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DECAYS OF η AND η'

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Abstract

Decays of the η and η' are suited to study symmetries and symmetry violations in QCD, e.g., the chiral anomalies and the axial U(1) anomaly in strong interactions, as well as isospin breaking due to the quark mass difference $m_u - m_d$. We present a theoretical framework which is suited to describe η, η' decays in a consistent and uniform manner. It is based on the combination of the chiral effective Lagrangian with a coupled-channels Bethe-Salpeter equation which satisfies unitarity constraints. Very good agreement with experimental data is achieved.

1 Introduction

Quantum chromodynamics (QCD) is the field theory of strong interactions. In low-energy hadron physics, however, it cannot be directly applied by means of a perturbative series in the strong coupling α_s which is large in this case [1]. In the non-perturbative regime of QCD one must thus resort to alternative model-independent approaches .

A promising approach is provided by lattice simulations which are a numerical solution to QCD, see e.g. [2]. But due to limited computing resources lattice QCD simulations must be performed at finite lattice spacings and volumes and at unphysically large pion masses. The various extrapolations to the physical world remain a challenging task and imply systematic uncertainties.

Chiral perturbation theory (ChPT), the effective field theory of QCD at low energies, constitutes an alternative model-independent framework [3–5]. ChPT is based on the observation that at low energies the relevant, effective degrees of freedom of the strong interactions are hadrons composed of confined quarks and gluons. The QCD Lagrangian is replaced by an effective Lagrangian which is formulated in terms of the effective degrees of freedom. At sufficiently low energies these are the pions, kaons and the eta.

The effective Lagrangian incorporates all relevant symmetries and symmetry-breaking patterns of QCD, in particular, the (approximate) $SU(3)_L\times SU(3)_R$ chiral symmetry which is broken down spontaneously to $SU(3)_V$.

The conventional SU(3) chiral effective Lagrangian is a function of the octet of Goldstone boson fields (π, K, η). Making use of the hypothesis of the partially conserved axial vector current (PCAC) and Goldstone boson pole dominance one concludes that the interaction between the Goldstone bosons vanishes at zero momentum. A low-energy expansion can be formulated wherein the Green's functions are expanded in powers of Goldstone boson masses and small momenta: a *chiral counting scheme* is established.

Although the classical QCD Lagrangian is invariant under the extended group $SU(3)_L\times SU(3)_R\times U(1)_A\times U(1)_V$ quantum effects destroy the $U(1)_A$ symmetry of the classical Lagrangian: a so-called *anomaly* occurs.[1] Under axial U(1) transformations the path integral picks up an additional contribution from the fermion determinant such that the full quantum theory does not exhibit the classical axial U(1) symmetry [6]. The axial U(1) anomaly in QCD prevents the η' from being a Goldstone boson which is phenomenologically manifested in its relatively large mass of 958 MeV—much larger than the masses of the Goldstone boson octet. Hence, the η' is not included explicitly in conventional SU(3) ChPT, albeit its effetcs are hidden in coupling constants [5, 7]. Nonetheless, ChPT can be extended to include the η' as a dynamical degree of freedom [8]. In addition to the chiral expansion one can impose an expansion in $1/N_c$—where N_c is the number of colors—which allows for a rigorous counting scheme also in the presence of the η' and a perturbative loopwise expansion can be carried out [9].

However, large unitarity corrections due to final-state interactions, which are already substantial in $\eta \to 3\pi$, must be accounted for in a non-perturbative fashion [10, 11]. Final state interactions are expected to be even more important in η' decays due to larger phase space and the presence of nearby resonances. This cannot be accomplished within ChPT wherein unitarity is restored only perturbatively in the chiral expansion, but one must rather resort to non-perturbative methods. To this aim, the combination of the chiral effective Lagrangian with a coupled-channels Bethe-Salpeter equation (BSE) has proven useful. In this approach, unitarity corrections are taken into account by deriving the interaction kernel for meson-meson scattering from the effective Lagrangian and iterating it to infinite order by means of the BSE. Such chiral unitary approaches have been applied quite successfully in many investigations, see e.g. [12].

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[1] The $U(1)_V$ symmetry corresponds with baryon number conservation and is usually not presented explicitly.

We have applied this framework to various decays of η and η' [13–18]. For example, to the isospin violating decays $\eta, \eta' \to 3\pi$ which can only occur due to an isospin breaking quark mass difference $m_u - m_d$ or electromagnetic effects [13, 16, 17]. While for most processes isospin violation of the strong interactions is masked by electromagnetic effects, these corrections are expected to be small for the three pion decays of η and η' (Sutherland's theorem) [19]. Neglecting electromagnetic corrections the decay amplitude is directly proportional to $m_u - m_d$.

The chiral unitary method has also been applied to the dominant decay mode of the η', $\eta' \to \eta\pi\pi$, where the contributions from the resonances $a_0(980), f_0(980)$ and the so-called σ can be studied [13, 16, 17].

Moreover, the decays $\eta, \eta' \to \gamma\gamma$ and $\eta, \eta' \to \pi^+\pi^-\gamma$ are phenomenological manifestations of the chiral anomaly of QCD and can provide important information on the violation of chiral symmetry in strong interactions [14, 15]. These investigations have recently been extended to the decays $\eta, \eta' \to \pi^+\pi^- l^+ l^-$ $(l = e, \mu)$ [18].

On the experimental side, there is renewed interest in η, η' decays which are investigated at WASA-at-COSY [20], MAMI [21], KLOE [22], CLEO at CESR [23] and by the VES collaboration [24, 25]. There is thus the necessity to provide a consistent and uniform theoretical description for these decays. We present here a framework which satisfies important theoretical requirements, while at the same time being in very good agreement with experimental data.

2 Outline of the approach

We start with the illustration of the hadronic decays $\eta, \eta' \to 3\pi$ and $\eta' \to \eta\pi\pi$ [13, 16]. The underlying idea of the approach is that the initial particle, i.e. the η or η', decays into three mesons and that two out of these rescatter (elastically or inelastically) an arbitrary number of times, see Fig. 1 for illustration. All occurring vertices are derived from the effective Lagrangian and are thus constrained by chiral symmetry. Interactions of the third meson with the pair of rescattering mesons are neglected which turns out to be a good approximation, particularly for the decays $\eta \to 3\pi$ and $\eta' \to \eta\pi\pi$ [16].

Such an infinite meson-meson rescattering process can be generated by application of the Bethe-Salpeter equation. To this aim, the partial wave interaction kernels for meson meson scattering, A_l, are derived from the chiral effective Lagrangian and iterated in a Bethe-Salpeter equation which generates the propagator for two interacting particles in a covariant fashion. For each partial wave l the matrix-valued solution T_l of the BSE with coupled

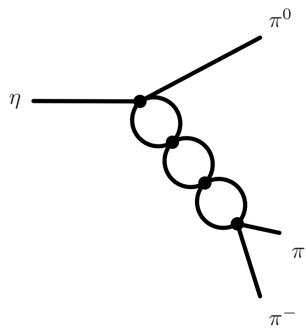

Figure 1: Shown is a possible contribution to final state interactions in the decay $\eta \to \pi^+\pi^-\pi^0$.

channels and on-shell interaction kernels is given by

$$T_l^{-1} = A_l^{-1} + G \ , \tag{1}$$

where the diagonal matrix G collects the scalar loop integrals of the different two-meson channels. The solution of the BSE satisfies exact unitarity for two-particle scattering and generates resonances dynamically by an infinite string of meson-meson rescattering processes, see also [12]. In the investigations discussed here we have restricted ourselves to s- and p-wave interaction kernels.

This approach can be extended to the anomalous decays $\eta, \eta' \to \gamma^{(*)}\gamma^{(*)}$ [14] and $\eta, \eta' \to \pi^+\pi^-\gamma$ [15] in a straightforward manner. In $\eta, \eta' \to \gamma^{(*)}\gamma^{(*)}$, e.g., the incoming pseudoscalar meson P can directly decay via a vertex of either the Wess-Zumino-Witten Lagrangian, which describes the chiral anomaly, or the unnatural-parity Lagrangian into one of the following three channels: two photons, a photon and two pseudoscalar mesons, or four pseudoscalar mesons. Pairs of mesons can then rescatter an arbitrary number of times before they eventually couple to a photon, see Fig. 2 for illustration. In order to implement the non-perturbative summation of loop graphs covered by the BSE in the decay processes under consideration, we treat the BSE T-matrix as an effective vertex for meson-meson scattering. The anomalous decays into $\pi^+\pi^-\gamma$ are treated in the same way [15]. Most recently, the decays $\eta, \eta' \to \pi^+\pi^- l^+ l^-$ ($l = e, \mu$) have been investigated in this approach, where the virtual photon couples to a lepton pair [18]. The results of [18] will soon be tested at KLOE at DAΦNE and WASA-at-COSY.

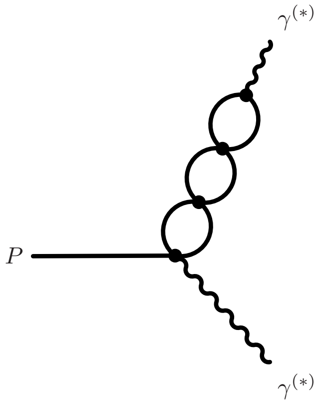

Figure 2: Sample rescattering process in the decay $P \to \gamma^{(*)}\gamma^{(*)}$.

3 Results

We now turn to the discussion of the numerical results, where for brevity we restrict ourselves to the hadronic decays [16], while the anomalous decays are discussed in [14,15]. The results of our calculation for the hadronic decays are obtained from a combined analysis of the decay widths, branching ratios, and slope parameters of the considered decays as well as phase shifts in meson-meson scattering. The widths of $\eta \to 3\pi$ and $\eta' \to \eta\pi\pi$ have been measured roughly at the 10 % precision level, while for $\eta' \to 3\pi^0$ the experimental uncertainty is considerably larger and only an upper limit exists for $\Gamma(\eta' \to \pi^+\pi^-\pi^0)$ [26]. Moreover, some of these decay widths are constrained by the well-measured branching ratios

$$r_1 = \frac{\Gamma(\eta \to 3\pi^0)}{\Gamma(\eta \to \pi^+\pi^-\pi^0)}\,, \qquad r_2 = \frac{\Gamma(\eta' \to 3\pi^0)}{\Gamma(\eta' \to \eta\pi^0\pi^0)}\,. \tag{2}$$

The squared absolute values of the decay amplitudes for $\eta' \to \eta\pi\pi$ and the charged decay modes of $\eta, \eta' \to 3\pi$, $|A(x,y)|^2$, are expanded in terms of the usual Dalitz variables x, y, as (see e.g. [16] for definitions of x and y)

$$|A(x,y)|^2 = |N|^2 \big[1 + ay + by^2 + cx^2 + dy^3 + \cdots\big]\,, \tag{3}$$

while for the decays into three identical particles Bose symmetry dictates the form

$$|A(x,y)|^2 = |N'|^2 \big[1 + g(y^2 + x^2) + \cdots\big]\,. \tag{4}$$

For the Dalitz plot parameters a, b, c of $\eta \to \pi^+\pi^-\pi^0$ the experimental situation is not without controversy. We employ the numbers of [27], since it is the most recent published measurement and the results appear to be consistent with the bulk of the other experiments listed by the Particle Data Group [26]. They differ somewhat from the new preliminary results of the KLOE Collaboration [28] that has found a non-zero value for the third order parameter d which was not included in previous experimental parametrizations. Very recently the Dalitz plot parameters of $\eta' \to \eta\pi^+\pi^-$ have been determined with high statistics by the VES experiment [25]. Note that the slope parameters of $\eta' \to 3\pi$ have not yet been determined experimentally, but such a measurement is intended at WASA-at-COSY [20, 29].

From the unitarized partial-wave T-matrix one may also derive the phase shifts in meson-meson scattering. Hence, our approach is further constrained by the experimental phase shifts for $\pi\pi \to \pi\pi, K\overline{K}$ scattering.

An intriguing feature of the fits is that they accommodate the large negative slope parameter g of the decay $\eta \to 3\pi^0$ measured by the Crystal Ball Collaboration [30] which could not be met by previous theoretical investigations, see e.g. [31]. In particular, it is in sharp disagreement with the most recent two-loop calculation in conventional chiral perturbation theory [32] which provides a positive slope parameter. The experimental value of [30] has now been confirmed by the more recent but yet preliminary g value of the KLOE Collaboration [33].

It is of interest, in this respect, to mention that the revised g parameter of the KLOE collaboration [33] replaces a smaller $|g|$ value of a previous analysis [22] which was in disagreement with Crystal Ball [30]. As as a matter of fact, our approach is not able to accomodate the previous g value together with the KLOE slope parameters for the charged decay $\eta \to \pi^0\pi^+\pi^-$ [22]. In this context, we have illustrated that utilizing the $\Delta I = 1$ selection rule which relates both $\eta \to 3\pi$ decays and taking the KLOE parametrization of the charged decay as input leads in a model-independent way to a g value not consistent with the previous KLOE g result [16]. This inconsistency appears to be settled now.

Let us turn our attention to the isospin-breaking quark mass difference $m_d - m_u$. An accurate way of extracting $m_d - m_u$ is, in principle, given by the isospin-violating decays $\eta, \eta' \to \pi^0\pi^+\pi^-$ and $\eta, \eta' \to 3\pi^0$. For this reason, it has been claimed in [34] that the branching ratio $r = \Gamma(\eta' \to \pi^0\pi^+\pi^-)/\Gamma(\eta' \to \eta\pi^+\pi^-)$ can be utilized in a very simple manner to extract the light quark mass difference $m_d - m_u$. To this aim, it is assumed that

a) the amplitude $A(\eta' \to \pi^0\pi^+\pi^-)$ is determined by the corresponding amplitude $A(\eta' \to \eta\pi^+\pi^-)$ via

$$A(\eta' \to \pi^0\pi^+\pi^-) = \epsilon\, A(\eta' \to \eta\pi^+\pi^-) \tag{5}$$

with $\epsilon = (\sqrt{3}/4)\,(m_d - m_u)/(m_s - \hat{m})$ the π^0-η mixing angle and $\hat{m} = (m_d + m_u)/2$. (Note that in [34] the difference $m_s - \hat{m}$ has been approximated by m_s in the denominator of ϵ.) Eq. (5) implies that the decay $\eta' \to \pi^0\pi^+\pi^-$ proceeds entirely via $\eta' \to \eta\pi^+\pi^-$ followed by π^0-η mixing.

b) both amplitudes are *"essentially constant"* over phase space (see the remark in front of Eq. (19) of Ref. [34]).

Based on these two assumptions one arrives at the relation

$$r = \frac{\Gamma(\eta' \to \pi^0\pi^+\pi^-)}{\Gamma(\eta' \to \eta\pi^+\pi^-)} \simeq (16.8)\frac{3}{16}\left(\frac{m_d - m_u}{m_s}\right)^2, \tag{6}$$

where the factor 16.8 represents the phase space ratio. Comparison with experimental data—for which, so far, only an upper limit exists—would then lead to a prediction for the quark mass ratio $(m_d - m_u)/(m_s - \hat{m}) \simeq (m_d - m_u)/m_s$.

Utilizing the developed chiral unitary approach, we are in a position to critically examine these two assumptions which lead to the simple relation in Eq. (6). Our results clearly indicate that the two underlying assumptions *a*) and *b*) of [34] in order to arrive at a relation between r and $(m_d - m_u)/(m_s - \hat{m})$ are not justified at all [17]. The results from the full chiral unitary approach are in plain disagreement with these two assumptions. This shows that sophisticated theoretical approaches, which take into account final-state interactions appropriately, are in general required to extract this quark mass ratio from η' decays.

From these discussions it becomes evident that in general more precise experimental data on η and η' decays are needed. An improvement of the experimental situation is foreseen in the near future due to the upcoming data from WASA-at-COSY [20], MAMI-C [21] and KLOE at DAΦNE [22].

4 Conclusions

To summarize, we have explained why η and η' decays provide a perfect opportunity to study symmetries and symmetry-breaking patterns of QCD. These are in more detail, isospin breaking due to different light quark masses $m_u \neq m_d$, explicit chiral symmetry breaking due to finite quark masses, the chiral anomalies as well as the axial U(1) anomaly which is closely related to the η'.

The simultaneous treatment of different η and η' decays poses tighter constraints on theoretical approaches. The recent VES data on the decay $\eta' \to \eta\pi^+\pi^-$, e.g., implies important consequences on the decay $\eta' \to \pi^0\pi^+\pi^-$

within our approach. In general, the importance of resonances in these decays can be studied unambiguously as they are generated dynamically via the Bethe-Salpeter equation and are not inserted by hand.

Moreover, the study of the η-η' system may shed some light into the gluonic degrees of freedom in low-energy hadronic physics due to the close link to the axial U(1) anomaly. Finally, very rare η and η' decays can provide a window to physics beyond the Standard Model. The search for unexpectedly large branching ratios into those channels which cannot be explained within the Standard Model is another main goal of experimental efforts at the various facilities.

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