leftrightarrow \varphi + \varphi$ was studied in [48]. The weak contribution has a significant frequency dependence. In Fig. 4 we show the results for an oscillation period $\tau = 2\pi/\omega = 1$ ms. We observe that at $T \simeq (1-10)$ MeV the bulk viscosity of CFL matter is comparable to that of unpaired quark matter. For $T < 1$ MeV, ζ_2 is strongly suppressed. Depending on the poorly known value for $\delta m \equiv m_{K^0} - \mu_s$ the pure φ contribution given in equ. (30) may dominate over the $K^0 \leftrightarrow \varphi + \varphi$ reaction at low enough temperatures. However, for $T < 0.1$ MeV the φ mean free path is on the order of the size of the star, i.e., the system is in the collisionless rather than in the hydrodynamic regime, and the result

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ceases to be meaningful.

The thermal conductivity of a CFL superfluid was studied by Braby et al. [49]. The calculation is subtle because κ vanishes for a system of quasi-particles with exactly linear dispersion relations [50]. The reason is that κ measures the rate of energy transport relative to the motion of the fluid, but in a gas of massless particles with linear dispersion one cannot transport energy without transporting momentum. As a consequence, thermal conductivity is sensitive to non-linearities in the dispersion relation. Braby et al. find [49]

$$\kappa \simeq 4.01 \times 10^{-2} \frac{\mu^8}{\Delta^6} \, \text{MeV}^2 \,. \tag{31}$$

They also estimate the contribution to κ from phonon scattering on kaons. This term grows as $\sqrt{T}$, but it is significantly smaller than the phonon contribution in the regime where the calculation is reliable. Non-linearities in the dispersion relation also play a role in determining the remaining two bulk viscosities, ζ_1 and ζ_3 [51]. Mannarelli and Manuel find $\zeta_1 \sim m_s^2/(\mu T)$ and $\zeta_3 \sim 1/(T\mu^2)$. Note that ζ_3 is non-zero even in the approximately conformal limit $m_s \to 0$.

4.3 Neutrino emissivity

In CFL quark matter the neutrino emissivity is dominated by reactions involving pseudo-Goldstone modes such as

$$\begin{aligned}
\pi^\pm, K^\pm &\rightarrow e^\pm + \overline{\nu}_e \,, \\
\pi^0 &\rightarrow \nu_e + \overline{\nu}_e \,, \\
\varphi + \varphi &\rightarrow \varphi + \nu_e + \overline{\nu}_e \,.
\end{aligned} \tag{32}$$

These processes were studied in [52, 53]. The decay rates of the massive mesons $\pi^\pm$, $K^\pm$, and π^0 are proportional to their number densities and are suppressed by Boltzmann factors $\exp(-E/T)$, where E is the energy gap of the meson. The emissivity from $\pi^\pm$ decay is

$$\epsilon_\pi = \frac{1}{8\pi} \left(G_F^2 \, f_\pi^2 \, m_e^2 \right) m_\pi^2 \, n_\pi \left(1 + 2 \left(1 - v_\pi^2 \right) + \frac{2 m_\pi T}{v_\pi^2 m_e^2} \left(1 - v_\pi^2 \right)^2 \right) \,, \tag{33}$$

where n_π is the number density of pions. Similar results can be derived for π^0 and $K^\pm$ decay. Since the pseudo-Goldstone boson energy gaps are on the order of a few MeV, the emissivities are strongly suppressed as compared to unpaired quark matter for temperatures below this scale. Neutrino emission from processes involving the φ is not exponentially suppressed, but it involves a very large power of T,

$$\epsilon_\nu \sim \frac{G_F^2 T^{15}}{f^2 \mu^4} \,, \tag{34}$$

and is numerically very small. Reddy et al. also studied the neutrino mean free path l_ν. For $T \sim 30$ MeV the mean free path is on the order of 1 m, but for $T < 1$ MeV, $l_\nu > 10$ km [53].

5   Outlook

There are a variety of issues that remain to be studied. While the calculation of transport coefficients in the CFL phase is now essentially complete, this is not the case for many of the less dense phases. There are calculations of shear and bulk viscosity as well as neutrino emissivity in the CFL-K phase [54, 55, 56], but there are essentially no results for the spatially inhomogeneous or anisotropic phases. There is also much work to be done in order to understand many phenomena that are relevant in compact stars, like mutual friction between the normal fluid and superfluid vortices [57, 58], or the role of the quark-hadron interface.

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