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PROGRESS IN $NN \to NN\pi$

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Abstract

We survey the recent developments in the reaction $NN \to NN\pi$ in effective field theory. We show that the proper construction of the production operator needs a careful separation of irreducible pieces from reducible ones. The result of this consideration is a complete cancellation of all loops in the production operator at NLO. Moreover, we show that this procedure brings the leading Weinberg-Tomozawa vertex on-shell, thus enhancing the corresponding contribution to the transition amplitude by a factor of 4/3 as compared to the commonly used value. We also discuss the role of the $\Delta(1232)$ for the s-wave pion production. Being relatively sizable individually the direct and rescattering mechanisms of the Δ excitation at NLO cancel each other to a large extent. Thus, we conclude that the net effect of the Δ at NLO is very small.

1 Introduction

Understanding the dynamics of pion production in nucleon-nucleon collisions near threshold is a challenge for theoreticians. Knowledge of the pion production mechanism in the isospin symmetric case is an important step to the study of isospin violation in few-nucleon processes [1, 2], which provides a test for chiral perturbation theory (ChPT). Furthermore, success in the description of the charged pion production reactions is a necessary condition for a calculation of the dispersive correction to πd scattering [3], which is one of the most uncertain and at the same time important corrections to this

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process. When accurate data for the total cross-section close to threshold appeared in 1990 [4], existing models [5, 6] failed to describe the data by a factor of five to ten for the channel $pp \to pp\pi^0$ and a factor of two for the channels $pp \to pn\pi^+$ and $pp \to d\pi^+$. To cure this discrepancy, many phenomenological mechanisms were proposed — for a recent review see Ref. [7]. Also various groups started to investigate $NN \to NN\pi$ using ChPT. As a big surprise, however, it turned out that using the original power counting proposed by Weinberg [8] leads to even larger discrepancy between data and theory at next–to–leading order (NLO) for $pp \to pp\pi^0$ [9] as well as for $pp \to d\pi^+$ [10]. Even worse, the corrections at one–loop order (next–to–next–to–leading order ($\mathrm{N^2LO}$) in the standard counting) turned out to be even larger than the tree level NLO corrections, putting into question the convergence of the chiral expansion [11, 12].

At the same time it was already realized that a modified power counting is necessary to properly take care of the large momentum transfer characteristic for pion production in NN collisions [13–16]. The expansion parameter in this case is

$$\chi = \frac{|\vec{p}_{\rm thr}|}{M_N} = \sqrt{\frac{m_\pi}{M_N}} \tag{1}$$

where $m_\pi\,(M_N)$ is the pion (nucleon) mass and $|\vec{p}_{\rm thr}|$ is the initial nucleon momentum at threshold. As a consequence the hierarchy of diagrams changes and some one-loop diagrams start to contribute already at NLO. In this presentation we discuss the charged pion production where the produced pion is in an s-wave relative to the final NN pair up to NLO in ChPT. In sec. 2 we discuss the pion production operators involving only pionic and nucleonic degrees of freedom. We start from the concept of reducibility that is necessary to distinguish between the production operator, which should consist of all irreducible pieces, and the NN wave functions. The proper treatment of this concept allows to avoid double counting in the calculation and thus is extremely important. We discuss in detail the special case of diagrams with the energy dependent vertices originating from, e.g., the Weinberg-Tomozawa (WT) term. In this case some diagrams that seem to be purely reducible from the general rules acquire an important irreducible contribution. In sec. 3 we discuss the role of the Delta resonance for the s-wave pion production near threshold. The corresponding diagrams start to contribute at NLO and thus are relevant for the present study. The main results are summarized in sec. 4.

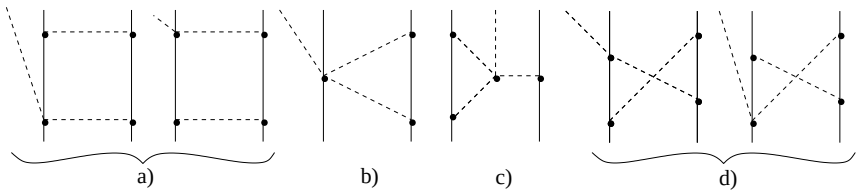

Figure 1: Leading loop diagrams for $NN \to NN\pi$. Here dashed lines denote pions and solid lines denote nucleons.

2 Nucleonic amplitudes up to NLO

A method how to calculate processes on few nucleon systems with external probes was proposed by Weinberg [8]: here the transition (production) operators are to be calculated using ChPT. Then those transition operators must be convoluted with the appropriate NN wave functions — in full analogy to the so–called distorted wave Born approximation traditionally used in phenomenological calculations [5].

Therefore it is necessary to disentangle those diagrams that are part of the wave function from those that are part of the transition operator. In complete analogy to NN scattering, the former are called reducible and the latter irreducible. The distinction stems from whether or not the diagram shows a two-nucleon cut. Thus, in accordance to this rule, the one-loop diagrams shown in Fig. 1(b)–(d) are irreducible, whereas diagrams (a) seem to be reducible. This logic was used in the paper by Hanhart and Kaiser [16] to single out the irreducible loops contributing at NLO. The findings of Ref. [16] were:

- For the channel $pp \to pp\pi^0$ the sum of diagrams (b)–(d) of Fig. 1 vanished due to a cancellation between individual diagrams
- For the channel $pp \to d\pi^+$ the same sum gave a finite answer[2]:

$$A^{b+c+d}_{pp\to d\pi^+} = \frac{g_A^3}{256 f_\pi^5}\,(-2+3+0)\,|\vec{q}| = \frac{g_A^3 |\vec{q}|}{256 f_\pi^5}. \tag{2}$$

The latter amplitude grows linearly with increasing final NN–relative momentum $|\vec{q}|$, which leads to a large sensitivity to the final NN wave function, once the convolution of those with the transition operators is evaluated. However, the problem is that such a sensitivity is not allowed in a consistent field

[2]The connection of the amplitude A to the observables is given, e.g., in Ref [17]

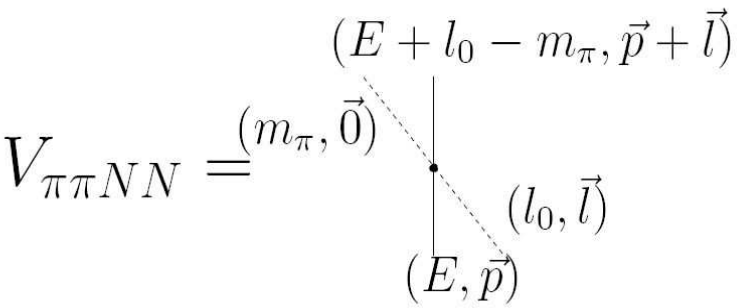

Figure 2: The $\pi N \to \pi N$ transition vertex: definition of kinematic variables as used in the text.

theory as was stated in Ref. [18]. The solution of this problem was presented in Ref. [17] and will be discussed in this presentation.

It is the main point of this section that diagrams (a) contain a genuine irreducible piece due to the energy dependence of the leading $\pi N \to \pi N$ vertex. Specifically, the energy dependent part of the WT vertex cancels one of the intermediate nucleon propagators, resulting in the additional irreducible contribution at NLO from diagrams (a). This additional contribution compensates the linear growth of diagrams (b)–(d) thus solving the problem. To demonstrate this, we write the expression for the $\pi N \to \pi N$ vertex in the notation of Fig. 2 as

$$\begin{aligned} V_{\pi\pi NN} &= l_0 + m_\pi - \frac{\vec{l}\cdot(2\vec{p}+\vec{l})}{2M_N} \\ &= \underbrace{2m_\pi}_{\text{on-shell}} + \underbrace{\left(l_0 - m_\pi + E - \frac{(\vec{l}+\vec{p})^2}{2M_N}\right)}_{(E'-H_0)=(S')^{-1}} - \underbrace{\left(E - \frac{\vec{p}^{\,2}}{2M_N}\right)}_{(E-H_0)=S^{-1}} . \end{aligned} \tag{3}$$

For simplicity we skipped the isospin part of the amplitude. The first term in the last line denotes the transition in on–shell kinematics, the second the inverse of the outgoing nucleon propagator and the third the inverse of the incoming nucleon propagator. Note that for on–shell incoming and outgoing nucleons, the $\pi N \to \pi N$ transition vertex takes its on–shell value $2m_\pi$ — even if the incoming pion is off–shell. This is in contrast to standard phenomenological treatments [5], where l_0 is identified with $m_\pi/2$ — the energy transfer in on–shell kinematics — and the recoil terms are not considered. Note, since $p_{thr}^2/M_N = m_\pi$ the recoil terms are to be kept. A second consequence of Eq. (3) is even more interesting: when the rescattering diagram

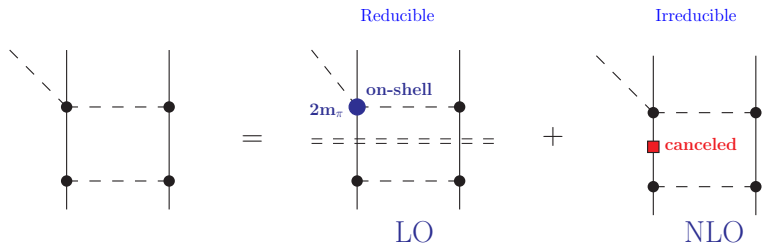

Figure 3: Reducible and irreducible parts of the box diagram. The square indicates that the corresponding nucleon propagator is canceled. The double dashed line shows the two-nucleon cut.

with the $\pi N \to \pi N$ vertex gets convoluted with NN wave functions, only the first term leads to a reducible diagram. The second and third terms, however, lead to irreducible contributions, since one of the nucleon propagators gets canceled. This is illustrated in Fig. 3 on the example of the second diagram of Fig. 1a. It was shown explicitly in Ref. [17] that those induced irreducible contributions cancel the finite remainder of the NLO loops ((b)-(d)) in the $pp \to d\pi^+$ channel. Thus, up to NLO only the diagrams

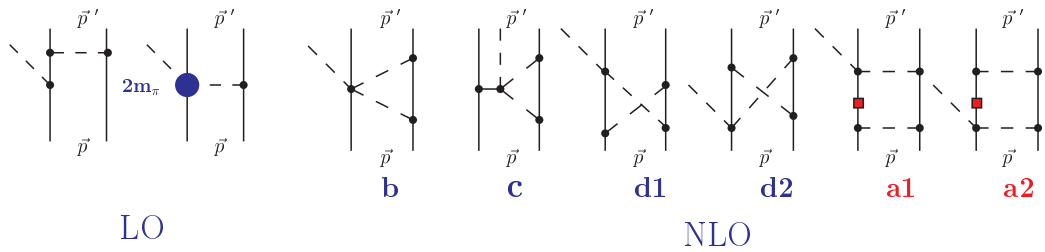

Figure 4: Complete set of nucleonic diagrams up to NLO. Note that sum of all loops at NLO vanishes.

appearing at LO, as shown in Fig. 4, contribute to $pp \to d\pi^+$, with the rule that the $\pi N \to \pi N$ vertex is put on–shell. This enhances the dominating isovector πN-rescattering amplitude by a factor of 4/3 as compared to the traditionally used value, which leads to a good description of the experimental data for $pp \to d\pi^+$. The result found in Ref. [17] is shown in Fig. 5 as the solid line whereas the dashed line is the result of the model by Koltun and Reitan [5] that basically corresponds to our LO calculation with $3/2m_\pi$ for the $\pi N \to \pi N$ vertex. The data sets are from TRIUMF [19], IUCF [20], COSY [21] and PSI [22].

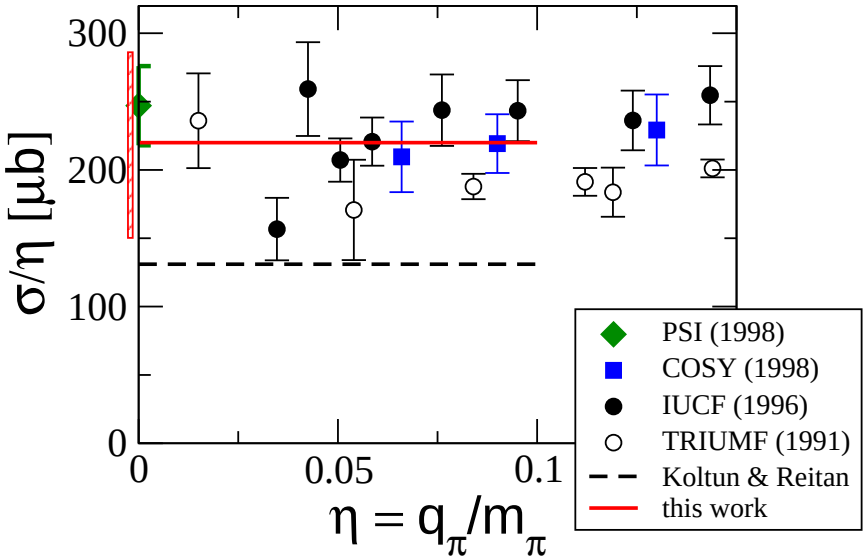

Figure 5: Comparison of our results to experimental data for $NN \to d\pi$ (see text for the details).

3 Role of the Delta resonance

In this section we would like to discuss the influence of the Δ resonance on the s-wave pion production cross section. The effect of the Delta has been extensively studied in the literature both in the phenomenological framework and using EFT. However, the current situation in the literature is quite contradictory. In particular, in phenomenological study by Niskanen [23] it was shown that the inclusion of the Δ isobar leads to an enhancement of the total cross section in $pp \to d\pi^+$ by almost a factor of 3. This enhancement is governed by the process where Δ in the intermediate state emits a p-wave pion which is then rescattered on the nucleon in an s-wave, i.e. the diagrams analogous to the box diagrams of Fig.1 a) but with the Δ instead of the nucleon in the intermediate state. However, the finding of Ref. [23] was not confirmed in the model calculation by Hanhart et al. [24]. The authors of this work have found that the direct pion emission from the Δ increases the cross section by about 30% whereas the rescattering process is negligible. The Jülich meson-nucleon model [25] was applied in Ref. [24] to generate the off-shell $\pi N \to \pi N$ transition T-matrix and the coupled channel CCF model [26] was employed to take into account the $N\Delta$ and NN distortions in the initial and final states. In contrary to these results the direct pion production from the Δ was shown to be negligible in the EFT calculation

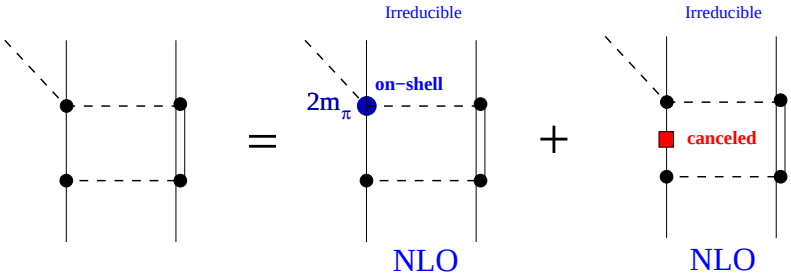

Figure 6: Rescattering diagram with the Δ isobar. Pattern of separation on two parts due to the energy dependent WT vertex.

by da Rocha et al. [27]. However, the $NN \to N\Delta$ transition in Ref [27] was approximated by one-pion exchange only, and the $NN \to N\Delta$ contact interaction that contributes at the same order was not taken into account. At the same time it is known from phenomenology that heavy ρ meson exchange which plays the role of this contact term in phenomenological calculations is significant [26]. In addition the static Δ propagator is used in Ref. [27] which leads to the large model dependence of the results. The similar problem with the use of the static Δ propagator for the πd scattering was investigated in Ref. [28]. Let us now discuss in more detail the πN rescattering diagrams with the Δ. First of all, due to the small mass difference between the nucleon and Δ, $\Delta M = M_\Delta - M_N \simeq 2m_\pi$, the $N\Delta$ propagator behaves as

$$\frac{1}{m_\pi - \Delta M - p^2/2M_{N\Delta}} \sim \frac{1}{m_\pi}, \tag{4}$$

where $M_{N\Delta}$ is the reduced mass of the $N\Delta$ system, i.e. in full analogy with the counting rules for the NN propagator. Secondly, these rescattering diagrams contain the energy dependent WT vertex, and thus the method developed in the previous section for the diagrams with nucleons can be applied here as well. In particular, these diagrams can be also divided into two parts, as demonstrated in Fig. 6: the first one goes with the on shell πN vertex and in the second one the nucleon propagator cancels the corresponding piece in the vertex. In contradistinction from the diagrams of Fig. 3 both these parts contribute at NLO and both are irreducible. Furthermore, in the recent paper by Hanhart and Kaiser [16] the full set of one-loop diagrams with the Δ contributing at NLO was studied in EFT. In particular, it was shown that those parts of the rescattering diagrams, in which the nucleon propagator is canceled, take part in a cancellation with other loop diagrams (see Fig. 4 in Ref. [16]). Thus, the only remainders contributing at NLO are the direct pion production and the rescattering process with the on shell πN

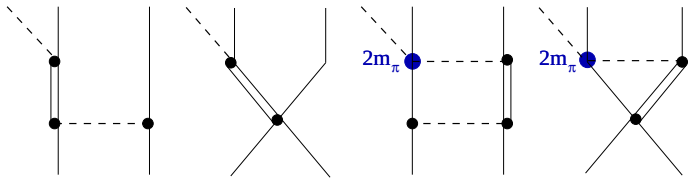

Figure 7: Non vanishing diagrams with the Δ isobar contributing at NLO.

vertex as shown in Fig. 7. We evaluated these diagrams in a manner similar to our recent study of the role of the Δ in pion-deuteron scattering [28]. The calculation revealed that each of these diagrams gives about a 10-15% correction to the transition amplitude but they enter with opposite signs – the direct contribution increases the cross section in line with the finding of Ref. [24] whereas the rescattering piece leads to a reduction by almost the same amount. Thus, there is a significant cancellation between different mechanisms involving the Δ excitation and the resulting contribution of the Δ resonance at NLO is very small. The calculation was done with the CCF [26] and the Hannover [29] coupled-channel NN models, and the pattern of cancellation was the same for both models although the individual contributions were slightly different.

4 Summary

We reported about recent developments in the reaction $NN \to NN\pi$ in the effective field theory. Within the counting scheme that acknowledges the large momentum transfer between the initial and the final nucleons we have calculated the transition operator for this reaction up to NLO. We discussed how to implement properly the reducibility concept for the pion production process. In particular it was shown how to identify the irreducible contribution of the formally reducible loop diagrams with the energy dependent vertices. As a result we obtained that the irreducible loops at NLO cancel altogether, and the leading order πN-rescattering amplitude is enhanced by a factor of 4/3 as compared to the commonly used value. This enhancement leads to a good description of the experimental data for $pp \to d\pi^+$. We also investigated the effect of the Δ isobar on the s-wave pion production. Being relatively sizable individually the direct and rescattering mechanisms of the Δ excitation at NLO cancel each other to a large extent. Thus, we conclude that the net effect of the Δ at NLO is very small.

The theoretical uncertainty of our NLO calculation was estimated con-

servatively using the dimensional arguments. The large uncertainty of about $2m_\pi/M_N \approx 30\%$ for the cross section is a consequence of the rather large expansion parameter. Thus, a computation at NNLO is necessary for drawing more solid conclusions on the pion production mechanism, especially if one wants to learn more about the charge-symmetry breaking effects.

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