B decay and CP violation: CKM angles and sides at the BABAR and BELLE B-Factories

Marc Verderi
LLR Ecole polytechnique/IN2P3-CNRS
Route de Saclay
91128 Palaiseau, FRANCE

1 The CKM matrix and Unitarity Triangle

In the Standard Model (SM), the Cabibbo-Kobayashi-Maskawa (CKM) [1] matrix represents the coupling of the u, c, t up-quarks to the d, s, b down-quarks in the charged current interactions. It is a 3 × 3 unitarity matrix that can be parameterized by three mixing angles and one CP-violating phase, which is the only source of CP violation in the SM. The popular Wolfenstein parameterization [2] expresses the CKM matrix in term of the $\lambda \simeq 0.22$, $A \simeq 0.83$, $\rho$ and $\eta$ parameters and reflects the matrix hierarchy by a development in power of $\lambda$. The parameters $\rho$ and $\eta$ describe the CP violation, $\eta$ being the CP-violating phase. In this representation the CKM angles are carried by the $V_{td} = |V_{td}|e^{-i\beta} = A\lambda^3 (1 - \rho - i\eta)$ and $V_{ub} = |V_{ub}|e^{-i\gamma} = A\lambda^3 (\rho - i\eta)$ elements, the third angle being $\alpha = \pi - \beta - \gamma$.

The Unitarity Triangle (UT) depicts the unitarity condition of the CKM matrix between the first and third columns, namely $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, by a triangle in the complex plane, which apex is $\rho + i\eta$ and which angles are the previously mentionned $\alpha(\phi_2)$, $\beta(\phi_1)$, $\gamma(\phi_3)$ angles in the BABAR(BELLE) convention. As all terms in the above sum are of the order $\lambda^3$, the UT angles are sizeable.

The sides of the UT are measureable with non-CP violating processes, as semileptonic $B$ decays, or $B^0\bar{B}^0$ mixing frequency. The angles are measured with CP violating processes, like $B^0 \rightarrow J/\Psi K_S^0$. The $\alpha$ and $\beta$ angles are measured with decays of neutral $B$ mesons as they undergo $B^0\bar{B}^0$ mixing which is sensitive to the phase of the (off-shell) t-quark related $V_{td}$ through box diagrams. The angle $\gamma$ can be measured with neutral and charged $B$ meson decays.

There are three types of CP violation (CPV). The first one, so-called “direct” CPV, results from the difference between the amplitudes for a process $B \rightarrow f$ and its

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1Note that the relative phase between CKM elements does not depend on the matrix representation.

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measure the coefficients \(|B^0| - |\bar{B}^0|\). It is possible for both neutral and charged \(B\) meson decays, and is the only possible CPV for charged \(B\) meson decays.

The second type of CPV is in mixing and results from \(\langle B^0|B^0\rangle \neq \langle \bar{B}^0|B^0\rangle\). With \(|B^0\rangle\) and \(|\bar{B}^0\rangle\) being the CP-eigenstates, the mass eigenstates \(|B_L\rangle\) and \(|B_R\rangle\) are given by the linear relations \(|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle\) and \(|B_R\rangle = p|B^0\rangle - q|\bar{B}^0\rangle\) where \(p\) and \(q\) are complex coefficients. In the SM, the CPV in mixing is expected to be small with \(|p/q|\) departing from 1 at the \(\sim 10^{-3}\) level, close to present experimental limits.

The last type of CPV happens between interference mixing and decay. When a same final state, \(f_{CP}\), with CP-eigenstate value \(\eta_{fCP}\), can be reached by both \(B^0\) and \(\bar{B}^0\) mesons, the total amplitude for \(B^0 \to f_{CP}\) is the sum of the direct amplitude \(A_{CP}(B^0 \to f_{CP})\) and the amplitude \(A_{mix}(B^0 \to \bar{B}^0) \times \bar{A}_{CP}(\bar{B}^0 \to f_{CP})\) for a \(B^0\) to oscillate to a \(\bar{B}^0\) and then to decay to \(f_{CP}\). With \(\Gamma(B^0(\Delta t) \to f_{CP})\) (resp. \(\Gamma(\bar{B}^0(\Delta t) \to f_{CP})\)) being the decay rate for a \(B\) meson of known flavor \(B^0\) (resp. \(\bar{B}^0\)) at \(\Delta t = 0\), to decay to \(f_{CP}\) at \(\Delta t\), the time-dependent asymmetry \(A_{fCP}(\Delta t) \equiv \frac{\Gamma(B^0(\Delta t) \to f_{CP}) - \Gamma(\bar{B}^0(\Delta t) \to f_{CP})}{\Gamma(B^0(\Delta t) \to f_{CP}) + \Gamma(\bar{B}^0(\Delta t) \to f_{CP})}\) = \(S_{fCP}\) \(\sin(\Delta m_{B^0}\Delta t)\) - \(C_{fCP}\) \(\cos(\Delta m_{B^0}\Delta t)\), where \(\Delta m_{B^0}\) is the mass difference of the neutral \(B\) meson mass eigenstates, allows to measure the coefficients \(C_{fCP} \equiv \frac{1-|\lambda_{fCP}|^2}{1+|\lambda_{fCP}|^2}\) and \(S_{fCP} \equiv \frac{2\lambda_{fCP}}{1+|\lambda_{fCP}|^2}\) which are functions of the parameter \(\lambda_{fCP} \equiv \eta_{fCP}\). A non-zero value for \(C_{fCP}\) signifies a direct CPV. Even in the absence of such direct CPV, the asymmetry can be non-zero, as \(S_{fCP}\) is sensitive to the phase of \(\lambda_{fCP}\). This is notably the case for \(B^0 \to J/\Psi K^0_S\).

The \(B\)-Factories design, with a boost of the \(e^+e^- \to \Upsilon(4S) \to B\bar{B}\) system along the \(z\) axis, allows to measure \(\Delta t \simeq \Delta z/(\beta\gamma)\) by measuring the distance \(\Delta z\) between the two \(B\) decay vertices, \(\beta\) and \(\gamma\) being the boost parameters (not to be confused with the \(CP\) angles...). The beam energies are \(E_{\text{beam}}^- = 9\) GeV and \(E_{\text{beam}}^+ = 3.1\) GeV for BABAR and \(E_{\text{beam}}^+ = 8\) GeV and \(E_{\text{beam}}^- = 3.5\) GeV for BELLE. The initial \(B^0\) or \(\bar{B}^0\) CP-flavor of the \(B\) meson decaying to a \(CP\) final state, \(B_{CP}\), is inferred by a semi-inclusive reconstruction and analysis of the decay products of the other \(B\) meson, \(B_{tag}\), as follows. The decay time of \(B_{tag}\) defines \(\Delta t = 0\). At this time, by total antisymmetry of the \(B\bar{B}\) system from the \(1^{-+} \Upsilon(4S)\) decay, \(B_{CP}\) is in a pure \(CP\) state, opposite to that of \(B_{tag}\). The performances of the tagging and vertexing algorithms are determined on large samples of \(B\bar{B}\) events with a self-tagging \(B\) decay meson which is used in place of the \(B_{CP}\) meson. The typical resolution on \(\Delta z\) is about 170\(\mu\)m, largely dominated by the vertex resolution of the semi-inclusive reconstruction of the \(B_{tag}\), for an average \(\Delta z\) of about 260\(\mu\)m. The effective tagging efficiency \(\epsilon(1-2w)^2\), that includes tagging efficiency \(\epsilon\) and mistag fraction \(w\), is at the 30\% level.

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The perturbative and non-perturbative corrections that enter the inclusive decay rate \( \Gamma(B \to X_qq\nu) = \frac{G_F^2m_b^7}{192\pi^3} |V_{qb}|^2 [1 + A_{EW} A_{pert.} A_{nonpert.}] \) are computed as \( \alpha_s \) and \( (1/m_b) \) expansions, respectively. The non-perturbative parameters have to be measured, and depend on the \( m_b \) definition. They can be extracted from the moments \( \langle E^n \rangle_{E>\text{cut}} = \frac{1}{E_q} \int \langle E_l \rangle n \frac{d^3\nu}{d^3E_l} dE_l \) and \( \langle m^n_X \rangle_{E>\text{cut}} = \frac{1}{m_X} \int m^n_X \frac{d\tau}{dm_X} dm_X \) of the lepton energy and hadronic mass spectra, with integration above a minimum lepton energy \( E_{\text{cut}} \). The “kinetic” [3] and “1S” [4] frameworks, based on Heavy Quark Expansion (HQE) and Operator Product Expansion (OPE) provide calculations.

Experimentally, beyond the reconstruction used on the \( B \) signal side, additional criteria can be applied to the other \( B \) meson, \( B_{\text{tag}} \), which allows some trade-off between efficiency and purity. The \( B_{\text{tag}} \) can simply be unused, leading to an “Untagged” analysis, or a “Semileptonic tag” can be used, with partial reconstruction of \( B \to D(\ast)\pi \nu \), or an “Hadronic tag” can be required, with full reconstruction of \( B \to D(\ast)\pi/K \). This last case allows to reconstruct the full kinematics –as the missing neutrino momentum on the signal side can be estimated– \( B \) charge and flavor.

### 2.1 \( V_{cb} \) inclusive and exclusive

On a sample of 232M \( B\bar{B} \) events sample, BABAR has measured in an inclusive analysis with hadronic tag, the hadronic mass moments \( \langle m^k_X \rangle, k = 1, \ldots, 6 \) and the mixed hadronic energy-mass moments \( \langle n^k_X \rangle, k = 2, 4, 6, m^2_X \equiv m^2_2 2\alpha^2 - 2\Lambda E_X + \Lambda^2, \Lambda = 0.65 \) GeV, for minimal lepton momenta ranging from 0.8 to 1.9 GeV/c [5]. The \( \langle n^k_X \rangle \) moments allow a more reliable extraction of higher-order HQE parameters. The moments are combined in the fit in the “Kinetic” scheme with lepton-energy moments from [6] and photon-energy moments from \( B \to X_{s\gamma} \) [7]. This yields to \( |V_{cb}| = (41.88 \pm 0.44_{\text{exp}} \pm 0.35_{\text{theo}} \pm 0.59_{\gamma_{s\gamma}}) \times 10^{-3} \) and \( m_b = 4.552 \pm 0.038_{\text{exp}} \pm 0.040_{\text{theo}} \) GeV, together with a set of non-perturbative parameters. This determines \( |V_{cb}| \) up to a 2% precision.

A recent exclusive measurement of \( V_{cb} \) has been done by BABAR [8] on a sample of 226M \( B\bar{B} \) events with \( B^- \to D^*e^-\bar{\nu}, D^* \to D^{0}\pi^0 \). The differential decay rate \( \frac{d\Gamma}{d\omega} \).
where \( w \) is the dot product of the \( B \) and \( D^* \) four velocities, is measured in order to fit for heavy quark effective QCD correction parameters. One of the parameter, \( \rho_{A_1}^2 \), is uncertain with previous measurements with \( B^0 \to D^{*+} l^- \nu \), and this measurement can help to clarify the situation. BABAR measures \( BR(B^- \to D^{0} e^- \bar{\nu}_e) = (5.56 \pm 0.08 \pm 0.41)\% \), \( \rho_{A_1}^2 = 1.16 \pm 0.06 \pm 0.08 \) --in the center of the range obtained with neutral \( B \) decays-- and \( F(1) |V_{cb}| = (35.9 \pm 0.6 \pm 1.4) \times 10^{-3} \), which, using \( F(1) = 0.919 \pm 0.033 \) from Lattice QCD [9], leads to \( |V_{cb}| = (39.0 \pm 0.6 \pm 2.0) \times 10^{-3} \).

The BELLE [10] and BABAR [11] experiments have measured the \( B \to D^{*+} l \nu \) branching ratio as the pollution from this channel is a source of systematic uncertainty in \( |V_{cb}| \) analyses. They both use an hadronic tag for a full \( B \) signal reconstruction. HQET predicts that the rate for the broad \( D_0 \) channel is \( \frac{1}{10} \) of the narrow \( D^0 \) narrow. On a 657M \( B \bar{B} \) events sample, BELLE [10] disproves this expectation measuring e.g. \( BR(B^+ \to D_0^{*+} l^+ \nu) = (0.24 \pm 0.04 \pm 0.06)\% \) and \( BR(B^+ \to D_2^{*+} l^+ \nu) = (0.22 \pm 0.03 \pm 0.04)\% \), which is the first observation of this decay mode.

### 2.2 \( V_{ub} \) inclusive and exclusive

The measurement of the inclusive \( B \to X_s l \nu \) decay is complicated by the high background from \( B \to X_c l \nu \) decay which has a rate \( \sim 50 \) higher. Taking advantage of \( m_u \ll m_c \), \( B \to X_u l \nu \) analyses select regions of phase space free from \( B \to X_c l \nu \) background. This however happens in regions where non-perturbative effects are important. These are related to the “Fermi motion”, i.e. \( b \)-quark motion inside the meson, which is parameterized as a “Shape Function” (SF), extracted from the \( \gamma \) energy spectrum of \( B \to X_s \gamma \).

With an hadronic tag technique, and using the \( u \)- wrt \( c \)-quark discriminating variables \( M_X \), hadronic mass system, \( q^2 \), lepton-neutrino system mass squared, and \( P_+ \equiv E_X - |\vec{P}_X| \), with the hadronic energy \( E_X \) and momentum \( \vec{P}_X \) calculated in the \( B \) rest frame, BELLE [12] measures on a 275M \( B \bar{B} \) event samples \( |V_{ub}| = (4.09 \pm 0.19_{\text{exp}} \pm 0.20_{\text{syst}}^{+0.14}_{-0.15} \pm 0.18_{\text{theo}}) \times 10^{-3} \). BABAR [13] has performed a similar analysis and provide a series of \( |V_{ub}| \) measurements for various theoretical calculations.

A possible systematic uncertainty is due to the weak annihilation (WA) as this process could enhance the decay rate near the endpoint, where the \( |V_{ub}| \) measurement is done. WA may happen for charged \( B \) mesons only. BABAR has compared the partial decay rates of \( B^0 \to X_u l \nu \) and \( B^+ \to X_u l \nu \) in the 2.3–2.6 GeV/c of the lepton momentum range [14]. Measuring the ratio \( R^{+0} = 1.18 \pm 0.35_{\text{stat}} \pm 0.17_{\text{syst}} \), compatible with one, BABAR does not spot significant WA contribution.

Exclusive \( |V_{ub}| \) measurements have been performed by the \( B \)-Factories, with an untagged analysis of \( B \to \pi l \nu \) by BABAR [15] on a 227M \( B \bar{B} \) events sample and a \( D^* l \nu \) tag analysis of \( B \to \pi l \nu \) and \( B \to \rho l \nu \) on a 275M \( B \bar{B} \) data sample by
Table 1: BABAR [17] and BELLE [19] results for $\sin 2\beta (= \sin 2\phi_1)$, $|\lambda|$ and $A (= -C_{fCP})$.

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<thead>
<tr>
<th>BABAR</th>
<th>BELLE</th>
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<td>$\sin 2\beta$</td>
<td>$0.714 \pm 0.032_{stat} \pm 0.018_{syst}$</td>
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<td>$</td>
<td>\lambda</td>
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BELLE [16]. Recent unquenched lattice QCD results for the form factors are used.

### 2.3 $|V_{td}/V_{ts}|$ from $B_d$ and $B_s$ mixing

An important result, coming from the Tevatron, is $|V_{td}/V_{ts}|$ with $B_s$ mixing. The D0 and CDF collaborations have searched for the $B_s$ oscillation with a 5 standard deviations observation for CDF on the Run II data. The neutral $B_{q=d,s}$ meson oscillation frequency is related to the mass difference $\Delta m_q = \frac{G_F^2 m_q^2 \eta S(x^2)}{6\pi^2} B_q f^2_{B_q} B_{B_q} |V_{tq}^* V_{tb}|^2$. The $f_{B_q}$ form factors and $B_{B_q}$ parameters are known to a $\sim 15\%$ precision from Lattice calculations, leading to systematical uncertainty on $V_{td}$. The ratio $\xi^2 = \frac{f^2_{B_d}}{f^2_{B_s}} = 1.210^{+0.047}_{-0.035}$ is however known to a 4% precision, making $|V_{td}|/|V_{ts}|$ a more stringent constrain in the $(\rho, \eta)$ plan than the individual $|V_{td}|$ or $|V_{ts}|$.

Using an “amplitude scan” technique, events are fitted for $\frac{1}{2}e^{-t/\tau}(1\pm A \cdot D \cos(\Delta m_s t))$ where the probe parameter $A$ becomes compatible with one when $\Delta m_s$ takes one the correct value during the scan. The CDF result $\Delta m_s = 17.77 \pm 0.10_{stat} \pm 0.07_{syst}$ ps$^{-1}$ leads to $\left| \frac{V_{td}}{V_{ts}} \right| = 0.2060 \pm 0.0007_{exp} \pm 0.008_{theo}$.

### 3 UT Angles Measurements

#### 3.1 Measurement of $\beta/\phi_1$

The measurement of $\beta$ consists in collecting the phase of the $B^0\bar{B}^0$ mixing amplitude. The angle $\beta$ is by far the best measured angle of the UT, with the golden channel $B^0 \to J/\Psi K_S^0$. For this channel and related $b \to c\bar{s}s$ quark level transition channels, we have $\lambda_{fCP} = \eta_{fCP} \frac{\bar{s}s}{\bar{c}c} = \eta_{fCP} e^{-2i\beta}$, leading to $|\lambda_{fCP}| = 1, C_{fCP} = 0$. $S_{fCP} = -\eta_{fCP} \sin 2\beta$.

The uncertainty due to penquin pollution is expected to be at the $\sim 1\%$ level.

The BABAR measurement on $J/\Psi K_S^0, J/\Psi K^{*0}, \Psi(2S)K_S^0, J/\Psi K_L^0, \eta_c K_S^0, \chi_{c1} K_S^0$ channels on a 383M $B\bar{B}$ sample is shown in table 1. On a 535M $B\bar{B}$ event sample, BELLE measures for the $J/\Psi K_S^0, J/\Psi K_L^0$ channels [18] $\sin 2\phi_1 = 0.642 \pm 0.031_{stat} \pm 0.017_{syst}$, $A (= -C_{fCP}) = -0.018 \pm 0.021_{stat} \pm 0.014_{syst}$. BELLE performs a new
measurement for $\Psi(2S)K_0^0$ on a 657M $B\bar{B}$ sample [19], sin $2\phi_1 = +0.72 \pm 0.09_{\text{stat}} \pm 0.03_{\text{syst}}$, $A = +0.04 \pm 0.07_{\text{stat}} \pm 0.05_{\text{syst}}$, and provides the new average shown in table 1.

Constrain to the penguin pollution in $J/\Psi K^0$ can be obtained by SU(3) studying the $B^0 \to J/\Psi \pi^0$ channel. This is a $b \to c\bar{s}d$ quark-level transition process which carries at the tree level the same weak phase than the $J/\Psi K^0$ process. If a significant penguin pollution exists, the $S_{f\bar{c}p}$ and $C_{f\bar{c}p}$ parameters will differ from the expected the $-\sin 2\beta$ and 0 values, respectively.

On a 466M $B\bar{B}$ sample, BABAR measures the branching ratio and $CP$ parameters [20] $BR(J/\Psi \pi^0) = (1.60 \pm 0.14_{\text{stat}} \pm 0.07_{\text{syst}}) \times 10^{-5}$, $S_{J/\Psi \pi^0} = -1.23 \pm 0.21_{\text{stat}} \pm 0.04_{\text{syst}}$, $C_{J/\Psi \pi^0} = -0.20 \pm 0.19_{\text{stat}} \pm 0.03_{\text{syst}}$, which is a 4$sigma$ evidence for CPV. This is a new measurement. BELLE measures on a 535M $B\bar{B}$ sample the $CP$ parameters [21] $S_{J/\Psi \pi^0} = -0.65 \pm 0.21_{\text{stat}} \pm 0.05_{\text{syst}}$, $C_{J/\Psi \pi^0} = -0.08 \pm 0.16_{\text{stat}} \pm 0.05_{\text{syst}}$. This is a 2.4$sigma$ effect from 0 for $S_{J/\Psi \pi^0}$.

### 3.2 Measurement of $\alpha/\phi_2$

Significant complications arise in the case of the $\alpha$ angle measurement because of penguin pollution. For the $B^0 \to \pi^+\pi^-$ channel, a pure tree level process would carry a phase $-2\beta$ from the $B^0\bar{B}^0$ mixing and $-2\gamma$ from the tree decay, leading to

$$\lambda_{f\bar{c}p} = \eta_{f\bar{c}p} \frac{2}{3} A = \eta_{f\bar{c}p} e^{-2i\beta} e^{-2i\gamma} = \eta_{f\bar{c}p} e^{2i\alpha}$$

and $S_{f\bar{c}p} = -\eta_{f\bar{c}p} \sin 2\alpha, C_{f\bar{c}p} = 0$. The penguin pollution amplitude carries a different weak phase, and is at the 30 to 60% level of the tree amplitude. Denoting by $T$ and $P$ the tree and penguin amplitudes, respectively, and by $\delta = \delta_T - \delta_P$ their relative strong phase, the $CP$ parameters become

$$\lambda_{f\bar{c}p} = \eta_{f\bar{c}p} \frac{2}{3} e^{2i\alpha} A_T e^{i\delta} = \eta_{f\bar{c}p} |\lambda_{f\bar{c}p}| e^{2i\alpha_{\text{eff}}}, S_{f\bar{c}p} = \eta_{f\bar{c}p} \sqrt{1 - C_{f\bar{c}p}^2} \sin 2\alpha_{\text{eff}},$$

$C_{f\bar{c}p} \propto \sin \delta$. The measurement of the time-dependent asymmetry would only lead to a measurement of $\alpha_{\text{eff}}$. Extraction of $\alpha$ from $\alpha_{\text{eff}}$ is possible in principle (up to a 8-fold ambiguity) with an isospin analysis that compares the triangles formed by the amplitudes and by the conjugate amplitudes of $B^+ \to h^+ h^0$, $B^0 \to h^+ h^-, h^0 h^0$ [22]. It requires a time-dependent analysis of $B^0 \to h^0 h^0$. Upper bounds on $\sin^2(\alpha_{\text{eff}} - \alpha)$ can also be obtained and are interesting if $BR(B^0 \to h^0 h^0)$ is small [23].

On a 383M $B\bar{B}$ sample, BABAR extracts $1139 \pm 49$ $B^0 \to \pi^+\pi^-$ events, and obtains the $CP$ parameters $S_{\pi^+\pi^-} = -0.60 \pm 0.11_{\text{stat}} \pm 0.03_{\text{syst}}, C_{\pi^+\pi^-} = -0.21 \pm 0.09_{\text{stat}} \pm 0.02_{\text{syst}}$ [24]. BELLE obtains $1464 \pm 65$ signal events out of a 535M $B\bar{B}$ events sample and measures $S_{\pi^+\pi^-} = -0.61 \pm 0.10_{\text{stat}} \pm 0.04_{\text{syst}}, C_{\pi^+\pi^-} = -0.55 \pm 0.08_{\text{stat}} \pm 0.05_{\text{syst}}$ [25]. Both experiments observe a more than 5$sigma$ effect on $S_{\pi^+\pi^-}$. This makes the CPV well established in this channel. The BABAR and BELLE measurements on $C_{\pi^+\pi^-}$ differs today by 2.1$sigma$.

The $B$-Factories perform an isospin analysis to extract $\alpha$, using $S_{\pi^+\pi^-}, C_{\pi^+\pi^-}, C_{\pi^0\eta^0}, BF_{\pi^+\pi^-}$ and $BF_{\eta^0\eta^0}$. The parameter $S_{\pi^0\eta^0}$ is not used as it would require a challenging time-dependent analysis of the $B^0 \to \pi^0\eta^0$ channel. The confidence
Table 2: BABAR and BELLE results for $B \to \rho \rho$ analyses. BABAR results for $\rho^+\rho^-$ are obtained from a 383M $B\bar{B}$ events sample [27]. The BELLE measurements are performed on 265 and 535M $B\bar{B}$ event samples [28]. The BABAR analysis of $B^0 \to \rho^0 \rho^0$ is using a 427M $B\bar{B}$ events sample. Limits from BELLE [30] for this channel are given in the text. The first error is statistical, the second one systematical.

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<th>$B^0 \to \rho^+\rho^-$</th>
<th>$B^0 \to \rho^0\rho^0$</th>
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<tr>
<td>$BR \times 10^6$</td>
<td>$(25.5 \pm 2.1^{+3.0}_{-3.9})$</td>
<td>$(22.8 \pm 3.8^{+2.3}_{-2.6})$</td>
</tr>
<tr>
<td>$f_L$</td>
<td>$0.992 \pm 0.024^{+0.032}_{-0.026}$</td>
<td>$0.941^{+0.034}_{-0.046} \pm 0.30$</td>
</tr>
<tr>
<td>$C_L$</td>
<td>$+0.01 \pm 0.15 \pm 0.06$</td>
<td>$-0.16 \pm 0.21 \pm 0.08$</td>
</tr>
<tr>
<td>$S_L$</td>
<td>$-0.17 \pm 0.20^{+0.05}_{-0.06}$</td>
<td>$+0.19 \pm 0.30 \pm 0.08$</td>
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level (CL) curves obtained for $\alpha$ show a series of ambiguities, as they must. Picking up the SM compatible solution, BABAR obtains $\alpha = (96^{+11}_{-6})^\circ$ [26] and BELLE $\alpha = (96 \pm 11)^\circ$ [25].

The $B$-Factories have studied the $CP$ asymmetries of $B^0 \to \rho^+\rho^-$. The advantages of this channel are its large branching ratio and small penguin pollution. It is however an $a$ priori non pure $CP$ channel because of the vector-vector nature of its final state; but the longitudinal polarization fraction was found to be close to one, making this channel an almost pure $CP$-even final state. Experimental complications arise with the presence of two $\pi^0$’s in the final state, and because of the large $\rho$ width. The BABAR and BELLE measurements for $B^0 \to \rho^+\rho^-$ are shown in table 2.

In contrast with $B^0 \to \pi^0\pi^0$, the time-dependent analysis of the $B^0 \to \rho^0\rho^0$ can be performed, as the $\rho^0\rho^0$ vertex can be reconstructed. This allows for a full isospin analysis of $B \to \rho\rho$. A preliminary study of the time-dependent analysis of $B^0 \to \rho^0\rho^0$ has been performed by BABAR (table 2). This is a new measurement. The low branching fraction indicates that the penguin pollution is small. The full isospin analysis favors $\Delta\alpha = 11.3^\circ$. BELLE has performed a new measurement of the $B^0 \to \rho^0\rho^0$ branching fraction on a sample of 657M $B\bar{B}$ events, and finds $BR(\rho^0\rho^0) = (0.4 \pm 0.4^{stat} \pm 0.2^{syst}) \times 10^{-6}$, which is turned into the limit $BR(\rho^0\rho^0) < 1.0 \times 10^{-6}$ at 90% CL [30].

An other $\alpha$ measurement is performed in a Dalitz analysis of the $B^0 \to (\rho\pi)^0 \to \pi^+\pi^-\pi^0$ channel. Three amplitudes, namely $B^0 \to \rho^+\pi^-$, $B^0 \to \rho^-\pi^+$ and $B^0 \to \rho^0\pi^0$ ones, contribute to the final state. Note that the dominant $B^0 \to \rho^+\pi^-$ is not a $CP$ eigenstate. An isospin analysis, as it requires the two additional amplitudes $B^+ \to \rho^+\pi^0$ and $B^+ \to \rho^0\pi^+$, leads to a difficult isospin pentagon analysis. An
other approach was proposed by Snyder and Quinn [31], based on time-dependent Dalitz analysis, assuming isospin symmetry. The amplitude for $B^0 \rightarrow \pi^+\pi^0\pi^0$, and related charge conjugate amplitude, are described by

$$A(B^0 \rightarrow \pi^+\pi^-\pi^0) = f_+ A(\rho^+\pi^-) + f_- A(\rho^-\pi^+) + f_0 A(\rho^0\pi^0)$$

$$\bar{A}(\bar{B}^0 \rightarrow \pi^+\pi^-\pi^0) = f_+ \bar{A}(\rho^+\pi^-) + f_- \bar{A}(\rho^-\pi^+) + f_0 \bar{A}(\rho^0\pi^0),$$

and, in the $(\rho^-\pi^+, \rho^+\pi^-)$ masses square Dalitz plan, interferences at equal masses provide information on the strong phases between resonances.

On a 375M $B\bar{B}$ events sample, BABAR [32] has performed a time-dependent Dalitz analysis of $B^0 \rightarrow (\rho\pi)^0 \rightarrow \pi^+\pi^-\pi^0$. 2067 ± 86 signal events were found. BABAR measures $\alpha = (87^{+45}_{-13})^\circ$ (with a mirror solution at $\alpha + 180^\circ$). BELLE [33] has performed both the time-dependent Dalitz and isospin analyses on a 349M $B\bar{B}$ events sample and obtain the range $68^\circ < \alpha < 95^\circ$ at 68% CL.

Additional channels to measure $\alpha$ are studied. The $B^0 \rightarrow a_1\pi$ channel is considered by both the BABAR and BELLE experiments. It is similar to the $B \rightarrow \pi\pi$ case as it is not a $CP$ eigenstate, and as a quasi two-body approach has to be followed. A quite high branching fraction is measured by both BABAR, $BR(B^0 \rightarrow a_1\pi) = (33.2 \pm 3.2_{\text{stat}} \pm 3.2_{\text{syst}}) \times 10^{-6}$ [34], and BELLE $BR(B^0 \rightarrow a_1\pi) = (29.8 \pm 3.2_{\text{stat}} \pm 4.6_{\text{syst}}) \times 10^{-6}$ [35]. BABAR extracts $\alpha_{\text{eff}}^{a_1\pi} = (78.6 \pm 7.3)^\circ$. To further constrain $\alpha - \alpha_{\text{eff}}$ by SU(3) symmetry ($\pi \leftrightarrow K$, $a_1 \leftrightarrow K_1$), studies of $B \rightarrow a_1K$ are done. BABAR measures $BR(B^0 \rightarrow a_1K^+) = (16.3 \pm 2.9_{\text{stat}} \pm 2.3_{\text{syst}}) \times 10^{-6}$ and $BR(B^0 \rightarrow a_1K^0) = (34.9 \pm 5.0_{\text{stat}} \pm 4.4_{\text{syst}}) \times 10^{-6}$ [36].

### 3.3 Measurement of $\gamma/\phi_3$

Measurements of $\gamma$ with charged $B$ meson decays (no results with neutral $B$ meson decays presented here) exploit the interferences between the color favored $B^- \rightarrow K^{(*)}D^{(*)0}$ and color suppressed $B^- \rightarrow K^{(*)}\bar{D}^{(*)0}$ amplitudes that arise when final states common to the $D^{(*)0}$ and $\bar{D}^{(*)0}$ mesons are selected. As no penguin pollution exists, these are theoretically clean measurements. The color favored and suppressed $B$ decay amplitudes are respectively proportional to $\lambda^3$ and $\lambda^3 r_B^{(*)} e^{-i\epsilon} e^{i\delta}$, with $\delta$ being their relative (unknown) strong phase, and $r_B^{(*)}$ the critical ratio of the suppressed to favored amplitudes, which ranges from 0.1 to 0.2. The angle $\gamma$ has to be determined together with previous parameters.

The three following methods are used [37]. The Gronau-London-Wyler (GLW) method considers $D^0/\bar{D}^0$ $CP$-eigenstate final states with $CP$-even states like $K^+K^-$, $\pi^+\pi^-$, or $CP$-odd states like $K_S^0\pi^0$, $K_S^0\omega$, $K_S^0\phi$. It is based on modes with branching ratio at the $10^{-6}$ level. The Atwood-Dunietz-Soni (ADS) method considers the $K^+\pi^-\pi^0$ final state for the $D^0/\bar{D}^0$ meson. By combining the favored $B^- \rightarrow K^-D^0$ amplitude with the doubly-Cabibbo suppressed $D^0 \rightarrow K^+\pi^-$ one and the suppressed
Belle provides the limits for the $\text{BR}(B^+ \rightarrow DK^+)$ and $\text{BR}(B^+ \rightarrow D^+K^+)$ phases. No significant signal is observed at this point in the suppressed mode, and the measurement of $\Delta m$ is $\pm 10^{+4}_{-2}$, which is consistent with $\Delta m = 0$ at the $90\%$ CL.

Table 3: Updated BABAR measurements of GLW observables on a 383M $B\overline{B}$ events sample. The $D^*$ results are preliminary. Errors are statistical and systematical.

<table>
<thead>
<tr>
<th>$B^+ \rightarrow DK^+$</th>
<th>$B^+ \rightarrow D^+K^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{CP^+}$</td>
<td>$0.27 \pm 0.09 \pm 0.04$</td>
</tr>
<tr>
<td>$A_{CP^-}$</td>
<td>$-0.09 \pm 0.09 \pm 0.02$</td>
</tr>
<tr>
<td>$R_{CP^+}$</td>
<td>$0.10 \pm 0.05 \pm 0.05$</td>
</tr>
<tr>
<td>$R_{CP^-}$</td>
<td>$0.10 \pm 0.12 \pm 0.04$</td>
</tr>
</tbody>
</table>

$B^- \rightarrow K^-\overline{D}^0$ with the favored $\overline{D}^0 \rightarrow K^+\pi^-$ one, the ADS method is targeting large $CP$ asymmetries. This is at the cost of branching ratios at the $10^{-7}$ level. The Giri-Grossman-Soffer-Zupan (GGSZ) considers three-body final states like $K_S^0 K^- \pi^-$, $K_S^0 K^- \pi^+$, and extracts parameters from a Dalitz analysis.

BABAR has provided updated results on the $B^+ \rightarrow D^{(*)+}K^+$ channels with the GLW method [38]. The GLW observables, $R_{CP^\pm} = \frac{\Gamma(B^+ \rightarrow D_{supp}^{CP^\pm} K^-)}{\Gamma(B^+ \rightarrow D_{favored} K^-)}$, $A_{CP^\pm} = \frac{\Gamma(B^+ \rightarrow D_{supp}^{CP^\pm} K^-) - \Gamma(B^+ \rightarrow D_{favored} K^-)}{\Gamma(B^+ \rightarrow D_{supp}^{CP^\pm} K^-) + \Gamma(B^+ \rightarrow D_{favored} K^-)}$, $\delta = \delta_B + \delta_D$, and extracts parameters from a Dalitz analysis.

BELLE has provided updated results for $B^+ \rightarrow DK^+$ with the ADS method, on a 657M $B\overline{B}$ events sample [39]. The observables in the ADS method are $R_{ADS} = \frac{\Gamma(B^- \rightarrow D_{supp} K^0)}{\Gamma(B^- \rightarrow D_{favored} K^0)} = \frac{1 + r_B^2 \pm 2 r_B \cos \delta \cos \gamma}{1 + r_B^2 \pm 2 r_B \cos \delta \cos \gamma}, \quad A_{ADS} = \frac{\Gamma(B^- \rightarrow D_{supp} K^0) - \Gamma(B^- \rightarrow D_{favored} K^0)}{\Gamma(B^- \rightarrow D_{supp} K^0) + \Gamma(B^- \rightarrow D_{favored} K^0)} = 2 r_B r_D \sin \gamma \sin \delta / R_{ADS}$. No significant signal is observed at this point in the suppressed mode, and BELLE provides the limits $BR(B \rightarrow D_{supp} K) < 2.8 \times 10^{-7}$ at 90% CL, $r_B < 0.19$ at 90% CL.

In the GGSZ method, the $CP$ amplitudes, $A_{\pm}(m_2^2, m_3^2)$, describing the Dalitz plan with coordinates $m_2^2 \equiv m^2(K_0^0 \pi^\pm)$ or $m^2(K_S^0 K^\pm)$ or $m^2(\pi^0 \pi^\pm)$, depending on the final state considered, are given by $A_{\pm}(m_2^2, m_3^2) = |A(B^+ \rightarrow \overline{D}^0/D^0 K^\pm)| \times [A_D(m_2^2, m_3^2) + r_B e^{i \delta_B} e^{\pm i \gamma} A_D(m_2^2, m_3^2)],$ where $A_D$ is the amplitude for describing the $D^0$ Dalitz plan. This method allows to extract $\gamma$ and $\delta_B$ up to the two-fold ambiguity $(\gamma, \delta_B) \leftrightarrow (\gamma + \pi, \delta_B + \pi)$.

Technically, the $\gamma, r_B$ and $\delta_B$ parameters are extracted using the cartesian coordinates $x_\pm \equiv \kappa r_B \cos(\delta_B \pm \gamma), y_\pm \equiv \kappa r_B \sin(\delta_B \pm \gamma)$, which are Gaussian-behaving and SM fit.
Table 4: BABAR [40] and BELLE [41] results for the GGSZ analysis obtained on samples of respectively 383 and 657M \(B\bar{B}\) events. Errors are statistical, systematical and, if present, due to the \(D\) Dalitz model.

<table>
<thead>
<tr>
<th>(B^\text{-} \to D K^\text{-})</th>
<th>(B^\text{-} \to D^\ast K^\text{-})</th>
<th>(B^\text{-} \to D^\ast K^\text{-})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_-, x^*_-)</td>
<td>+0.090 ± 0.043 ± 0.015 ± 0.011</td>
<td>-0.111 ± 0.069 ± 0.014 ± 0.004</td>
</tr>
<tr>
<td>(y_-, y^*_-)</td>
<td>+0.053 ± 0.056 ± 0.007 ± 0.015</td>
<td>-0.051 ± 0.080 ± 0.009 ± 0.010</td>
</tr>
<tr>
<td>(x_+, x^*_+)</td>
<td>-0.067 ± 0.043 ± 0.014 ± 0.011</td>
<td>+0.137 ± 0.068 ± 0.014 ± 0.005</td>
</tr>
<tr>
<td>(y_+, y^*_+)</td>
<td>-0.015 ± 0.055 ± 0.006 ± 0.008</td>
<td>+0.080 ± 0.102 ± 0.010 ± 0.012</td>
</tr>
<tr>
<td>(r_B)</td>
<td>0.086 ± 0.035</td>
<td>0.135 ± 0.051</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(B^\text{-} \to D K^\ast-) (BABAR)</th>
<th>(B^\text{-} \to D K^\ast-) (BABAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{s-}, x_{s+})</td>
<td>+0.115 ± 0.138 ± 0.039 ± 0.014</td>
</tr>
<tr>
<td>(y_{s-}, y_{s+})</td>
<td>+0.226 ± 0.142 ± 0.058 ± 0.011</td>
</tr>
<tr>
<td>(\kappa r_s)</td>
<td>0.163^{+0.088}_{-0.105}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BABAR</th>
<th>BELLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>((76 ± 22 ± 5 ± 5)^\circ)</td>
</tr>
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</table>

make the likelihood unbiased. The total decay rate to fit for, is then \(\Gamma_{\pm}(m^2_+, m^2) \propto |A_{D^\pm}|^2 + r_B^2 |A_{D^\ast}|^2 + 2\eta \{x_\pm \text{Re}[A_{D^\pm} A_{D^\ast}^\pm] + y_\pm \text{Im}[A_{D^\pm} A_{D^\ast}^\pm]\} + |A_{D^\ast}|^2 + 2\eta \} \) where \(\kappa\) accounts for the \(K^\ast\) width and \(\eta\) for the parity of \(D^\ast\to D^0\gamma\) wrt \(D^0\pi^0\).

An accurate description of the \(D^0\) Dalitz plan is needed. High statistics samples of \(D^{++} \to D^0\pi^+\) are used, tagging the \(D^0/\bar{D}^0\) flavor with the companion pion charge. Updated parameterizations from BABAR and BELLE are detailed in [40, 41] for the \(D^0 \to K_S^0\pi^+\pi^-\) decay, and from BABAR for \(D^0 \to K_S^0 K^+K^-\). Parameterizations are based on an isobar approach consisting of a coherent sum of two-body amplitudes and non-resonant contributions.

The BABAR and BELLE results are shown in table 4, and 3\(\sigma\) and 3.5\(\sigma\) evidences for direct CPV are observed, respectively. It can be noticed that statistical errors on \(\gamma\) are significantly lower for BELLE than for BABAR, despite similar precision on the \(x^{(s)}_\pm\) and \(y^{(s)}_\pm\) quantities. This is due to the larger \(r^{(s)}_B\) values obtained by BELLE.
4 Conclusion

A remarkable success has been achieved by the $B$-Factories, going beyond expectation in some field, like the measurement of $\gamma$. BABAR has now finished its data taking, leaving BELLE alone in the “race”, but still many analyses are going on.

The CKM UT is constrained by both measurements of $CP$-conserving and $CP$-violating quantities, leading to a picture of the CKM sector consistent with the SM. Measurements of semi-leptonic decays benefit from improving experimental techniques and more precise theoretical computations. The angle $\beta$ is a precision measurement, reaching accuracy of SM calculation. The angle $\alpha$ will ultimately be limited by penguin pollution. The measurement of $\gamma$ is reaching the $13^\circ$ precision.

References

Marc Verderi  B decay & CP violation: CKM angles & sides at BABAR & BELLE

[38] B.Aubert at al. Phys. Rev. D 77, 111102 (2008); The $D^*$ results were presented at FPCP08, by D. Kirkby, “Hot topics”, BABAR.

