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ON NATURE OF THE f_0 AND f_2 MESONS

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Abstract

In approach, based on analyticity and unitarity, experimental data on the isoscalar S- and D-waves of $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ have been analyzed jointly for studying the f_0- and f_2-mesons. Assignment of scalar and tensor mesons to lower nonets is proposed. The $f_0(1500)$ and $f_2(2000)$ are interpreted as glueballs.

1 Analysis of the isoscalar-scalar sector

First we analyze the S-waves of $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ by the uniformizing variable method [1] applicable only in the 2- and 3-channel cases. Therefore, we have carried out two variants of the 3-channel analysis: (I) the one of $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, and (II) of $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta'$.

The 3-channel S-matrix is determined on the 8-sheeted Riemann surface with the right cuts, starting with $4m_\pi^2$, $4m_K^2$, and $4m_\eta^2$ (or $(m_\eta + m_{\eta'})^2$), and the left cuts. In the elements $S_{\alpha\beta}$, $\alpha, \beta = 1(\pi\pi), 2(K\overline{K}), 3(\eta\eta$ or $\eta\eta')$. The surface sheets are numbered according to the signs of analytic continuations of $k_1 = (s/4 - m_\pi^2)^{1/2}$, $k_2 = (s/4 - m_K^2)^{1/2}$, $k_3 = (s/4 - m_\eta^2)^{1/2}$ (or $k_3' = (s - (m_\eta + m_{\eta'})^2)^{1/2}/2$) as follows: signs $(\mathrm{Im}k_1, \mathrm{Im}k_2, \mathrm{Im}k_3) = +++, -++, --+, +-+, +--, ---, -+-, ++-$ correspond to sheets I, II,$\cdots$, VIII, respectively. (All, related to variant II, is denoted by prime.) The 3-channel resonances are described by seven types of pole clusters (of poles and zeros on the Riemann surface) [1] according to seven possible cases when there are resonance zeros on sheet I only in (**a**) S_{11}, (**b**) S_{22}, (**c**) S_{33}, (**d**) S_{11} and S_{22}, (**e**) S_{22} and S_{33}, (**f**) S_{11} and S_{33}, and (**g**) S_{11}, S_{22} and S_{33}. The cluster kind is related to the nature of state.

We use the Le Couteur–Newton relations (CNR) [2] which express the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1, \cdots, k_n)$ that is a real analytic function with the only branch-points at $k_i = 0$. The branch points are taken into account in an uniformizing

variable. Neglecting the $\pi\pi$ threshold, it is [1] $w = (k_2 + k_3)/[m_\eta^2 - m_K^2]^{1/2}$ for variant I, and $w' = (k_2' + k_3')/[(m_\eta + m_{\eta'})^2/4 - m_K^2]^{1/2}$ for variant II. CNR on the w-plane are given in Ref. [1]. In $d(w) = d_B d_{res}$ the resonance part is $d_{res} = w^{-\frac{M}{2}} \prod_{r=1}^{M}(w + w_r^*)$ with M the number of resonance zeros; the background part in variant I is $d_B = \exp[-i \sum_{n=1}^{3}(k_n/m_n)(\alpha_n + i\beta_n)]$, where $\alpha_n = a_{n1} + a_{n\sigma}\ (s - s_\sigma)/s_\sigma\ \theta(s - s_\sigma) + a_{nv}\ (s - s_v)/s_v\ \theta(s - s_v)$ and $\beta_n = b_{n1} + b_{n\sigma}\ (s - s_\sigma)/s_\sigma\ \theta(s - s_\sigma) + b_{nv}\ (s - s_v)/s_v\ \theta(s - s_v)$ with s_σ the $\sigma\sigma$ threshold, s_v the combined threshold of the $\eta\eta'$-, $\rho\rho$-, $\omega\omega$-channels.
In variant II, we add to α_n' and β_n' the terms $(s - 4m_\eta^2)/4m_\eta^2\ a_{n\eta}'\ \theta(s - 4m_\eta^2)$ and $(s - 4m_\eta^2)/4m_\eta^2\ b_{n\eta}'\ \theta(s - 4m_\eta^2)$, allowing for influence of the $\eta\eta$-channel.

References to sources of data, in variant I, can be found in [1]. In variant II, the data on $\pi\pi \to \eta\eta'$ are taken from [3]. In variant I, the analysis prefers the case when the $f_0(600)$ is described by the (**a**) cluster with poles on the complex energy plane $\sqrt{s}$ (in MeV) $683.5 - i589$ on sheet II, $673.3 - i589$ on sheet III, $593.5 - i589$ on sheet VI, $603.7 - i589$ on sheet VII; $f_0(1370)$ – (**c**) with poles $1398.3 - i287.5$ on sheet V, $1398.3 - i270.5$ on sheet VI, $1398.3 - i154.9$ on sheet VII, $1398.3 - i171.9$ on sheet VIII; $f_0(1500)$ – (**g**) with poles $1502.6 - i357.1$ on sheet II, $1479.1 - i140.2$ on sheet III, $1502.6 - i238.7$ on sheet IV, $1497 - i139.8$ on sheet V, $1497.5 - i191.8$ on sheet VI, $1496.7 - i87.35$ on sheet VII, $1502.6 - i356.5$ on sheet VIII; $f_0(1710)$ – (**b**) with poles $1708.3 - i142.3$ on sheet III, $1708.3 - i160.3$ on sheet IV, $1708.3 - i323.1$ on sheet V, $1708.3 - i305.1$ on sheet VI; the $f_0(980)$ is represented only by poles $1013.4 - i32.8$ on sheet II and $984.1 - i57.5$ on sheet III. The poles on sheets IV, VI, VIII and V, related to the $f_0(1500)$, are of the 2nd and 3rd order, respectively. A satisfactory description is obtained: the total χ^2/NDF for all three processes is 1.36. The background parameters are: $a_{11} = 0.2006$, $a_{1\sigma} = 0.0141$, $a_{1v} = 0$, $b_{11} = 0$, $b_{1\sigma} = -0.01025$, $b_{1v} = 0.04898$, $a_{21} = -0.7039$, $a_{2\sigma} = -1.4213$, $a_{2v} = -5.951$, $b_{21} = 0.0447$, $b_{2\sigma} = 0$, $b_{2v} = 6.787$, $b_{31} = 0.6456$, $b_{3\sigma} = 0.3348$, $b_{2v} = 0$; $s_\sigma = 1.638$ GeV^2, $s_v = 2.084$ GeV^2.

In variant II, the $f_0(600)$ is described by the (**a**$'$) cluster with poles $655.94 - i606$ on sheet II, $651.9 - i606$ on sheet III, $594.46 - i606$ on sheet VI, $598.5 - i606$ on sheet VII; $f_0(1370)$ – (**b**$'$) with poles $1391.2 - i246.3$ on sheet III, $1391.2 - i263.7$ on sheet IV, $1411.2 - i263.7$ on sheet V, $1411.2 - i246.3$ on sheet VI; $f_0(1500)$ – (**d**$'$) with poles $1504.1 - i198.7$ on sheet II, $1499 - i239$ on sheet III, $1504.1 - i193.3$ on sheet IV, $1504.1 - i199$ on sheet V, $1493.8 - i193.8$ on sheet VI, $1504.1 - i193.3$ on sheet VII; $f_0(1710)$ – (**c**$'$) with poles $1721.2 - i142.3$ on sheet V, $1721.2 - i109.3$ on sheet VI, $1721.2 - i82.3$ on sheet VII, $1721.2 - i115.3$ on sheet VIII; the $f_0(980)$ is represented by poles $1012.8 - i31.82$ on sheet II and $986.3 - i57.7$ on sheet III. The poles on sheets

IV and V, related to the $f_0(1500)$, are of the 2nd order. The total χ^2/NDF is ≈ 1.18. The background parameters are: $a'_{11} = 0.02411$, $a'_{1\eta} = -0.0638$, $a'_{1\sigma} = 0$, $a'_{1v} = 0.0916$, $b'_{11} = b'_{1\eta} = b'_{1\sigma} = 0$, $b'_{1v} = 0.0388$, $a'_{21} = -3.4384$, $a'_{2\eta} = -0.5377$, $a'_{2\sigma} = 1.695$, $a'_{2v} = -4.953$, $b'_{21} = 0.1193$, $b'_{2\eta} = -0.7953$, $b'_{2\sigma} = 2.5315$, $b'_{2v} = 2.925$, $b'_{31} = 0.6731$, $s_\sigma = 1.638$ GeV^2, $s_v = 2.126$ GeV^2.

These types of clusters and coupling constants from our 2-channel analysis [1], suggest that $f_0(1370)$ and $f_0(1710)$ have a dominant $s\overline{s}$ component; $f_0(1500)$, the dominant glueball one. The $f_0(980)$ is described by a pole on sheet II and shifted pole on sheet III under the $\eta\eta$ threshold without the corresponding poles on sheets VI and VII, as it was expected for standard clusters. This corresponds to the description of the $\eta\eta$ bound state [1].

Masses and total widths, calculated from the pole positions, are, respectively (in MeV): 869 and 1178 for $f_0(600)$, 1013.4 and 65.6 for $f_0(980)$, 1408.8 and 344 for $f_0(1370)$, 1544 and 713 for $f_0(1500)$, 1714.2 and 285 for $f_0(1710)$.

2 Analysis of the isoscalar-tensor sector

Analyzing data [4] on the isoscalar D-waves of $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ with an allowance also for channel $(2\pi)(2\pi)$ $(i = 4)$, we used the Breit–Wigner forms in CNR. In $d(k_1, k_2, k_3, k_4) = d_B d_{res}$, $d_{res}(s) = \prod_r [M_r^2 - s - i \sum_{j=1}^4 \rho_{rj}^5 R_{rj} f_{rj}^2]$ with $\rho_{rj} = 2k_i/(M_r^2 - 4m_j^2)^{1/2}$ and f_{rj}^2/M_r the partial width. $R_{rj}(s, M_r, s_j, r_{rj})$ is a Blatt–Weisskopf barrier factor [5] with s_j the channel threshold, and with radii $r_{rj} = 0.943$ fm for all resonances in all channels, except for $f_2(1270)$ and $f_2(1960)$ for which they are: for $f_2(1270)$, 1.498, 0.708 and 0.606 fm respectively in channels $\pi\pi$, $K\overline{K}$ and $\eta\eta$, and for $f_2(1960)$, 0.296 fm in channel $K\overline{K}$. The background part is $d_B = \exp[-i \sum_{n=1}^3 (2k_n/\sqrt{s})^5 (a_n + ib_n)]$, where $a_1 = \alpha_{11} + (s - 4m_K^2)/s\ \alpha_{12}\ \theta(s - 4m_K^2) + (s - s_v)/s\ \alpha_{10}\ \theta(s - s_v))$ and $b_n = \beta_n + (s - s_v)/s\ \gamma_n\ \theta(s - s_v)$ $(n = 1, 2, 3)$ with $s_v \approx 2.274$ GeV^2 the combined threshold of channels $\eta\eta'$, $\rho\rho$, $\omega\omega$.

We obtain a satisfactory description (the total χ^2/NDF ≈ 1.56) already with ten resonance $f_2(1270)$, $f_2(1430)$, $f'_2(1525)$, $f_2(1580)$, $f_2(1730)$, $f_2(1810)$, $f_2(1960)$, $f_2(2000)$, $f_2(2240)$ and $f_2(2410)$ [7]. Since the analysis of $p\overline{p} \to \pi\pi, \eta\eta, \eta\eta'$ [6] requires one more resonance ($f_2(2020)$), we have made also the analysis considering it. Description is the same one as in the previous case: the total χ^2/NDF ≈ 1.58. The resonance parameters are shown in Table 1. The background parameters are $\alpha_{11} = -0.0755$, $\alpha_{12} = 0.0225$, $\alpha_{10} = -0.2344$, $\beta_1 = -0.0782$, $\gamma_1 = -0.05215$, $\beta_2 = -0.985$, $\gamma_2 = 0.7494$, $\beta_3 = -0.5162$, $\gamma_3 = 0.786$.

Table 1: The f_2-resonance parameters (in MeV).

State	M	f_{r1}	f_{r2}	f_{r3}	f_{r4}	Γ_{tot}
$f_2(1270)$	1276.3±1.8	468.9±5.5	201.6±11.6	89.9±4.79	7.2±4.6	≈210.5
$f_2(1430)$	1450.5±18.8	128.3±45.9	562.3±144	32.7±18.6	8.2±63	>230
$f_2'(1525)$	1534.7±8.6	28.5±8.5	253.9±79	89.5±12.5	51.6±155	>49.5
$f_2(1565)$	1601.5±27.9	75.5±19.6	315±50.6	388.9±28.6	127±190	>170
$f_2(1730)$	1719.8±6.2	78.8±43	289.5±62.6	460.3±545.	108.6±76.	>182.4
$f_2(1810)$	1760±17.6	129.5±14.8	259±32.	469.7±25.2	90.3±89.5	>177.6
$f_2(1960)$	1962.2±29.8	132.6±23.3	331±61.5	319±42.8	62.4±91.3	>118.6
$f_2(2000)$	2006±22.7	155.7±24.4	169.5±95.3	60.4±26.7	574.8±211	>193
$f_2(2020)$	2027±25.6	50.4±24.8	441±196.7	58±50.8	128±190	>107
$f_2(2240)$	2202±45.4	133.4±32.6	545±150.4	381±116	168.8±103	>222
$f_2(2410)$	2387±33.3	175±48.3	395±197.7	24.5±68.5	462.8±211	>168

3 Summary

A confirmation of the σ-meson with the mass 869 MeV is obtained in the model-independent analysis of data on $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$.

Indication for $f_0(980)$ to be the $\eta\eta$ bound state is obtained.

The $f_0(1370)$ and $f_0(1710)$ have the dominant $s\overline{s}$ component.

The $f_0(1500)$ is supposed to be practically the 8th component of octet mixed with a glueball being dominant in this state.

We propose a following assignment of scalar mesons to lower nonets, excluding $f_0(980)$ as the $\eta\eta$ bound state. The lowest nonet: $a_0(980)$, $K_0^*(900)$, and $f_0(600)$ and $f_0(1370)$. The Gell-Mann–Okubo (GM-O) formula $3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$ gives $m_{f_8} = 880$ MeV. Our result: $m_\sigma = 869 \pm 14$ MeV. In relation for masses of nonet $m_\sigma + m_{f_0(1370)} = 2m_{K_0^*}$, the left side is about 26 % bigger than the right one.
For the next nonet we find: $a_0(1450)$, $K_0^*(1450)$, and $f_0(1500)$ and $f_0(1710)$. From the GM-O formula, $m_{f_8} \approx 1450$ MeV. In formula $m_{f_0(1500)} + m_{f_0(1710)} = 2m_{K_0^*(1450)}$, the left side is about 12 % bigger than the right one.
This assignment moves a number of questions, stood earlier, and does not put the new ones.

In the tensor sector, we performed two analysis – without and with the $f_2(2020)$. We do not obtain $f_2(1640)$, $f_2(1910)$, $f_2(2150)$ and $f_2(2010)$, however, we see $f_2(1450)$ and $f_2(1730)$.

Usually one assigns to the ground tensor nonet the $f_2(1270)$ and $f_2'(1525)$. To the 2nd nonet, one could assign $f_2(1600)$ and $f_2(1767)$ though the isodoublet member yet is not discovered. If $a_2(1730)$ is the isovector of this octet and if $f_2(1601)$ is almost its 8th component, then, from the GM-O formula, we expect this isodoublet mass at about 1635 MeV. Then the relation for masses of nonet would be well fulfilled. In the experiment [8] one had observed the strange isodoublet with the mass 1629 ± 7 MeV. This state might be the tensor isodoublet of the 2nd nonet.
The $f_2(1963)$ and $f_2(2207)$ together with the $K_2^*(1980)$ could be put into the 3rd nonet. In the relation $M_{f_2(1963)} + M_{f_2(2207)} = 2M_{K_2^*(1980)}$, the left

side is only 5.3 % bigger than the right one. If $f_2(1963)$ is the 8th component of octet, then the GM-O formula $M_{a_2} = 4M_{K_2^*(1980)} - 3M_{f_2(1963)}$ gives $M_{a_2} = 2031$ MeV. This value coincides with the one (2030 MeV) for a_2-meson obtained in the analysis [9].

As to $f_2(2000)$, in the analysis with ten resonances, the ratio of its $\pi\pi$ and $\eta\eta$ widths is in the limits obtained in Ref. [6] for the tensor glueball. However, its $K\overline{K}$ width is too large for the glueball. At practically the same description with the consideration of eleven resonances as in the case of ten, their parameters have varied not much, except for the ones of $f_2(2000)$ and $f_2(2410)$. The mass of the latter has decreased by about 40 MeV. As to $f_2(2000)$, now all the obtained ratios of the partial widths are in the limits corresponding to the glueball.

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