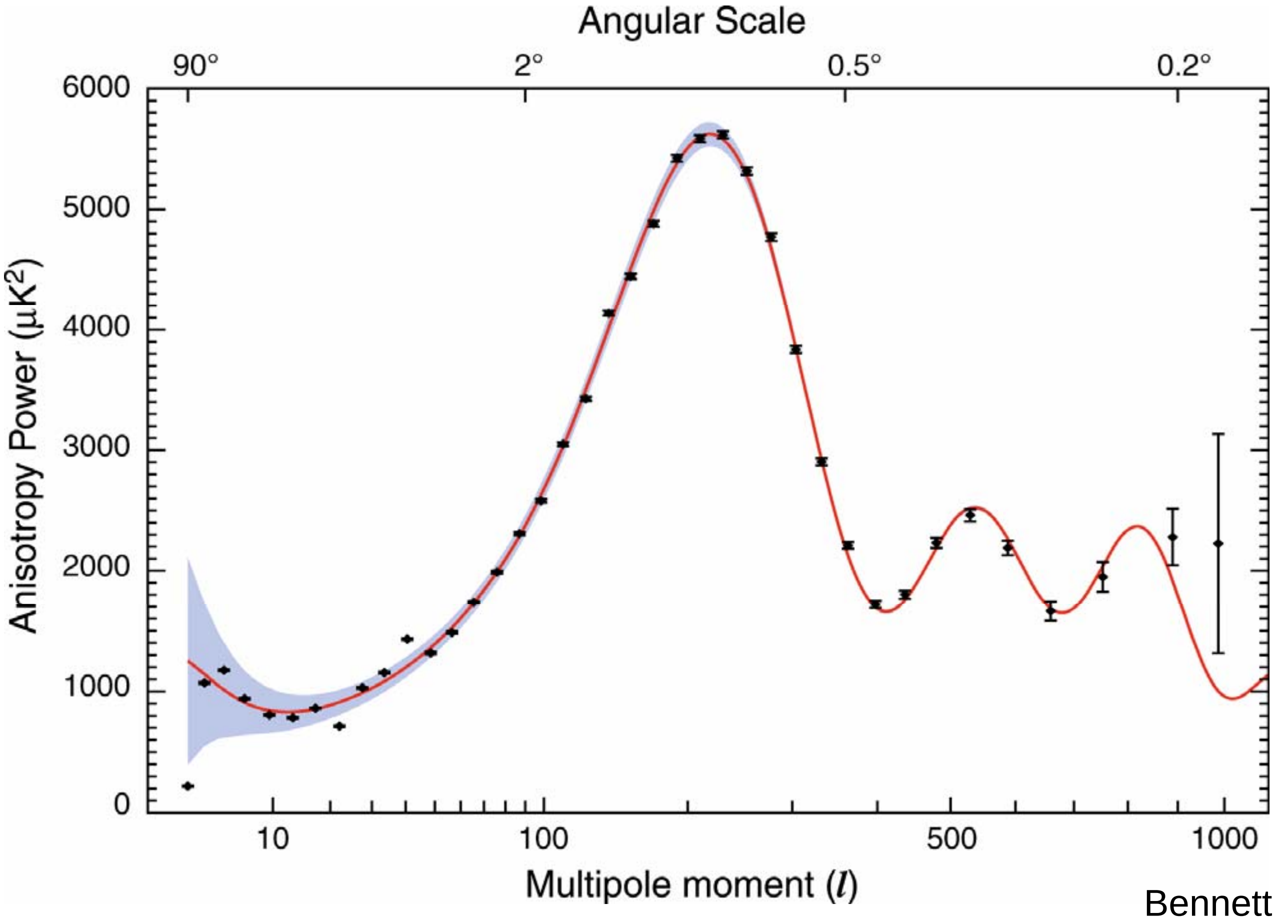
Cosmology for Particle Physicists: Anisotropies in the Cosmic Microwave Background

- Acoustic Oscillations
- Damping
- Inhomogeneities to Anisotropies
- Ramifications

Goal: Explain the Physics and Ramifications of this Plot



Bennett et al. 2006

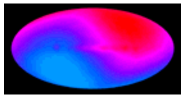
Photon Distribution

- Distribution Θ depends on position **x** (or wavenumber **k**), direction **n** and time t (or η).
- Moments $$\Theta_{lm}(\vec{x},\eta) = \int d^2n Y^*_{lm}(\hat{n})\Theta(\vec{x},\hat{n},\eta)$$

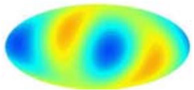
$$\langle\Theta_{lm}\rangle = 0 \quad ; \quad \langle\Theta_{lm}\Theta_{l'm'}\rangle = \delta_{mm'}\delta_{ll'}C_l$$

Monopole: Θ_0

Dipole: Θ_1

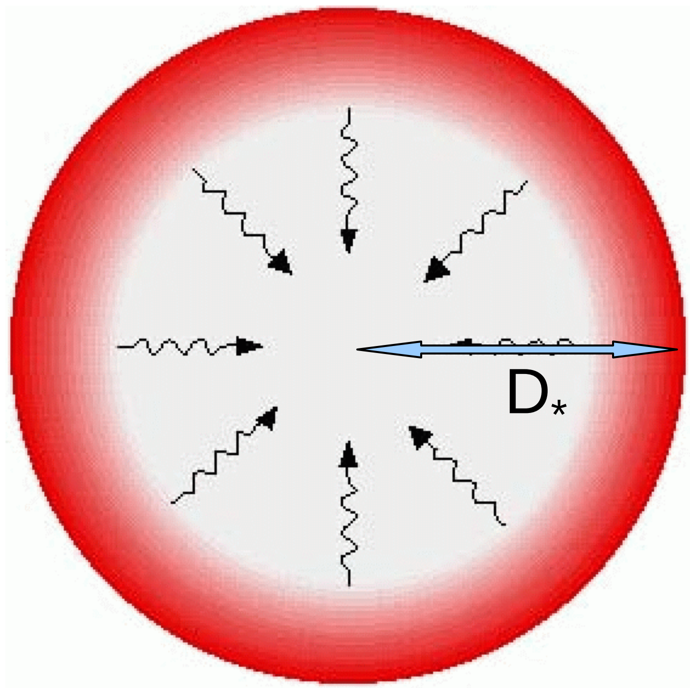


Quadrupole: Θ_2



- You might think we care only about Θ at our position because we can't measure it anywhere else, but ...

We see photons today from last scattering surface at z=1100



Ψ accounts for redshifting out of potential well

D_* is distance to last scattering surface

$$\Theta(\vec{x}=0,\hat{n},\eta_0) \simeq [\Theta_0 + \Psi](D_*\hat{n},\eta_*)$$

Pre-recomb Oscillations

- Pressure of ra clumping
- If a region get e acts to reduce the density: **re**



Before recombination, electrons and photons are tightly coupled; equations reduce to

Temperature perturbation

$$\frac{\partial^2 \Theta_0}{\partial \eta^2} - c_s{}^2 \nabla^2 \Theta_0 = F$$

Very similar to …

Displacement of a string

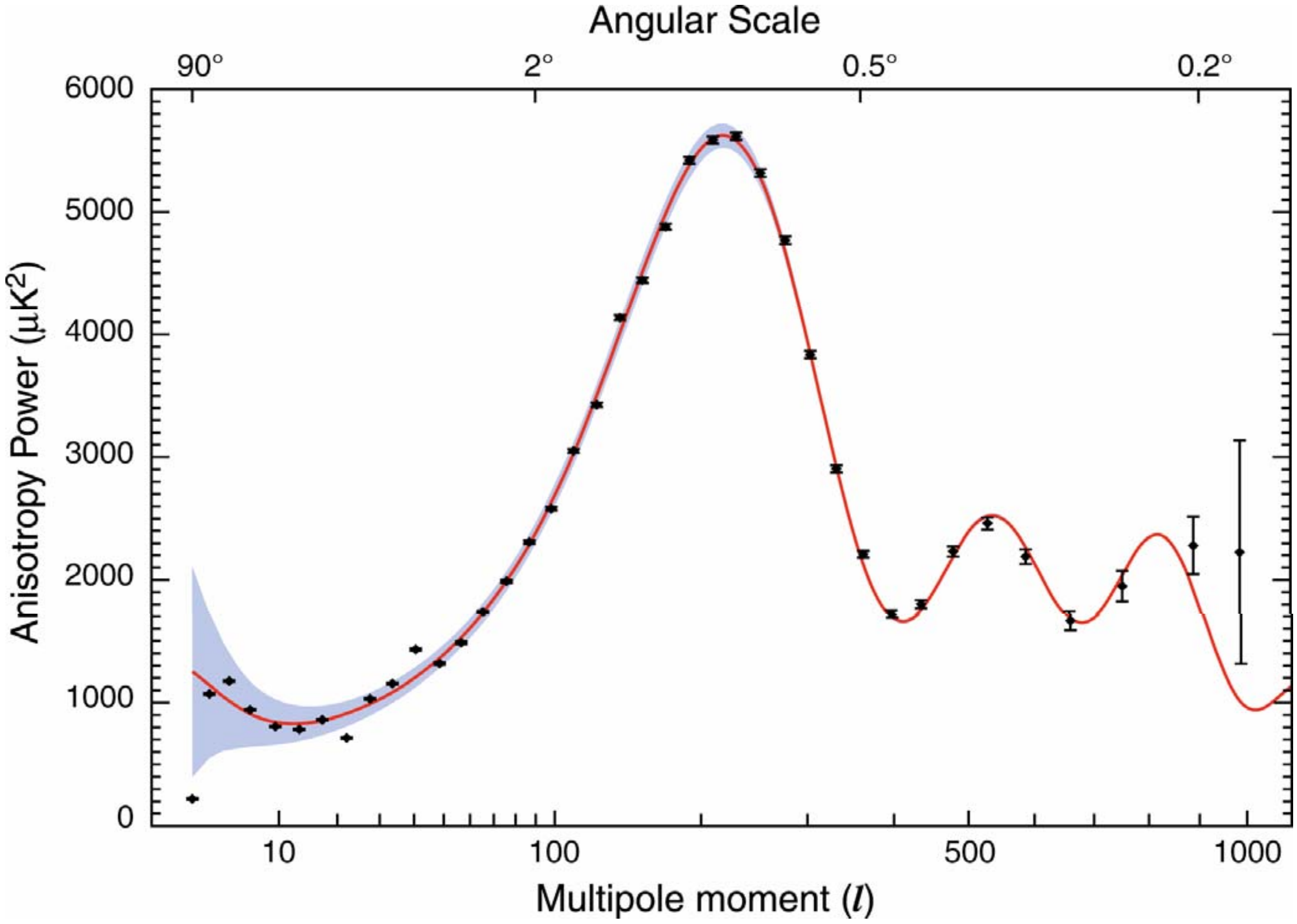
$$\frac{\partial^2 y}{\partial t^2} - c_s^2 \frac{\partial^2 y}{\partial x^2} = F$$

What spectrum is produced by a stringed instrument?

Middle C on a ukelele

Audacity.exe

Compare the piano spectrum to CMB spectrum



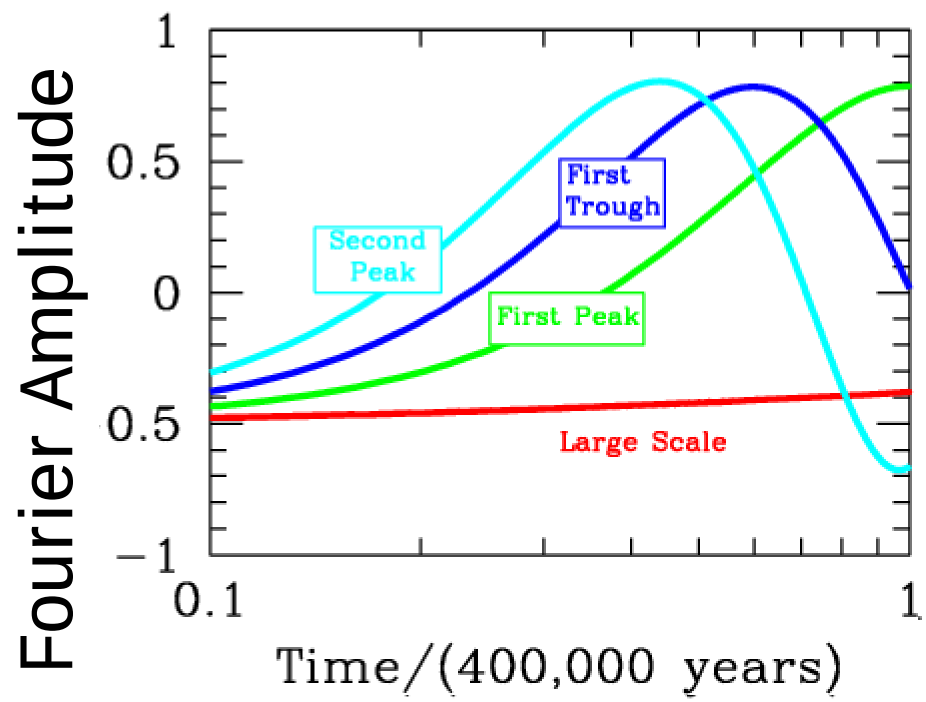
Bennett et al. 2006

CMB is different because ...

- Fourier Transform of spatial, not temporal, signal
- Time scale much longer (400,000 yrs vs. 1/260 sec)
- No finite length: all k allowed; ends not tied down?!

Why peaks and troughs?

- **Vibrating String:** Characteristic frequencies because ends are tied down
- **Temperature in the Universe:** Small scale modes begin oscillating earlier than large scale modes



Acoustic Oscillations

Solutions of the form: $$\Theta_0 \propto \cos(kr_s)$$

With the sound horizon defined as:

$$r_s(\eta) \equiv \int_0^\eta d\eta' \, c_s(\eta')$$

Expect a series of peaks at

$$k_p = \frac{n\pi}{r_s}$$

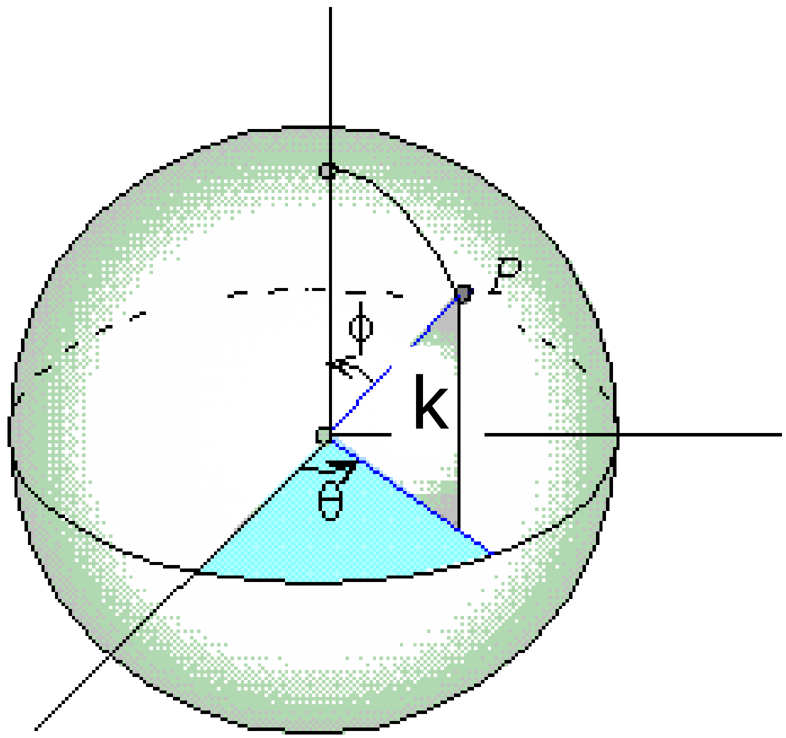
Puzzle: Second Order Differential Eqn has Two Solutions

The perturbation corresponding to each wavevector can be either a **cosine** mode (zero initial velocity) or a **sine** mode (zero initial amplitude)

We implicitly assumed that each $\Theta(\mathbf{k})$ started in the cosine mode.

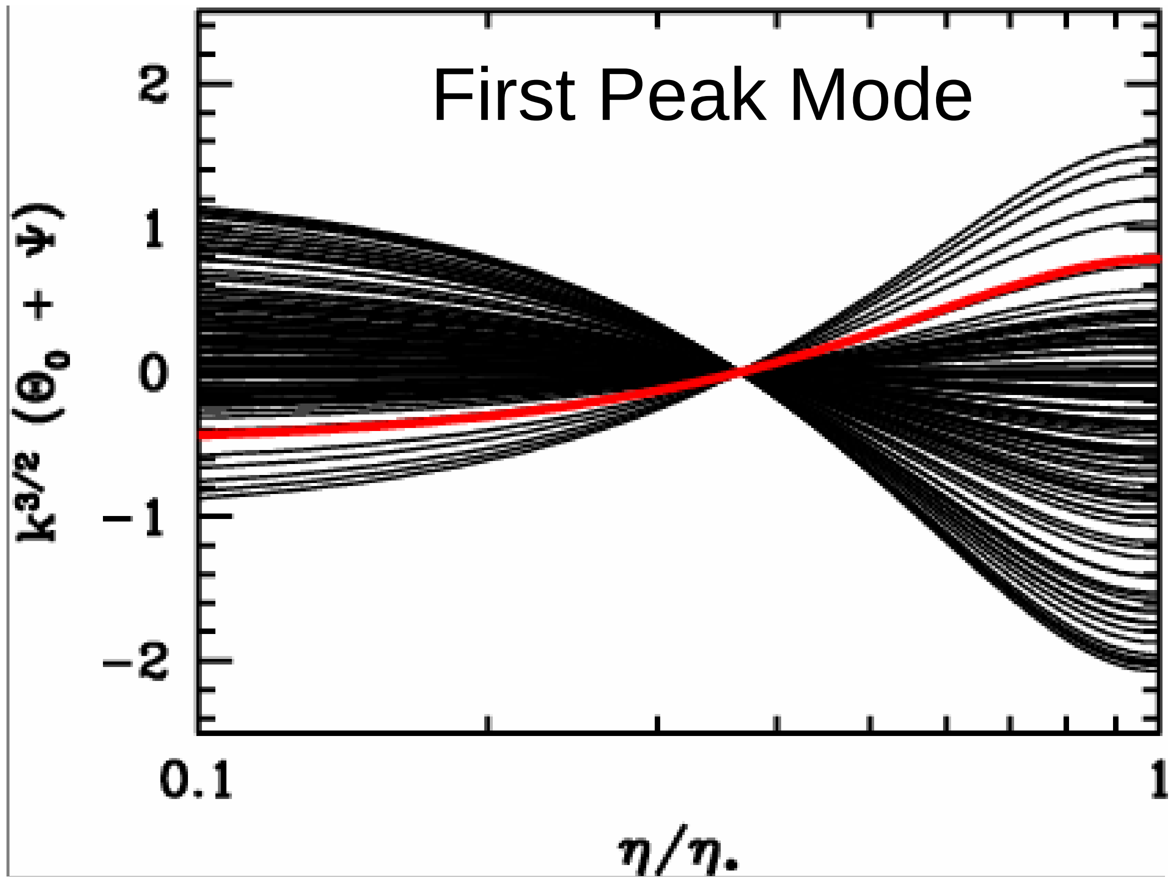
It is worse than this: each wavenumber has an infinite number of wavevectors associated with it

Do all of these Fourier modes have **cosine** initial conditions?



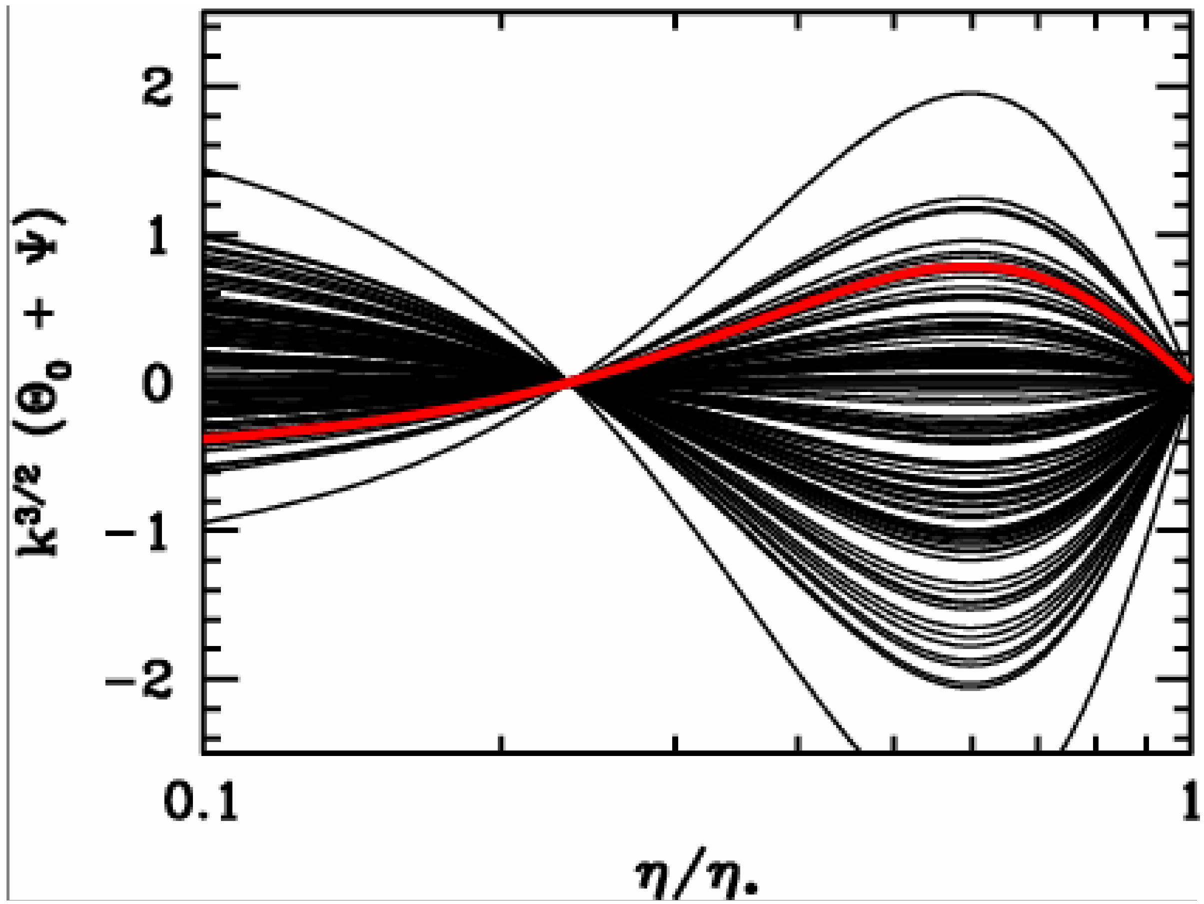
If they do all start out with the same phase …

First peak will be well-defined



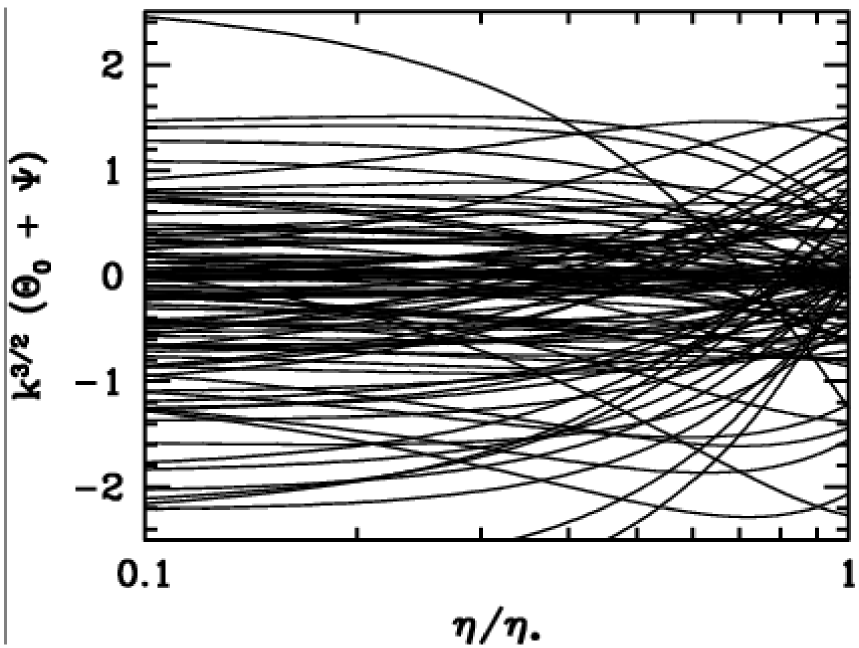
As will first trough ...

And all subsequent peaks and troughs

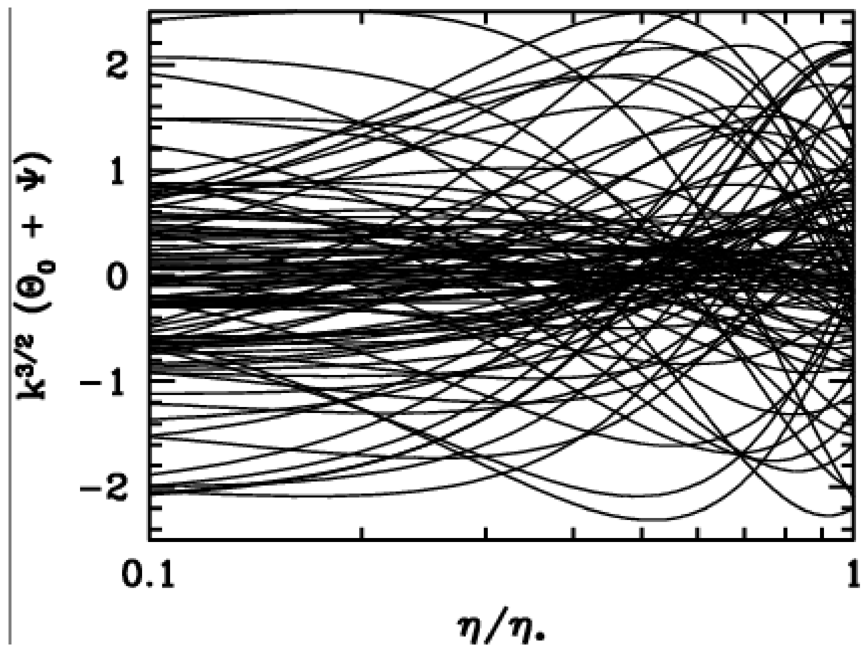


If all modes are **not** synchronized though

First "Peak"

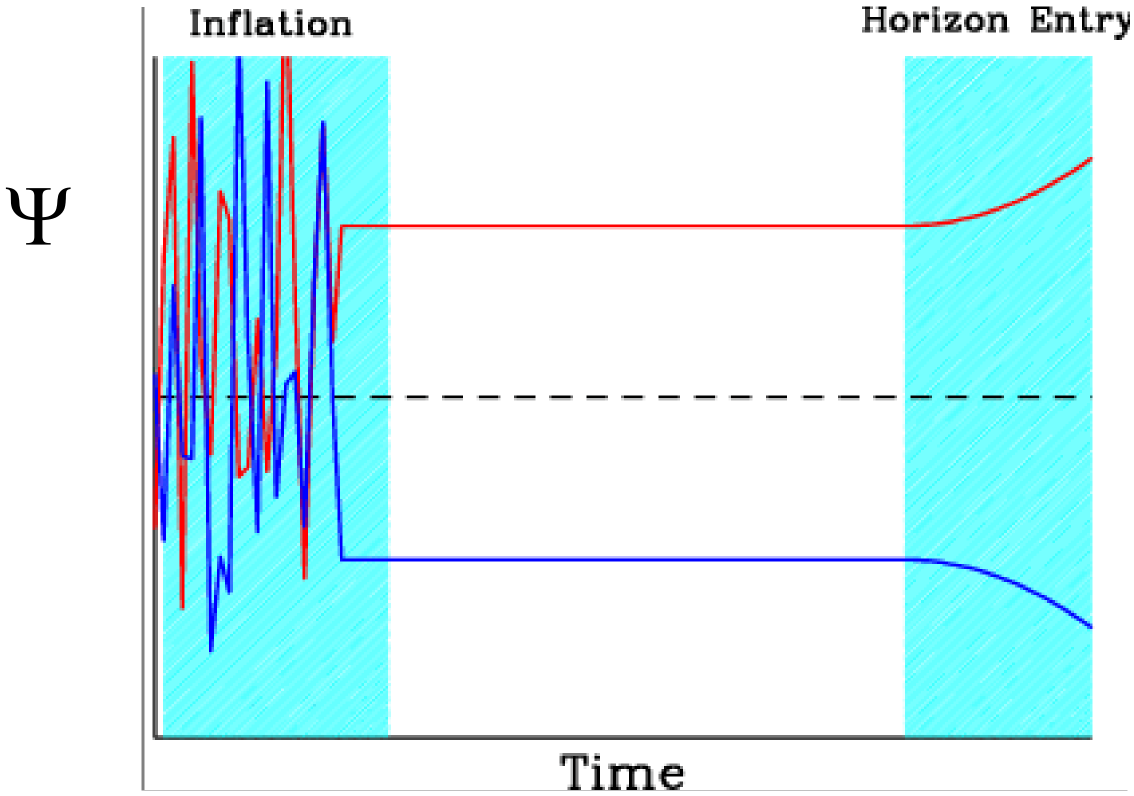


First "Trough"



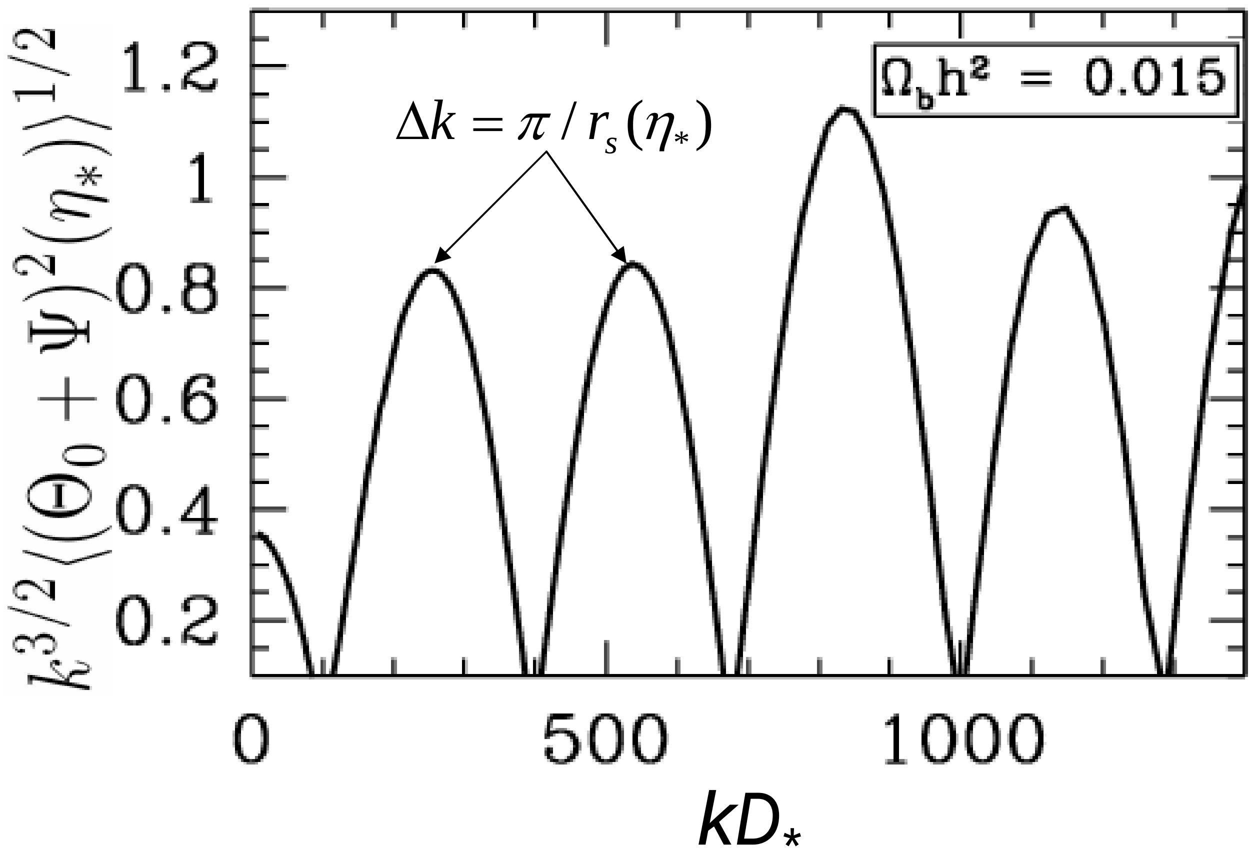
We will NOT get series of peaks and troughs!

Inflation gives a beautiful explanation of synchronization



When modes leave the horizon, they cease to evolve; when they re-enter, only the **cosine** mode remains

The spectrum at last scattering is:



Damping on small scales

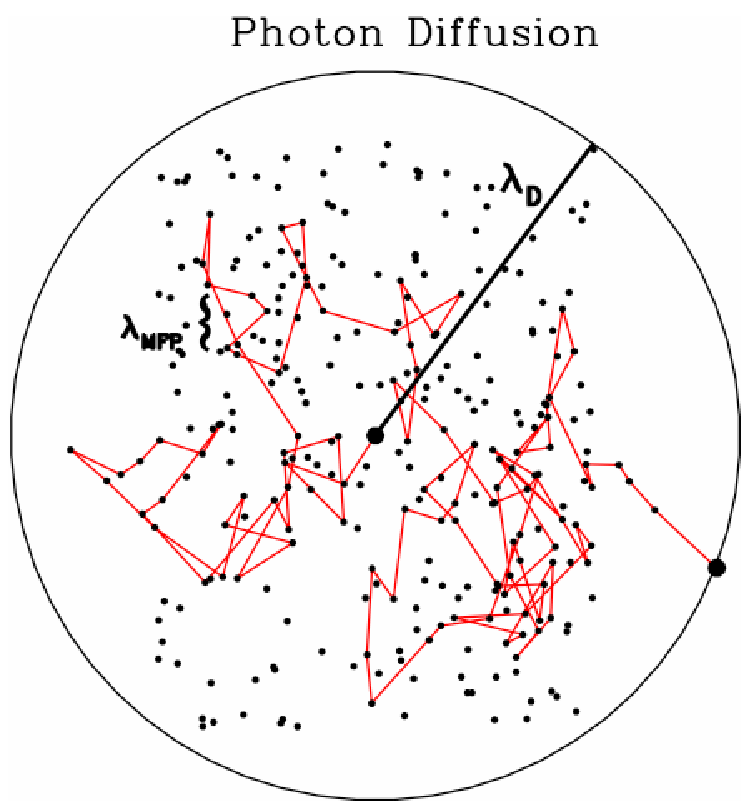
$$\lambda_D \sim \lambda_{\rm MFP} N^{1/2}$$

But

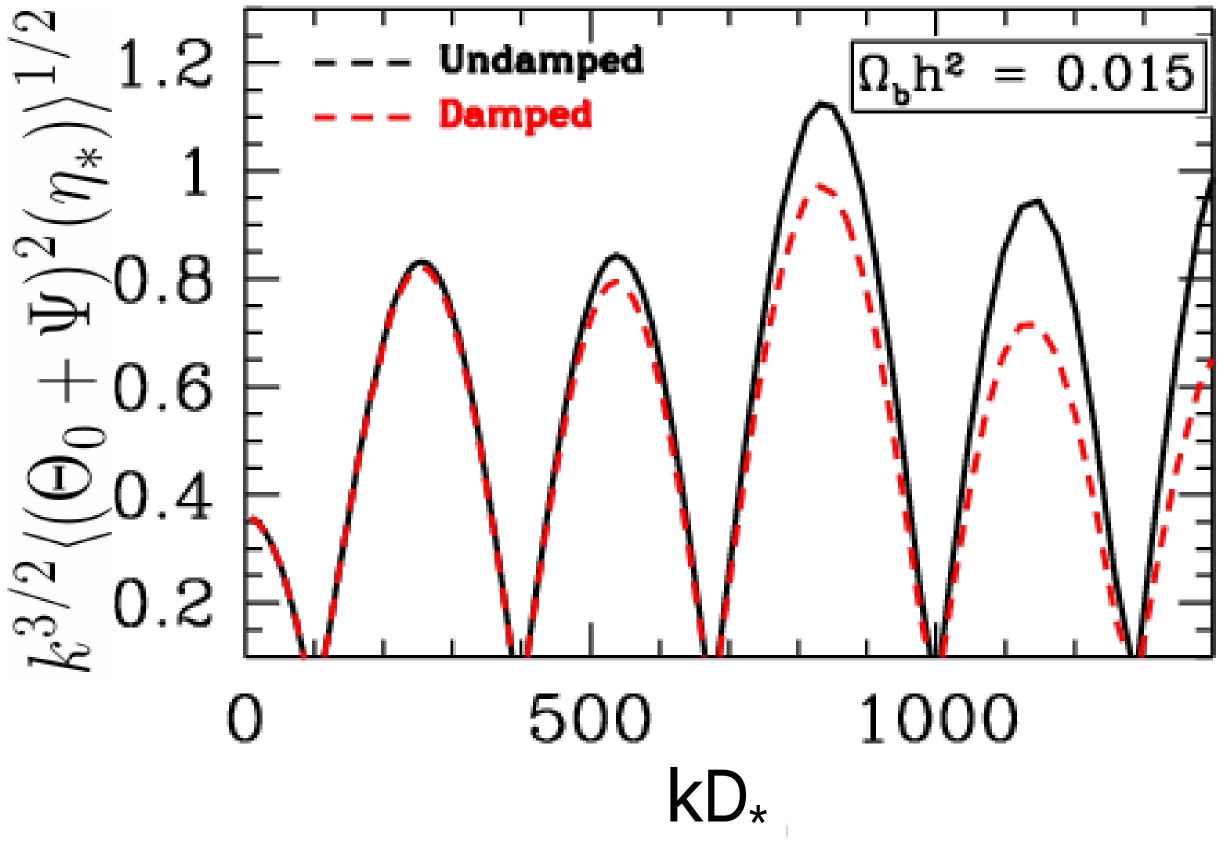
$$N \sim \frac{c}{\lambda_{\rm MFP}} \times H^{-1}$$

So

$$\lambda_D \sim \left(\frac{\lambda_{\rm MFP}}{cH} \right)^{1/2}$$

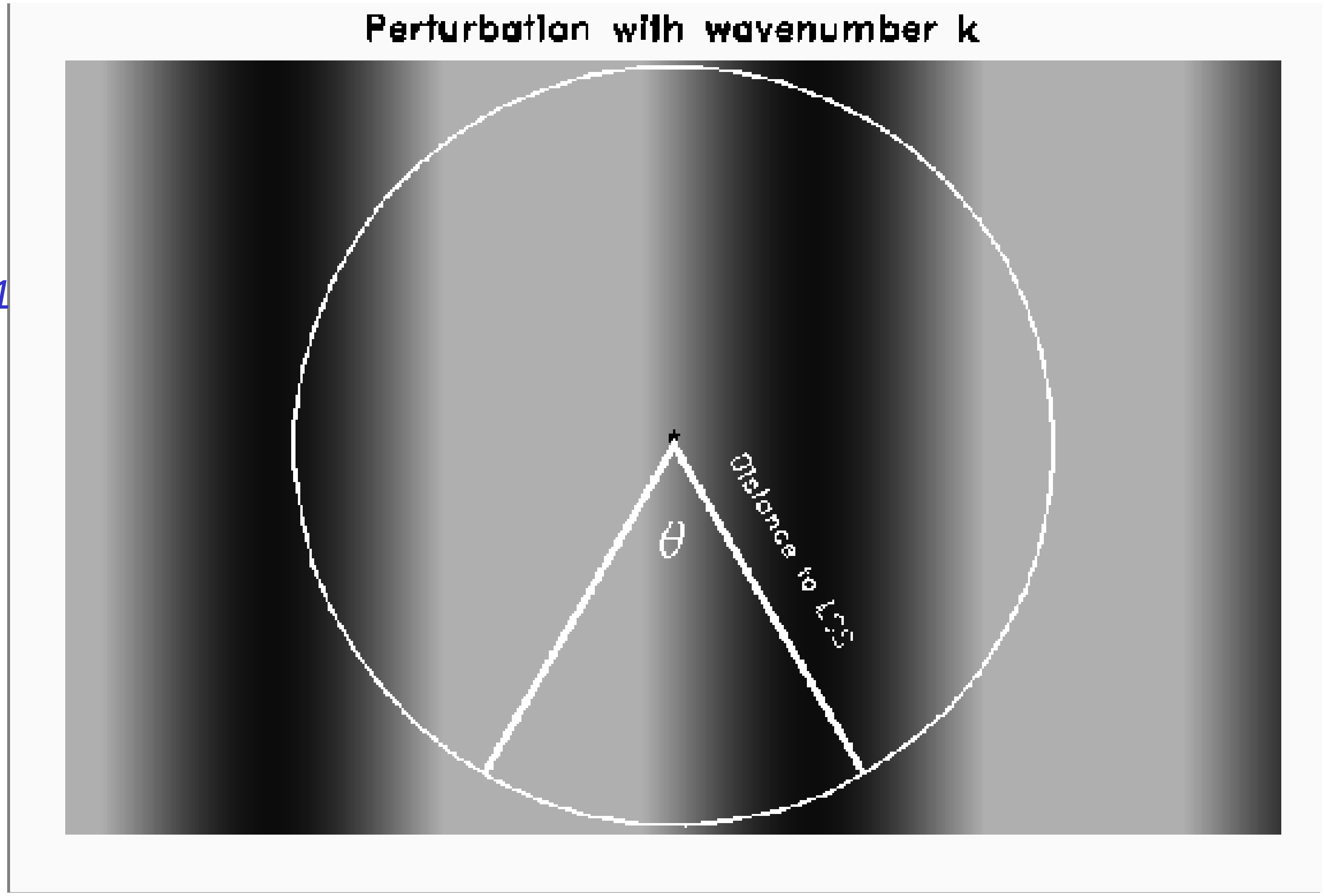


On scales smaller than λ_D (or $k>k_D$) perturbations are damped



How do inhomogeneities at last scattering show up as anisotropies today?

Perturbation w/ wavelength k^{-1} shows up as anisotropy on angular scale $\theta \sim k^{-1}/D_* \sim l^{-1}$



Quantitatively

$$\begin{aligned}\Theta(\hat{n},\eta_0) &\simeq \left[\Theta_0+\Psi\right](D_*\hat{n},\eta_*)\\ &= \int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot D_*\hat{n}}\left[\tilde{\Theta}_0+\tilde{\Psi}\right](\vec{k},\eta_*)\end{aligned}$$

Use identity

$$e^{i\vec{k}\cdot\vec{x}} = 4\pi\sum_{lm} i^l j_l(kx) Y_{lm}(\hat{n}) Y^*_{lm}(\hat{k})$$

leading to

$$\Theta(\hat{n},\eta_0) = 4\pi\sum_{lm} Y_{lm}(\hat{n})\left\{(-i)^l\int\frac{d^3k}{(2\pi)^3}j_l(kD_*)\left[\tilde{\Theta}_0+\tilde{\Psi}\right](\vec{k},\eta_*)Y^*_{lm}(\hat{k})\right\}$$

So the moments are

$$\Theta_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} j_l(kD_*) \left[\tilde{\Theta}_0 + \tilde{\Psi}\right](\vec{k},\eta_*) Y^*_{lm}(\hat{k})$$

Each Θ_{lm} is drawn from a Gaussian distribution with mean zero and variance

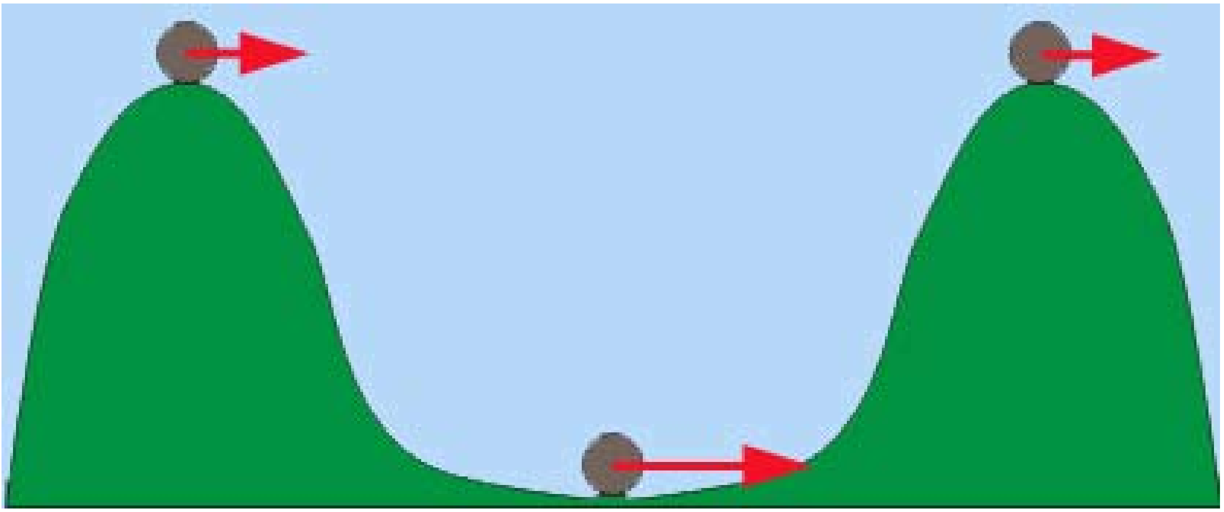
$$16\pi^2 \int \frac{d^3k}{(2\pi)^3} Y^*_{lm}(\hat{k}) \int \frac{d^3k'}{(2\pi)^3} Y_{l'm'}(\hat{k}') j_l(kD_*) j_{l'}(k'D_*) \langle \left[\tilde{\Theta}_0 + \tilde{\Psi}\right](\vec{k}) \left[\tilde{\Theta}^*_0 + \tilde{\Psi}^*\right](\vec{k}') \rangle$$

The expectation value sets **k=k'**; then, by orthogonality of Y_{lm}, the angular integral sets l=l' and m=m' leaving a variance

$$C_l = 4\pi \int_0^\infty \frac{dk}{k} j_l^2(kD_*) \frac{k^3 P_{\Theta_0+\Psi}(k,\eta_*)}{2\pi^2}$$

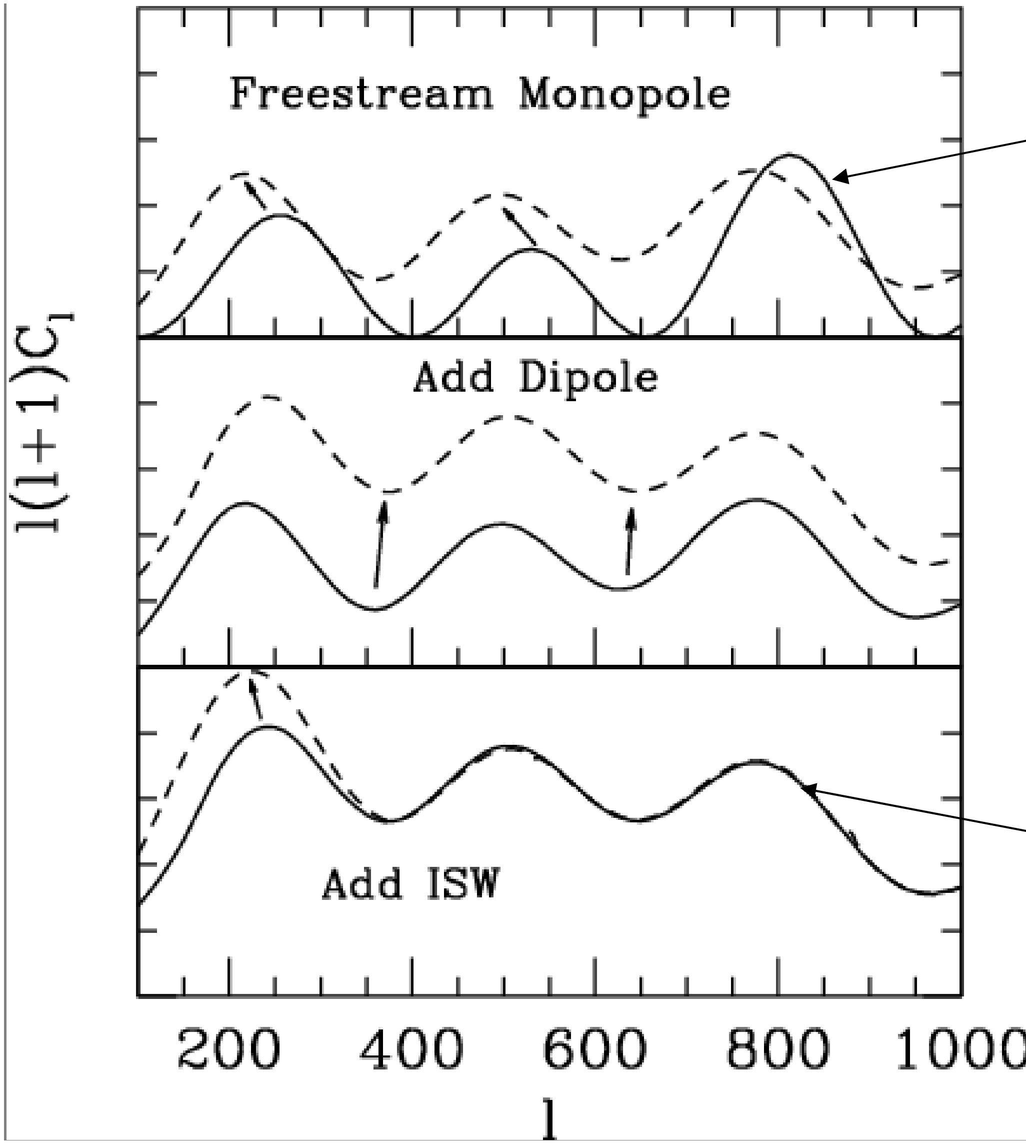
j_l^2 sharply peaked at $kD^* \sim l$ with amplitude l^{-2}

Post Last Scattering: Integrated Sachs-Wolfe Effect



If potential wells decay, photons gain more energy falling in than they lose going out.

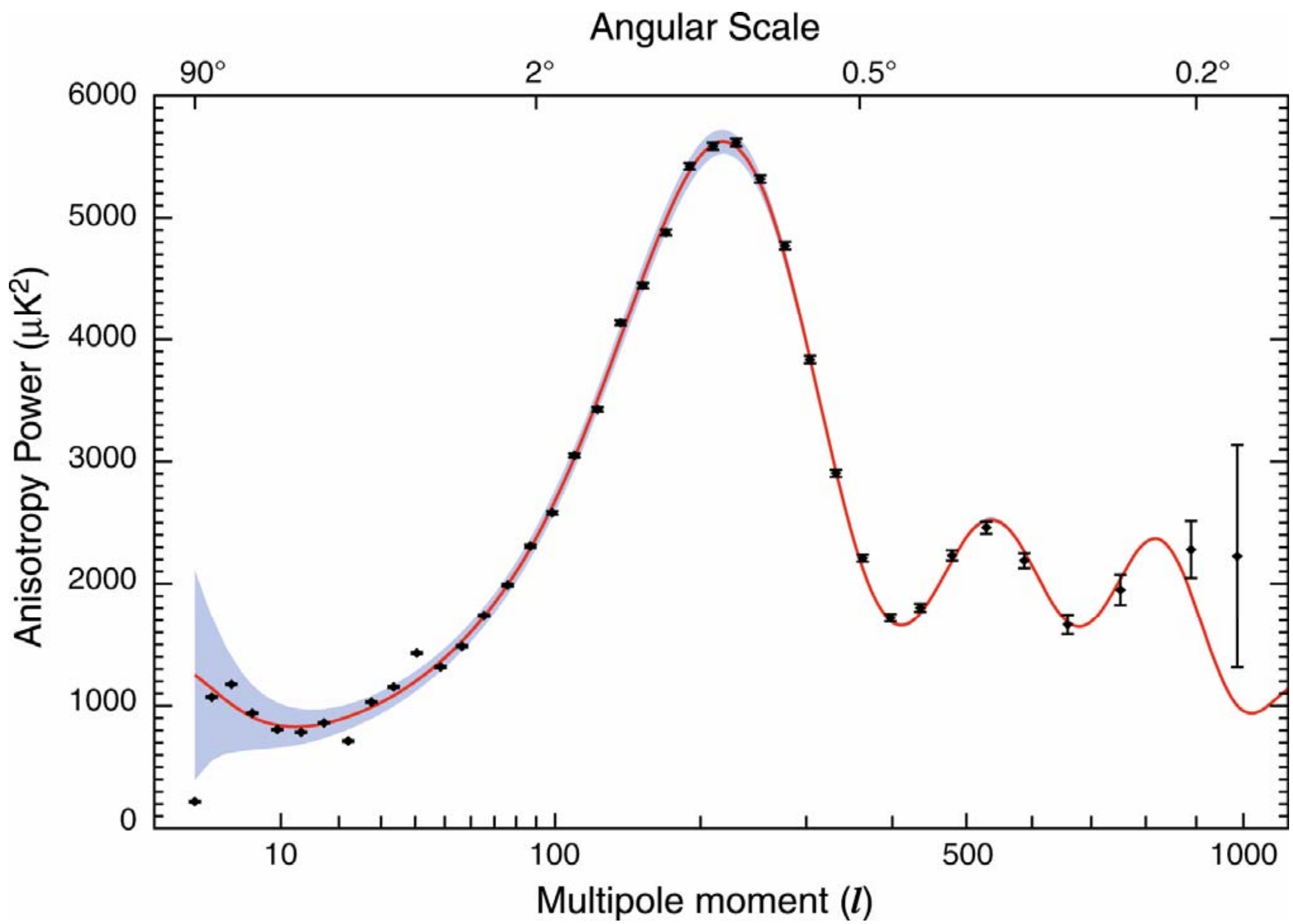
C_l simply related to $[\Theta_0+\Psi]^2(k=l/D^*)$



Fourier transform of temperature at Last Scattering Surface

Anisotropy spectrum today

When we see this, we conclude that modes were synchronized during inflation!



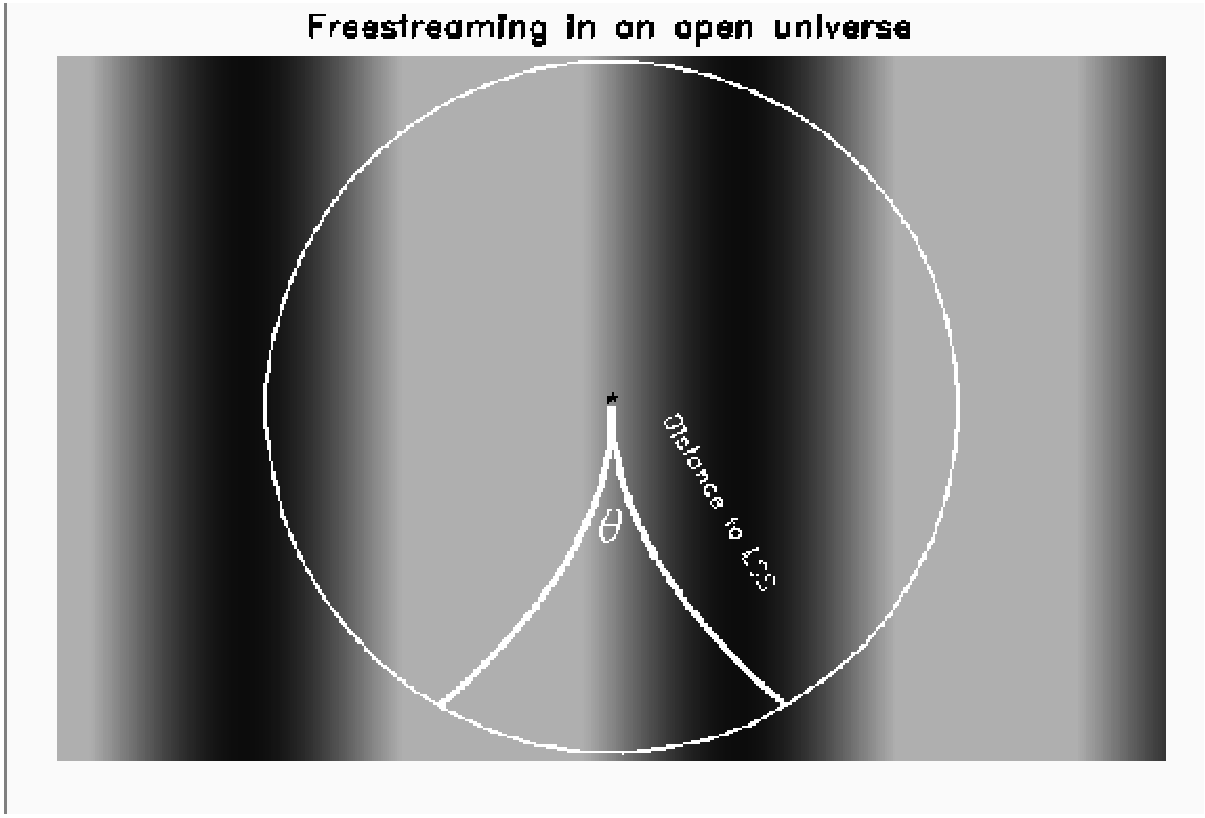
Bennett et al. 2006

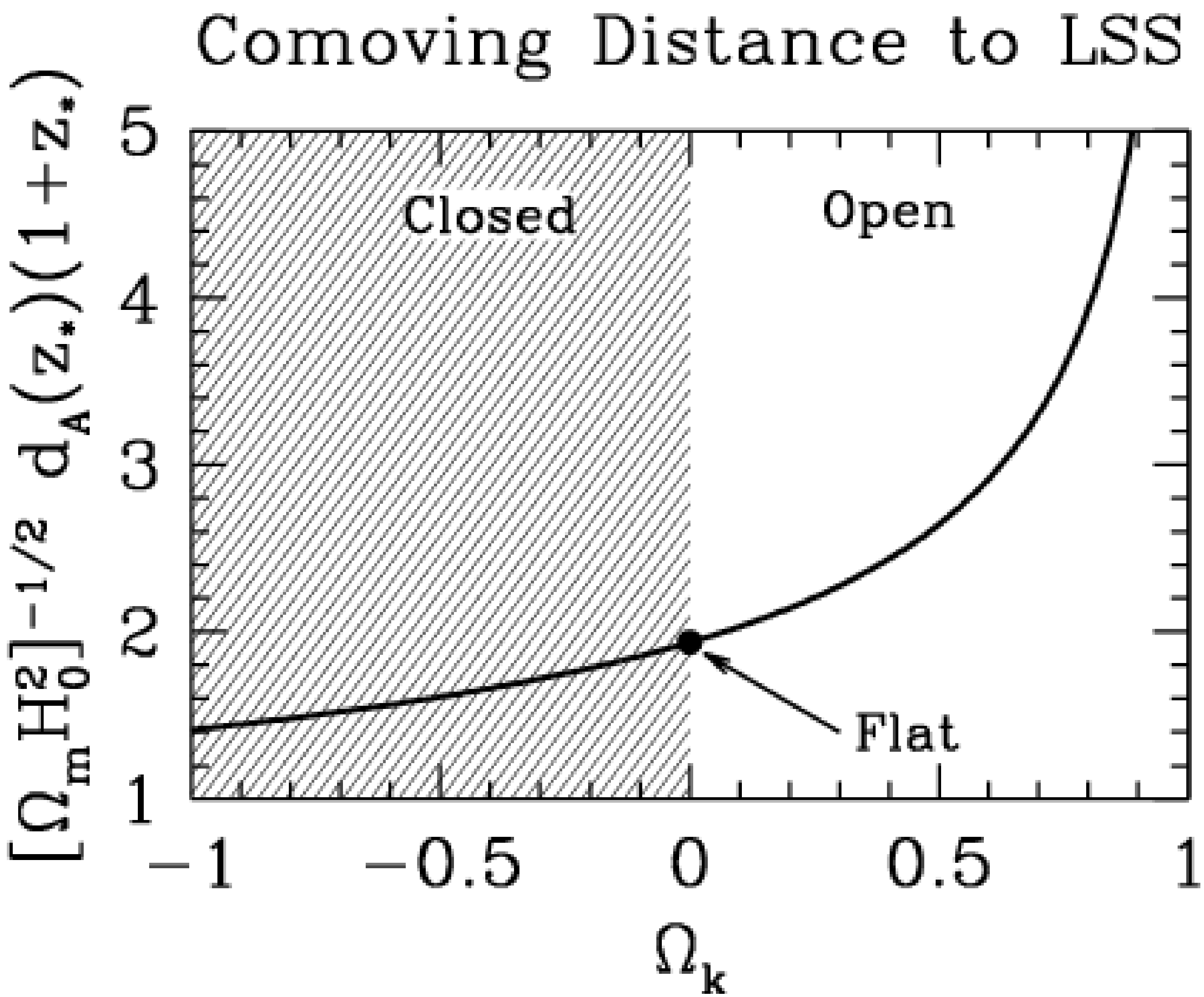
We have good reason to believe we are working with the correct model

So let the fun begin: fit for cosmological parameters!

Parameter I: Curvature

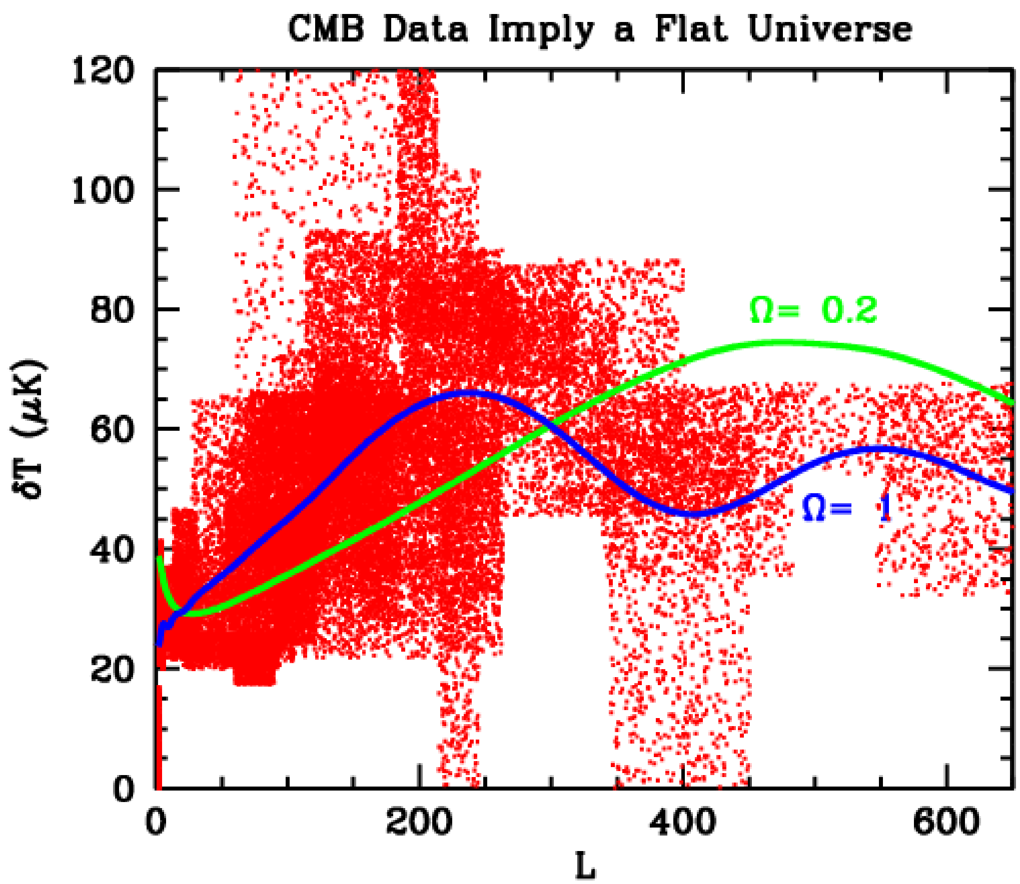
- ❑ Same wavelength subtends smaller angle in an open universe
- ❑ Peaks appear on smaller scales in open universe



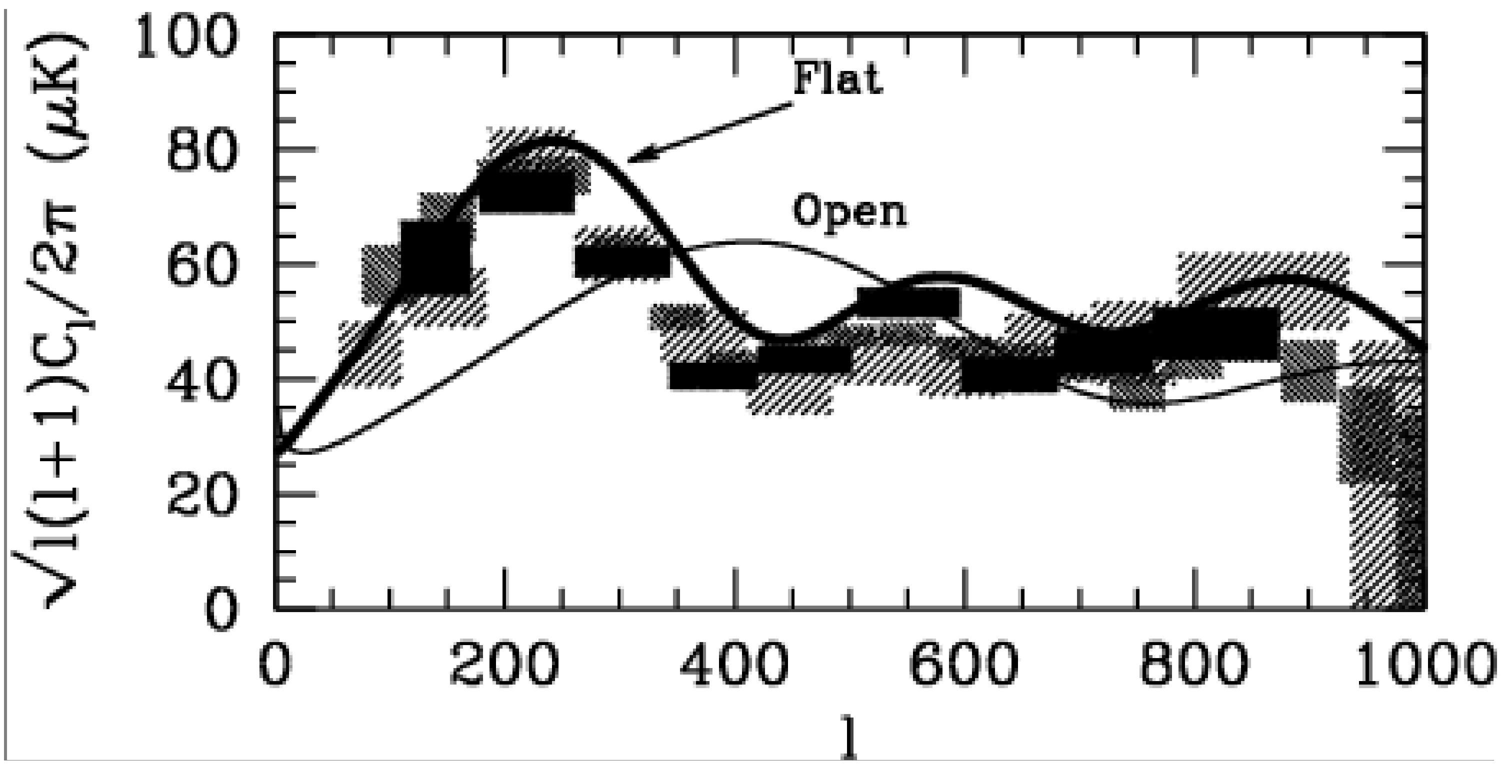

Comoving Distance to LSS
Closed
Open
Flat
$[\Omega_m H_0^2]^{-1/2}\ d_A(z_*)(1+z_*)$
Ω_k
5
4
3
2
1
−1
−0.5
0
0.5
1

Second Chance?

As early as 1998, observations favored flat universe

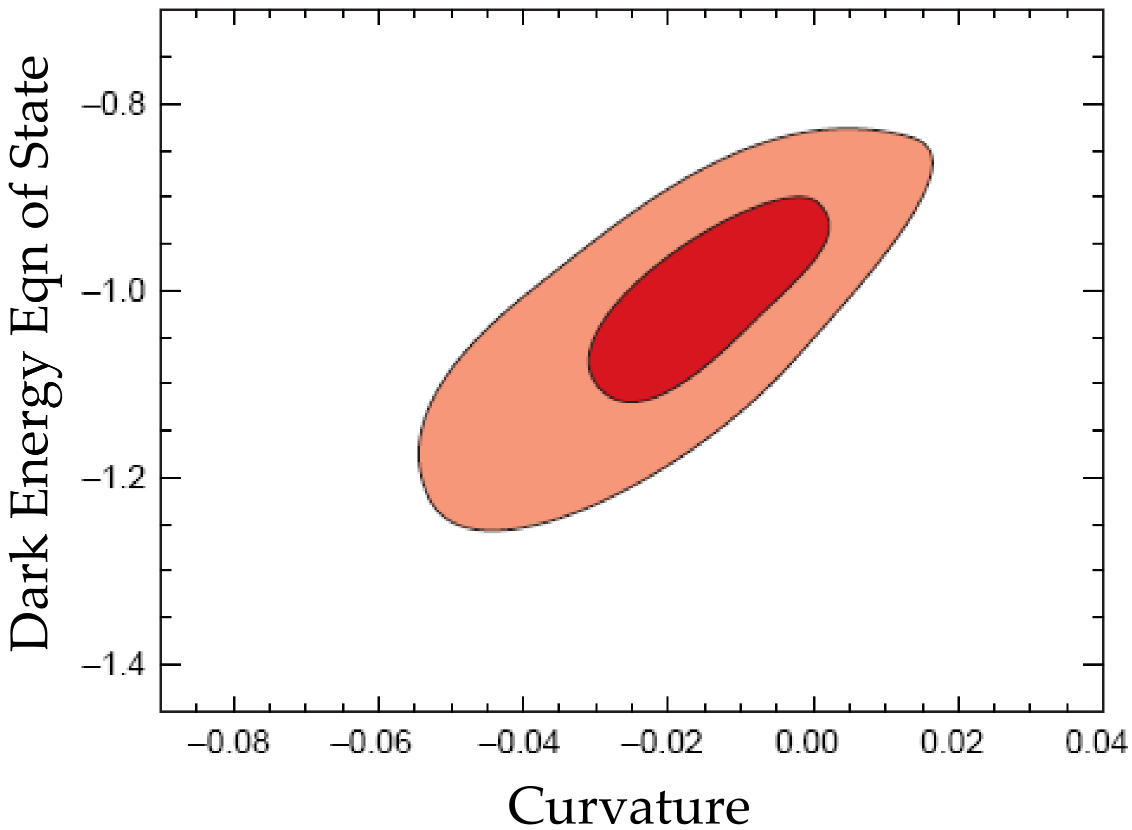


By 2001, the case was closed



DASI, Boomerang, Maxima (2001)

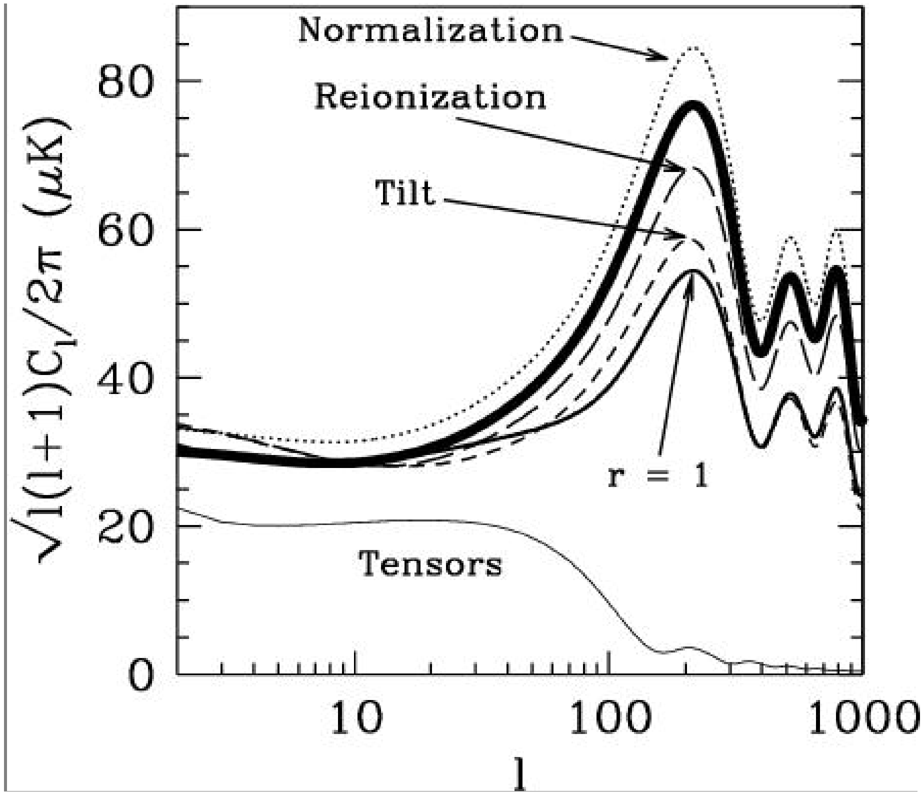
The Universe is Flat!



Spergel et al. 2006

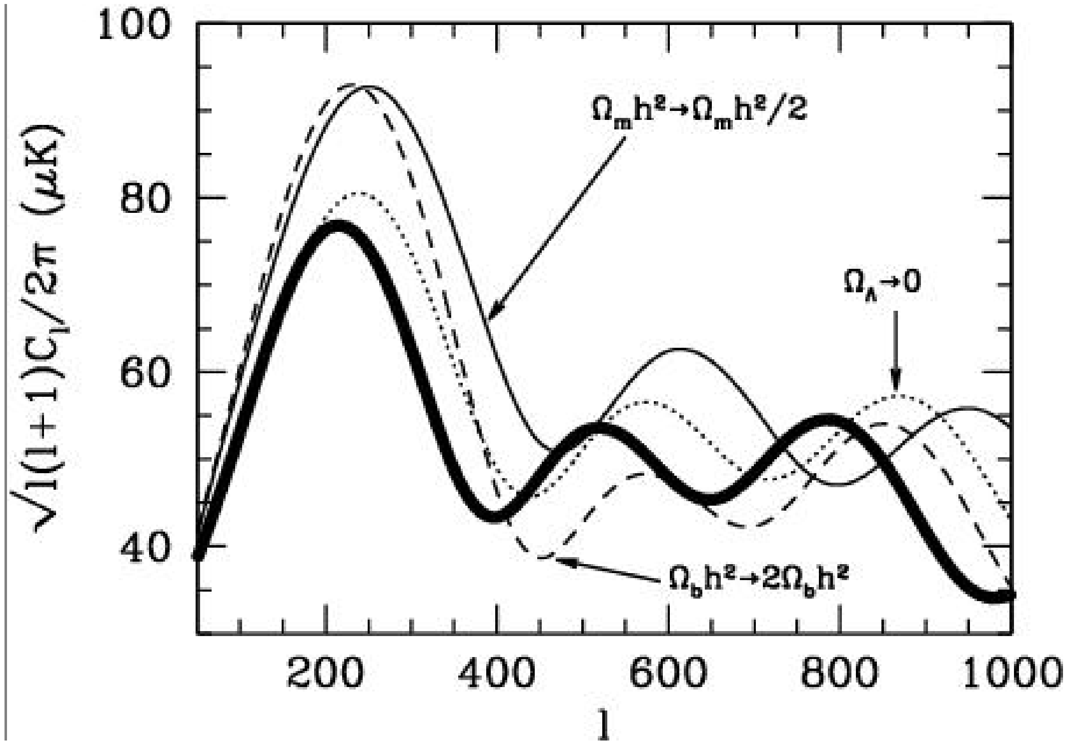
Parameters II

- Reionization lowers the signal on small scales
- A tilted primordial spectrum (n<1) increasingly reduces signal on small scales
- Tensors reduce the scalar normalization, and thus the small scale signal



Parameters III

- Baryons accentuate odd/even peak disparity
- Less matter implies changing potentials, greater driving force, higher peak amplitudes
- Cosmological constant changes the distance to LSS



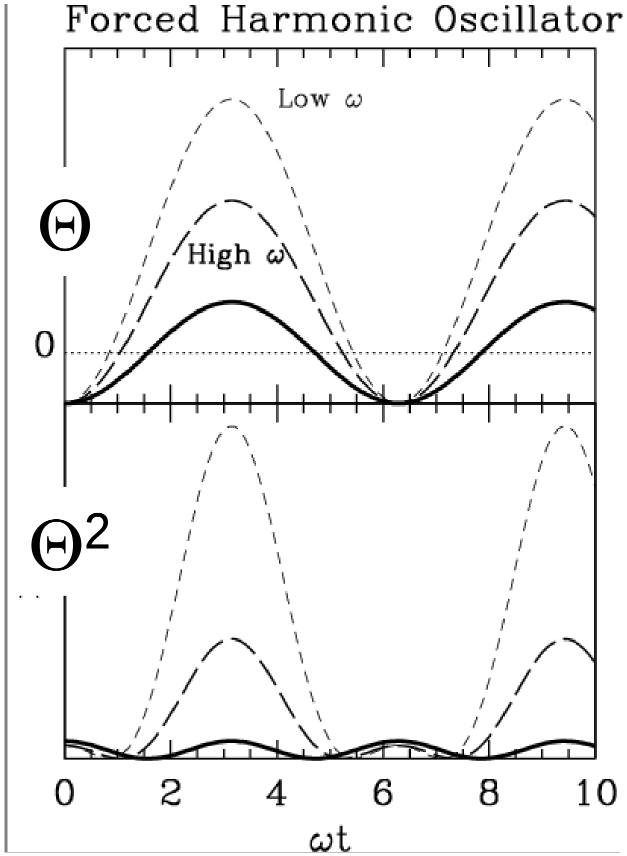
E.g.: Baryon density

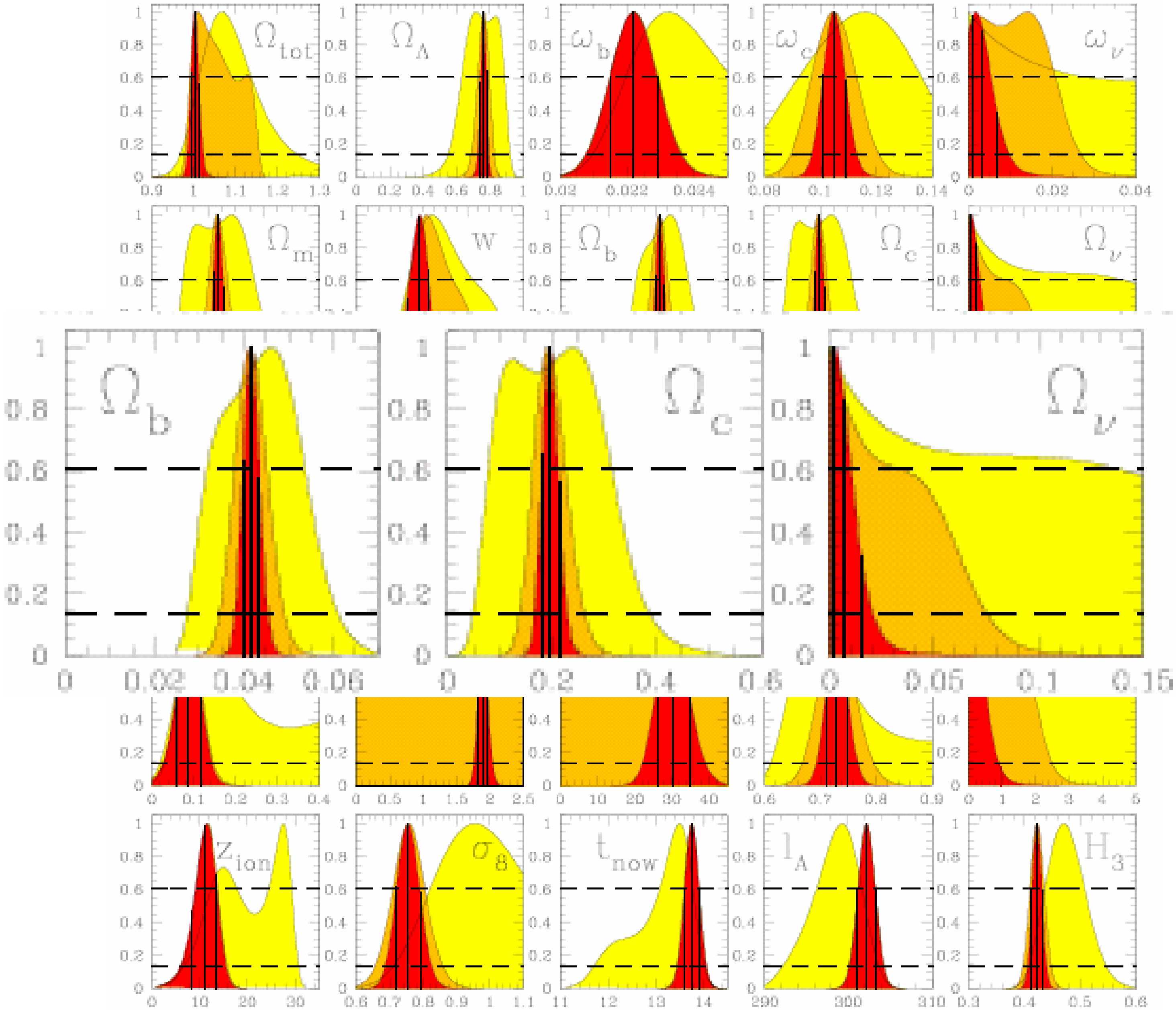
$$\ddot{\tilde{\Theta}} + \omega^2 \tilde{\Theta} = F$$

Here, F is forcing term due to gravity.

$$\omega = kc_s = \frac{k}{\sqrt{3[1 + 3\rho_b/4\rho_\gamma]}}$$

As baryon density goes up, frequency goes down. Greater odd/even peak disparity.





Tegmark et al. 2006

Penultimate Slide

If you want to get your hands dirty check out …

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http://cosmologist.info/cosmomc/
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Meet me at the bar tonight if you have a good idea about …

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Atomic physics of recombination
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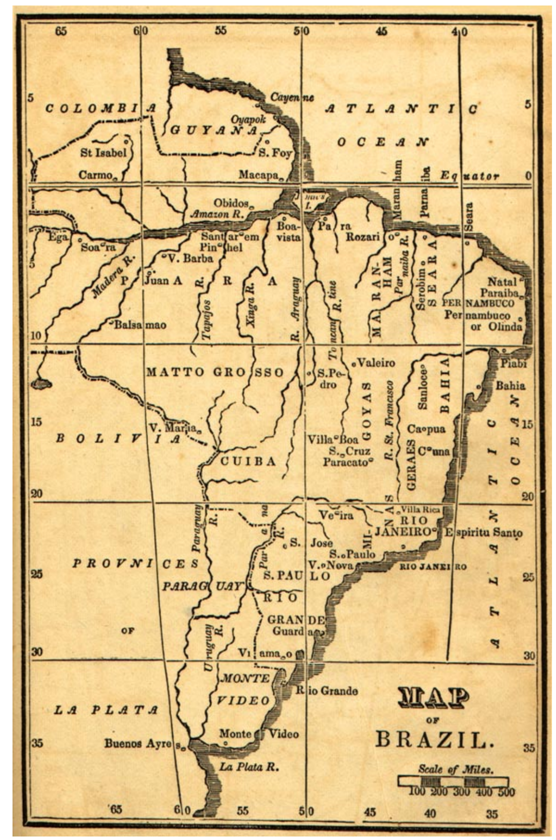
Available at www.amazon.com

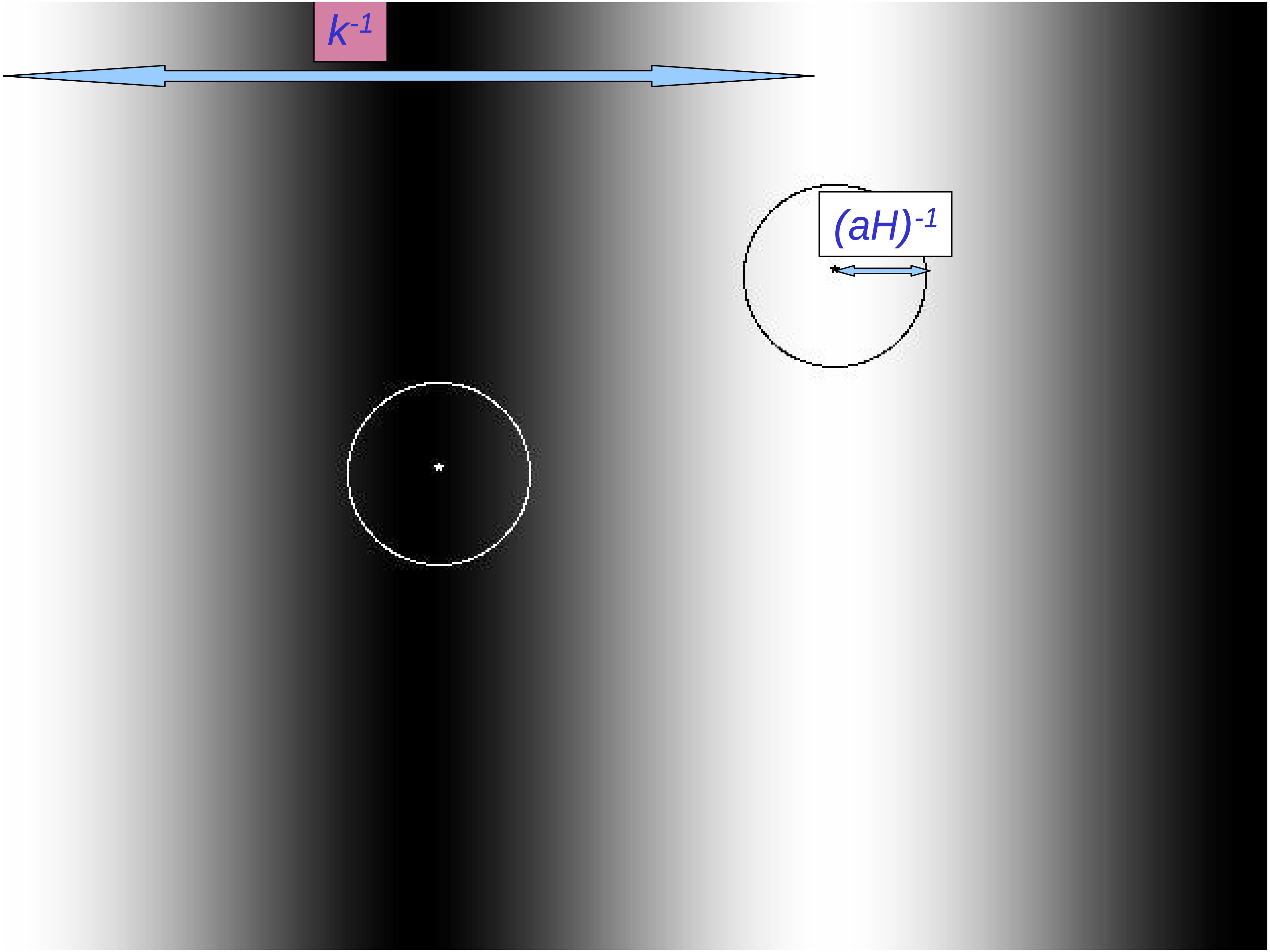
Fable

If you receive a letter in Rio from Sao Paolo on Monday, you know that it was sent on Sunday. Similarly if you receive a letter on Tuesday, you know it was sent from Sao Paolo on Sunday. In fact, since mail carriers cannot traverse the full distance between the two cities on bicycle during the week, even if you receive a letter in Rio on Friday, you know that it must have been sent from Sao Paolo on Sunday.

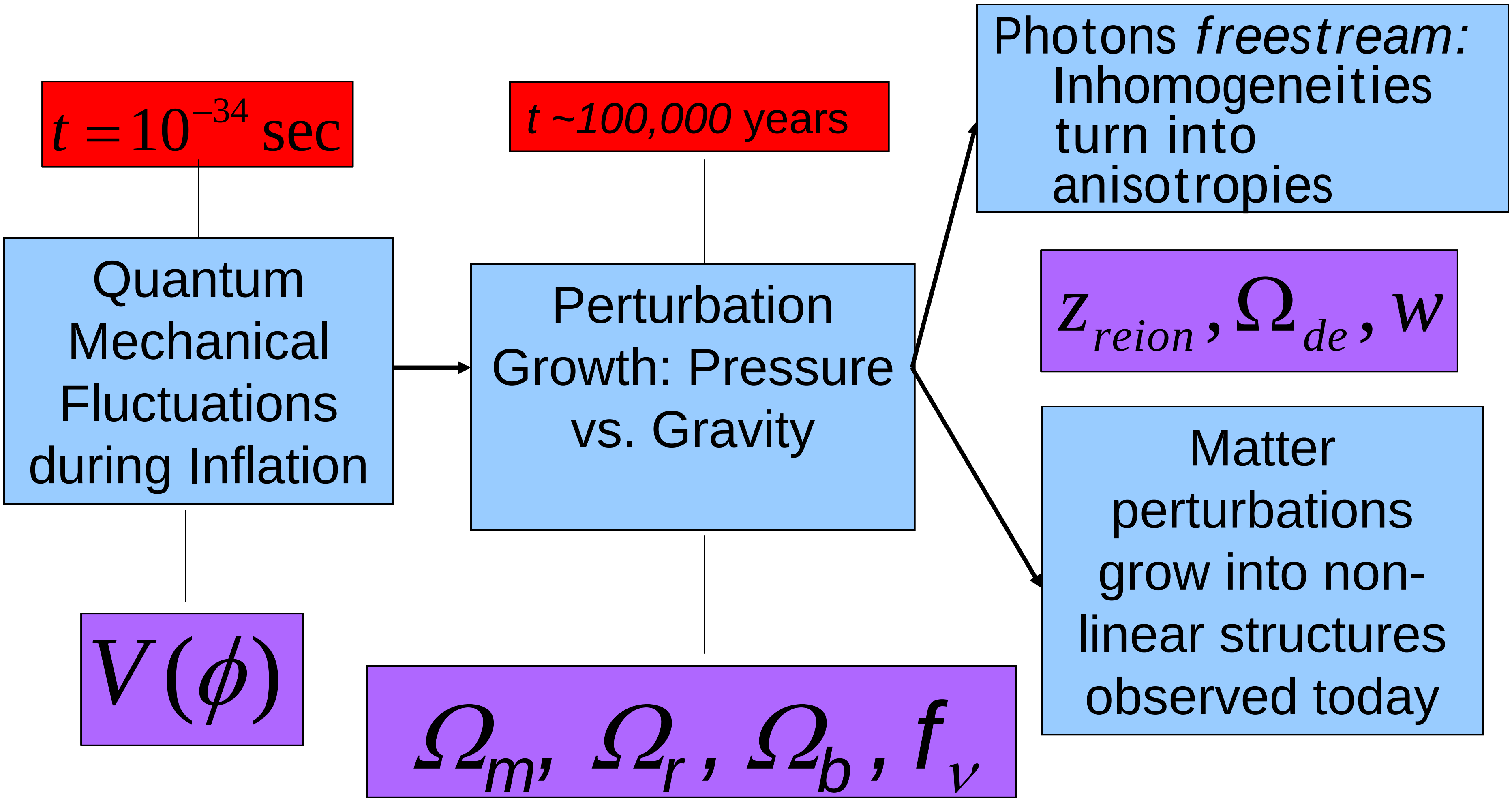
Consider a week in a Land with 2 cities: Sao Paolo and Rio. In this land, people may travel only by bicycle on Monday, Tuesday,… Friday so can travel only ~50 km on a weekday. On Saturday and Sunday, however, people may travel in cars or planes.

If, however, you receive a letter on Saturday, you cannot say for sure whether your lover in Sao Paolo sent it on the previous Sunday or on this very day, Saturday.

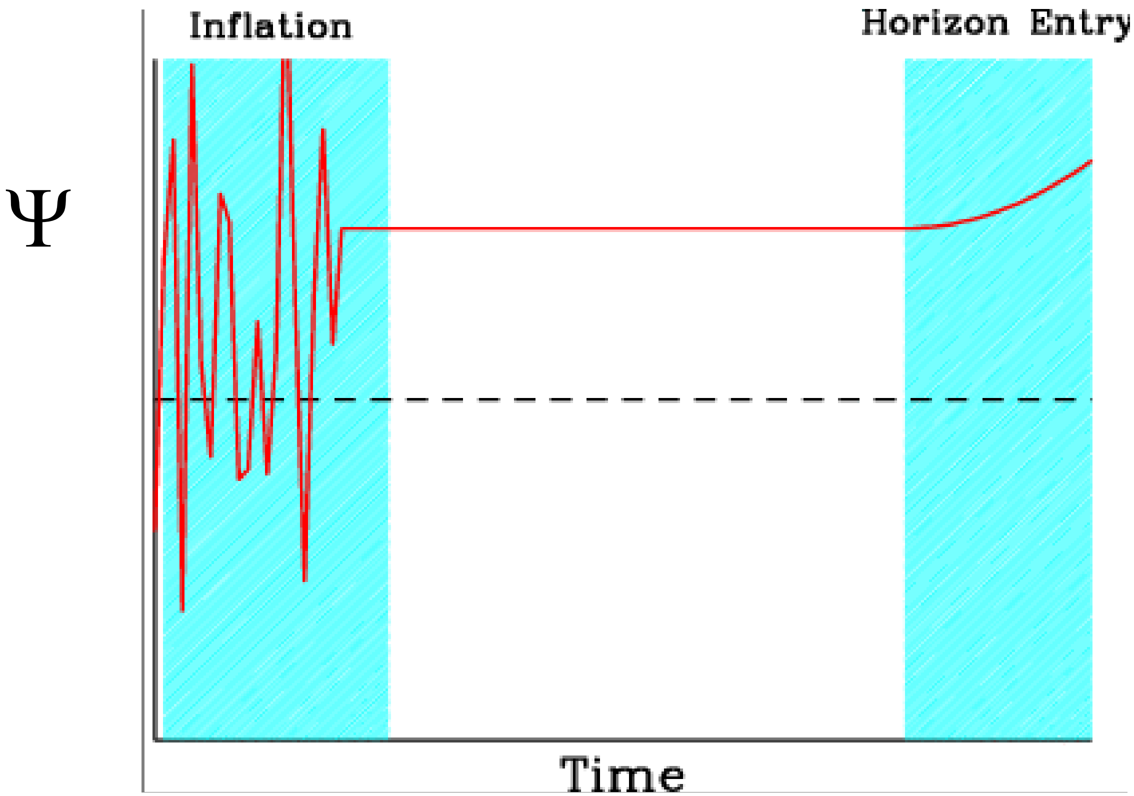



k^{-1}
$(aH)^{-1}$

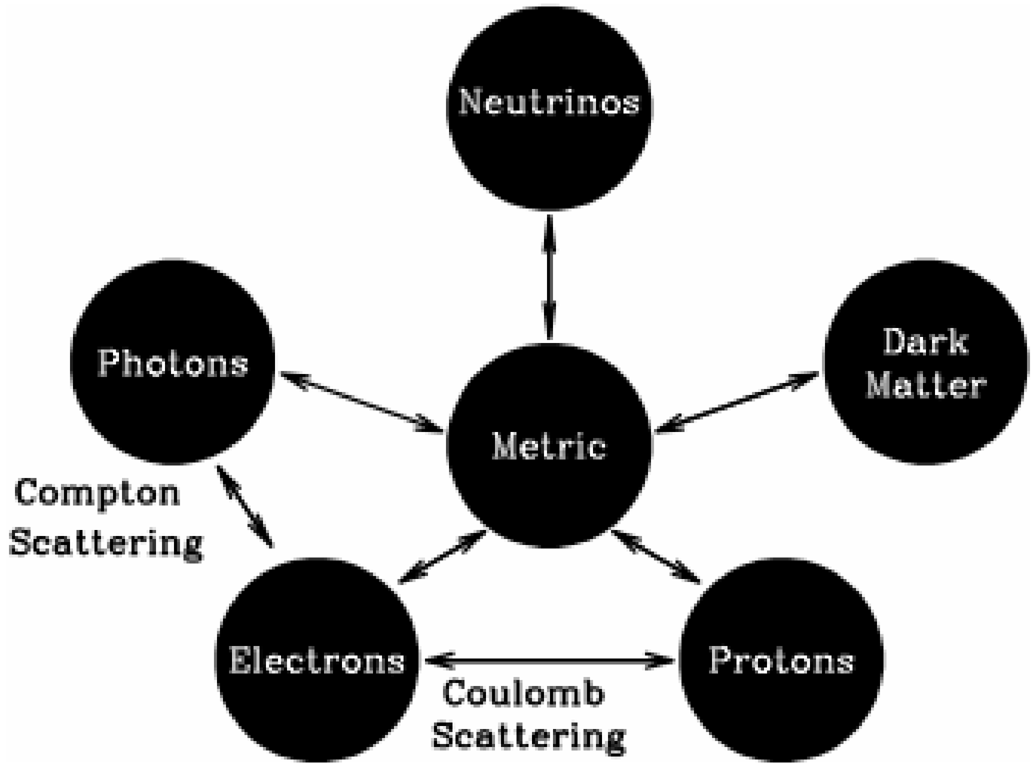
Coherent picture of formation of structure in the universe



What happens to photon perturbations when they re-enter the horizon?



To see how perturbations evolve, need to solve an infinite hierarchy of coupled differential equations



Perturbations in metric induce photon, dark matter perturbations