

# Clustered Quark Matter Calculation for Strange Quark Matter

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## Abstract

Motivated by the need for a solid state strange quark matter to better explain some observational phenomena, we discussed possibility of color singlet cluster formation in cold strange quark matter by a rough calculation following the excluded volume method proposed by Clark et al (1986) and adopted quark mass density dependent model with cubic scaling. It is found that 70% to 75% of volume and 80% to 90% of baryon number is in clusters at temperature from 10MeV to 50MeV and 1 to 10 times nuclear density.

## 1 Introduction

Many believe that neutron stars might be hybrid star with quark matter at its core or even composed entirely of quark matter with 2 or 3 flavor quark matter. However due to non-perturbative nature of QCD, property of quark matter at relevant temperature and baryon number density for neutron star is still far from clear. Even though at high enough density when asymptotic freedom sets in, quark matter should appear in color superconducting phase (CFL or 2SC) as is calculate rigorously from first principles [1], no one can be sure about to what degree this sector can stretch toward lower density on the QCD phase diagram. On astrophysics side, there are some hints such as the need for large amount of energy during bursts of SGRs and possible precession signals of some radio pulsars suggesting a solid phase of pulsar interior [9]. Therefore, in order to have a quark matter phase with regular lattice like normal solid seen on earth it is intriguing to discuss the possibility of quark clustering at moderate densities where quark clusters can serve as lattice points just like positive ions in metal.

## 2 A Simple Calculation

We first perform some simple calculation to incorporate clustering in three flavor quark matter modeled by non-interacting relativistic Fermi gas. Consider color-singlet spin-1/2 clusters with 3 quarks since quarks interact strongly attractively in this channel. As a first approximation we can fix the cluster mass at  $M_{cl} = 1000\text{MeV}$  which is roughly the average mass of baryon octet. Then we use Quark Mass Density Dependent (QMDD) model with parametrization following Peng et al. [7]

$$M_u = M_d = \frac{D_0}{\nu^{1/3}} \quad (1)$$

$$M_s = m_{s0} + \frac{D_0}{\nu^{1/3}} \quad (2)$$

to simulate asymptotic freedom and confinement. Suppose clusters can dissolve into free quarks in the Fermi sea and vice versa, we have chemical equilibrium relation

$$\mu_c = 3\mu_q \quad (3)$$

and treating clusters as a new ingredient of the Fermi sea, we now have

$$\nu = \int \frac{d^3p}{(2\pi)^3} \sum_{i=c,u,d,s} \frac{g_i}{\exp\left(\frac{\sqrt{p^2+M_i^2}-\mu_i}{T}\right) + 1} \quad (4)$$

where the degeneracy factors are  $g_u = g_d = g_s = 6$  and  $g_c = 16$  for all settings of in the baryon octet. Now we can solve equation (1), (3) and (4) to get  $\mu_q$  for fixed temperature  $T$  and baryon number density  $\nu$ . Fig 1,2 shows how chemical potential  $\mu_q$  and cluster fraction which is the fraction of baryon number in clusters as a function of baryon number density  $\nu$  in unit of nuclear density  $\nu_0 = 0.159\text{fm}^{-3}$  at relatively low temperature  $T = 10 \sim 50\text{MeV}$  (Here we extend the temperature up to  $50\text{MeV}$  to show the effect of temperature although for neutron stars after a few second old we should have a temperature lower than  $10\text{MeV}$ ). The parameters in QMDD model equation (1) were adopted from [7] to be  $D_0 = (80\text{MeV})^2$  and  $m_{s0} = 150\text{MeV}$ . As we can see  $\mu_q$  first rise rapidly before reaching about  $300\text{MeV}$ , while cluster fraction rise rapidly after  $\mu_q$  has reached  $300\text{MeV}$  then gradually saturate.

The above simple calculation have two shortcomings

1. Cluster mass is fixed, while it is expected to rise with increasing density since quark mass would gradually grow in QMDD model.
2. Interaction between clusters and quarks is not taken into account

To introduce the density dependence of cluster mass we can let the cluster mass be the sum of mass of two u or d quarks and an s quark:

$$M_{cl}(\nu) = 2M_u(\nu) + M_s(\nu) \quad (5)$$

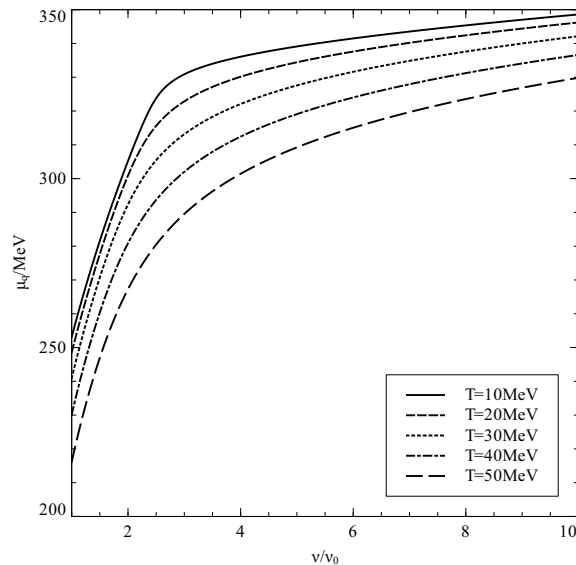


Figure 1: Chemical potential with fixed cluster mass  $M_{cl} = 1000\text{MeV}$  at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$

The resulting chemical potential and cluster fraction in total baryon number are shown in Fig 3 and 4 respectively. As we can see from the plot, the chemical potential is significantly lowered because of much lower cluster mass as sum of density dependent quark masses (since QMDD quark mass in this density range is must less than  $\sim 300\text{MeV}$ ), thus from relatively low densities cluster fraction is already very high.

### 3 Excluded Volume Method

One way to remedy the second shortcoming is to introduce an excluded volume method i.e. consider influence of cluster's finite volume on momentum space integral. Similar method was first used by Clark et al [3] at zero temperature and latter by Bi and Shi [2] for finite temperature partly attempting to explanation to the EMC effect. Here we do not introduce running coupling constant to account for interaction among deconfined quarks, but use QMDD.

Still consider color singlet spin-1/2 3-quark clusters, interaction between free quarks, between clusters and quarks and among clusters come in similar to hard-ball potential: as a factor of available volume multiplied to every momentum integral.

$$\eta = 1 - \frac{N_{cl}V_{cl}}{V} \quad (6)$$

To self-consistently solve cluster mass we will need a relation between volume

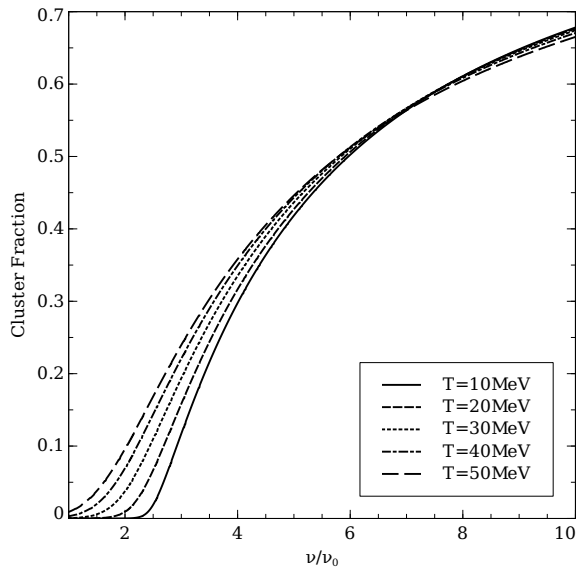


Figure 2: Cluster fraction with fixed cluster mass  $M_{cl} = 1000\text{MeV}$  at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$

and mass. Use the MIT bag model with a simplified form of total energy just as in [3]

$$M(R) = BV + \frac{c}{R} \quad (7)$$

and adopting Bag constant  $B = 161.5\text{MeV} \cdot fm^{-3} = (187.7\text{MeV})^4$  from Saito & Thomas [8] from a fit to baryon octet we can get  $c$  by requiring  $M(R)$  to have minimum value<sup>1</sup> of  $M_\Lambda = 1115\text{MeV}$  as roughly the average mass in baryon octet.

$$c = \left( \frac{3}{4} \frac{M_\Lambda}{(4\pi B)^{1/4}} \right)^{4/3} \approx 3.15 \quad (8)$$

Since hadronic vacuum has been moved out of the entire region of quark matter, compared to the environment, the cluster (as a MIT bag) would have an energy  $M_{cl}(R) = c/R$  which gives a  $M - V$  relation

$$V_{cl} = \frac{4\pi}{3} \left( \frac{c}{M_{cl}} \right)^3 \quad (9)$$

In addition, the pressure of cluster (which is also pressure of quark matter) is

$$P = -\frac{\partial M_{cl}}{\partial V_{cl}} = \frac{M_{cl}^4}{4\pi c^3} \quad (10)$$

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<sup>1</sup>actually the dependence for hadrons with strangeness is not  $c/R$  (T. DeGrand et al 1975 [4]) but the difference is small. Thus we continue to use parametrization in the massless case.

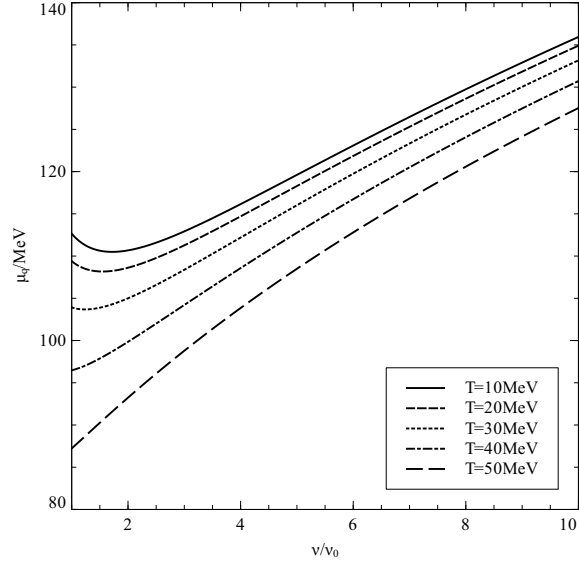


Figure 3: Chemical potential with cluster mass  $M_{cl} = 2M_u + M_s$  at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$

We also assume the relativistic equation of state following [3]:

$$P = \frac{1}{3}\epsilon \quad (11)$$

where the energy density  $\epsilon$  do not include vacuum energy  $B$ . Baryon number density, energy density and available volume factor  $\eta$  can be written as

$$\nu = \frac{\eta}{(2\pi)^3} \sum_{i=c,u,d,s} \int dp \frac{4\pi p^2 g_i B_i}{\exp\left(\frac{\sqrt{p^2 + M_i^2} - \mu_i}{T}\right)} \quad (12)$$

$$\epsilon = \frac{\eta}{(2\pi)^3} \sum_{i=c,u,d,s} \int dp \frac{4\pi p^2 g_i \sqrt{p^2 + M_i^2}}{\exp\left(\frac{\sqrt{p^2 + M_i^2} - \mu_i}{T}\right)} \quad (13)$$

$$\eta = 1 - \frac{4\pi}{3} \left(\frac{c}{M_{cl}}\right)^3 \int dp \frac{4\pi p^2 g_c}{\exp\left(\frac{\sqrt{p^2 + M_{cl}^2} - \mu_c}{T}\right) + 1} \quad (14)$$

Then chemical potential  $\mu_q$ , cluster mass  $M_{cl}$  and radius  $R_{cl}$  can be solved from equation (10), (11) and (12). Fig 5,6, 7 and 8 shows various quantities with temperature  $T = 10 \sim 50\text{MeV}$  as a function of baryon number density in the range  $1 \sim 10\nu_0$ .

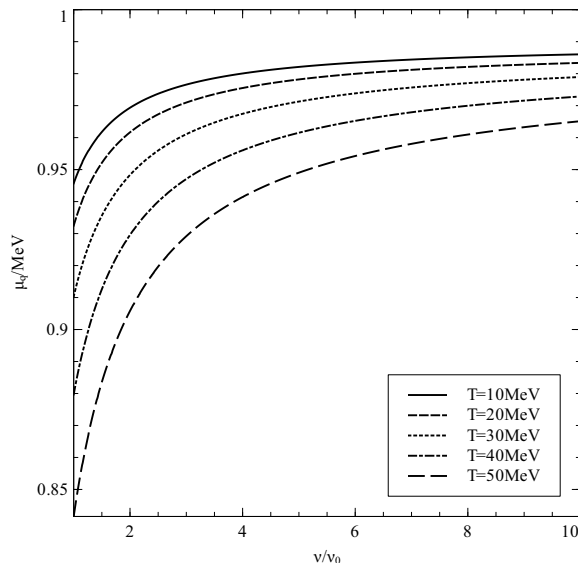


Figure 4: Cluster fraction with cluster mass  $M_{cl} = 2M_u + M_s$  at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$

## 4 Results and Discussion

For baryon number density in the range of about  $1 \sim 10$  times normal nuclear density and temperature not too high  $T = 10 \sim 50\text{MeV}$ , our treatment gets the result that

1. Cluster mass  $M_{cl}$  is in the range  $500 \sim 1100\text{MeV}$  and increase with increasing density.
2. Quark chemical potential is in the range  $200 \sim 400\text{MeV}$  and increase with increasing density
3. Available volume is about  $15\% \sim 30\%$  while  $80\% \sim 90\%$  of baryon number is in clusters. In other words, most volume and baryon number is in clusters at moderate densities similar to Clark's two flavor system. Above some temperature between  $30\text{MeV}$  to  $40\text{MeV}$  cluster fraction decrease with increasing density while below this temperature the behavior is the opposite.

As mentioned above, the excluded volume treatment interaction is taken into account only through influence of finite volume on phase space integral which is a very rough approximation. However, it suggests a possibility that clusters can appear also in strange quark matter and in which clusters takes most volume and baryon number. This in turn favors the picture that clusters immersed in small amount of free quarks which somehow resembles positive ion of normal

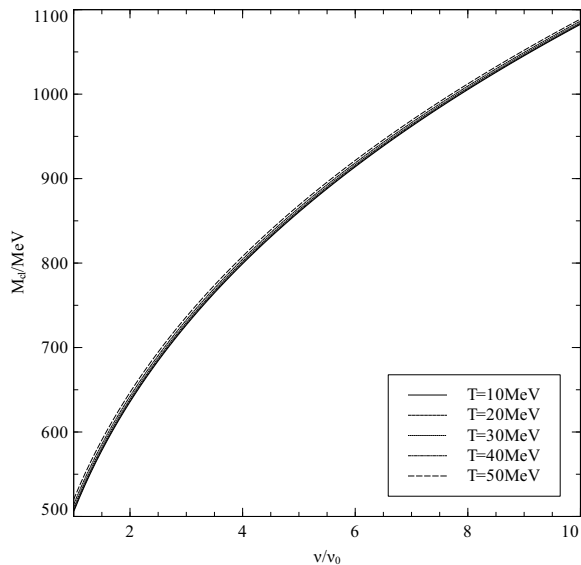


Figure 5: Cluster mass  $M_{cl}$  in excluded volume method at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$

metal immersed in electron gas.

On the other hand, while positive ions in metal are considered classical particles, in excluded volume method 3 quark clusters are still too light to be treated classically. In the future we plan to calculate clustering which involves more heavier species such as H-dibaryon [5] or even ‘quark alpha’ [6] with 18 quarks (six of each flavor). For heavy clusters of mass  $m$  GeV with baryon number  $B$  we can simply compare the non-relativistic expression of thermal wavelength  $\lambda = \sqrt{2\pi\hbar^2/(mk_B T)}$  to mean particle separation  $l = n^{-1/3} = (\nu/B)^{1/3}$  to work out a temperature scale above which the wave packets of clusters no longer strongly overlap. It can be show that this temperature is

$$\frac{T}{\text{MeV}} \simeq 72B^{-2/3} \frac{\text{GeV}}{m} \left( \frac{\nu}{\nu_0} \right)^{2/3} \quad (15)$$

which means that for clusters of mass that equals  $6m_\Lambda \simeq 6\text{GeV}$  (ignoring a possible binding energy which would reduce this mass) at density  $3 \sim 4\nu_0$  when temperature grows well above  $7 \sim 9\text{MeV}$  cluster would behave like classical particle which makes them capable of forming lattice.

I am grateful to pulsar group here at Peking University and Professor Efrain J. Ferrer and Vivian de la Incera from University of Texas El Paso for inspiring discussions.

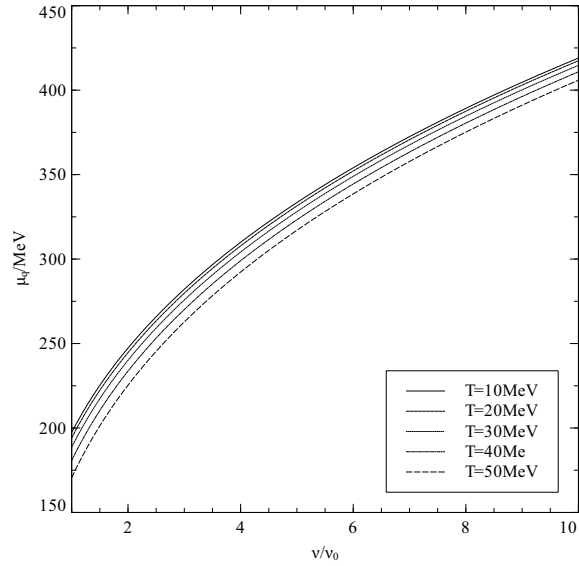


Figure 6: Chemical potential  $\mu_q$  in excluded volume method at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$

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## Discussion

**Prof. Qi-Ren Zhang (Peking University):** Do you think hadrons are clusters, if not what is the difference between hadrons and clusters? The second



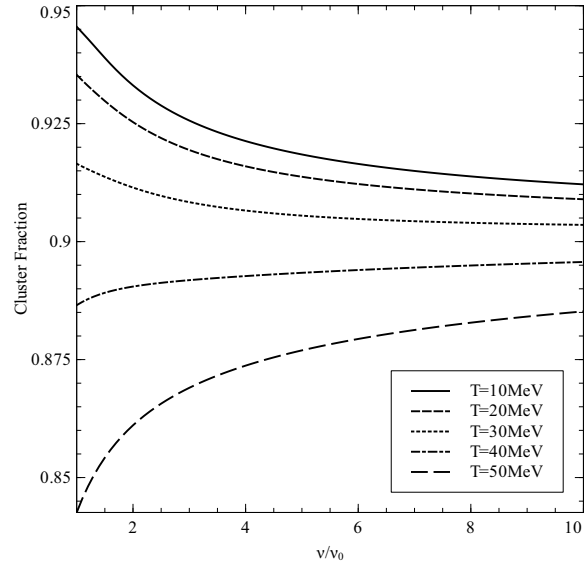


Figure 7: Cluster fraction in excluded volume method at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$

question is that there are models for nuclear matter based on the quark crystal model or bag crystal model. Is there any relations between your ideas and this model? This model was worked out about 20 years ago and some are quite impressive results for example they can reproduce nuclear data.

**Xuesen Na:** For the first question, the hadronic vacuum have been move out of the entire region which can be seen from equation of state we used.

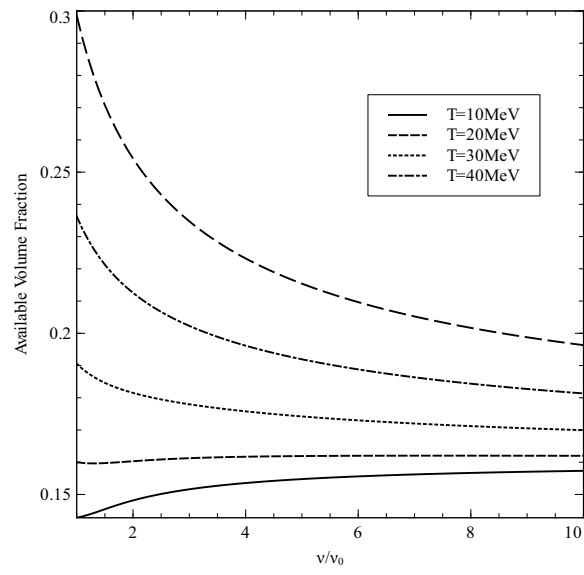


Figure 8: Available volume fraction in excluded volume method at  $1 \sim 10\nu_0$  and  $T = 10 \sim 50\text{MeV}$