Recoilless Resonant Emission and Detection of Electron Antineutrinos:
Mössbauer Antineutrinos

Walter Potzel
Technische Universität München
Physik-Department E15, Garching, Germany

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Outline

I) Bound-state β-decay: resonant character

II) Example: ^3H – ^3He system

III) Recoilless $\bar{\nu}_e$ emission and detection:
Mössbauer Anti-neutrinos

1) Recoilfree fraction, lattice expansion and contraction
2) Linewidth: homogeneous and inhomogeneous broadening
3) Relativistic effects: Second-order Doppler shift
    a) temperature
    b) zero-point motion

IV) Requirements for successful experiment

V) Interesting experiments

VI) Conclusions

I) β-decay

I) Bound-state β-decay

J. N. Bahcall, Phys. Rev. 124, 495 (1961)

$$A(Z-1) \rightarrow \boxed{A(Z) + e^-} + \bar{\nu}_e$$

$\bar{\nu}_e - source$
mono-energetic

Bound-state atomic orbit.
Not a capture of e⁻ initially created in a continuum state (less probable).

2-body process, $\bar{\nu}_e$ has a fixed energy:

$$\boxed{E_{\bar{\nu}_e} = Q + B_z - E_R}$$ where

$Q = (M_{Z-1} - M_Z)c^2$ end-point energy

B_z binding energy

E_R recoil energy $\boxed{{}^3He+e^-}$ recoils

Example:

$$^3H \rightarrow \boxed{^3He + e^-} + \bar{\nu}_e$$

Atomic orbit in ^{3}He

I) β-decay

Reverse process (absorption):

$$\overline{\nu}_e + \boxed{\mathrm{A(Z)} + \mathrm{e}^-} \rightarrow A(Z-1)$$

in atomic orbit

target for $\overline{\nu}_e$

Example:

$$\overline{\nu}_e + \boxed{{}^3He + \mathrm{e}^-} \rightarrow {}^3H$$

in atomic orbit of ^{3}He

energy required for $\overline{\nu}_e$:

$$\boxed{E_{\overline{\nu}_e}{}' = Q + B_z + E_R'}$$

${}^3H$ recoils

Bound-state β-decay has a resonant character which is (partially) destroyed by the recoil in source and target.

II) Example: ^{3}H-^{3}He system

$Decay$	$E_{\bar{\nu}_e}^{res}$	$ft_{1/2}$	$B\beta \,/\, C\beta$
$^3H \rightarrow {}^3He$	18.60 keV	1132 sec	6.9×10^{-3} (80% ground state, 20% excited states)

^{3}H (source) and ^{3}He (target): gases at T=300K
→thermal motion, Doppler energy profile

$$FWHM(300K): 2\Delta = 2E_{\bar{\nu}_e}^{res}(2k_B T / Mc^2)^{1/2} \approx 0.16eV$$

→ resonant spectral density ρ≈10^6/0.16≈6×10^6

recoil energy:

$$E_R = \frac{(E_{\bar{\nu}_e}^{res})^2}{2Mc^2} \approx 0.06eV$$

$2E_R$≈0.12eV; 2Δ≈0.16eV
→reduced overlap

Resonance cross section: σ ≈ 1×10^{-42} cm^2

To observe bound-state β-decay: 100-MCi sources (^{3}H) and kg-targets (^{3}He) would be necessary

III) Recoilless antineutrino emission and detection: Mössbauer neutrinos

1) Recoilfree fraction

Stop thermal motion!
Make E_R negligibly small!

recoil energy:

$$E_R = \frac{(E_{\bar{\nu}_e}^{res})^2}{2Mc^2}$$

^3H as well as ^3He in metallic lattices:
freeze their motion →no Doppler broadening.
$M \rightarrow M_{lattice} >> M$
Leave lattice unchanged, leave phonons unchanged.

zero-point energy

Energy of lattice with N particles: $E_L = \sum_{s=1}^{3N} (n_s + 1/2)\hbar\omega_s$ $\quad (n_s = 0,1,2,\ldots)$

3N normal modes

$$E_L = \int_0^{\omega_{max}} \left(\overline{n(\omega)} + 1/2\right)\hbar\omega \cdot Z(\omega)\, d\omega$$ with $\overline{n(\omega)} = 1/\left(\exp(\hbar\omega/k_B T) - 1\right)$

$Z(\omega) \cdot d\omega$: number of oscillators with frequency *ω* between *ω* and *ω* + *dω*

III) Recoilless antineutrino …

Recoilfree fraction f: $f = e^{-\left(\frac{E}{\eta c}\right)^2 \cdot \langle x^2 \rangle} \longrightarrow f < 1$

$$f(T) = \exp\left\{ -\frac{E^2}{2Mc^2} \cdot \frac{1}{3N} \cdot \int_0^{\omega_{\max}} \frac{Z(\omega)}{\eta\omega} \left[\frac{2}{\exp\{\eta\omega / k_B T\} - 1} + 1 \right] d\omega \right\}$$

Debye model:

$T \rightarrow 0$: $f(T \rightarrow 0) = \exp\left\{ -\frac{E^2}{2Mc^2} \cdot \frac{3}{2k_B \Theta} \right\}$

f depends on: transition energy E
mass M of the atom
Debye temperature Θ

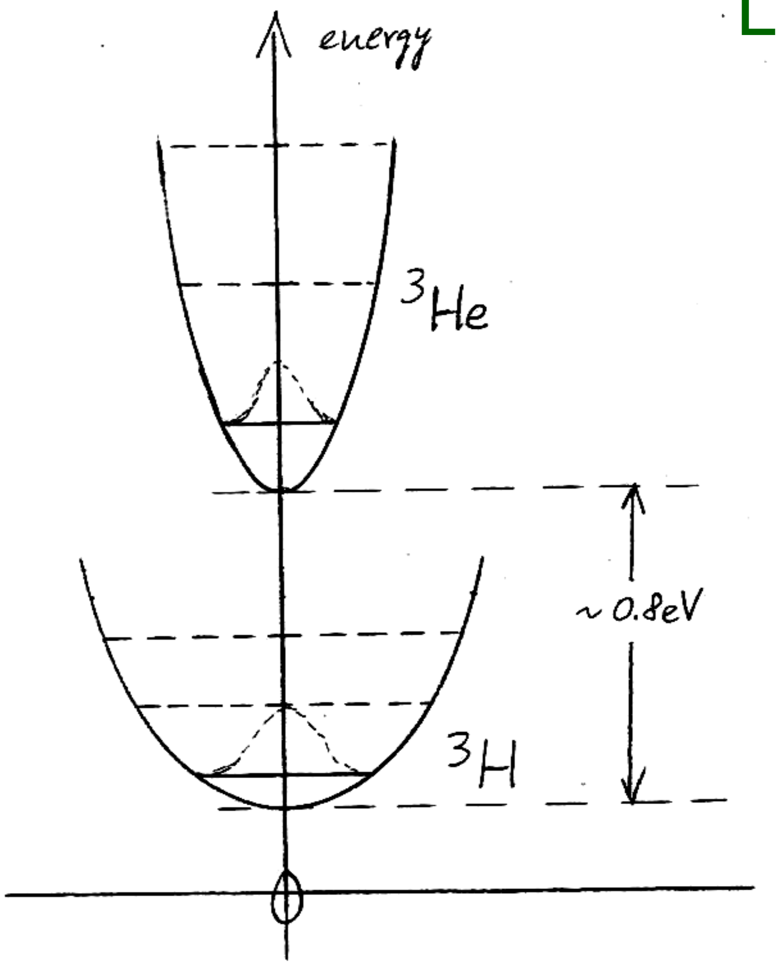
Example: $^3H - {}^3He$

typically: $f(0) \approx 0.27$ for Θ≈800K

Emission and absorption:

$f^{^3H} \cdot f^{^3He} \approx 0.07$ for T→0

III) Recoilless antineutrino …



Lattice expansion and contraction

^{3}He and ^{3}H use different amount of lattice space. ^{3}H is more strongly bound than ^{3}He. Will this cause lattice excitations?

The antineutrino picks up (delivers) most of the 18.6 keV transition energy. The energy of the electron is very small.

Adiabatic approximation: when ^{3}He forms, it pushes neighboring atoms away; lattice energy is "stored " until it is released again when by antineutrino absorption a ^{3}H is formed.

→ Theoretical calculations

III) Recoilless antineutrino …

2) Linewidth

minimal width (natural width): $\Delta E^{nat} = \Gamma = \eta / \tau$ τ : lifetime

^{3}H: τ = 17.81 y $\longrightarrow$ $\Delta E^{nat} = \Gamma = 1.17 \cdot 10^{-24} eV$ (extremely narrow)

Two types of line broadening:

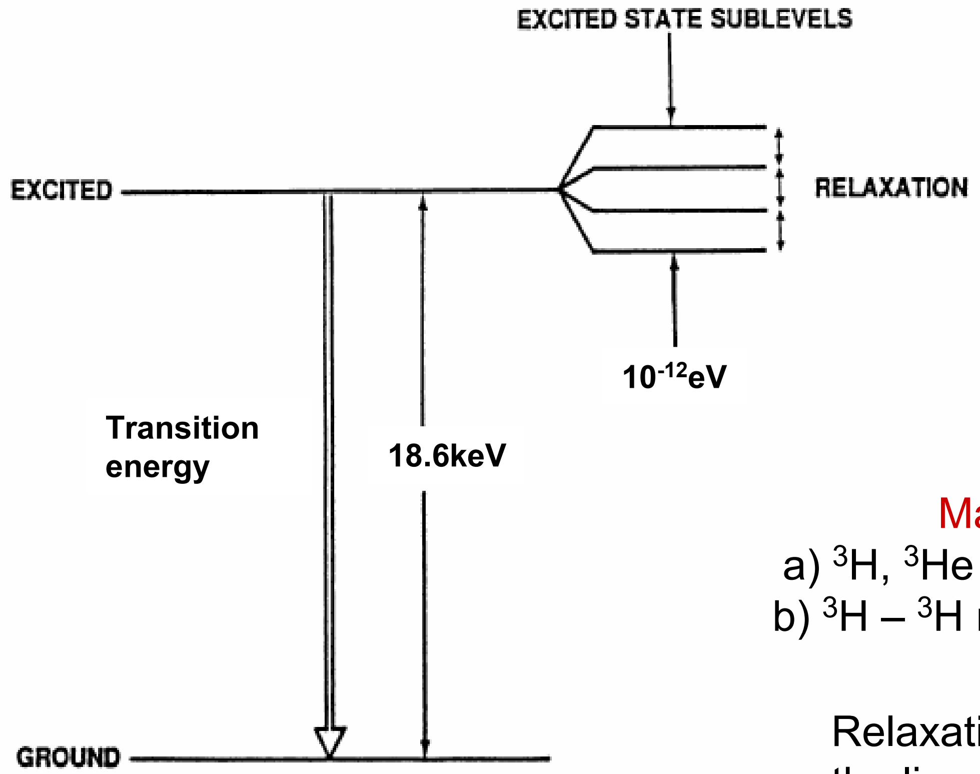
a) homogeneous broadening $\downarrow$ due to fluctuations, e. g. of magnetic fields

b) inhomogeneous broadening $\downarrow$ due to stationary effects, e.g. impurities, lattice defects

How big are these broadening effects?

III) Recoilless antineutrino …

a) homogeneous broadening



Measurements: ^{3}H (Pd), ^{3}H (Ti-H), NbH

Typical relaxation times: T_2~2ms, 79μs

$\Delta E \sim 8.6\text{x}10^{-12}\text{eV} = \Gamma_H$
$\sim 7\text{x}10^{12}\ \Gamma$

Magnetic interactions:
a) ^{3}H, ^{3}He with atoms (nuclei) of metallic lattice
b) ^{3}H – ^{3}H magnetic dipolar spin-spin interaction

Relaxation between the sublevels affects the lineshape and the total linewidth.

The linewidth is determined by the relaxation rate.

2) Linewidth

^{109m}Ag: gravitational spectrometer

$\Gamma \approx 1.2 \cdot 10^{-17}\, eV \qquad \tau \approx 40 s$

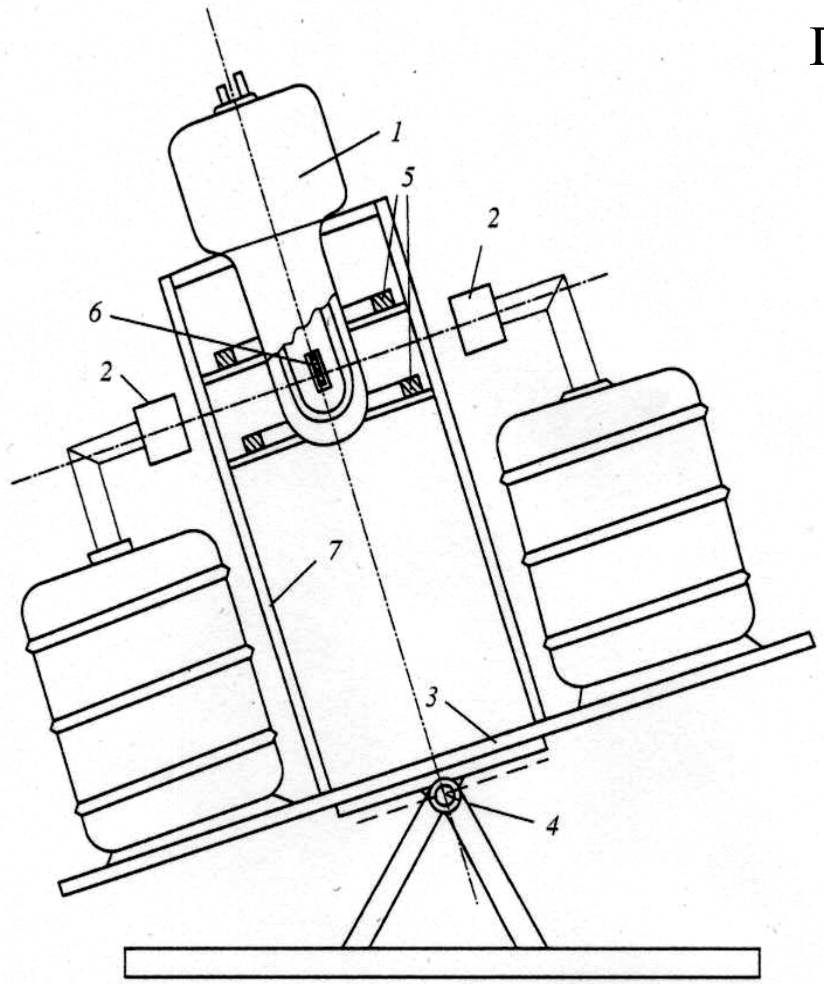
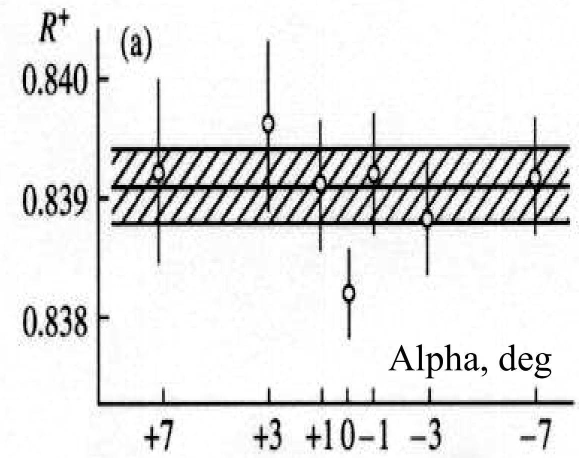
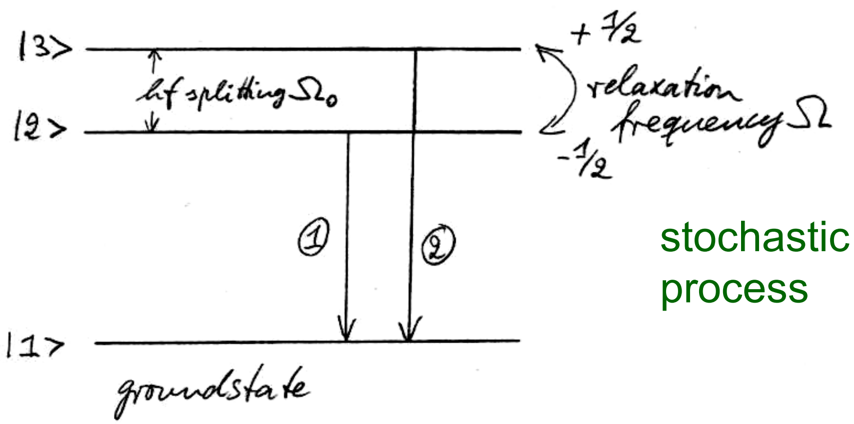


Fig. 1. Scheme of the gravitational gamma spectrometer: (*1*) cryostat, (*2*) germanium gamma detectors, (*3*) rotating platform, (*4*) support of cryostat and Helmholtz coils, (*5*) Helmholtz coils, (*6*) gamma sources, and (*7*) rotation axic of the platform.



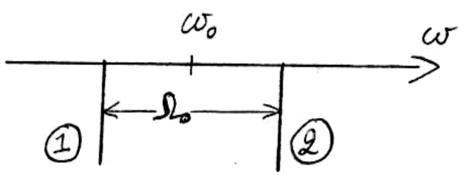
V.G. Alpatov et al., Laser Physics 17 (2007) 1067

Homogeneous Broadening: Magnetic Relaxation



stochastic process

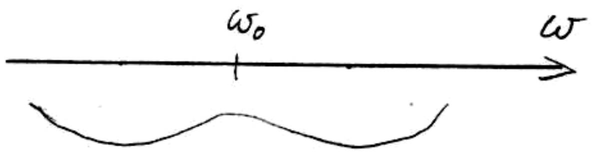
Simplest magnetic relaxation model consists of a three-level system



$\Omega << \Omega_0$

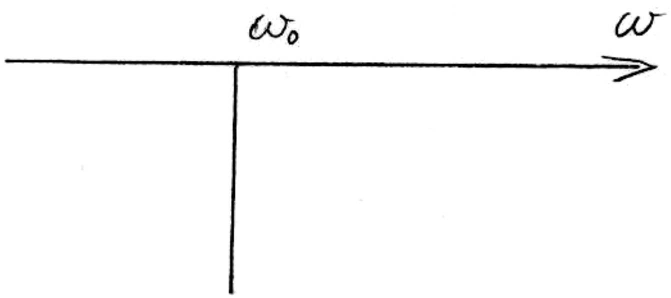
Two lines of (almost) natural width;
With increasing Ω, the lines broaden
→effective lifetime

Typical for resonances in Ag and for the $^3H/^3He$ system. For Ag: $\Omega_0 \sim 10^5\ s^{-1}$ and $\Omega \sim 10\ s^{-1}$



$\Omega \geq \Omega_0$

Intensity is distributed over a broad pattern, which extends over the total hf splitting Ω_0 as suggested by the time-energy uncertainty principle

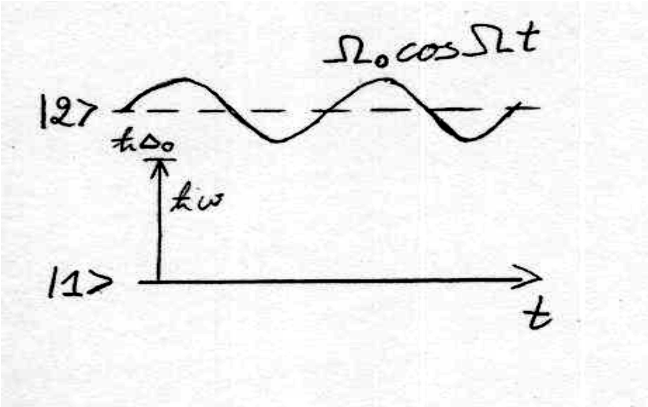


$\Omega >> \Omega_0$

Motional narrowing: one line at the center of the hf splitting of practical natural width. Stochastic frequency changes: between lines 1 and 2. Averaging process over short parts of the lifetime. **Not for Ag and $^3H/^3He$**.

Homogeneous Broadening: Frequency Modulation

M. Salkola and S. Stenholm, Phys. Rev. A **41**, 3838 (1990)



$$A \propto \sum_{k=-\infty}^{k=+\infty} J_k^2(\eta) \frac{1}{\left[(\Delta_0/\Gamma) - k\xi\right]^2 + 1}$$

sum of Lorentzians, located at $\omega=\omega_0 \pm k\Omega$

Ω_0 : max. freq. deviation

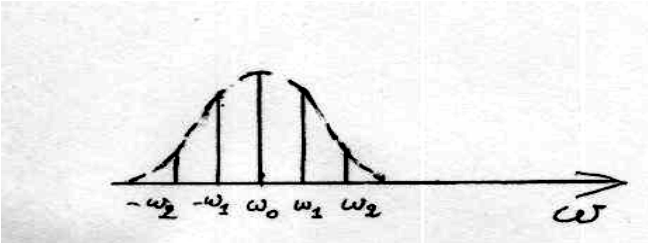
Ω : relaxation frequency

$\eta = \dfrac{\Omega_0}{\Omega}$: modulation index

$\Delta_0 = \omega_0 - \omega$

Γ : linewidth

$\xi = \dfrac{\Omega}{\Gamma}$

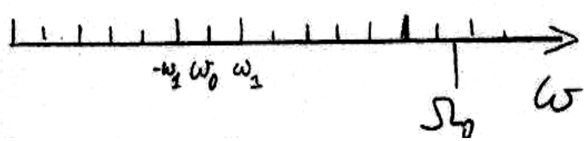


$\eta \approx 1 \Rightarrow \Omega \approx \Omega_0$

$\Gamma << \Omega$

motional narrowing: $\Omega >> \Omega_0 \Rightarrow \eta \approx 0$

only center line at ω_0 survives



$\Omega_0 >> \Omega \Rightarrow \eta >> 1$

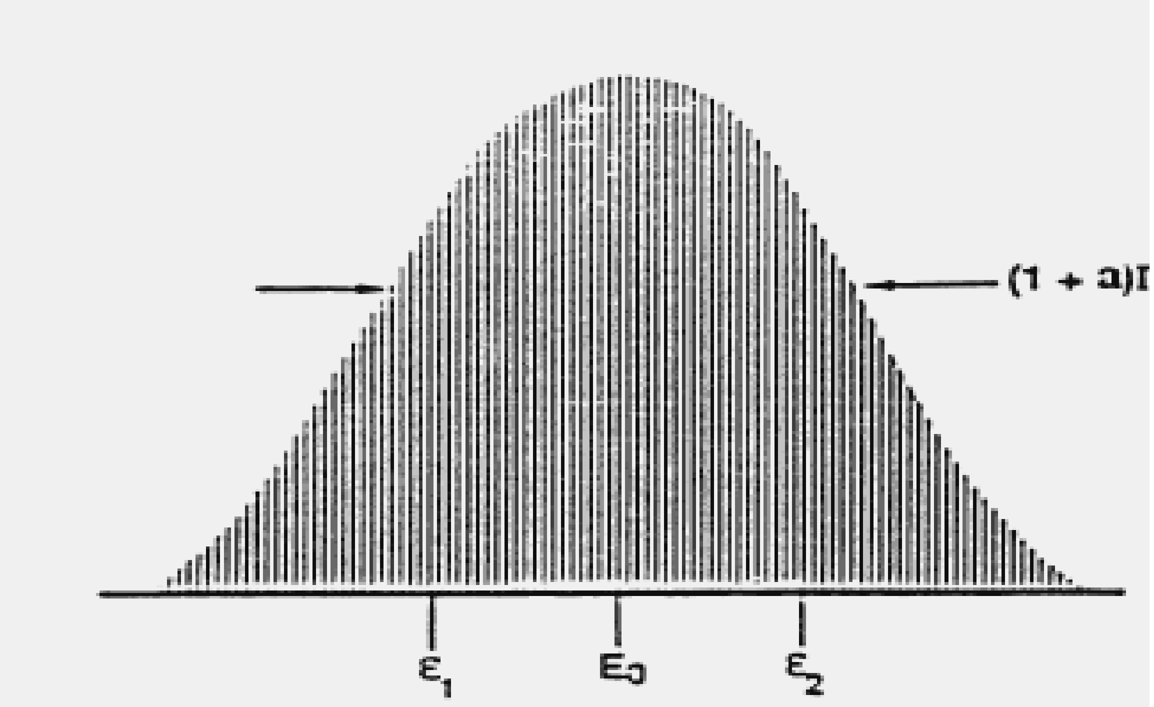
many sidebands→at ω_0 very little intensity

motional narrowing: not possible

Typical for resonaces in Ag and for the $^3H/^3He$ system. For Ag: Ω_0~10^5 s^{-1} and Ω~10 s^{-1}

III) Recoilless antineutrino …

b) inhomogeneous broadening



Many individual resonances displaced from the nonperturbed resonance energy E_0

In the best single crystals: $(1 + a)\Gamma \sim 10^{-13}$ eV corresp. to $10^{11}\,\Gamma$ or larger

Both types of broadening reduce the resonant reaction intensity

III) Recoilless antineutrino …

3) Relativistic effects

Second-order Doppler shift due to mean-square atomic velocity <V^2>

moving system

Time-dilatation effect: $$\Delta t = \frac{\Delta t'}{\sqrt{1-(V/c)^2}}$$

stationary system

Frequencies: $$\nu = \nu' \sqrt{1-(V/c)^2} \approx \nu' \left(1 - \frac{V^2}{2c^2}\right)$$

Second-order Doppler shift: $$\Delta\nu = \nu - \nu' = -\frac{V^2}{2c^2}$$

Reduction of frequency (energy)

III) Recoilless antineutrino …

Within the Debye model:

$$\frac{\Delta E}{E} = \frac{9k_B}{16Mc^2}(\Theta_s - \Theta_t) + \frac{3k_B}{2Mc^2}\left[T_s \cdot f\left(\frac{T_s}{\Theta_s}\right) - T_t \cdot f\left(\frac{T_t}{\Theta_s}\right)\right]$$

Zero-point energy

where

$$f\left(\frac{T}{\Theta}\right) = 3\left(\frac{T}{\Theta}\right)^3 \cdot \int_0^{\Theta/T} \frac{x^3}{\exp x - 1} dx$$

If $|T_s - T_t| = 1$ degree $\rightarrow \Delta E / E \approx 10^{-13} \longrightarrow \Delta E \approx 200 \cdot \Gamma_{\exp}$

Low temperatures: $T_s \approx T_t \approx 0 \longrightarrow [.....] \approx 0$

However, zero-point energy remains!

If $|\Theta_s - \Theta_t| = 1$ degree $\rightarrow \Delta E / E \approx 2 \cdot 10^{-14} \longrightarrow \Delta E \approx 40 \cdot \Gamma_{\exp}$

The Debye temperature for ^3H has to be the same in source and target. The same holds for ^3He. The Debye temperatures of ^3H and ^3He in the metal matrix do <u>not</u> have to be equal.

IV) Requirements for experiment

A) Preparation of source and target

Source:

^{3}H chemically loaded into metals to form hydrides (tritides), e.g., Nb: in tetrahedral interstitial sites (IS).

Target:

^{3}He accumulates with time due to the tritium trick:

$$Nb^3H_x \xrightarrow{\text{time=200d}} Nb^3H_{x\text{-}y}{}^3He_y \xrightarrow{\text{remove}} Nb^3He_y$$

Remove ^{3}H by isotopic exchange $^3H \rightarrow D$

R. S. Raghavan, hep-ph/0601079 v3, 2006

IV) Requirements for experiment

How much metal for source and target?

Source:

1 kCi of ^3H (~100mg ^3H): ~3g of Nb^3H

for NMR studies: 0.5 kCi ^3H in 2.4g $PdH_{0.6}$

Target:

100mg of ^3He implies ~100g of Nb^3H aged for 200 d

IV) Requirements for experiment

B) Event rates for ^3H – ^3He recoilless resonant capture of antineutrinos

Base line	^3H	^3He	Antineutrino capture per day	$R\beta(\Delta t=65d)$ per day
5 cm	1 kCi	100 mg	$\sim 40 \times 10^3$	~40
10 m	1 MCi	1 g	$\sim 10^3$	~10

$R\beta(\Delta t)$/day: Reverse β-activity rate after growth period $\Delta t=65d=0.01\tau$

R. S. Raghavan, hep-ph/0601079 v3, 2006

IV) Requirements for experiment

C) Recoilless emission and detection of Mössbauer Anti-neutrinos

1) Recoilfree fraction f:

Use low temperatures (liquid He) to make f large.

How are ^{3}H and ^{3}He bound in the metallic matrix?
Can the lattice expansion and contraction be assumed to occur adiabatically?

Difficult problems. Inelastic neutron scattering experiments are necessary to determine the recoilfree fraction of ^{3}H and ^{3}He.

IV) Requirements for experiment

2) Source and target should be as similar as possible:

Use low temperatures (liquid He) to avoid effects caused by temperature drifts.

However:

Source contains ^3H, whereas target contains mainly ^3He.

Interaction (chemical bonds) of ^3H and ^3He with the metal atoms may be different because of different neighborhood

$\longrightarrow \quad \Theta_s \neq \Theta_t$

IV) Requirements for experiment

3) Linewidth

Two types of line broadening:

a) homogeneous broadening

↓

due to fluctuations, e. g. of magnetic fields, single-line spectrum highly unlikely

b) inhomogeneous broadening

↓

due to stationary effects, e.g. impurities, lattice defects, different atomic neighbors

Mössbauer Anti-neutrinos:

Energy width: $\Gamma_{\exp} = 8.6 \cdot 10^{-12} eV$

Cross section: $\sigma_{res} \approx 3 \cdot 10^{-33} cm^2$

V) Interesting experiments

1) Do Mössbauer neutrinos oscillate?

2) Determination of mass hierarchy and oscillation parameters
Δm^2_{32} and Δm^2_{12}: 0.6% and $\sin^2 2\theta_{13}$: 0.002

3) Search for sterile neutrinos

4) Gravitational redshift experiments (Earth).

V) Interesting experiments

1) Do Mössbauer neutrinos oscillate?

S.M. Bilenky et al., Phys. Part. Nucl. **38**, 117 (2007)
S.M. Bilenky, arXiv: 0708.0260
S.M. Bilenky et al., J. Phys. **G34**, 987 (2007)
E.Kh. Akhmedov et al., arXiv: 0802.2513; JHEP 0805 (2008) 005
S.M. Bilenky et al., arXiv: 0803.0527 v2
E.Kh. Akhmedov et al., arXiv: 0803.1424
S.M. Bilenky et al., arXiv: 0804.3409

V) Interesting experiments

2) If Mössbauer neutrinos do oscillate:

Ultra-short base lines for neutrino-oscillation experiments

Oscillatory term: $\sin^2(\pi L / L_0)$

Oscillation length: $L_0 = 4\pi\hbar c \dfrac{E}{|\Delta m^2|} \approx 2.480 \dfrac{E / MeV}{|\Delta m^2| / eV}$ [m]

A) Determination of Θ_{13}: *E=18.6 keV* instead of *3 MeV.*

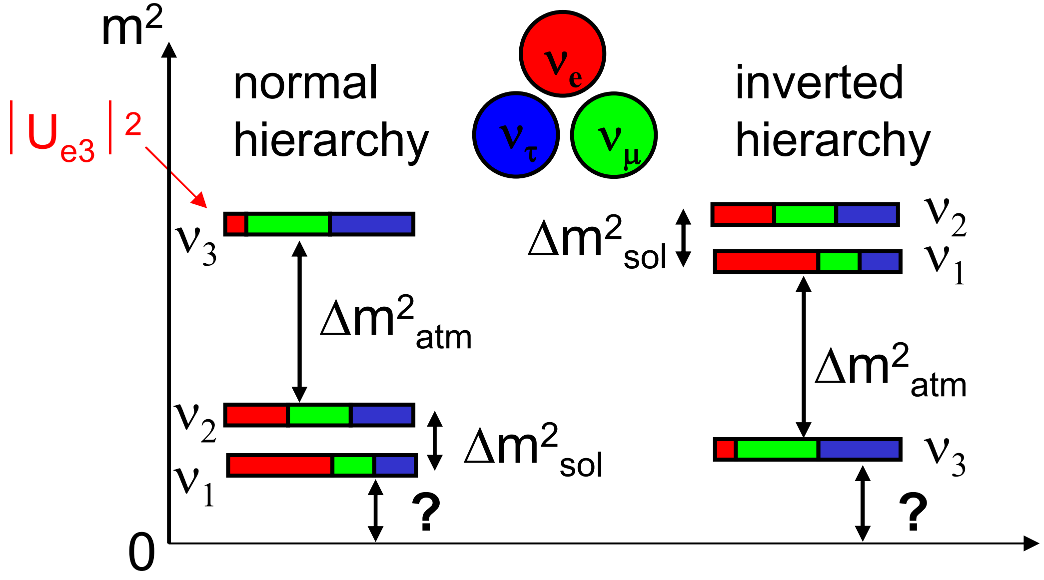
Δm_{23}^2 observed with *atmospheric* neutrinos

Chooz experiment: $\sin^2 2\Theta_{13} \leq 2 \cdot 10^{-1}$ Oscillation base line: L_0/2~9.3 m

$\longrightarrow$ Base line *L* of *9.3 m* instead of *1500 m*

V) Interesting experiments

B) Mass hierarchy and oscillation parameters



H. Nunokawa et al., hep-ph/0503283

To determine mass hierarchy:
Measure Δm^2 in reactor-neutrino and muon-neutrino (accelerator long-baseline) disappearance channels to better than a fraction of 1%

H. Minakata et al., hep-ph/0602046

For $\sin^2 2\theta_{13}$=0.05 and 10 different detector locations one can reach uncertainties:
in Δm^2_{31} and Δm^2_{12}: 0.6%,
in $\sin^2 2\theta_{13}$: 0.002

V) Interesting experiments

3) Search for conversion of $\bar{\nu}_e \longrightarrow \nu_{sterile}$

LSND experiment: $\Delta m^2 \approx 1 eV^2$ and $\sin^2 2\theta \sim 0.1$ to 0.001

(largely excluded by MiniBooNE)

Possibility: $\bar{\nu}_e \longrightarrow \nu_{sterile}$

V. Kopeikin et al. : hep-ph/0310246

Test: Disappearance experiment with 18.6 keV antineutrinos

$\longrightarrow$ Oscillation length L_0 ~5cm!

$\longrightarrow$ Ultra-short base line, difficult to reach otherwise

V) Interesting experiments

4) Gravitational redshift experiments (Earth)

Gravitational redshift: $\frac{\delta E}{E} = \frac{gh}{c^2}$

Experimental linewidth: $\Gamma_{\exp} = \Delta = 8.6 \cdot 10^{-12} eV$

$\Delta = \frac{\eta\omega}{c^2} g h_\Delta$ where h_Δ is height corresponding to 1 experimental linewidth

$\longrightarrow \quad h_\Delta \approx 4.25m$

Can not be used to determine the neutrino mass

Gravitational spectrometer

VI) Conclusions

1) Recoilless resonant emission and detection of antineutrinos:

^3H – ^3He system is the prime candidate.

2) Experiment is very difficult:

a) Recoilfree fraction might be smaller than expected.

Adiabatic lattice expansion and contraction?

b) Temperature difference between source and target (temperature shift)

c) Different Debye temperatures in source and in target (chemical shift)

d) Homogeneous and inhomogeneous broadening of linewidth

3) If successful, very interesting experiments become possible:

a) Do Mössbauer neutrinos oscillate?

b) Mass hierarchy and accurate determination of oscillation parameters

c) Search for sterile neutrinos (LSND experiment)

d) Gravitational redshift experiments (Earth).

Extra slides

Papers

Earlier papers:

W. M. Visscher, Phys. Rev. 116, 1581 (1959)

W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

More recent papers:

R. S. Raghavan, hep-ph/0601079 v3, 2006

W. Potzel, Phys. Scrip. T127, 85 (2006);

S. M. Bilenky, F. von Feilitzsch, and W. Potzel,
J. Phys. G: Nucl. Part. Phys. **34**, 987 (2007);

E. Kh. Akhmedov, J. Kopp, and M. Lindner, 0802.2513 (hep-ph)

e^-

I) β-decay

1) Usual β-decay

$$A(Z-1) \rightarrow A(Z) + \underbrace{e^- + \bar{\nu}_e}_{\text{occupy states in continuum}}$$

neutron transforms into a proton

3-body process: $e^-, \bar{\nu}_e$ show (broad) energy spectra

Maximum $\bar{\nu}_e$ energy: $E_{\bar{\nu}_e}^{\max} = Q$

where $Q = (M_{Z-1} - M_Z)c^2$

is the end-point energy

I) β-decay

Resonance cross section

$$\sigma = 4.18 \cdot 10^{-41} g_0^2 \cdot \frac{\rho(E_{\bar{\nu}_e}^{res})}{ft_{1/2}} \left[cm^2\right]$$

L.A. Mikaélyan, et al.: Sov. J. Nucl. Phys. 6, 254 (1968)

$$g_0 = 4\pi\left(\frac{\eta}{mc}\right)^3 |\Psi|^2 \approx 4\left(\frac{Z}{137}\right)^3$$

for low Z, hydrogen-like ψ
m: electron mass
|ψ|²: probability density of e in A(Z)

$\rho(E_{\bar{\nu}_e}^{res})$: resonant spectral density, i.e., number of $\bar{\nu}_e$ in an energy interval of 1MeV around $E_{\bar{\nu}_e}^{res}$

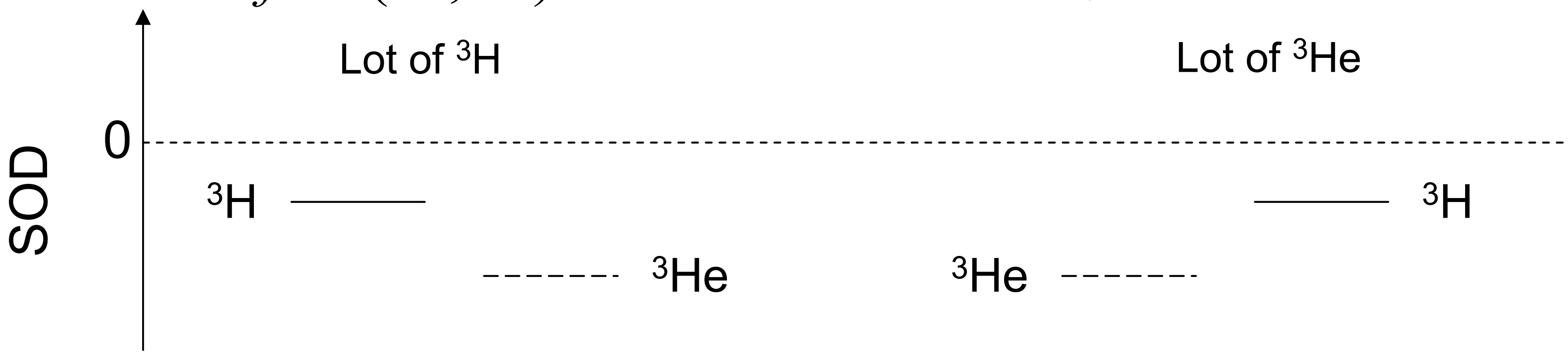
$ft_{1/2}$ value: reduced half-life of decay

$ft_{1/2} \approx 1000$: super-allowed transition

III) Recoilless antineutrino …

What does this mean for the effective values Θ_s and Θ_t ?

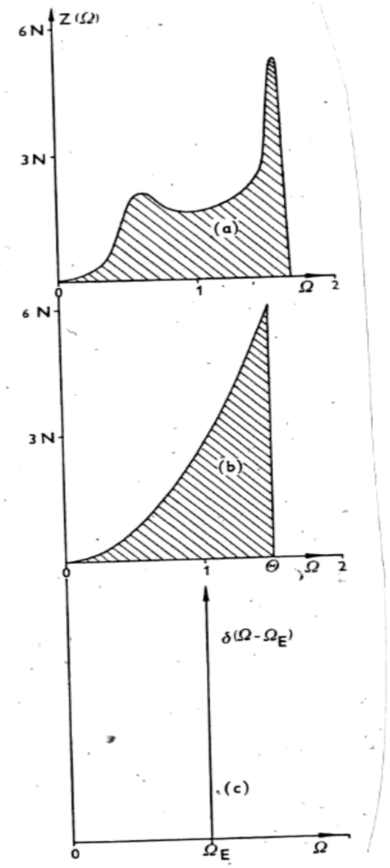
Source	Target
$^3H \longrightarrow {}^3He$	$^3He \longrightarrow {}^3H$
$f^{^3He}(M,{}^3H)$	$f^{^3H}(M,{}^3He)$
Lot of ^3H	Lot of ^3He

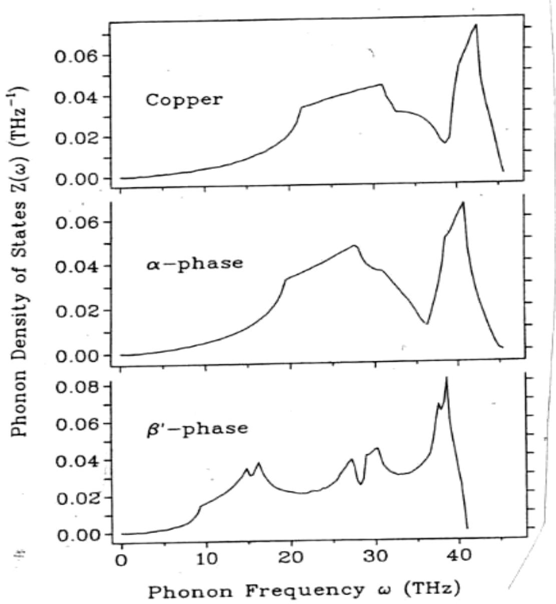


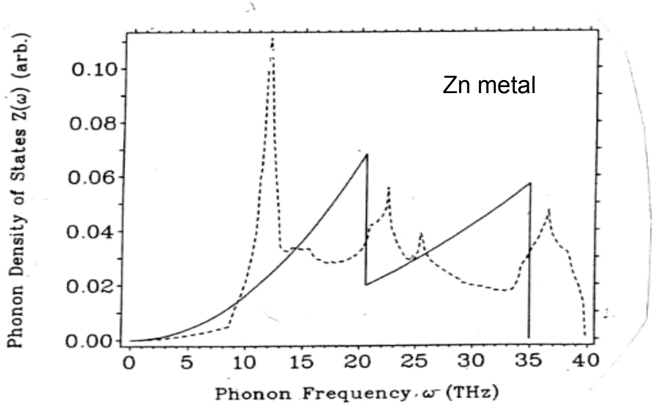
The differences of these SOD values in source and target have to be the same. In a practical experiment this means:

The Debye temperature for ^3H has to be the same in source and target. The same holds for ^3He. The Debye temperatures of ^3H and ^3He in the metal matrix do <u>not</u> have to be equal.

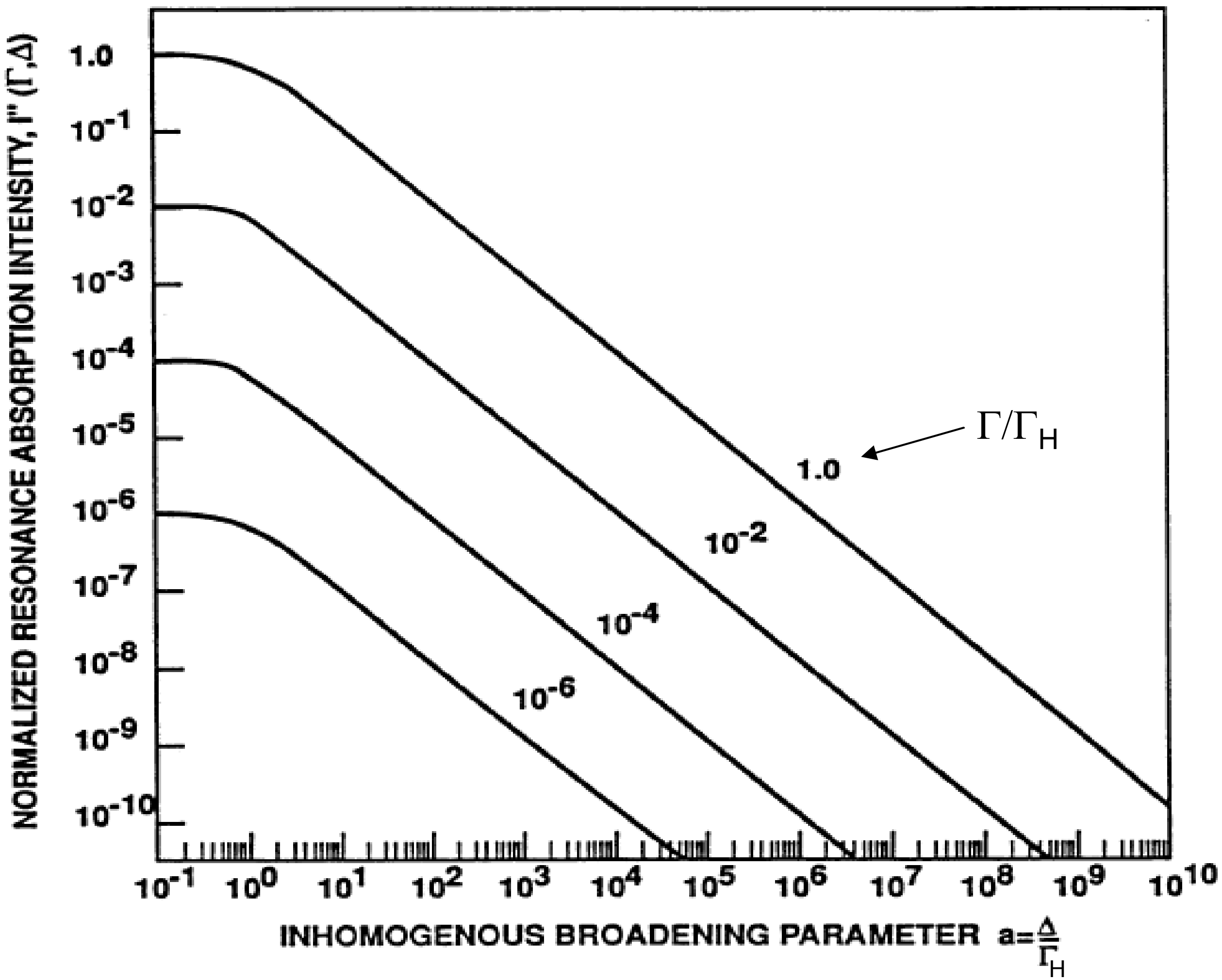
Phonon density of states







III) Recoilless antineutrino …



B. Balko, I. W. Kay, J. Nicoll, J. D. Silk, and G. Herling, Hyperfine Interactions **107**, 283 (1997).

Candidates for recoilless neutrino absorption

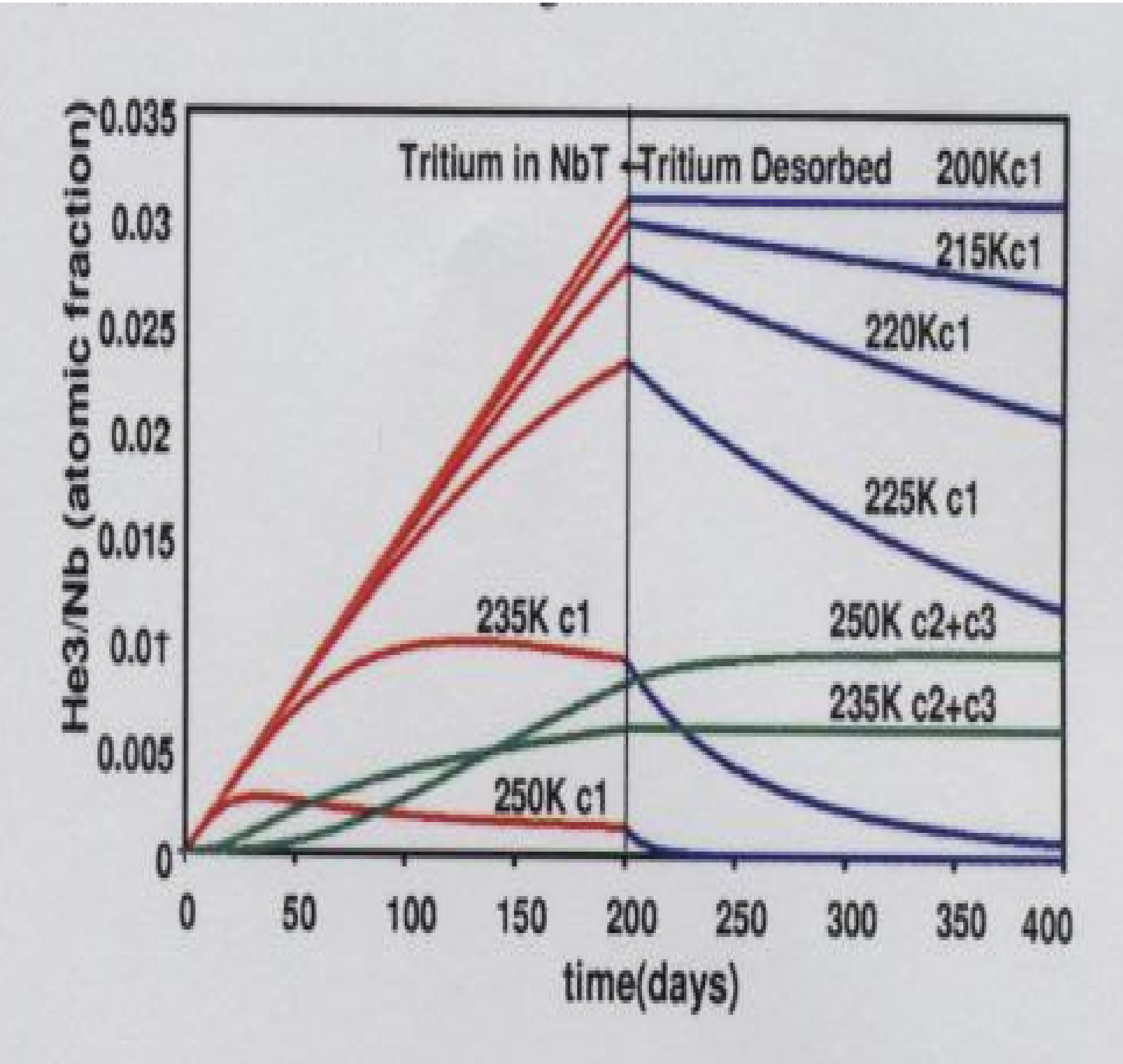
TABLE I. Candidates for recoilless neutrino absorption.

Nuclide	Q (keV)	τ (yr)	f_R[a] Recoilless fraction	α (10^{-4})	γ (10^{-16}) Line broadening	σ_{eff} $(10^{-36}\ \text{cm}^2)$	σ_{eff}/τ[b]
^{3}H	18.6	12.3	0.40	200[c]	8	0.1	1.0
^{63}Ni	68	92	0.07	1	1	10^{-9}	10^{-9}
^{93}Zr	60	1.5×10^{6}	0.18	1	7×10^{-5}	10^{-12}	10^{-16}
^{107}Pd	33	6×10^{6}	0.62	1	2×10^{-5}	10^{-11}	10^{-16}
^{151}Sm	76	90	0.11	1	1	10^{-9}	2×10^{-9}
^{171}Tm	97	1.9	0.04	1	50	5×10^{-9}	3×10^{-7}
^{187}Re	2.6	4×10^{10}	1.0	1000[d]	10^{-9}	2×10^{-7}	10^{-15}
^{193}Pt	61	50	0.29	1	2	3×10^{-8}	8×10^{-8}
^{157}Tb	58	150	0.29	0.4[d]	0.7	2×10^{-9}	10^{-9}
^{163}Ho	2.6	7000	1	73[d]	0.01	7×10^{-3}	1×10^{-4}
^{179}Ta	115	1.7	10^{-2}	0.5[d]	60	10^{-10}	6×10^{-9}
^{205}Pb	60	1.4×10^{7}	0.3	8[d]	10^{-5}	10^{-11}	10^{-16}

[a] Recoilless fraction calculated for effective Debye temperatures assuming that the nuclei are imbedded in *W*, and that the simple approximations in the text are valid.
[b] Normalized to 1.0 for ^{3}H.
[c] From Ref. 4.
[d] Estimated from atomic wave function calculations of the relevant shells.

W. P. Kells and J. P. Schiffer, Phys. Rev. C 28, 2162 (1983)

IV) Consequences …



^3He generated in Nb: c1: concentration in interstitial sites for different temperatures and times. The He in the T-free absorber below 200K is almost all interstitial.

R.S. Raghavan: hep-ph/0601079 revised v3; calculations: Sandia Natl. Lab., USA

IV) Consequences …

Table 1 He transport parameters in NbT at 200K

M_1T_1	E1 eV	E2 eV	E3 eV	D/cm^2 s
M=Nb	0.9[a]	0.13[b]	0.43[b]	1.1E-26[c]

[a] Ref. 7; [b] Ref. 9; [c] Assumes *tritium* pre-exponential D_0 (ref. 6)

Table 2. Theoretical (Ref. 7) EST & ZPE for T and ^3He in Nb interstitial sites (IS)

Site	EST (eV)		ZPE (eV)	
	T	He	T	He
TIS	-0.133	-0.906	0.071	0.093
OIS	-0.113	-0.903	0.063	0.082

Table 3. Nearest neighbor (NN) Displacements(%) and measured[6] activation energies Eac(eV) in NbIS (Ref. 7)

	1st NN Displacement			2nd NN Displacement.		
	H	D	T	H	D	T
TIS	4.1	3.9	3.9	-0.37	-0.36	-0.35
OIS	7.7	7.5	7.4	0.2	0.19	0.19
Eac[6]	0.106	0.127	0.135			

6 TIS
3 OIS

EST: self-trapping energy
ZPE: zero-point energy

Little difference between Deuterium and Tritium

theoretical

experimental activation energies

V) Interesting experiments

1) Do Mössbauer neutrinos oscillate?
Different approaches to neutrino oscillations

CC weak process, $\left|\nu_l\right\rangle = \sum_k U_{lk}^{*}\left|\nu_k\right\rangle$ U: unitary PMNS matrix
Pontecorvo, Maki, Nakagawa, Sakata

Transition probability: $P(\nu_l \rightarrow \nu_{l'}) = \left|\sum_{k=1}^{3} U_{l'k} e^{-i\Delta m_{1k}^{2}\frac{L}{2E}} U_{lk}^{*}\right|^{2}$

For only two flavors: $P(\nu_a \rightarrow \nu_b) = \sin^2 2\Theta \cdot \sin^2(\pi L / L_0)$

Amplitude: $\sin^2 2\Theta$ Oscillatory term: $\sin^2(\pi L / L_0)$

Oscillation length: $L_0 = 4\pi\hbar c \frac{E}{\left|\Delta m^2\right|} \approx 2.480 \frac{E / MeV}{\left|\Delta m^2\right| / eV}$ [m]

V) Interesting experiments

Question: What will be the state of the neutrino after some time (at some distance L)?

A) Evolution in time

Schrödinger equation for evolution of any quantum system:

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \longrightarrow |\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

$$P(\nu_l \to \nu_{l'}) = \left|\sum_{k=1}^{3} U_{l'k} e^{-i(E_k - E_i)t} U^{*}_{lk}\right|^2$$

No matter what the neutrino momenta are !

If $E_k = E_i$, there will be no neutrino oscillations: $P(\nu_l \to \nu_{l'}) = \delta_{l'l}$
The neutrino state is stationary

If E_k are different, neutrino state is non-stationary.
→time-energy uncertainty relation holds: $\Delta E \cdot \Delta t \geq 1$

Δt is the time interval during which the state of the system is significantly changed

If $E_k \neq E_i$, the uncertainty relation takes the form: $(E_k - E_1)\cdot t \approx \frac{\Delta m^2_{1k}}{2E} t$

V) Interesting experiments

B) Evolution in time and space

Mixed neutrino state at space-time point $x = (t, \vec{x})$:

$$|\nu_l\rangle_x = \sum_{k=1}^{3} e^{-ip_k x} U_{lk}^* |\nu_k\rangle \longrightarrow P(\nu_l \to \nu_{l'}) = \left| \sum_{k=1}^{3} U_{l'k} e^{-i(p_k - p_1)x} U_{lk}^* \right|^2$$

with $(p_k - p_1) = \dfrac{E_k^2 - E_1^2}{E_k + E_1} t - (p_k - p_1) L$ and $E_i^2 = p_i^2 + m_i^2$

a) $t \approx L \longrightarrow (p_k - p_1)x \approx \dfrac{\Delta m_{1k}^2}{2E} L$ oscillatory phase

b) neutrinos: different masses have the *same* energy

$\longrightarrow$ neutrino state is stationary

$$\longrightarrow p_k \neq p_i: \quad (p_k - p_i)x = \frac{\Delta m_{1k}^2}{2E} L \qquad P(\nu_l \to \nu_{l'}) = \left| \sum_{k=1}^{3} U_{l'k} e^{-i\Delta m_{1k}^2 \frac{L}{2E}} U_{lk}^* \right|^2$$

V) Interesting experiments

Mössbauer neutrinos:

Energy width: $\Gamma_{\exp} = 8.6 \cdot 10^{-12} eV$

a) $(E_3 - E_2) \approx \dfrac{\Delta m_{23}^2}{2E} \approx 6.5 \cdot 10^{-8} eV$ $\quad\Delta m_{23}^2$ observed with *atmospheric* neutrinos

→ Mössbauer neutrinos take a long time to change significantly

→ Time-energy uncertainty: Extremely long "oscillation " length

Determination of Θ_{13}: *E=18.6 keV* instead of *3 MeV.*

Chooz experiment: $\sin^2 2\Theta_{13} \le 2 \cdot 10^{-1}$ Oscillation base line: $L_0/2 \sim 9.3$ m

b) Δm_{12}^2 observed with *solar* neutrinos

$$(E_2 - E_1) \approx \frac{\Delta m_{12}^2}{2E} \approx 2.1 \cdot 10^{-9} eV$$

Amplitude: $\sin^2 2\Theta_{12} \approx 0.82$

Oscillation base line: $L_0/2 \sim 300$ m

Oscillation length: $L_0 = 4\pi\eta c \dfrac{E}{\left|\Delta m^2\right|} \approx 2.480 \dfrac{E/MeV}{\left|\Delta m^2\right|/eV}$ [m]

V) Interesting experiments

5) Real-time, ^{3}H-specific signal of $\bar{\nu}_e$ resonance

a) sudden change of the magnetic moment from
-2.1nm (^{3}He)→+2.79nm (^{3}H)

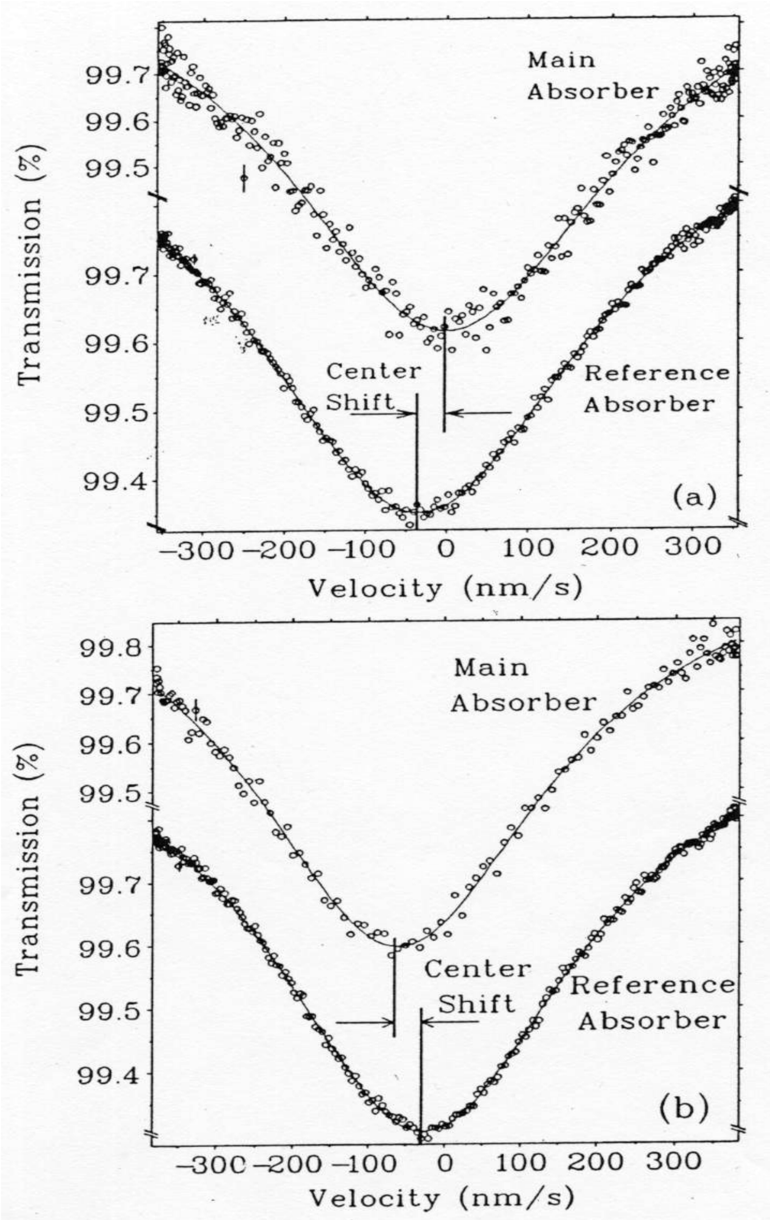
→ transient (~0.1ms) magnetic field which couples to electron moment of ^{3}H via hyperfine interaction

→ Read-out by SQUID

b) new electrons appear in the Nb bands when ^{3}H is formed. These electrons cause additional specific heat that grows linearly with ^{3}H concentration.

→ detectable by ultra-sensitive (micro)-calorimeters ?

Red(blue)shift ^{67}ZnO-Mössbauer exp.



gravitational redshift

difference in height: 1m
in gravitational field of Earth

gravitational blueshift

accuracy: $(\Delta E/E) \leq 1 \times 10^{-18}$

W. Potzel et al., Hyp. Interact. 72, 197 (1992)

Gravitational Redshift Experiment

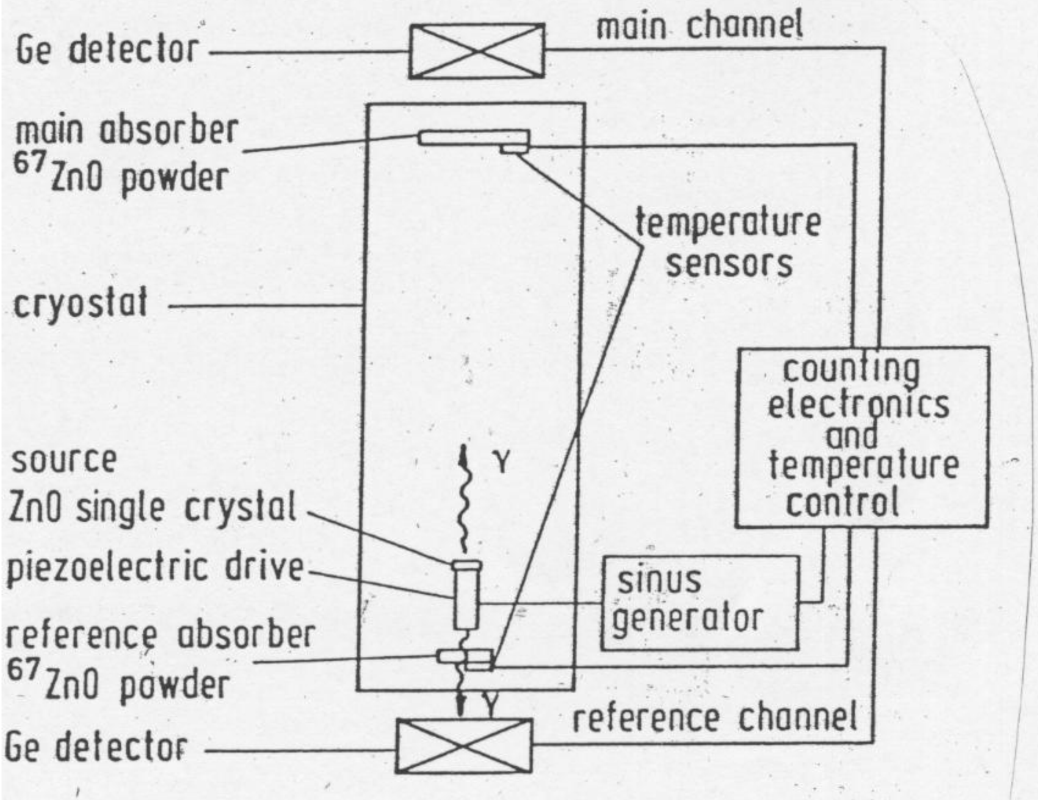


Fig. 3. Basic set-up of the Mössbauer gravitational redshift experiment. Two transmission experiments are carried out simultaneously: through the reference absorber and through the main absorber. A piezoelectric drive moves the source sinusoidally with respect to both absorbers.