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# SOLITON MODEL APPROACH TO KAON–NUCLEON SCATTERING IN THE PENTAQUARK CHANNEL

H. Weigel
Fachbereich Physik
Siegen University
D–57068 Siegen

**Abstract**

In soliton models for baryons the congruous description of hadronic decays of baryon resonances that are described as collective excitations has been a long standing problem. Here I present a solution to this problem for the pentaquark channel in the Skyrme model. The resulting kaon–nucleon scattering amplitude satisfies large $N_C$ consistency conditions and falsifies estimates of tiny pentaquark widths that are based on axial current matrix elements. Rather this specific soliton model predicts the width to be about 50MeV and a mass of about 1800MeV, which are typical hadronic scales.

## 1 Introduction

Soliton models for baryons are based on action functionals of meson fields, $\Gamma = \Gamma[\Phi]$. These action functionals contain classical (static) soliton solutions, $\Phi_{\rm cl}$, that are identified as baryons. Their interaction with mesons is described by the (small) meson fluctuations about the soliton: $\Phi = \Phi_{\rm cl} + \phi$. By pure definition, the expansion of $\Gamma[\Phi]$ about $\Phi_{\rm cl}$ does not contain a term that is linear in $\phi$ to be interpreted as Yukawa interaction. As in experiment, resonance properties must be extracted from meson baryon scattering amplitudes. Two–meson processes acquire contributions from $\Gamma^{(2)}$, the $\mathcal{O}(\phi^2)$ piece in the expansion of $\Gamma[\Phi]$ about $\Phi_{\rm cl}$. It simultaneously represents the systematic (but not accurate) expansion in $N_C$, the number of color degrees of freedom: $\Gamma = \mathcal{O}(N_C)$, $\Gamma^{(2)} = \mathcal{O}(N_C^0)$ while terms $\mathcal{O}(\phi^3)$ vanish as $N_C \to \infty$. Hence $\Gamma^{(2)}$ contains all large–$N_C$ information about hadronic decays of resonances. Conversely, this sheds light on *ad hoc* computations of hadronic decay widths in soliton models: The large–$N_C$ limit of their results *must* be identical to those obtained from $\Gamma^{(2)}$. Unfortunately, the most prominent baryon resonance, the $\Delta$, is stable for $N_C \to \infty$ and its decay is not subject

to this litmus–test. Soliton models in flavor $SU(3)$ are more challenging: In the so–called *rigid rotator approach* (RRA) resonances emerge that dwell in the anti–decuplet representation of flavor $SU(3)$. The most discussed (and disputed) such state is the $\Theta^+$ pentaquark with zero isospin and strangeness $S = +1$. The decay properties of anti–decuplet states are indeed $\mathcal{O}(N_C^0)$. For $S = -1$ the equations of motion for $\phi$ yield a $P$-wave bound state that serves to describe the ordinary hyperons, whence the notion *bound state approach* (BSA). The litmus–test requires that the BSA and RRA give identical results for the $\Theta^+$ properties as $N_C \to \infty$. This did not seem true and it was prematurely argued that pentaquarks were a mere artifact of the RRA [1]. Hence the comparison between the BSA and RRA serves (i) to verify that the pentaquark is a genuine prediction of soliton models and (ii) to provide an unambiguous computation of pentaquark widths. Details of these Skyrme model studies are reported in ref. [2]. Ref. [3] may be consulted for a review on $SU(3)$ soliton models.

## 2 Rotation–Vibration Approach & $\Theta^+$ width

Restricting the $P$–wave fluctuations to the modes spanned by rigid rotations yields two bound states of strangeness $S = \pm 1$ with energies

$$\omega_\pm = \frac{1}{2}\left[\sqrt{\omega_0^2 + \frac{3\Gamma}{2\Theta_K}} \pm \omega_0\right] \quad \text{where} \quad \omega_0 = \frac{N_C}{4\Theta_K}\,. \tag{1}$$

Here $\Theta_K$ is the moment of inertia for the rotation of the soliton into strangeness direction and $\Gamma$ is the functional that measures flavor symmetry breaking. Both functionals are $\mathcal{O}(N_C)$ so that $\omega_\pm = \mathcal{O}(N_C^0)$. While $\omega_-$ is the energy of the above mentioned bound state describing ordinary hyperons, $\omega_+$ is eventually utilized to construct pentaquark states. When the restriction that BSA modes $\eta$ are spanned by the rigid rotation is waived, $\omega_- < m_K$ corresponds to a true bound state but $\omega_+$ turns into a continuum state. Thus, a pronounced resonance structure would be expected around $\omega = \omega_+$. Unfortunately, this is not the case, as is seen from fig. 1 [1].

In the RRA the collective coordinates $A(t) \in SU(3)$ that parameterize the flavor orientation of the soliton are canonically quantized. The resulting Hamiltonian is exactly diagonalized for arbitrary (odd) $N_C$ [2] and symmetry breaking [4]. The resulting mass difference between the states that for $N_C = 3$ correspond to the $\Lambda$ ($\Theta^+$) and the nucleon, approaches $\omega_-$ ($\omega_+$) as $N_C \to \infty$.

The ultimate comparison requires to generalize the RRA to the *rotation–vibration approach* (RVA)

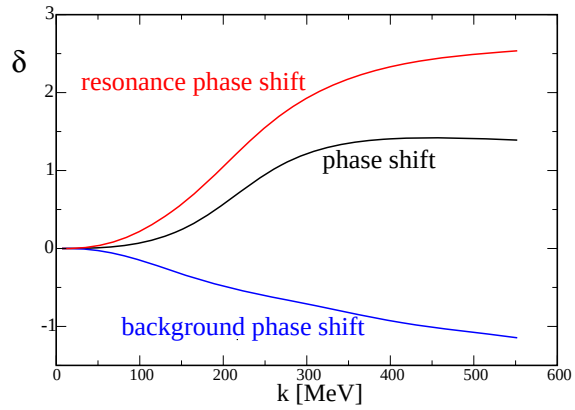


Figure 1: Phase shift in the BSA (middle, black line) and the resonance phase (top, red line) shift after removal of the background (bottom, blue line) contribution in the RVA that emerges from the constraint $\langle \eta | z \rangle = 0$.

$$U(\mathbf{x}\,,t) = A(t)\xi_0(\mathbf{x}) \exp\left[\frac{i}{f_\pi}\sum_{\alpha=4}^{7}\lambda_\alpha \widetilde{\eta}_\alpha(\mathbf{x}\,,t)\right]\xi_0(\mathbf{x})\,A^\dagger(t)\,, \tag{2}$$

where $\xi_0(\mathbf{x}) = \exp\big[i\hat{\mathbf{x}}\cdot\tau F(|\mathbf{x}|)/2\big]$ is the chiral field representation of the soliton ($\Phi_{\rm cl}$). Avoidance of double counting enforces the constraint $\langle \widetilde{\eta} | z \rangle \stackrel{!}{=} 0$, where $z \sim \sin(F/2)$ is the collective mode wave–function. Eq. (2) generates a term in $\Gamma^{(2)}$ that is linear in both $\widetilde{\eta}$ and the collective modes, parameterized by $A(t)$. This contribution corresponds to Yukawa exchanges and induces a separable potential $V_Y$ for $\widetilde{\eta}$. In ref. [2] we have shown that in the limit $N_C \to \infty$ the resulting equation of motion is solved by $\widetilde{\eta} = \eta - \langle z | \eta \rangle z$.

Thus the BSA and RVA are indeed equivalent in the large $N_C$ limit since the phase shifts extracted from $\eta$ and $\widetilde{\eta}$ are identical as $z(|\mathbf{x}|)$ is localized in space. Furthermore, the RVA provides a distinction between resonance and background contributions. Applying the $R$–matrix formalism shows that for $N_C \to \infty$ $V_Y$ *exactly* contributes the resonance phase shift shown in fig. 1. Thus pentaquarks are also predicted by the BSA; just well hidden. Yet, collective coordinates are mandatory to obtain finite $N_C$ corrections: *e. g.*, for $m_K = m_\pi$ the mass difference $M(\Theta) - M(N)$ increases by a factor two from $\omega_0$ to $(N_C+3)/4\Theta_K$. Skyrme model calculations indicate that $M(\Theta) - M(N)$ might be as large as 900MeV [2]. The $R$–matrix formalism

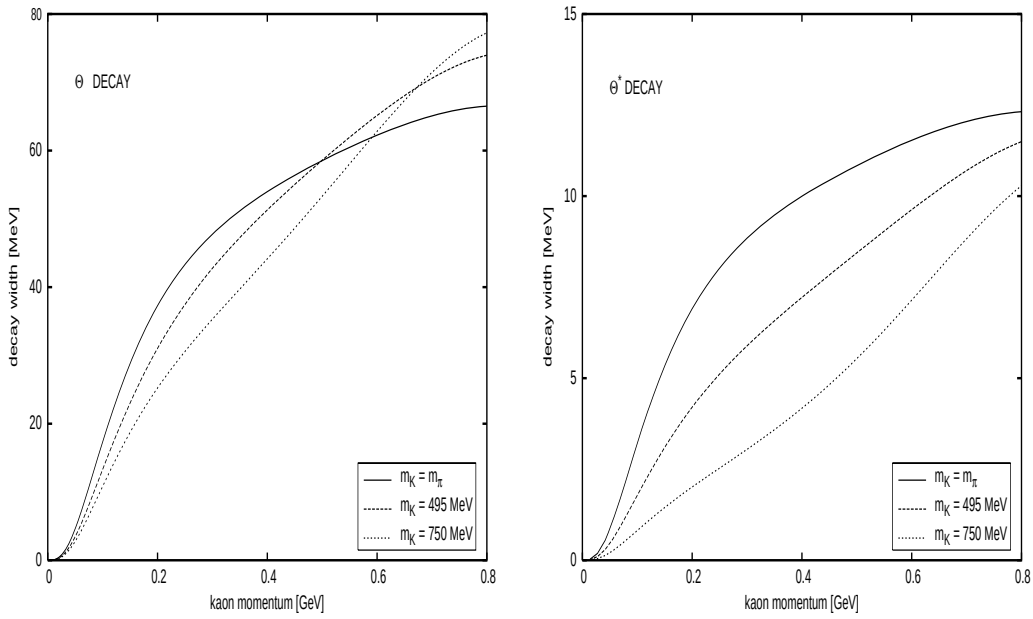


Figure 2: *Skyrme model prediction for the decay width, $\Gamma(\omega)$ of $\Theta^+$ and its iso–partner $\Theta^{*+}$ for $N_C = 3$ as function of the kaon momentum $k = \sqrt{\omega^2 - m_K^2}$, cf. eq. (3).*

finally yields the $\Theta^+$ width as a function of the kaon momentum [2]

$$\Gamma(k) = 2k\omega_0 \left| X_\Theta \int_0^\infty r^2 dr\, z(r) 2\lambda(r) \overline{\eta}_{\omega_k}(r) \right|^2 + \mathcal{O}\left(m_K^2 - m_\pi^2\right) . \qquad (3)$$

Here $\overline{\eta}_{\omega_k}(|\mathbf{x}|)$ is the P–wave background wave–function ($\overline{\eta} \hat{=} \widetilde{\eta}$ for $V_Y \equiv 0$) with energy $\omega_k = \sqrt{k^2 + m_K^2}$, $\lambda(|\mathbf{x}|)$ is a radial function that stems from the Wess–Zumino term and $X_\Theta$ is the nucleon–$\Theta^+$ transition matrix element of a collective coordinate operator. The resulting width is shown for $N_C = 3$ in fig. 2. Typical – though model dependent – results yield a $\Theta^+$ width of around 50MeV [2]. Remarkably, the width of the $\Theta^{*+}$, the $I = 1$ partner of $\Theta^+$, turns out smaller. In any event, it seems very unlikely that chiral soliton models predict a light long–living pentaquark.

# 3 Conclusion

Here I have compared the BSA and RRA to chiral soliton models. While the former gives the exact model results in the large $N_C$ limit and thus serves as a litmus–test, the latter incorporates substantial finite $N_C$ corrections. This comparison requires the consideration of harmonic oscillations in the

RRA as well. They are incorporated via the RVA which clearly shows that pentaquarks are genuine resonances within chiral soliton models.

In the flavor symmetric case the interaction Hamiltonian contains only a *single* structure ($X_\Theta$ in eq. (3)) of $SU(3)$ matrix elements for the $\Theta^+ \to KN$ transition. This proves earlier soliton model approaches [5] incorrect that adopted any possible structure that could contribute and fitted coefficients from a variety of hadronic decays under the assumption of $SU(3)$ relations.

Since the presented analysis merely concerns the treatment of kaon degrees of freedom, the qualitative results are valid for *any* chiral soliton model. In essence, these models models do not predict very light and very narrow pentaquark resonances.

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