A DEDICATED TORSION BALANCE TO DETECT NEUTRINOS BY COHERENT SCATTERING ON HIGH DEBYE TEMPERATURE MONOCRYSTALS

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Introduction

One of the most intriguing proposal of the last decades in physics is represented by Webber's idea of detecting neutrinos by coherent scattering on stiff crystals [1,2]. Neutrinos (in fact antineutrinos) are generated by intense β− sources such as ^{3}T or ^{60}Co at a rate equal to source activity, by Sun at a fluency debit of 6.6×10^{10} cm^{-2} s^{-1} or by nuclear reactors at a higher global fluency debits of about 10^{12} cm−2 s−1. It is worth mentioning that in the case of β− sources the neutrinos average energy is equal to average electrons energy while in the case of nuclear reactors or of the Sun, neutrinos average energies varies between 1 MeV and 300 keV respectively.

Coherent Interactions of Neutrinos with Stiff Targets

It is well known the fact that neutrinos interact very weak with matter, the cross-section of interaction between a neutrino having the energy E_n and a nucleon being given by the following expression:

$$\sigma_{nucleon} \approx \frac{G^2 E_n^2}{\hbar^2 c^2} \tag{1}$$

Where G is Fermi's weak interaction constant ($G \sim 10^{-4}$ MeV fm^3, fm = 10^{-15} m) such that for an 1 MeV neutrino this section becomes equal to 10^{-20} barn). To this it must be added the fact that in the case of a nucleus of mass A and atomic number Z the scattering cross-sections of protons and neutrons are of opposite signs such that the total cross section for such nucleus becomes:

$$\sigma_{nucleus} \approx \frac{G^2 E_n^2}{\hbar^2 c^2}\left[N - Z\left(1 - 4\sin^2\theta\right)\right]^2 \tag{2}$$

where N stands for the neutron number while $(1 - 4\sin^2\theta)$ appears as a result of mixing of scattering amplitude described by Weinberg's mixing angle θ.

For that reason, the actual neutrino detectors needs tremendous volumes of liquid and a corresponding number of photomultipliers, that makes such detectors very voluminous and expensive to build as well as to exploit [3,4].

Indeed, the transition probability within Born's approximation is

$$dw \approx \frac{1}{h}\left|H_1\right|^2 \delta\left(E - E'\right) dv' \tag{3}$$

where: $dv' = V p'^2 dp' d\Omega / h^3$ and H_1 stands for the matrix element of interaction operator.

We follow here the theoretical treatment given by Apostol [5].

In the case of a periodic lattice with characteristic length a and described by a periodic array of δ functions, eq. (3) becomes:

$$dw = \frac{u^2 a^6}{V c^3 h^4}\left|\sum_i e^{i(\mathbf{p}-\mathbf{p}')r_i/\hbar}\right|^2 E'^2 d\Omega \tag{4}$$

where: $\sum_i e^{i(\mathbf{p}-\mathbf{p}')r_i/\hbar}$ represents the well known form-factor $F(\mathbf{p}-\mathbf{p}')$ of the crystal and u is the strength of interaction.

Thus, the problem now consists of estimating this form factor for an enough stiff crystal on which a multiple coherent scatter of neutrinos to take place. Accordingly, this form factor can be written as a product of three components, each of them corresponding to one axis of coordinates, i.e.:

$$F(\mathbf{p}-\mathbf{p}') = F_1(p_1 - p_1')F_2(p_2 - p_2')F_3(p_3 - p_3') \tag{5}$$

Where: $F_i = \sin\left[(p_i - p_i')N^{1/3}a/\hbar\right]/\sin\left[(p_i - p_i')a/\hbar\right]$, i = 1,2,3, and N represents the number of cell in crystals.

By taking into account that $\boldsymbol{G}$ is the reciprocal vector of the lattice and in the limit of large N the form factor (5) becomes:

$$F = \pi \sum_G \delta\left(\frac{\boldsymbol{p} - \boldsymbol{p}'}{\hbar} + \boldsymbol{G}\right) \tag{6}$$

It is worth to point out that in eq. (6) that describes the well-known Laue's diffraction, summation is performed over all reciprocal vectors that satisfy Bragg's condition: $\boldsymbol{p}' = \boldsymbol{p} + \hbar\boldsymbol{G}$ and hence $\hbar\boldsymbol{G} = 2p\sin\theta/2$, where θ is the scattering angle between $\boldsymbol{p}'$ and $\boldsymbol{p}$. In the case of neutrinos, the scatter angle varies from a central peak to higher order diffraction peaks where $\boldsymbol{p} = \boldsymbol{p}'$ that corresponds to maximum neutrino energy. For energies of 1 MeV and higher, the number of diffractions peaks can be approximated by $(p/\hbar g)2 \sim (ap/\hbar)2$ as $g \sim 1/a$.

By taking into account eq. (5), the maximum value of F form-factor becomes $F_{max} = N$ that implies the corresponding maximum cross-section to be: $d\sigma_{max} = \sigma_0 N^2 d\Omega$, i.e. $d\sigma_{max} \sim N^2$. It is worth mentioning that this type of dependence is a general characteristic of coherent diffraction or coherent scattering and it is not influenced by phonons, lattice defects or other additional lattice modulations. Although long range phonons could contribute by diffuse scattering which is proportional to N, the scattering at the center of diffraction spot remains proportional to N^2. At the same time, in the case of dispersion of incident particles by energy as well as by incoming directions, the lateral dispersion peaks decrease in intensity by respect to the central one.

By estimating from eq. (4) the spread of diffraction peaks, $\delta p' \sim \hbar/a N^{1/3}$ as well as the subsequent area $\sim \hbar^2/a^2 N^{2/3}$ and solid angle $\Delta\Omega \sim \hbar^2/p^2 a^2 N^{2/3}$ it results immediately that the one peak cross-section becomes:

$$\sigma_{one-peak} \cong \sigma_0 N^2\left(\frac{\hbar^2}{p^2 a^2 N^{2/3}}\right) \sim N^{4/3} \tag{7}$$

By multiplying one peak cross-section with the total number of peaks $N_{peaks} \sim (pa/\hbar)^2$ finally it results that the total cross-section of coherent neutrino scattering becomes:

$$\sigma_{total} \cong \sigma_0 N^{4/3} \tag{8}$$

Although apparently total cross-section will increase indefinitely with the number N of crystal cells, by increasing the number of cells, the target area ceased to be perpendicular on incident particle direction that finally introduces a correction factor proportional to $N^{2/3}$ limiting in fact the validity of eq. (8) such that $N \ll (a/\sigma_0)^{2/3}$.

Also it must be pointed out that in the case amorphous targets (gas, liquid or vitreous state), the form-factor reduces to central peak only that corresponds to $G = 0$, and therefore the total cross-section (8) is diminished by a correction factor equal to $(\hbar/pa)^2$ which explain why to detect neutrino by coherent scattering it needs only solid state crystalline targets.

Some additional comments could be made. If the target is not fixed, it will receive from an incoming particle both momentum and energy, the collision becomes inelastic and the final states of scatterer must be taken into account. If at an elementary act of collision between an incoming particle and a crystalline solid, the particle will transfer both a momentum δp and an energy $\delta\varepsilon$, such that $p' = p - \delta p$ and $\delta\varepsilon = c(p' - p)$. These momentum and energy transfers are shared by different motions in the target that manifest both by vibrations of solid (phonons) and target motion as a whole. In this case, in eq. (5) the form-factor must be correspondingly modified, i.e. $p - p'$ must be replaced by $p - p' - \delta p$ such that it must estimate the new form-factor:

$$F = \sum_i e^{i(\mathbf{p}-\mathbf{p}'-\delta\,\mathbf{p})r_i/\hbar} \tag{9}$$

and average it over all phonon states.

Moreover, it must estimate under what conditions this transfer of momentum and energy is possible to a solid target as a whole.

As Weber stated [1,2], as neutrino interacts not with atomic electrons but with nuclei (in fact with their quark constituents), these nuclei may move coherently provided that the crystalline target has enough stiffness comparable with the strong-force between interacting nucleons in nucleus.

To prove this statement, first of all the energy transfer should be written as a sum of two parts, i.e. $\delta\varepsilon = \varepsilon_0 + \varepsilon_{ph}$ where $\varepsilon 0$ is the energy transferred to target as a whole and ε_{ph} represents the energy of excited phonons. On the other hand, the incoming particle can transfer a maximum momentum $2p$ to target having the mass M provided that there is no phonon excitation, momentum to whom corresponds the maximum energy $\varepsilon_0 = 2p^2/M$. By amplifying this expression with c^2 it results immediately $\varepsilon_0 = 2(cp)^2/Mc^2 = 2E^2_{inc}$, where $E_{inc} = cp$ represents the energy of incident particle. This is a small amount of energy so it can be neglected by respect to energy transferred to phonons such that $\delta\varepsilon = \varepsilon 0 + \varepsilon_{ph}$ becomes $\delta\varepsilon = \varepsilon 0 + \varepsilon_{ph} \sim \varepsilon_{ph}$. In this case it must be pointed out that the average energy transferred to phonons is of the order of Debye temperature Θ which in crystalline solids can reach higher values as in the case of diamond for which $\Theta \sim 2000$ K (0.2 eV). At the same time the energy transfer to phonons is a statistical process that takes places with the maximization of entropy, described by probability distribution function of the form $\sim e^{-\lambda\varepsilon_{ph}}$ By considering that in this case $\overline{\varepsilon}_{ph} = \Theta$ it results immediately that $\lambda = \Theta^{-1}$ and consequently the distribution function becomes: $1/\Theta\, e^{-\varepsilon/\Theta} \cong 1/\Theta\, e^{-\varepsilon_{ph}/\Theta}$ It is now obvious that the probability of energy transfer to phonons tends toward unity if crystalline lattice stiffness as described by Debye temperature tends to infinite.

At the same time, from the energy conservation $\delta\varepsilon = c(p - p')$ and from the fact that that $\delta\varepsilon/cp \sim \Theta/cp \ll 1$ It results that the momentum modulus practically remains unchanged during such process, by consequence $\delta p = 2p$. In this case, the momentum transfer from incident particle to phonons varies between zero and $2p$ with an average value of p. As the momentum transferred to phonons during an elementary act of interaction is of the order of the elementary crystal reciprocal vector $h/2\pi a$ which at its turn is much smaller than the total average momentum transfer, it results that the absolute majority of the momentum transfer goes to target crystal as en entire, i.e. $\Delta p \sim p \sim p_0$. As the average momentum transferred to phonons vanishes, contrary to energy transfer that goes mainly to phonons, the momentum transfer goes mainly to crystalline target and hence can be measured by using standard methods.

According to eq. (5) and (9) for an momentum transfer $dp = p_0$, the diffraction figure consists of $(ap')^2\hbar^{-2}$ peaks, a number equal to $\hbar^2/(ap')^2/N^{3/2}$ allowed reciprocal vectors, each of solid angle whose maximum number is equal to N^2, and therefore the corresponding cross-section is multiplied by a factor of $N^2/N^{2/3}$. As crystal movements are averaged, the diffraction figure can be considered as smeared out over a range proportional to p_0. For $p_0 \sim p, p'$ the vector p_0 acquires N different orientations to which correspond $N^{2/3}$ subtended solid angles and therefore the coherent cross-section should be multiplied by a factor equal to $N^2/N^{2/3} \cdot N^{2/3} = N^2$.

For an incident kinetic energy cp = 1 Mev, for a cross-section of 10^{-22} barn and for a mol of target containing about $N \sim 10^{23}$ nuclei, the total cross-section becomes of about 10^{24} barn which is an appreciable value, very close to those reported Weber [1].

By knowing now the magnitude of neutrino coherent scattering cross-section, the total force acting upon detector can be easily estimated. In this case the force is equal to produce between incident particle momentum, and the collision rate, the latter being equal to the fluency density of incoming particles and total cross-section, i.e.:

$$F = p\varphi\sigma \tag{10}$$

Accordingly for an incident energy of 1 MeV to which corresponds a momentum of 10^{-17} dyn·s ad a fluency density of 10^{12} nutrino/cm^2·s, the force acting varies between 10^{-3} dyn (for $\sigma \sim 10^{26}$ barn) and 10^{-9} dyn (for $\sigma \sim 10^{20}$ barn), comparable with the force of 10^{-5} reported by Weber for a 0.1 mol sapphire crystal ($\Theta \sim 1000$ K).

Consequently, the best method of detecting such weak forces, comparable with the gravity force acting between two masses of 100 g each placed at 1 cm distance, consists of using a torsion balance provided with two detectors, on of sapphire and the other with the same mass and density but consisting of another material with a very low Debye temperature, such as lead, and hence unable to interact coherently with neutrinos [5].

Even though Weber's theory is doubted, the only way to test it will be to go through new dedicated experiments. It is significant and necessary that new designed experiments test Weber's theory as well as Weber's experimental results.

It is not the aim of our experiment to strictly repeat Weber's one using a tremendously improved experimental setup. We have prepared a robust, simple and dedicated torsion balance, designed to evidence interacting neutrinos with a force sensitivity varying between 10^{-6} and 10^{-8} dyn, i.e. comparable with those reported by Weber and also to have the capacity to install it in significantly different locations.

Experimental

The torsion balance we have built uses two alternatives for the pendulum assembly with wofram and molybdenum gold plated annealed wires having diameters of 25 µm and 50 µm. To observe, monitor and measure the torsion we have chosen an electronic high precision autocollimator system with 0.1 arcsec accuracy and 0.005 arcsec resolution. In this way, the noise of the "swing" and "wobble" modes of the pendulum are minimized while the sensitivity to balance excursions are maximized, allowing sensitivity better than 10^{-7} dyn. Because the high Debye temperature is a critical condition, as neutrino detector we use a set of high Debye temperature ($\Theta \approx 1000$ K) sapphire monocrystals.

Concluding Remarks

A torsion pendulum containing a set of sapphire crystals (high Debye temperature) and lead rings (low Debye temperature) is the best instrument for testing Weber's theory of enhanced neutrino scattering cross-sections in a periodic structure.

Consequently, to achieve this goal, we have projected a high sensitivity torsion balance was designed and assembled will in relatively short time be commissioned.

Preliminary measurements performed on a model balance has shown that for such a pendulum, a torsion balance provided with a variable length tungsten/molybdenum wire and an electronic optical autocollimator, is able to measure small rotation angles of about 0.1 arcsec could be constructed by using current technique and materials.

Special arrangements of sapphire crystals/lead rings permit more extensive characterization of a physical neutrino field.

Figure 1 A photograph of the assembled torsion balance together its blueprint (inset). The laser autocollimator together with the dedicated computer (lower middle and right) are illustrated too.

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