

# LATTICE APPROACH TO LIGHT SCALARS

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## Abstract

I report on lattice QCD calculations that study the properties of the  $a_0$  and  $f_0$  mesons.

## 1 Introduction

I review the recent lattice results for the light  $J^{PC} = 0^{++}$  scalar mesons. The interpretation of many  $0^{++}$  mesons in terms of quark and glue degrees of freedom is still not clear [1,2]. The  $0^{++}$  mesons potentially contain glueball, tetraquark, meson molecule or even quark-antiquark degrees of freedom. I have recently written a review [3] of light meson spectroscopy from lattice QCD, that contains more detail on many of the topics covered here.

### 1.1 Background to lattice QCD

The physical picture behind lattice QCD calculations is that an interpolating operator creates a hadron in the QCD vacuum and after a specific time interval the hadron is destroyed. The choice of interpolating operator is particularly important for hadrons where it is not clear how the hadron is built out of quarks and gluons.

For example, to create a light flavour singlet  $0^{++}$  hadron, possible interpolating operators are

$$O_1 = \bar{q}q \tag{1}$$

$$O_2 = \bar{q}\gamma_5 q \bar{q}\gamma_5 q \tag{2}$$

$$O_3 = U_{plaq} \tag{3}$$

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Group	$n_f$	$m_{K_0}$ GeV
Prelovsek et al. [11]	2	$1.6 \pm 0.2$
McNeile and Michael [12]	2	$1.1 - 1.2$
Mathur et al. [13]	0	$1.41 \pm 0.12$
SCALAR [10]	0	$\sim 1.7$

Table 1: Lightest strange-light  $0^+$  meson from lattice QCD.

where  $U_{plaq}$  is a spatial plaquette of gauge fields with  $0^{++}$  symmetry, and  $q$  is a light quark operator.

The majority of recent lattice QCD calculations include the dynamics of sea quarks and have pion masses as low as 300 MeV [3]. The results I will present for scalar mesons largely use the last generation of lattice QCD calculations that are quenched or dynamical QCD calculations with pion masses above 500 MeV [4, 5].

There are a number of reasons that lattice calculations of the light scalar mesons are challenging. The lattice QCD correlators for scalar mesons are more noisy than for  $\rho$  and  $\pi$  mesons. The light scalar mesons decay via S-wave decays, and current lattice QCD calculations are in the quark mass regime where some decay channels to two mesons are open.

Eventually, the issue of dealing with resonances in lattice QCD will be dealt with by Lüscher's formalism [6] that produces scattering phase shifts. This year Lüscher's technique for resonances was applied to the  $\rho$  meson for the first time, by the CP-PACS collaboration [7].

## 1.2 The flavour non-singlet $0^{++}$ and $0^+$ mesons.

Although I am going to loop through the lattice results for the lightest  $0^{++}$  and  $0^+$  mesons, it is important to classify the states into SU3 multiplets or some other classification based on tetraquarks for example.

In the PDG the lightest strange-light  $0^+$  meson is the  $K_0(1430)$  [8]. There have also been claims that experimental data is consistent with  $0^+$   $I = 1/2$  meson called the  $\kappa$  with a mass of 660 MeV [9]. The existence of the  $\kappa$  is controversial, see [1, 2, 10] for a discussion.

In table 1 I collect results for the mass of the lightest  $0^+$   $\bar{s}q$  meson from lattice QCD calculations. The lattice results in table 1 are consistent with experimental mass of the  $K_0^*(1430)$ , but mostly miss the controversial  $\kappa$  particle. All the lattice calculations used  $\bar{s}q$  interpolating operators, so may have missed the  $\kappa$  state, if it is mostly a tetraquark state, with no overlap with  $\bar{s}q$  interpolating operators.

Experimentally the lightest  $I = 1$   $0^{++}$  mesons are the  $a_0(980)$  and the

Group	$n_f$	$m_{a_0}$ GeV
Bardeen et al. [14]	0	1.34(9)
Burch et al. [15]	0	$\sim 1.45$
Hart et al. [16]	2P	1.0(2)
Prelovsek et al. [11]	2	1.58(34)
Prelovsek et al. [11]	2P	1.51(19)
Mathur et al. [13]	0	1.42(13)

Table 2: A collection of results from lattice QCD for the mass of the lightest non-singlet  $0^{++}$  meson. The P stands for a partially quenched analysis.

$a_0(1450)$  [8]. There have been speculations that the  $a_0(980)$  meson is a molecule or tetraquark state [1, 2], so it is interesting to see whether lattice QCD calculations with  $\bar{q}q$  interpolating operators couple to the  $a_0(980)$  meson. In quenched QCD there is a ghost contribution [14], due to the  $\eta\pi$  contribution, to the scalar correlator that needs to be subtracted off the lattice data. I collect together some recent results for the mass of the light  $0^{++}$  meson from lattice QCD in table 2. I only include quenched data where the  $\eta\pi$  contribution has been corrected for [14].

McNeile and Michael [12], in an unquenched lattice QCD calculation focused on the mass difference (in the hope that systematics cancel), between the  $1^{+-}$  and the  $0^{++}$  mesons. The lattice calculation used gauge configurations from UKQCD [4] and CP-PACS [5]. The mass of the light  $1^{+-}$  state was always higher than the  $0^{++}$  meson. The final result was  $m_{b_1} - m_{a_0} = 221(40)$  MeV, compared to the experimental result of 245 MeV. Lang et al. recently reported masses for the lightest flavour non-singlet  $0^{++}$  consistent with the mass of the  $a_0(980)$  meson, from an unquenched lattice QCD calculation using chirally improved fermions [17].

The previous lattice QCD calculations, discussed in this section, were in a regime where the quark masses were large enough that the decay  $a_0 \rightarrow \eta\pi$  was forbidden. Now I discuss the new lattice QCD calculations where the decay  $a_0 \rightarrow \eta\pi$  is energetically allowed.

The MILC collaboration [18] originally claimed that they had evidence for  $a_0$  decay to  $\pi\eta$  from their calculations with improved staggered fermions. Other decays are discussed in [19]. Later work by the MILC [19] and UKQCD [20] collaborations showed that the lightest state in the flavour non-singlet  $0^{++}$  channel was actually below the  $\pi\eta$  threshold, with improved staggered fermions. This was puzzling, because experimentally the  $a_0 \rightarrow \pi\pi$  decay is forbidden by G parity.

In [21], Prelovsek explained the behaviour of the flavour non-singlet  $0^{++}$  correlator with improved staggered fermions using staggered chiral pertur-

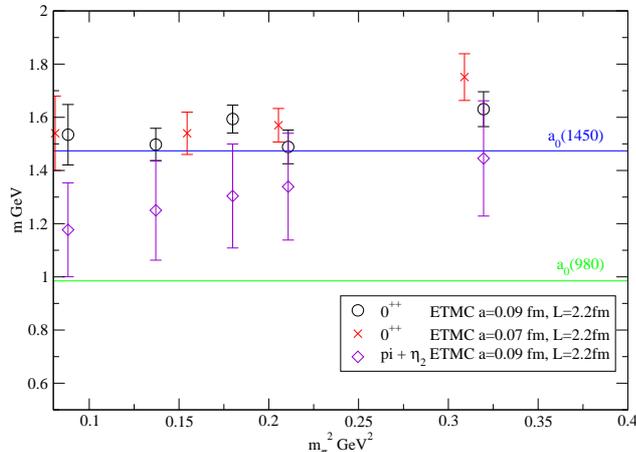


Figure 1: Mass of lightest state in  $0^{++}$  channel with  $\pi\eta_2$  decay threshold.

bation theory. Bernard, DeTar, Fu, and Prelovsek [22] extended the original analysis by Prelovsek, and also applied it to the flavour singlet  $f_0$  meson. A larger study, with more sea quark masses, is required to say something specific about the mass of the lightest  $a_0$  meson.

The ETM collaboration have preliminary results for the mass of the light  $0^{++}$  meson from a  $n_f=2$  unquenched lattice QCD calculation with twisted mass fermions [23–25]. In figure 1 I plot the mass of the light  $0^{++}$  meson and the  $\pi + \eta_2$  decay threshold as a function of the square of the pion mass. The mass of the  $\eta_2$  was computed by Michael and Urbach [24]. Figure 1 shows some evidence for the mass of the  $0^{++}$  tracking the  $\pi + \eta_2$  threshold, or at least for it being an open decay channel. The mass of  $0^{++}$  state is relatively independent of mass and consistent with the mass of  $a_0(1450)$  meson. As noted by Mathur et al. [13], the mass independence of  $0^{++}$  is consistent with closeness of the experimental masses of the  $a_0(1450)$  and  $K_0(1430)$ , if they are both part of the same  $SU3$  octet.

Although the preliminary results from the ETM collaboration are inconsistent with the results by Michael and McNeile [12], some caution is required. We are only just starting to deal with mesons with open decay channels in unquenched lattice QCD calculations. There is a of order 250 MeV difference between the mass of the lightest flavour singlet pseudoscalar meson in lattice QCD calculations with  $n_f = 2$  and  $n_f = 2 + 1$  sea quark flavours [3] and this will be important for the decay thresholds.

There are other quantities, other than masses, that can help determine the quark and glue content of scalar mesons. For example, Narison [26]

proposed to use the leptonic decay constant of the non-singlet  $0^{++}$  mesons to determine the structure of the  $a_0$  meson. The  $f_{a_0}$  decay constant of the light flavour non-singlet  $0^{++}$  meson has been computed using unquenched lattice QCD [12].

$$\langle 0 | \bar{q}q | a_0 \rangle = M_{a_0} f_{a_0} \quad (4)$$

See [12, 27, 28] for a further discussion of this decay constant and the connection with the electroweak current.

A molecule of two mesons should have a very small "wave-function" at the origin, hence  $f_{a_0}$  should be small. The definition of  $f_{a_0}$  is similar to that of the pion decay constant. Hence we mean "small" relative to 130 MeV. The other measured decay constants of pseudoscalar mesons are within a factor of 2.5 to the pion decay constant [8]. The only exception is the decay constant of the  $\pi(1300)$  that is suppressed [29, 30]. A large value for decay constant  $f_{a_0}$  does not rule out a  $\bar{q}q\bar{q}q$  multi-quark meson.

Using gauge configurations from UKQCD and CP-PACS, McNeile and Michael computed  $f_{a_0} \sim 480$  MeV. Sum rule and model estimates find  $f_{a_0}$  in the range 290 to 440 MeV [26, 28, 31, 32]. The  $f_{a_0}$  decay constant depends on the scale and this should be specified for a more detailed comparison.

Computing the decay width of a hadron is also very a valuable way of identifying a state on the lattice. In [12], it was reported that the experimental hadron coupling for the decays  $a_0(980) \rightarrow K\bar{K}$  and  $a_0(1450) \rightarrow K\bar{K}$  were 0.9 and 0.5 respectively. A lattice calculation [12] found that the lightest hadron in the  $0^{++}$  correlator had a coupling to  $K\bar{K}$  of  $\approx 1$ , thus providing additional evidence that the lightest state was the  $a_0(980)$ .

Pennington [33] has recently extracted the two photon decay width of the  $\sigma$  from experiment to be  $\Gamma(\sigma \rightarrow \gamma\gamma) \sim 4$  keV. Pennington notes that value of  $\Gamma(\sigma \rightarrow \gamma\gamma)$  can depend quite sensitively on the quark content of the  $\sigma$  [33]. Recently a formalism to compute two photon widths on the lattice has been developed [34]. Dudek and Edwards have computed  $\Gamma(\chi_0 \rightarrow \gamma\gamma) = 2.4 \pm 1.0$  keV, from a quenched QCD calculation [34]. It would be interesting to do a similar calculation for light scalar mesons.

### 1.3 Flavour singlet $0^{++}$ mesons

The spectrum of the light flavor singlet  $0^{++}$  mesons is where the  $0^{++}$  glueball is thought to be hiding out. The lightest flavor singlet  $0^{++}$  mesons listed in the PDG [8] are:  $f_0(600)$ ,  $f_0(980)$ ,  $f_0(1370)$ ,  $f_0(1500)$ , and  $f_0(1710)$ . There are claims that the  $f_0(980)$  is a molecule or tetraquark [8], so it may not couple to  $\bar{q}q$  interpolating operators.

Morningstar and Peardon [35] obtained  $M_{0^{++}} = 1730(50)(80)$  MeV for

the mass of the lightest  $0^{++}$  glueball from quenched QCD. Chen et al. [36] recently found  $M_{0^{++}} = 1710(50)(80)$  MeV. The quark model predicts that there should only be two  $0^{++}$  mesons between 1300 and 1800 MeV, so if the mixing between the glueball and  $\bar{q}q$  operators is weak, then the  $0^{++}$  glueball is hidden inside the  $f_0(1370)$ ,  $f_0(1500)$  and  $f_0(1710)$  mesons.

Weingarten and Lee [37] used quenched lattice QCD to estimate the mixing matrix between the glue and  $\bar{q}q$  states. Weingarten and Lee [37] predicted that the  $f_0(1710)$  meson was 74(10)%  $0^{++}$  glueball, and hence the mixing between the  $0^{++}$  glueball and  $\bar{q}q$  states was weak.

There are claims [38] that continuum phenomenology is more consistent with a sizable contributions from the  $0^{++}$  glueball to the  $f_0(600)$  and  $f_0(980)$  mesons.

The SESAM collaboration studied the glueball spectrum on unquenched lattices [39]. McNeile and Michael studied the light  $0^{++}$  spectrum with unquenched QCD [40] at a coarse lattice spacing and found the mass of the lightest flavour singlet  $0^{++}$  meson was very light. Using  $0^{++}$  glueball operators, Hart and Teper [41] found that

$$M_{0^{++}UNquenched} = 0.85(3)M_{0^{++}Quenched} \quad (5)$$

at a fixed lattice spacing of 0.1 fm. The UKQCD collaboration [20] separately studied  $0^{++}$  glueball and  $0^{++}$   $\bar{q}q$  operators on improved staggered gauge configurations, however higher statistics and an analysis similar to the one by Bernard et al. is required [22].

Unfortunately, the existing unquenched lattice QCD calculations of the flavour singlet  $0^{++}$  mesons don't have the range of lattice spacings where a continuum extrapolation can be attempted. In quenched QCD it was found that the lattice spacing dependence of the mass of the  $0^{++}$  glueball was strong. The use of a Symanzik improved gauge action by Chen et al. [36] and, Morningstar and Peardon [35], produced a slightly smaller slope with lattice spacing of the scalar  $0^{++}$  glueball mass, than for calculations that used the Wilson plaquette action. This is relevant to unquenched calculations, because any suppression of the mass of the flavour singlet  $0^{++}$  mass may be due to lattice spacing effects.

The SCALAR collaboration [42], used unquenched lattice QCD, with Wilson fermions and the Wilson gauge action, to study the  $0^{++}$  mesons. At a single lattice spacing a  $\sim 0.2$  fm, with  $\bar{q}q$  interpolating operators only, they obtain  $m_{\bar{q}q} \sim m_\rho$ . The lattice spacing dependence of this result needs to be quantified.

In unquenched QCD, both glue and  $\bar{q}q$  states will couple to singlet  $0^{++}$  mesons, so it is better to do a variational fit with both types of operators

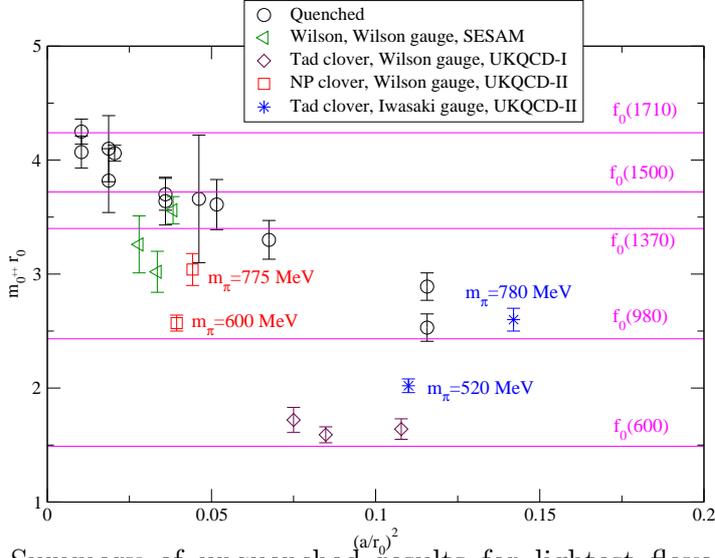


Figure 2: Summary of unquenched  $0^{++}$  results for lightest flavor singlet mesons from [43]. The unquenched results are from SESAM [39], UKQCD-I [40], and UKQCD-II [43].

as basis interpolating operators. The variational technique analysis of the singlet  $0^{++}$  mesons was done by Hart et al. [43]. A combined fit to  $0^{++}$  glue and  $\bar{q}q$  interpolating operators with two types of spatial smearing sources was done. The calculation used the non-perturbative improved clover action at a single lattice spacing [4]. Configurations from CP-PACS [5] with the Iwasaki gauge action and tadpole improved clover action were also used in the analysis, because this calculation should be less affected by lattice artifacts. A summary plot of the results, in units of  $r_0$  ( $1/r_0 \sim 400$  MeV) is in figure 2 (updated from [43]). The data with the bursts and squares (with the pion masses written near them) in figure 2 shows an additional reduction of the mass of the  $0^{++}$  state over the pure glueball operators, as used by Hart and Teper [41].

Mathur et al. [13] recently claimed to get a result for the mass of the  $f_0(600)$  ( $\sigma$ ) from quenched lattice QCD with pion masses as low as 180 MeV. Using the interpolating operator  $\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$  they obtain  $m_{f_0(600)} \sim 550$  MeV. The key part of this work is a three state fit ( $\pi(p=0)\pi(p=0)$ ,  $f_0(600)$ ,  $\pi(p=\frac{2\pi}{L})\pi(p=\frac{-2\pi}{L})$ ) using the Bayes adaptive curve fitting algorithm [44]. They studied the finite volume effects to distinguish the signal for the resonance from the  $\pi\pi$  scattering states [45]. Mathur et al.'s [45] calculation is discussed in slightly more detail in [3]. The effect of sea quarks on this calculation needs to be quantified.

There has also been a recent quenched QCD study [46] of light  $0^{++}$  states with  $qq\bar{q}\bar{q}$  interpolating operators that did not see resonant states in the quark mass regime they explored.

In [43] an attempt was made to compute the decay width for  $f_0$  decay to two pions. Unfortunately much higher statistics will be required to obtain an accurate value for that width.

## 2 Conclusions

There is still no consensus as to whether  $\bar{q}q$  operators in lattice QCD calculations are coupling to the  $a_0(980)$  meson. To clear up the many questions about the spectrum of the  $0^{++}$  scalar mesons, unquenched lattice QCD calculations with tetraquark interpolating operators are required. There is “some” evidence that the flavour singlet  $0^{++}$  interpolating operators, in unquenched lattice QCD calculations, are coupling to states around or below 1 GeV [43]. Although a continuum extrapolation is required for definite results. Recent lattice QCD calculations that include the dynamics of the sea quarks are working with light enough quarks that the two meson decays of some scalar mesons are allowed [3, 24, 25].

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