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DELTA-RESONANCE CONTRIBUTIONS TO THE NUCLEAR FORCE

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Abstract

The Δ-isobar is well known to play an important role in nuclear physics due to its low excitation energy, $\Delta \equiv m_\Delta - m_N \simeq 293$ MeV, and the strong coupling to the πN system. I discuss the implications of treating the Δ as a dynamical degree of freedom for two- and three-nucleon forces.

1 Introduction

Nuclear forces and few-nucleon dynamics based on chiral effective field theory (EFT) have been studied in great detail over the last decade, see [1, 2] for recent review articles. Most of the calculations in the few-nucleon sector carried out in this framework are based on the effective Lagrangian for pions and nucleons chirally coupled to external sources. The two-nucleon force (2NF) has been worked out up to next-to-next-to-next-to-leading order ($\mathrm{N^3LO}$) in the chiral expansion and demonstrated to provide an accurate description of the low-energy two-nucleon observables [3, 4]. On the other hand, the chiral expansion for the 2NF is known to exhibit a somewhat unnatural convergence pattern. In particular, by far the most important two-pion exchange (TPE) contribution arises at next-to-next-to-leading order ($\mathrm{N^2LO}$) [5]. The corresponding attractive central potential turns out to be one order of magnitude stronger than the (formally) dominant TPE contributions at at next-to-leading order (NLO).

The origin of the unnaturally strong subleading TPE potential can be traced back to the large values of the dimension-two low-energy constants (LECs) $c_{3,4}$ which are also responsible for the numerical dominance of the subleading three-pion exchange [6] and charge-symmetry breaking TPE 2NF [7] over the corresponding leading contributions. The large values of the LECs c_i are well understood in terms of resonance saturation [8]. In particular, the

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Δ-isobar provides the dominant (significant) contribution to c_3 (c_4). Given its low excitation energy, $\Delta \equiv m_\Delta - m_N = 293$ MeV, and strong coupling to the πN system, the Δ-isobar is known to play an important role in nuclear physics. One can, therefore, expect that the explicit inclusion of Δ in EFT will allow to resum a certain class of important contributions and improve the convergence as compared to the delta-less theory, provided a proper power counting scheme such as the small scale expansion (SSE) [9] is employed. The SSE is a phenomenological extension of chiral perturbation theory in which the delta-nucleon mass splitting is counted as an additional small parameter. This improved convergence has been explicitely demonstrated for pion-nucleon scattering where the description of the phase shifts at third order in the SSE comes out superior (inferior) to the third (fourth) order chiral expansion in the delta-less theory [10].

In this talk I describe the structure of the nuclear force in EFT with explicit delta degrees of freedom and compare it with the one resulting in the delta-less theory. The manuscript is organized as follows. In section 2, I discuss the power counting and the effective Lagrangian. The determination of the relevent LECs is described in section 3. The contributions to the 2NF and three-nucleon force (3NF) up to N^{2}LO are considered in sections 4 and 5, respectively. I end with a summary and outlook.

2 Power counting and the effective Lagrangian

Here and in what follows, I use Weinberg's power counting [11] utilizing the SSE. The low-momentum scale Q is set by external three-momenta of the nucleons, pion masses and the nucleon-delta mass splitting, $Q \in \{p, M_\pi, \Delta\}$. The irreducible[2] contributions to the scattering amplitude give rise to the nuclear force and can be obtained using the variety of schemes, see e.g. [2, 5, 11]. They are ordered according to the power ν of the expansion parameter Q/Λ with Λ being the pertinent hard scale. For an irreducible N-nucleon diagram, the power ν is given by [11]:

$$\nu = -2 + 2N + 2(L - C) + \sum_i V_i \Delta_i \, , \qquad \Delta_i = d_i + \frac{1}{2} b_i - 2 \, . \tag{1}$$

Here, L, C and V_i refer to the number of loops, separately connected pieces and vertices of type i, respectively. Further, b_i is the number of baryon field operators and d_i is the number of derivative and/or insertions of M_π. The

[2]These are the contributions which are not generated through iteration of the dynamical equation.

nucleon mass m_N is counted in a special way via $Q/m_N \sim Q^2/\Lambda^2$ which implies that $m_N \gg \Lambda$ [2, 11]. Notice further that according to the above counting rules, the momentum scales associated with the real pion and delta production are treated as the hard scale, $\sqrt{m_N M_\pi} \sim \sqrt{m_N \Delta} \sim \Lambda$, and thus do not need to be explicitly kept track of. Clearly, such a framework is only applicable at energies well below the pion production threshold.

The effective Lagrangian needed to derive the 2NF up to N^{2}LO ($\nu = 3$) has the following form in the heavy-baryon formulation:

$$
\begin{aligned}
\mathcal{L}^{(0)} &= \frac{F_\pi^2}{4}\langle u_\mu u^\nu + \chi_+\rangle + \overline{N}\big[iv\cdot D + g_A u\cdot S\big]N - \frac{1}{2}C_S(\overline{N}N)(\overline{N}N) \\
&\quad + 2C_T(\overline{N}S_\mu N)(\overline{N}S^\mu N) - \overline{T}^i_\mu\big[iv\cdot D^{ij} - \Delta\delta^{ij} + \ldots\big]g^{\mu\nu}T^j_\nu\,, \\
\mathcal{L}^{(1)} &= \overline{N}\big[c_1\langle\chi_+\rangle + c_2(v\cdot u)^2 + c_3 u\cdot u + c_4[S^\mu,\, S^\nu]u_\mu u_\nu\big]N \\
&\quad + \Big((b_3+b_8)\,\overline{T}^i_\mu i P^{\mu\nu}\omega^i_{\nu\rho}v^\rho N + \text{ h.c.}\Big)\,, \qquad (2)
\end{aligned}
$$

where the superscripts of $\mathcal{L}$ refer to the dimension Δ_i defined in Eq. (1), N denotes the large component of the nucleon field and T^i_μ with μ (i) being the Lorentz (isospin) index is the large component of the delta field. Further, $U(x) = u^2(x)$ collects the pion fields, $u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger)$, $\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u$ includes the explicit chiral symmetry breaking, $\langle\ldots\rangle$ denotes a trace in flavor space and $D_\mu (D^{ij}_\mu)$ is the chiral covariant derivative for the nucleon (delta) fields. Furthermore, $P_{\mu\nu}$ is the standard projector on the 3/2-components, $P_{\mu\nu} = g_{\mu\nu} - v_\mu v_\nu - 4S_\mu S_\nu/(1-d)$, with v_μ the four-velocity, S_μ the covariant spin vector and d the number of space-time dimensions. We also have $w^i_\alpha = \langle\tau^i u_\alpha\rangle/2$ and $w^i_{\alpha\beta} = \langle\tau^i[\partial_\alpha, u_\beta]\rangle/2$. The LECs in the lowest-order Lagrangian involve the pion decay constant $F_\pi = 92.4$ MeV, the nucleon axial-vector coupling $g_A = 1.27$ and the $\pi N\Delta$ axial coupling h_A and the constants $C_{S,T}$ accompanying nucleon-nucleon contact interactions. At subleading order, five further LEC contribute, namely the subleading πN LECs c_i ($i = 1, 2, 3, 4$) and the combination of $\pi N\Delta$ LECs $b_3 + b_8$. Notice further that the 2NF at N^{2}LO also involves subleading nucleon-nucleon contact interactions from $\mathcal{L}^{(2)}$ which are not shown explicitly. It should also be emphasized that the derivative-less $NNN\Delta$ contact interaction in $\mathcal{L}^{(0)}$ vanishes due to the Pauli principle [12]. For more details on the notation, the reader is referred to [9, 10], see also [13] for a recent review article.

3 Determination of the LECs

The values of the LECs c_i are different in the delta-less and delta-full theories and can be naturally extracted from πN scattering. At subleading order, which is sufficient for our purpose, the determination of c_i from the πN S- and P-wave threshold coefficients yields in the delta-less theory [14]

$$c_1 = -0.57, \quad c_2 = 2.84, \quad c_3 = -3.87, \quad c_4 = 2.89, \tag{3}$$

where only central values are given and the units are GeV^{-1}. The above values are somewhat smaller in magnitude than the ones obtained at higher orders, see e.g. [15]. Including the contributions form the Δ, one finds

$$c_1 = -0.57, \quad c_2 = -0.25, \quad c_3 = -0.79, \quad c_4 = 1.33, \quad b_3 + b_8 = 1.40. \tag{4}$$

Notice that the LECs $c_{2,3,4}$ are strongly reduced in magnitude when the Δ-isobar is included. It should also be emphasized that the values of these LECs depend sensitively on h_A, which in the above case was set to $h_A = 3g_A/(2\sqrt{2})$ from SU(4) (or large N_c). The results for the threshold coefficients and the TPE potential are, however, rather stable [14]. Notice further that the description of the P-wave threshold parameters improves significantly upon inclusion of the delta-isobar.

4 Two-nucleon force

The structure of the 2NF in the delta-full and delta-less theories up to N^{2}LO is depicted in Fig. 1. I will first briefly overview the various contributions in EFT without explicit Δ.

- The lowest-order 2NF is due to the OPE potential and the leading contact interactions[3]:

$$V_{2N}^{(0)} = -\frac{g_A^2}{(2F_\pi)^2}\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} + C_S + C_T\,\vec{\sigma}_1 \cdot \vec{\sigma}_2\,, \tag{5}$$

  where $\vec{q} \equiv \vec{p}\,' - \vec{p}$ is the nucleon momentum transfer with $\vec{p}\,(\vec{p}\,')$ being the initial (final) nucleon momenta in the center-of-mass system and $\vec{\sigma}_i$ ($\boldsymbol{\tau}_i$) denote the Pauli spin (isospin) matrices of the nucleon (i). Here and in what follows, the expressions for the potential are to be understood as operators in spin and isospin spaces and matrix elements with respect to momenta.

- The first corrections arise at order $\nu = 2$ from TPE and subleading

[3] Alternative counting schemes for contact interactions are currently being explored.

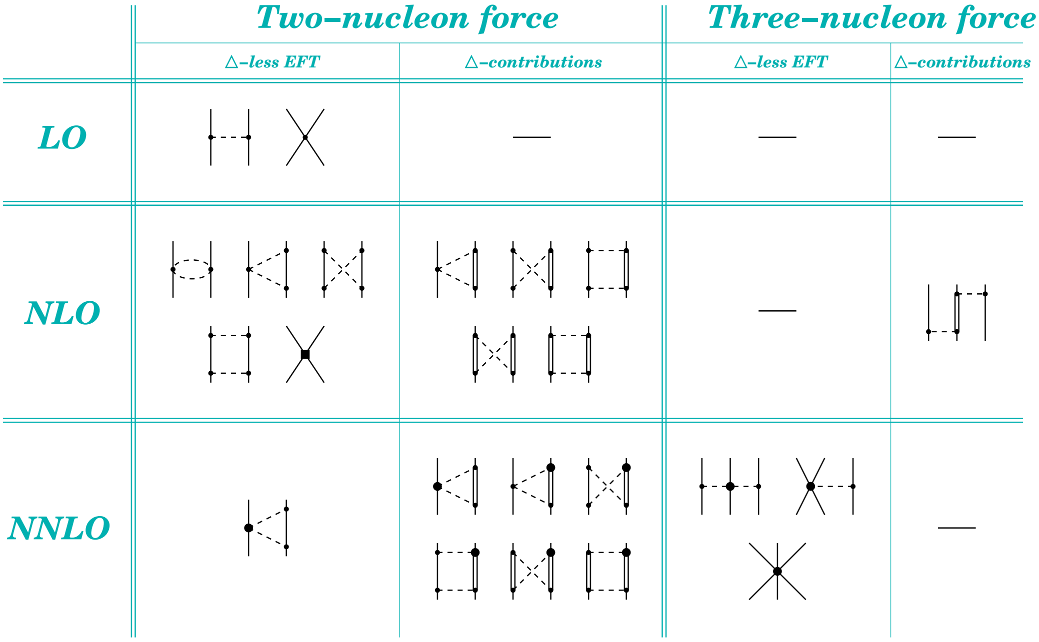

Figure 1: Two- and three-nucleon forces in the delta-less and delta-full EFT. Dashed, solid and double-solid lines represent pions, nucleons and delta isobars, respectively. Solid dots, filled circles and squares denote vertices with $\Delta_i = 0$, 1 and 2. Only one representative topology is depicted in each case.

contact terms

$$
\begin{aligned}
V_{2N}^{(2)} &= -\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{384\pi^2 F_\pi^4} L^\Lambda(q) \bigg[4M_\pi^2 (5g_A^4 - 4g_A^2 - 1) + q^2 (23 g_A^4 - 10 g_A^2 - 1) \\
&\quad + \frac{48 g_A^4 M_\pi^4}{4M_\pi^2 + q^2} \bigg] - \frac{3 g_A^4}{64\pi^2 F_\pi^4} \, L^\Lambda(q) \left[\vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, q^2 \right] \\
&\quad + C_1 \, (\vec{p} - \vec{p}\,')^2 + \frac{C_2}{4} \, (\vec{p} + \vec{p}\,')^2 + \ldots \,. \qquad (6)
\end{aligned}
$$

Here, the ellipses refer to further contact terms, C_i denote the LECs and the loop function $L^\Lambda(q)$ has the form

$$
L^{\tilde{\Lambda}}(q) = \frac{\omega}{2q} \ln \frac{\tilde{\Lambda}^2 \omega^2 + q^2 s^2 + 2\tilde{\Lambda} q \omega s}{4M_\pi^2 (\tilde{\Lambda}^2 + q^2)}, \quad \omega^2 = q^2 + 4M_\pi^2, \quad s^2 = \tilde{\Lambda}^2 - 4M_\pi^2,
$$

where $\tilde{\Lambda}$ is the cutoff in the spectral-function representation. Further contributions at this order resulting from renormalization of the OPE potential and the leading contact interactions only lead to shifts in the corresponding LECs.

- At N^{2}LO, one has to take into account the subleading TPE potential

$$
\begin{aligned}
V_{2N}^{(3)} &= -\frac{3g_A^2}{16\pi F_\pi^4}(2M_\pi^2(2c_1 - c_3) - c_3 q^2)(2M_\pi^2 + q^2)A^{\tilde{\Lambda}}(q) \\
&- \frac{g_A^2 c_4}{32\pi F_\pi^4}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q} - q^2\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\right] (4M_\pi^2 + q^2)A^{\tilde{\Lambda}}(q)\,,
\end{aligned}
\tag{7}
$$

with the loop function $A^{\tilde{\Lambda}}(q)$ given by

$$
A^{\tilde{\Lambda}}(q) = \frac{1}{2q}\, \arctan \frac{q(\tilde{\Lambda} - 2M_\pi)}{q^2 + 2\tilde{\Lambda} M_\pi}\,.
$$

Similarly to the previously considered case, OPE diagrams at this order only generate shifts in the corresponding LECs.

The inclusion of the Δ-isobar as an explicit degree of freedom in the SSE leads to additional contributions to the TPE potential which are listed in Fig. 1. The leading TPE diagrams were first discussed by Ordóñez et al. [16] using old-fashioned time-ordered perturbation theory. These contributions were then calculated by Kaiser et al. [17] using the Feynman graph technique. The corrections at N^{2}LO have also been worked out recently [14]. I do not list here all expressions for the Δ-contributions but show the results for the isovector tensor TPE potential W_T, defined according to $V_{2N} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, \vec{\sigma}_1 \cdot \vec{q}\, \sigma_2 \cdot \vec{q}\, W_T$, which may be regarded as a representative example:

$$
\begin{aligned}
W_T^{(2)} &= -\frac{h_A^2}{1296\pi^2 F_\pi^4 \Delta}\left[9\pi g_A^2 \omega^2\, A^{\tilde{\Lambda}}(q) + h_A^2\big(2L^{\tilde{\Lambda}}(q) + (4\Delta^2 + \omega^2)D^{\tilde{\Lambda}}(q)\big)\right], \\
W_T^{(3)} &= -\frac{h_A^2 \Delta}{648\pi^2 F_\pi^4}\Big[\big(2(b_3 + b_8)g_A(\omega^2 - 12\Delta^2) - 9c_4(\omega^2 - 4\Delta^2)\big)D^{\tilde{\Lambda}}(q) \\
&\quad + 6\big(3c_4 - 2(b_3 + b_8)h_A\big)L^{\tilde{\Lambda}}(q)\Big]\,.
\end{aligned}
\tag{8}
$$

Here, the new loop function $D^{\tilde{\Lambda}}(q)$ is defined via

$$
D^{\tilde{\Lambda}}(q) = \frac{1}{\Delta}\int_{2M_\pi}^{\tilde{\Lambda}} \frac{d\mu}{\mu^2 + q^2}\, \arctan \frac{\sqrt{\mu^2 - 4M_\pi^2}}{2\Delta}\,.
\tag{9}
$$

The complete results for the Δ-contributions can be found in Refs. [14, 17]. It is instructive to verify the consistency between the delta-full and delta-less theories which requires the contributions due to intermediate Δ-excitations, expanded in powers of $1/\Delta$, to be absorbable into a redefinition of the LECs in the delta-less theory. This implies e.g. that the nonpolynomial (in momenta)

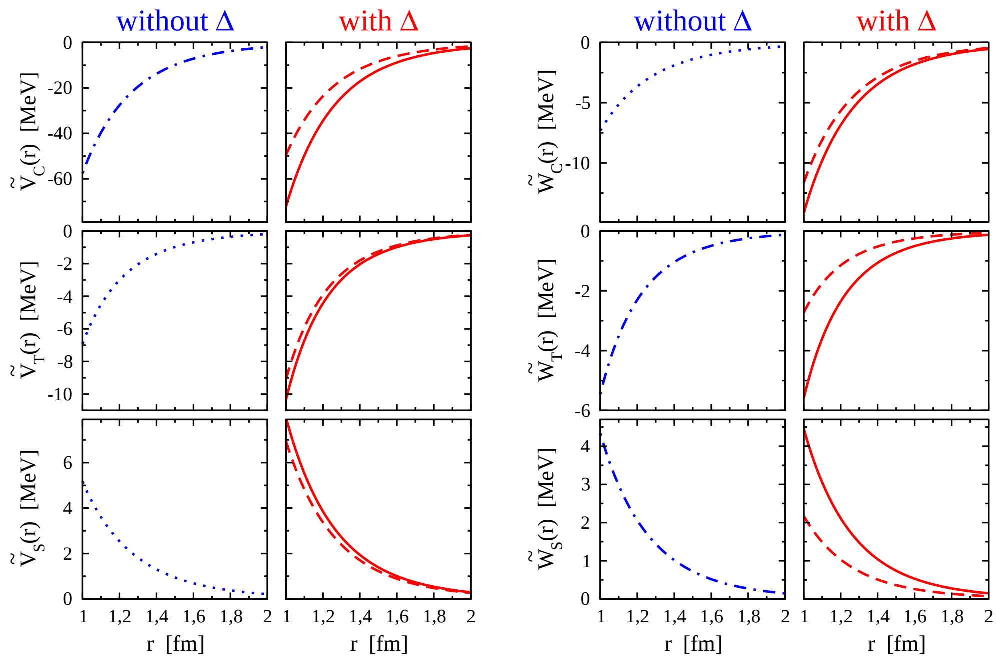

Figure 2: TPE potential in coordinate space for $\tilde{\Lambda} = 700$ MeV. Dashed and solid (dotted and dashed-dotted) lines refer to the NLO and N^{2}LO results in the delta-full (delta-less) theory, respectively. There are no contributions to $\tilde{V}_C$ and $\tilde{W}_{T,S}$ ($\tilde{V}_{T,S}$ and $\tilde{W}_C$) at NLO (N^{2}LO) in the delta-less theory.

contributions up to N^{2}LO resulting from such an expansion must have the same form as expressions in Eqs. (6, 7). This indeed turns out to be the case: all expanded nonpolynomial terms up to N^{2}LO are exactly reproduced by the shift in the LECs $c_{3,4}$, $c_3 = -2c_4 = -4h_A^2/(9\Delta)$ in Eqs. (6, 7).

To get more insight into the strength of various contributions, it is useful to switch to coordinate space. The TPE potential can then be written as

$$\tilde{V}(r) = \tilde{V}_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \tilde{W}_C + \left[\tilde{V}_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \tilde{W}_S \right] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \left[\tilde{V}_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \tilde{W}_T \right] S_{12} \,, \quad (10)$$

where $S_{12} = 3 \vec{\sigma}_1 \cdot \hat{r} \, \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$ is the tensor operator. The scalar functions $\tilde{V}_i(r)$ and $\tilde{W}_i(r)$ are plotted in Fig. 2 using the values for the LECs specified in section 3. As expected, one observes a much more natural convergence pattern in the theory with explicit delta with the N^{2}LO contributions yielding typically only modest corrections to the NLO result. This is, clearly, not the case in the delta-less theory where the entire contributions to $\tilde{V}_C$ and $\tilde{W}_{T,S}$ are generated at N^{2}LO. On the other hand, the N^{2}LO TPE potential in the delta-less theory provides a surprisingly good approximation to the

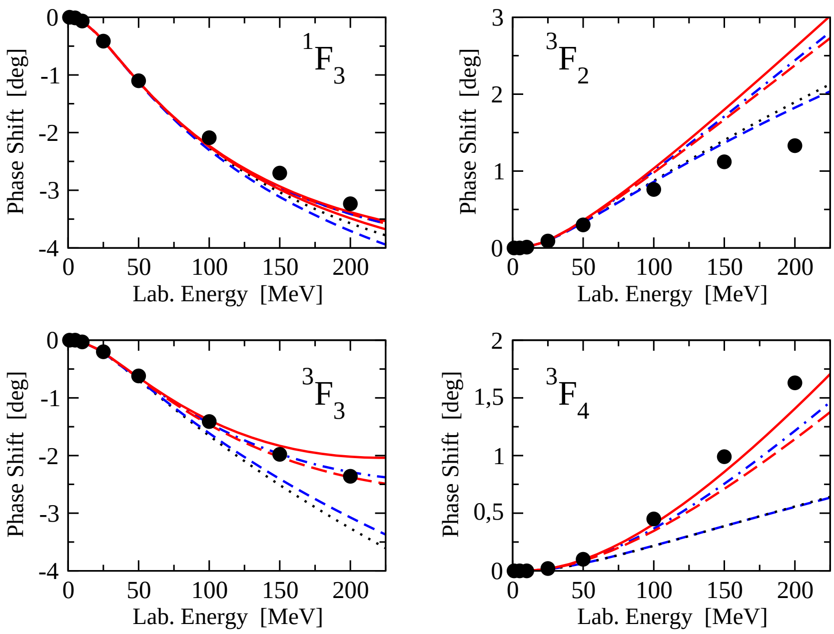

Figure 3: F-wave NN phase shifts for $\tilde{\Lambda} = 700$ MeV. The dotted curve is the LO prediction, long-dashed (short-dashed) and solid (dashed-dotted) lines show the NLO and N^{2}LO results with (without) the explicit Δ-contributions. The filled circles depict the results from the Nijmegen PWA [18].

potential resulting at the same order in the delta-full theory. This indicates that the saturation of the LECs $c_{3,4}$ is the most important effect of the Δ-isobar at the considered order. The results for NN F- and other peripheral waves calculated using the Born approximation also clearly demonstrate the improved convergence in the theory with explicit Δ, see Fig. 3.

5 Three-nucleon force

Chiral power counting in Eq. (1) implies that the 3NF first appears at NLO, $\nu = 2$. It is, however, well known that the corresponding terms in the delta-less theory vanish [1, 2], so that the 3NF only starts to contribute at N^{2}LO, see Fig. 1. The situation is different in the delta-full theory where the leading contribution of the TPE type shows up already at NLO giving rise to the so-called Fujita-Miyazawa 3NF [19]. Notice that the expression for the Δ-contribution to the TPE 3NF is exactly reproduced at N^{2}LO in the delta-less theory via resonance saturation of the LECs $c_{3,4}$. There are no short-range 3NFs with intermediate Δ-excitation since the corresponding

$NNN\Delta$ interaction is Pauli forbidden, see section 2. Interestingly, there are also no Δ-contributions at N^{2}LO. The TPE diagrams with one insertion of the subleading $\pi N\Delta$ vertex $\propto b_3 + b_8$ lead to $1/m_N$-suppressed contributions due to the time derivative which enters this vertex. Despite the fact that both delta-full and delta-less theories produce the same expressions for the TPE 3NF at N^{2}LO, one should keep in mind that their strength might be different[4]. We found, however, that this difference is rather small ($\sim 7\%$) [12].

6 Summary and outlook

In this talk, I discussed the structure of the nuclear force in EFT with and without delta degrees of freedom. Explicit expressions for two- and three-nucleon forces in the delta-full theory are currently available at the N^{2}LO in the SSE. As expected, EFT with explicit delta shows a more natural convergence pattern for the long-range nuclear forces, which is clearly visible in peripheral NN phase shifts. On the other hand, the obtained results demonstrate that the contributions of the Δ-isobar to the 2NF and 3NF at N^{2}LO are well approximated by the shifts of the LECs $c_{3,4}$ in the delta-less theory. In the future, these studies should be extended to low NN partial waves and few-nucleon observables. In addition, it would be very interesting to go to N^{3}LO. Based on the available results for the three-pion exchange 2NF [6], one would expect significant Δ-contributions to the corresponding three- and four nucleon forces at this order.

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