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State of matter for quark stars

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Abstract

It depends on the state of matter at supra-nuclear density to model pulsar's structure, which is unfortunately not certain due to the difficulties in physics. In cold quark matter at realistic baryon densities of compact stars (with an average value of $\sim 2-3\rho_0$), the interaction between quarks is so strong that they would condensate in position space to form quark-clusters. We argue that quarks in quark stars are grouped in clusters, then we apply two phenomenological models for quark stars, the polytropic model and Lennard-Jones model. Both of the two models have stiffer EoS, and larger maximum mass for quark stars (larger than 2 $M_\odot$). The gravitational energy releases during the AIQ process could explain the observed energy of three supergiant flares from soft gamma-ray repeaters ($\sim 10^{47}$ ergs).

1 Introduction

It depends on the state of matter at supra-nuclear density to model pulsar's structure, which is unfortunately not certain due to the difficulties in physics although some efforts have been made for understanding the behavior of quantum chromo-dynamics (QCD) at high density. Of particular interest is whether the density in such compact stars could be high enough to result in unconfined quarks (quark matter). Stars composed of quarks (and possible gluons) as the dominant degrees of freedom are called quark stars, and there is possible observational evidence that pulsar-like stars could be quark stars(see reviews, e.g., [1, 2]). But it is still a problem to model a realistic quark star for our lack of knowledge about the real state of quark matter.

The study of cold quark matter opens a unique window to connect three active fields: particle physics, condensed matter physics, and astrophysics. Many possible states(see, e.g., [3]) of cold quark matter are proposed in effective QCD models as well as in phenomenological models. An interesting suggestion is that quark matter could be in a solid state [4, 5, 6, 7], since the strong interaction may render quarks grouped in clusters and the ground state of realistic quark matter might not be that of Fermi gassee a recent discussion given by [2]). If the residua interaction between quark clusters is stronger than their kinetic energy, each quark cluster could be trapped in the potential well and cold quark matter will be in a solid state. Solid quark stars still cannot be ruled out in both astrophysics and particle physics [5, 6]. Additionally, there is evidence that the interaction between quarks is very strong in hot quark-gluon plasma(i.e., the strongly coupled quark-gluon plasma, [8]), according to the recent achievements of relativistic heavy ion collision experiments. When the temperature goes down, it is reasonable to conjecture that the interaction between quarks should be stronger than that in the hot quark-gluon plasma.

Because of the difficulty to obtain a realistic state equation of cold quark matter at a few nuclear densities, we apply two phenomenological models, the polytropic model [9] and Lennard-Jones model [10], which would have some implications about the properties of QCD at low energy scale if the astronomical observations can provide us with some limitations on such models.

This paper is arranged as follows. The polytropic model is described in Section 2. The Lennard-Jones model is described in Section 3. We make conclusions and discussions in Section 4.

2 The polytropic model

Because one may draw naively an analogy between the clusters in quark matter and the nucleus in normal matter, we apply a polytropic equation of state to quark stars, with different polytropic indices, n. This model could be regarded as an extension to the quark star model with a linear equation of state. We are going to model quark stars in two separated ways.

(i) The vacuum inside and outside of a quark star is assumed the same, i.e., quark stars have no QCD vacuum energy. In this case, the equation of state for a quark star is the standard polytropic model, with a non-zero surface density, representing the strong confinement between quarks. Stars of perfect fluid in general relativity was discussed by [12], with an equation of state,

$$P = K{\rho_g}^{\Gamma}, \tag{1}$$

$$\epsilon = \rho_g c^2 + nP, \tag{2}$$

where ρ_g is that part of the mass density which satisfies a continuity equation and is therefore conserved throughout the motion, and $\Gamma = 1 + 1/n$. Because the quark clusters in quark stars are non-relativistic particles, the equation of state can be written as

$$P = K{\rho_g}^{\Gamma}, \tag{3}$$

$$\epsilon = \rho_g c^2. \tag{4}$$

The mass and radius of a star are evaluated at the point when the density reaches the surface density, which is non-zero because of the strong interaction.

(ii) The vacuum energy inside and outside of a quark star are different. In this case, the equation of state is

$$P = K\rho_g^{1+\frac{1}{n}} - \Lambda, \tag{5}$$

$$\epsilon = \rho_g c^2 + \Lambda. \tag{6}$$

The density at surface (where pressure is zero) should also be non-zero.

The key difference between polytropic quark star and normal star models lies on the surface density ρ_s ($\rho_s > 0$ for the former but $\rho_s = 0$ for the latter), since a quark star could be bound not only by gravity but also by additional strong interaction due to the strong confinement between quarks. The non-zero surface density is also natural in the case with the linear equation of state, where the binding effect is represented by the bag constant, B (and then $\rho_s = 4B$).

For stars of perfect fluid, the hydrostatic equilibrium condition reads

$$\frac{1 - 2GM(r)/c^2 r}{P + \rho c^2} r^2 \frac{\mathrm{d}P}{\mathrm{d}r} + \frac{GM(r)}{c^2} + \frac{4\pi G}{c^4} r^3 P = 0, \tag{7}$$

where

$$M(r) = \int_0^R \epsilon/c^2 \cdot 4\pi r^2 \mathrm{d}r. \tag{8}$$

For stars with anisotropic pressure, the hydrostatic equilibrium condition is different. To simplify the problem, we only consider the case of spherical symmetry, that the tangential and radial pressure are not equal. In this case the hydrostatic equilibrium condition reads(e.g., [13])

$$\frac{1 - 2GM(r)/c^2 r}{P + \rho c^2} (r^2 \frac{dP}{dr} - 2\varepsilon r p) + \frac{GM(r)}{c^2} + \frac{4\pi G}{c^4} r^3 P = 0, \tag{9}$$

where ε is defined by $P_\perp = (1 + \varepsilon)P$, and P is the radial pressure and $P_\perp$ is the tangential one. Combine the hydrostatic equilibrium condition and equation of state, one can calculate the structures of quark stars with and without QCD vacuum energy.

2.1 Mass-radius curves

The mass-radius relation for different polytropic indices, n, with surface density $\rho_s = 1.5\rho_0$ (ρ_0 is the nuclear matter density) is shown in Fig 1.

It is evident from the calculation that the maximum mass of quark star decreases as the index, n, increases. This is understandable. A small n means a large Γ, and

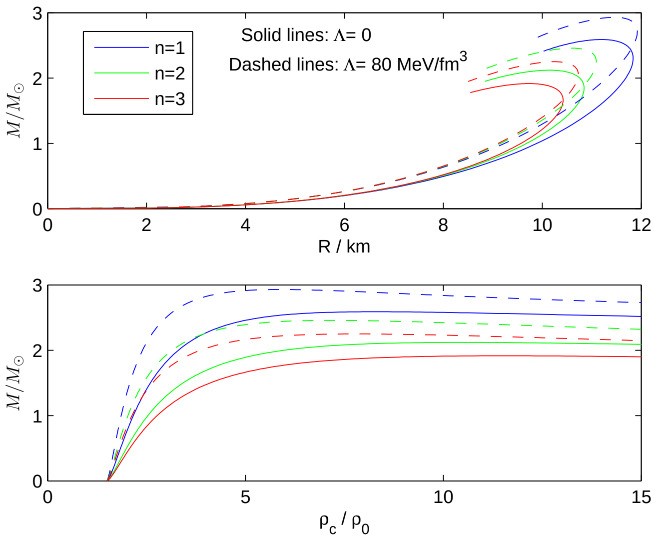

Figure 1: Mass-radius relations for different polytropic indices, n, with $\rho_s = 1.5\rho_0$. Solid lines are $\Lambda = 80\mathrm{MeVfm}^{-3}$, and dash-dotted lines for $\Lambda = 0$. Here and in the following figures, ρ_0 is the nuclear saturation density.

the pressure is relatively lower for higher values of n. Lower pressure should certainly support a lower mass of star.

On the gravitational stability. A polytropic star, with a state equation of $P \propto \rho^\Gamma$, supports itself against gravity by pressure, PR^2 (note: the stellar gravity $\propto M/R^2 \propto \rho R$). Certainly, a high pressure (and thus large Γ or small n) is necessary for a gravitationally stable star, otherwise a star could be unstable due to strong gravity. Actually, in the Newtonian gravity, a polytropic normal star (with $\rho_s = 0$) is gravitationally unstable if $n > 3$, but the star should be still unstable if $n = 3$ when the GR effect is included [11].

A polytropic quark star with non-zero surface density or with QCD vacuum energy, however, can still be gravitationally stable even if $n \geq 3$. A quark star with much low mass could be self-bound dominantly, and the gravity is negligible (thus not being gravitationally unstable). As the stellar mass increases, the gravitational effect becomes more and more significant, and finally the star could be gravitationally unstable when the mass increases beyond the maximum mass. The allowed region for central densities of stable stars are very narrow ($<\sim 5\rho_0$) for the chosen values of index n.

2.2 Gravitational energy released during a star quake

The gravitational energy difference between stars with $\varepsilon \neq 0$ and with $\varepsilon = 0$ are shown in Fig 2 for $n = 1$.

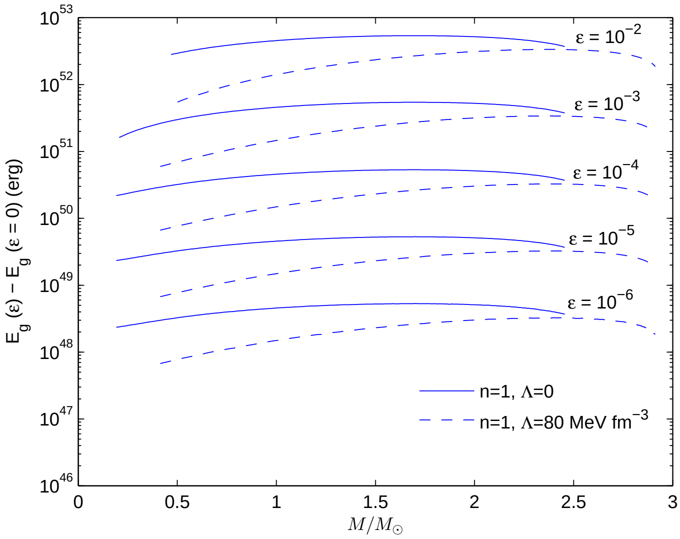

Figure 2: The gravitational energy difference between stars with and without anisotropic pressures, which may be released during sequential star quakes.

Three supergiant flares from soft γ-ray repeaters have been observed, with released photon energy being order of $\sim 10^{47}$ ergs. Our numerical results imply that for all the parameters we chosen, the released energy could be as high as the observed.

3 Lennard-Jones model

The inter-cluster potential could be written as the form of the potential between two inert gas molecules [14]

$$u(r) = 4U_0[(\frac{r_0}{r})^{12} - (\frac{r_0}{r})^6], \tag{10}$$

where U_0 is the depth of the potential and r_0 can be considered as the range of interaction. This form of potential has the property of short-distance repulsion and long-distance attraction.

We suppose that clusters form the simple-cubic structure, and then from the inter-cluster potential we can get the total energy density for cold quark matter

$$\epsilon_{\rm q} = 2U_0(A_{12}r_0^{12}n^5 - A_6 r_0^6 n^3) + \frac{9}{8}(6\pi^2)^{\frac{1}{3}}\hbar v n^{\frac{4}{3}} + n m_{\rm c} c^2, \tag{11}$$

where the first term comes from the potential between clusters, the second term comes from lattice vibration, and the last term comes from rest mass energy. m_c is the mass of each quark-cluster, $A_{12} = 6.2$ and $A_6 = 8.4$. The pressure can be derived as

$$\begin{aligned} P_q &= n^2 \frac{d(\epsilon_q/n)}{dn} \\ &= 4U_0(2A_{12}r_0^{12}n^5 - A_6 r_0^6 n^3) + \frac{3}{8}(6\pi^2)^{\frac{1}{3}}\hbar v n^{\frac{4}{3}}. \end{aligned} \tag{12}$$

The model of quark stars composed of Lennard-Jones matter is much different from the conventional models (e.g., MIT bag model) in which the ground state is of Fermi gas. In the former case the quark-clusters are non-relativistic particles, whereas in the the latter case quarks are relativistic particles. Consequently, the equations of state in this two kinds of models are different, and we find that the Lennard-Jones model has some more stiffer equations of state, which lead to higher maximum masses for quark stars. The mass-radius curves and mass-central density curves (the central density only includes the rest mass energy density), as are shown in Fig 3.

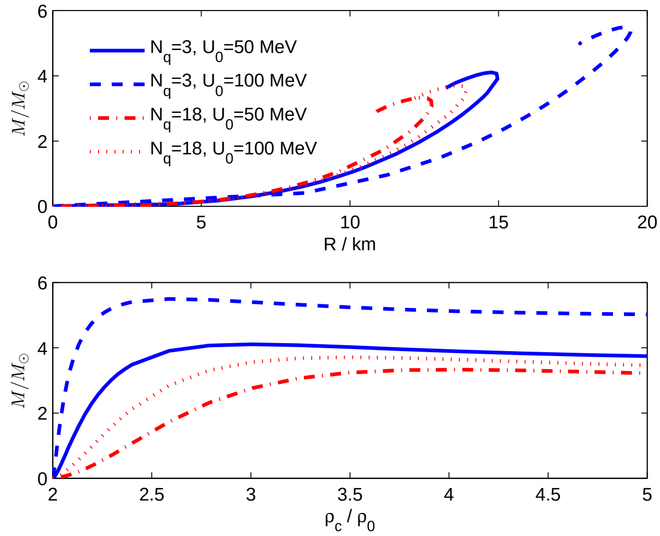

Figure 3: The mass-radius and mass-central density (rest-mass energy density) curves, in the case $N_q = 3$, including $U_0 = 50$ MeV (blue solid lines) and $U_0 = 100$ MeV (blue dashed lines), and the corresponding case $N_q = 18$ with $U_0 = 50$ MeV (red dash-dotted lines) and $U_0 = 100$ MeV (red dotted lines), for a given surface density $\rho_s = 2\rho_0$.

Because of stiffer equations of state, the maximum masses of quark stars in our model could be higher. In Fig.2, we can see that (i) a deeper potential well U_0 means a higher maximum mass; (ii) if there are more quarks in a quark-cluster, the maximum mass of a quark star will be lower.

A stiffer equation of state leading to a higher maximum mass could have very important astrophysical implications. Though we have not definitely detected any pulsar whose mass is higher than $2M_\odot$ up to now, the Lennard-Jones quark star model could be supported if massive pulsars ($> 2M_\odot$) are discovered in the future.

Comparison with the MIT bag model. In the MIT bag model, quark matter is composed of massless up and down quarks, massive strange quarks, and few electrons. Quarks are combined together by an extra pressure, denoted by the bag constant B. In our model, quarks are grouped in clusters and these clusters are non-relativistic particles. If the inter-cluster potential can be described as the Lennard-Jones form, the equation of state can be very stiff, because at a small inter-cluster distance (i.e., the number density is large enough), there is a very strong repulsion. Whereas in MIT bag model quarks are relativistic particles (at least for up and down quarks). For a relativistic system, the pressure is proportional to the energy density, so it cannot have stiff equation of state.

4 Conclusions and Discussions

In cold quark matter at realistic baryon densities of compact stars (with an average value of $\sim 2-3\rho_0$), the interaction between quarks is so strong that they would condensate in position space to form quark-clusters. Like the classical solid, if the inter-cluster potential is deep enough to trap the clusters in the potential wells, the quark matter would crystallize and form solid quark stars. This picture of quark stars is different from the one in which quarks form cooper pairs and quark stars are consequently color super-conductive.

We argue that quarks in quark stars are grouped in clusters, then we apply two phenomenological models for quark stars, the polytropic model and Lennard-Jones model. Both of the two models have stiffer EoS, and larger maximum mass for quark stars (larger than 2 $M_\odot$). The gravitational energy releases during the AIQ process could explain the observed energy of three supergiant flares from soft gamma-ray repeaters ($\sim 10^{47}$ ergs).

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