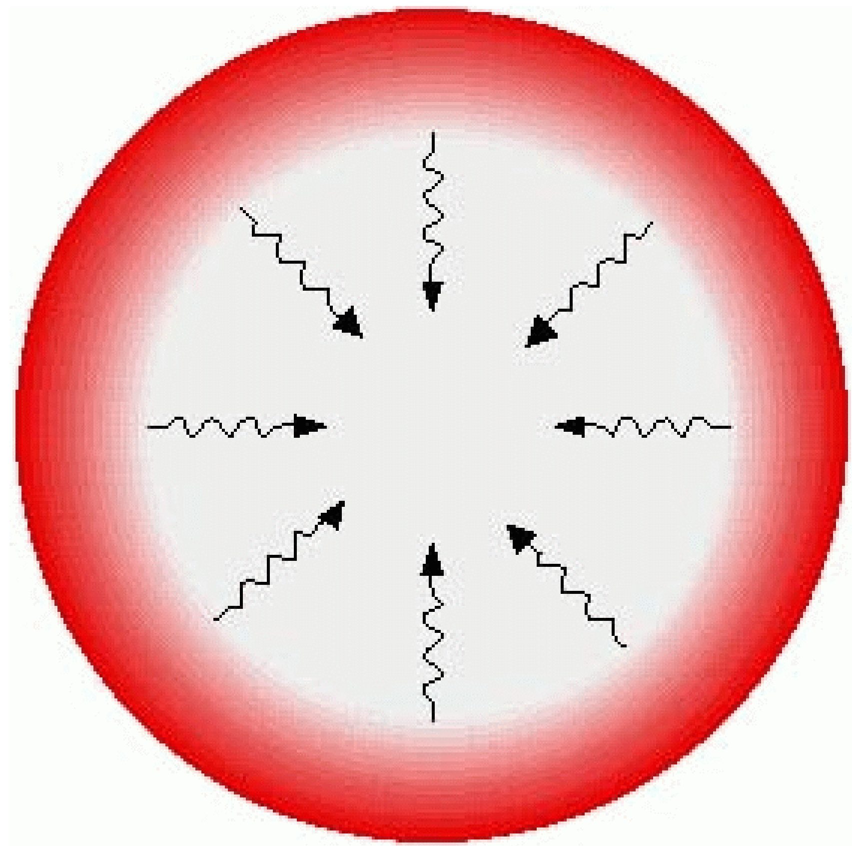
Cosmology for Particle Physicists II: Inflation and Large Scale Structure

- Horizon Problem/Inflation
- Generation of Perturbations
- Evolution of Perturbations
- Perturbation Probes: Constraints on Dark Matter

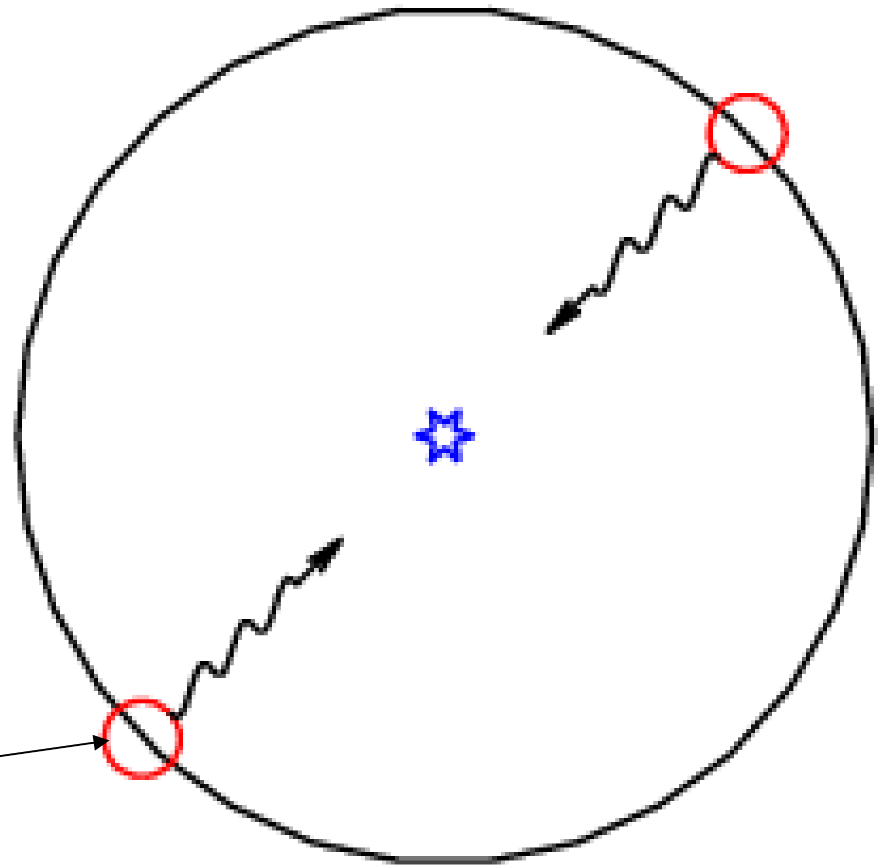
We see photons today from last scattering surface when the universe was just 400,000 years old

The temperature of the cosmic microwave background (CMB) is very nearly the same in all directions.



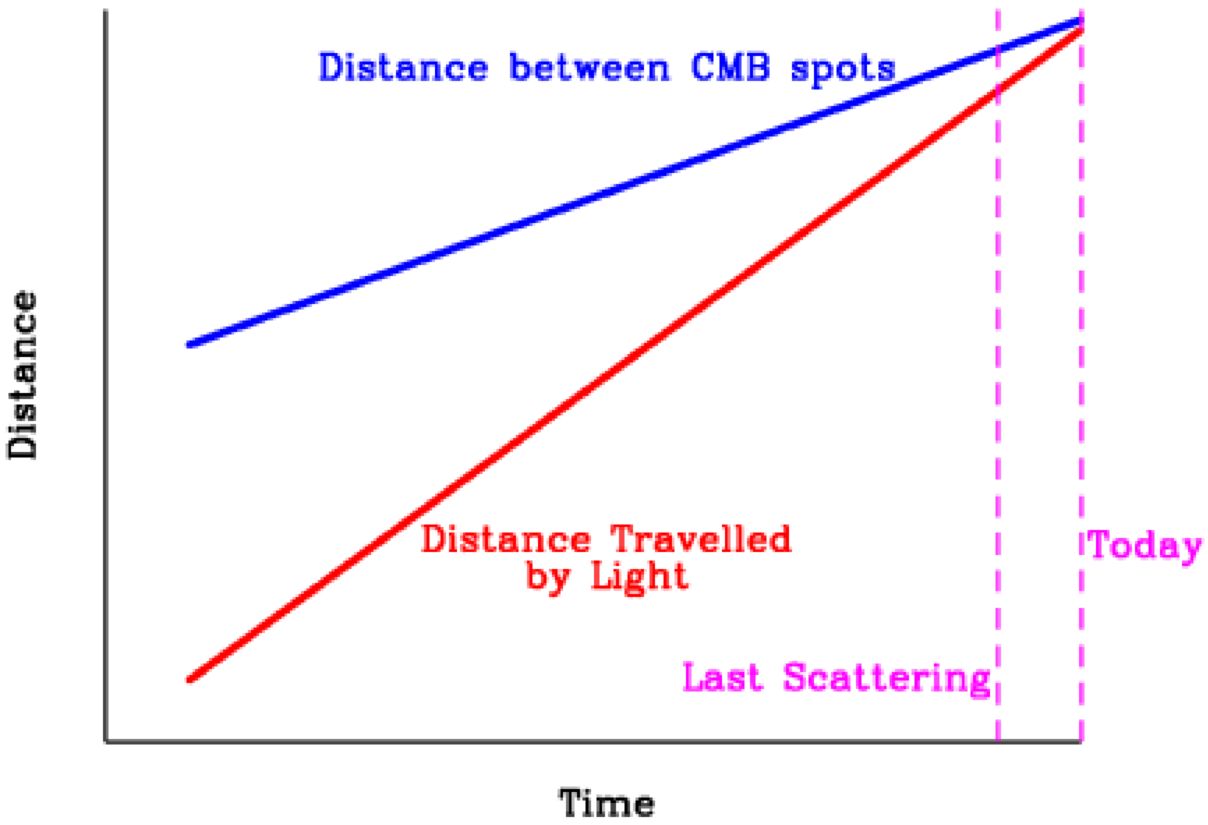
The Horizon Problem: Take 1

Why are the temperature of these two regions the same? They could not have communicated with one another!



Causally connected region at last scattering

The Horizon Problem: Take 2



Comoving Horizon

For particles which move at the speed of light $ds^2=0$, so $dx=dt/a$. Integrating up to time t gives the total comoving distance traveled by light since the beginning of expansion.

$$\eta = \int dx = \int_0^t \frac{dt'}{a(t')}$$

η is called the *comoving horizon.*

Hubble Radius

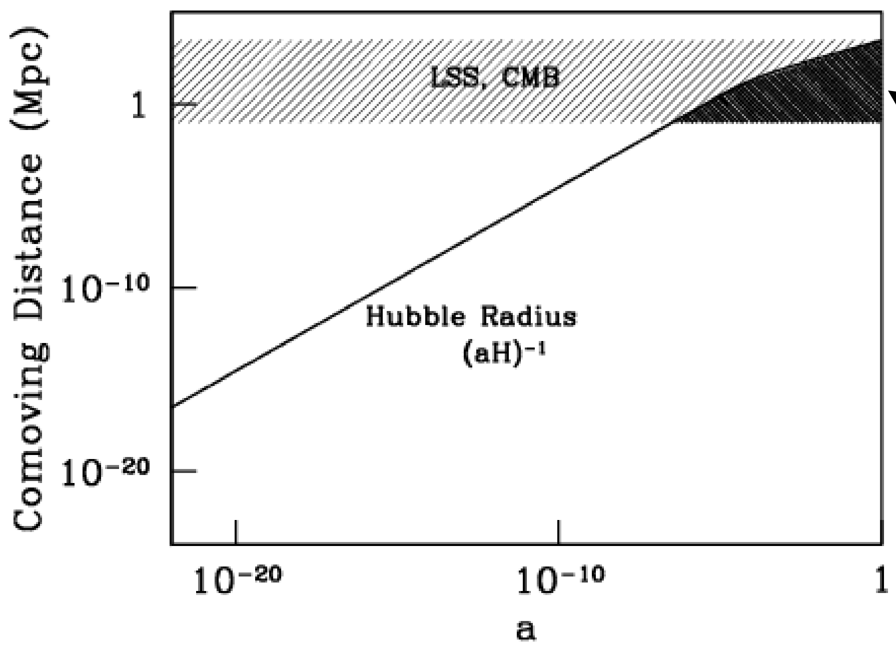
Can rewrite η as integral over *comoving Hubble radius* *$(aH)^{-1}$*, roughly the comoving distance light can travel as the universe expands by a factor of 2.

$$\int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{da'/dt'}\frac{1}{a'} = \int_0^a \frac{da'}{a'}\frac{1}{a'H(a')}$$

The horizon can be large even if the Hubble radius is small: e.g. if most of the contribution to η came from early times.

Horizon Problem: Take 3

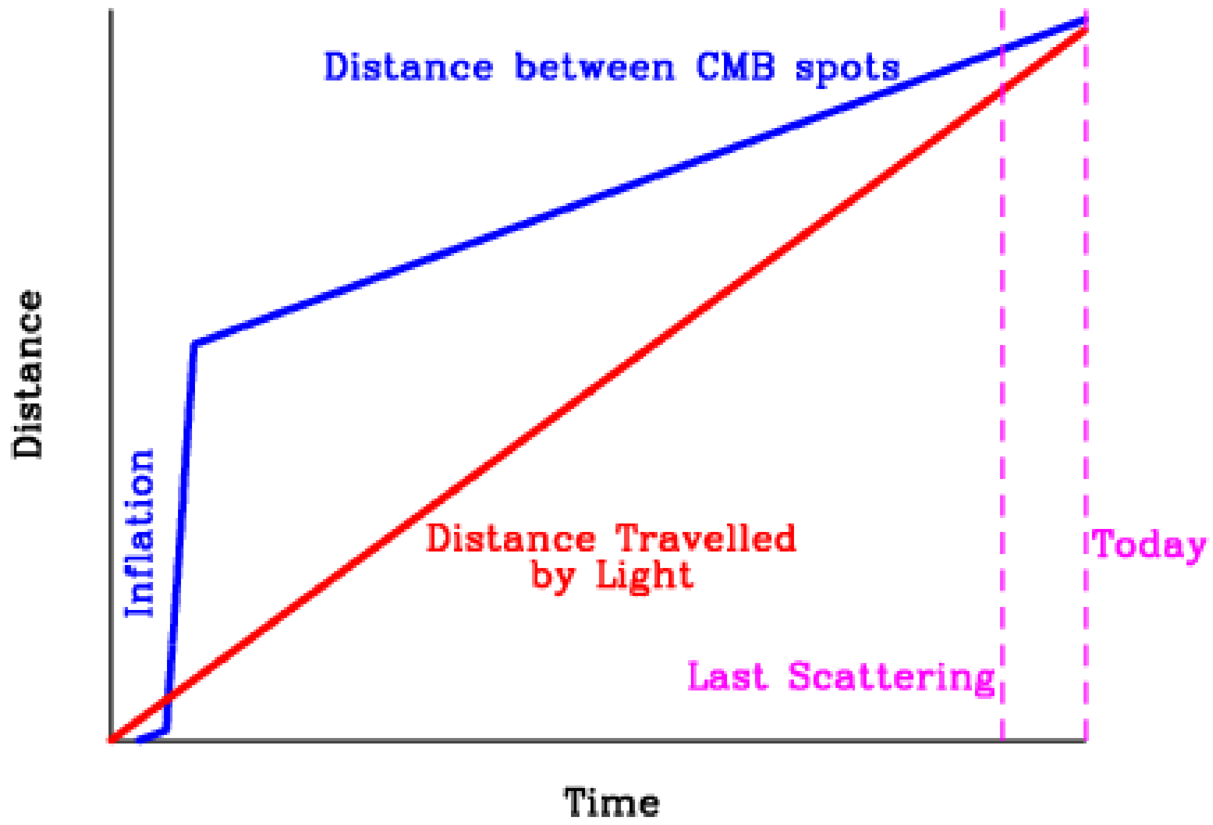
Compare comoving Cosmological Scales with comoving Hubble Radius



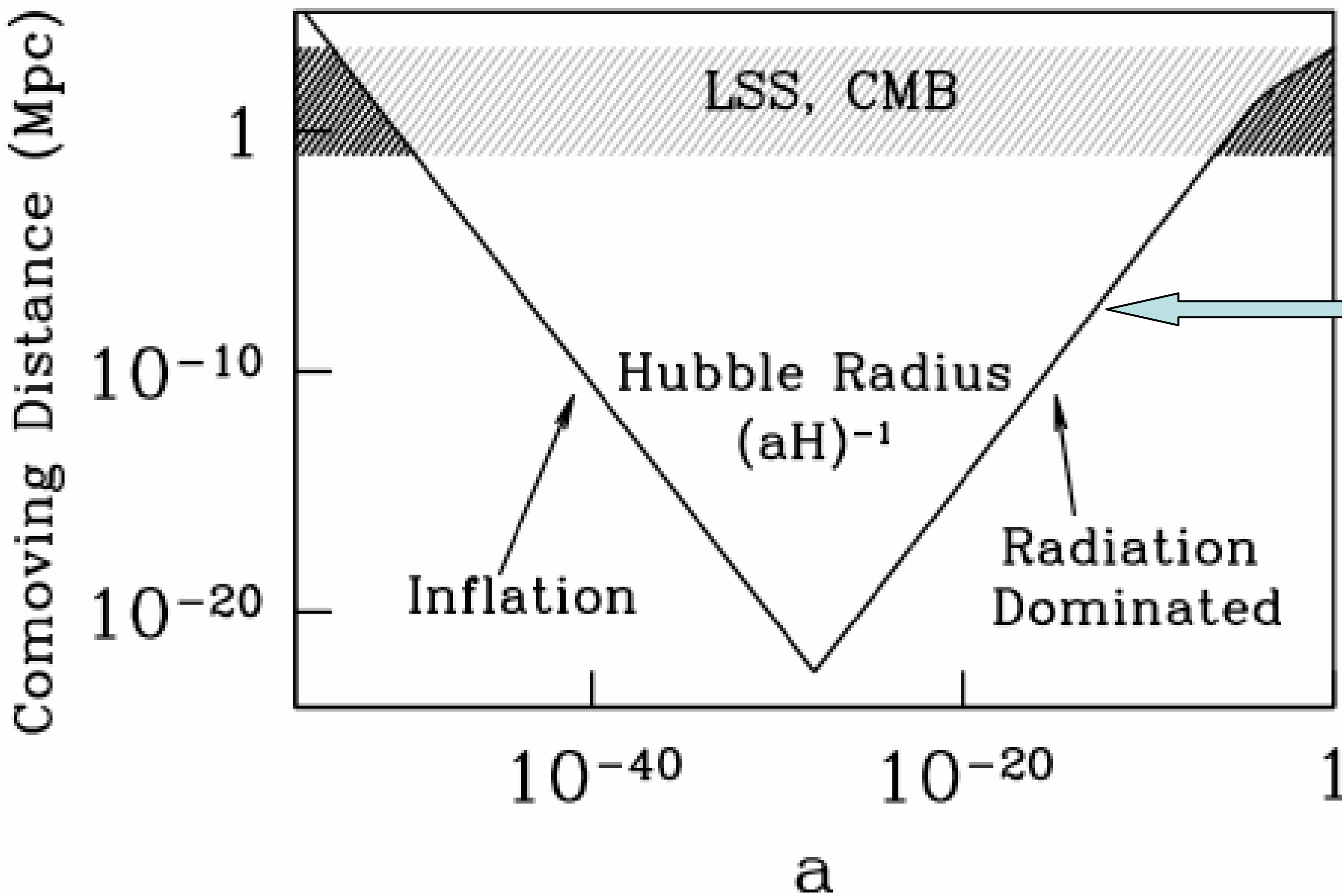
These scales entered the horizon too late to have equilibrated

Note: The scaling of *H* with *a* is dictated by the dominant component in the universe. For $a < 0.001$, I assume radiation domination --> $H \sim a^{-2}$.

Inflation in terms of Physical Distances



Inflation in terms of Comoving Distances



Cosmological scales *outside the horizon*

Early Dark Energy

Inflation correspond to an epoch in which the comoving Hubble radius decreases.

$$\frac{d}{dt}\left(aH\right)^{-1} = \frac{d}{dt}\left(\dot{a}\right)^{-1} = \frac{-\ddot{a}}{\dot{a}^2} < 0$$

$$\Rightarrow \ddot{a} > 0$$

Inflation is an epoch early in which dark energy dominated the universe. This early dark energy has a density ~10^{100} times larger than late dark energy.

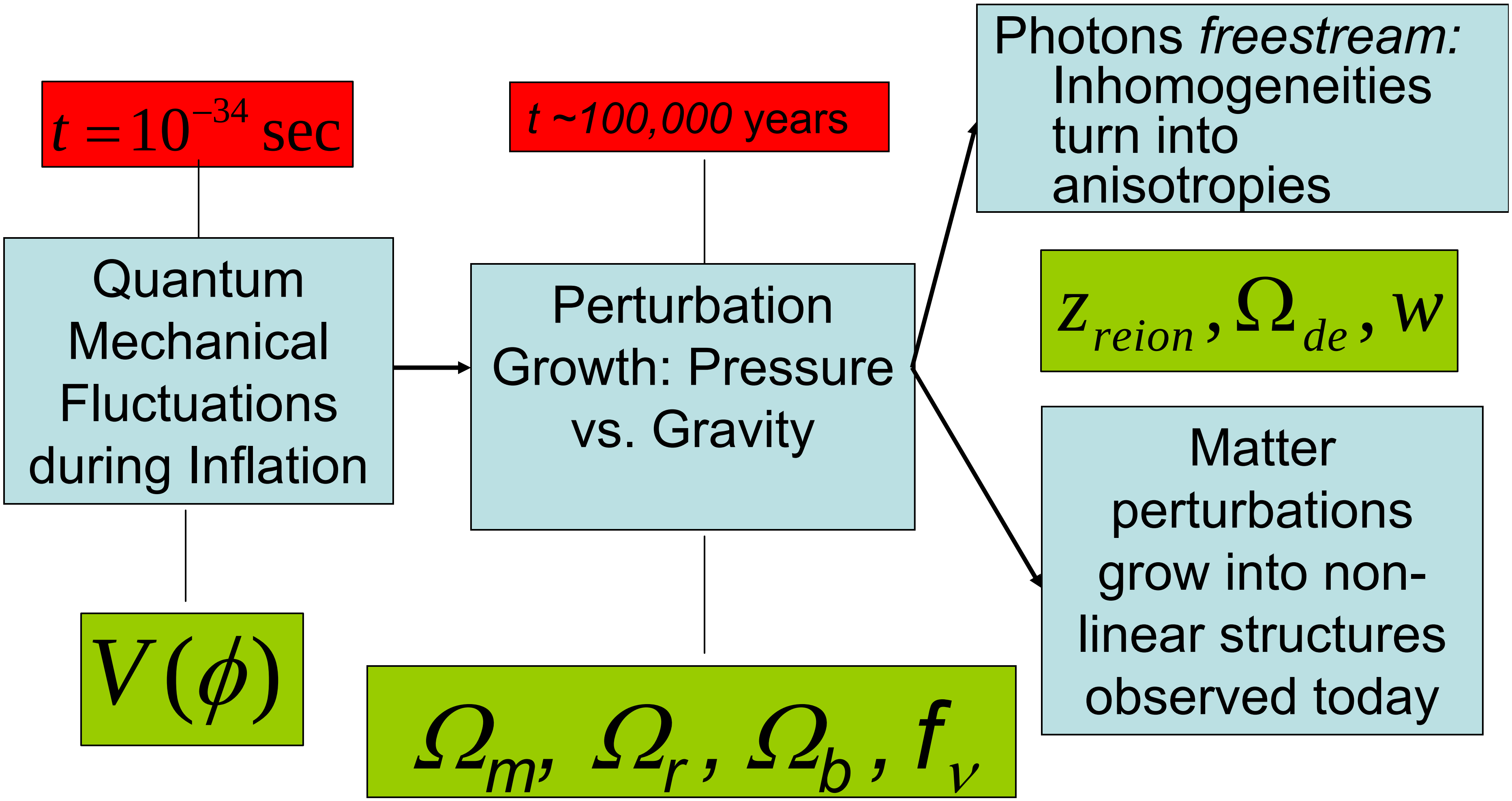
Typically model inflation with scalar field

Require:

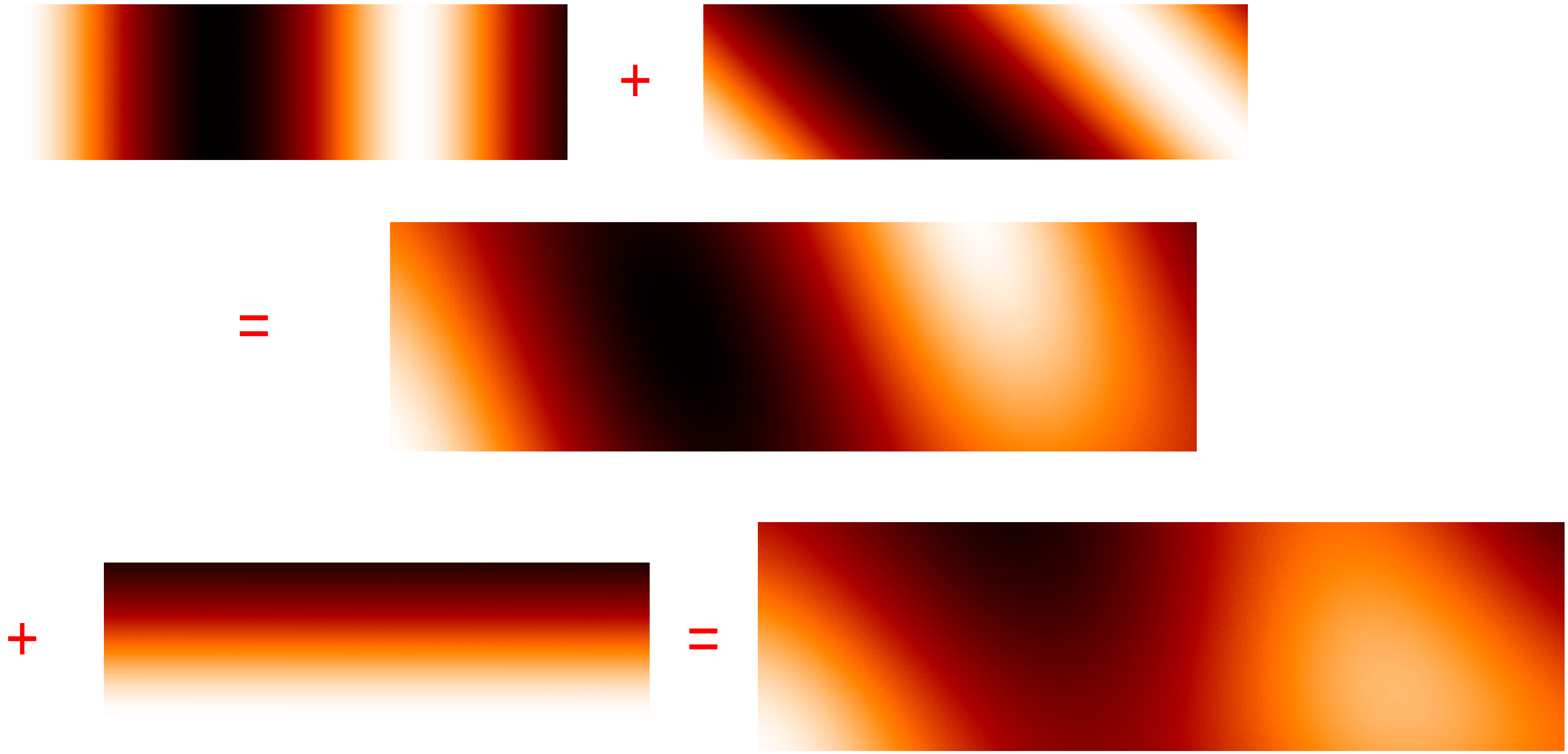
$$\underbrace{\left(\frac{1}{2}\dot{\varphi}^2 + V\right)}_{\rho} + 3\underbrace{\left(\frac{1}{2}\dot{\varphi}^2 - V\right)}_{P} < 0$$

$$\Rightarrow V > \dot{\varphi}^2$$

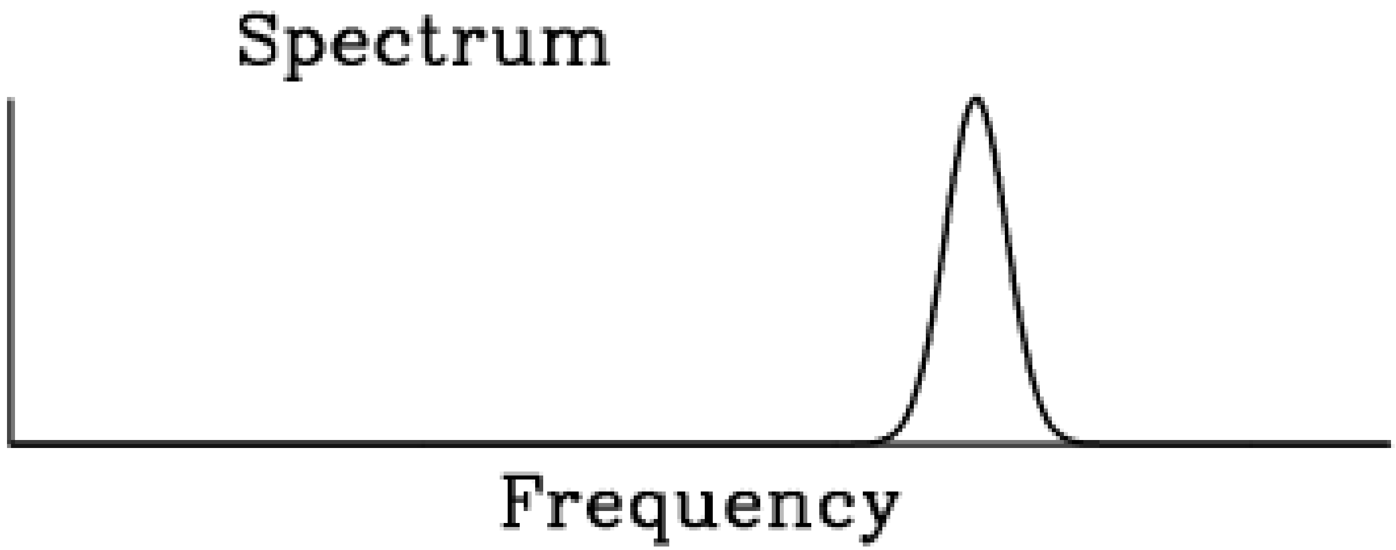
Coherent picture of formation of structure in the universe



To analyze quantitatively, decompose into Fourier modes

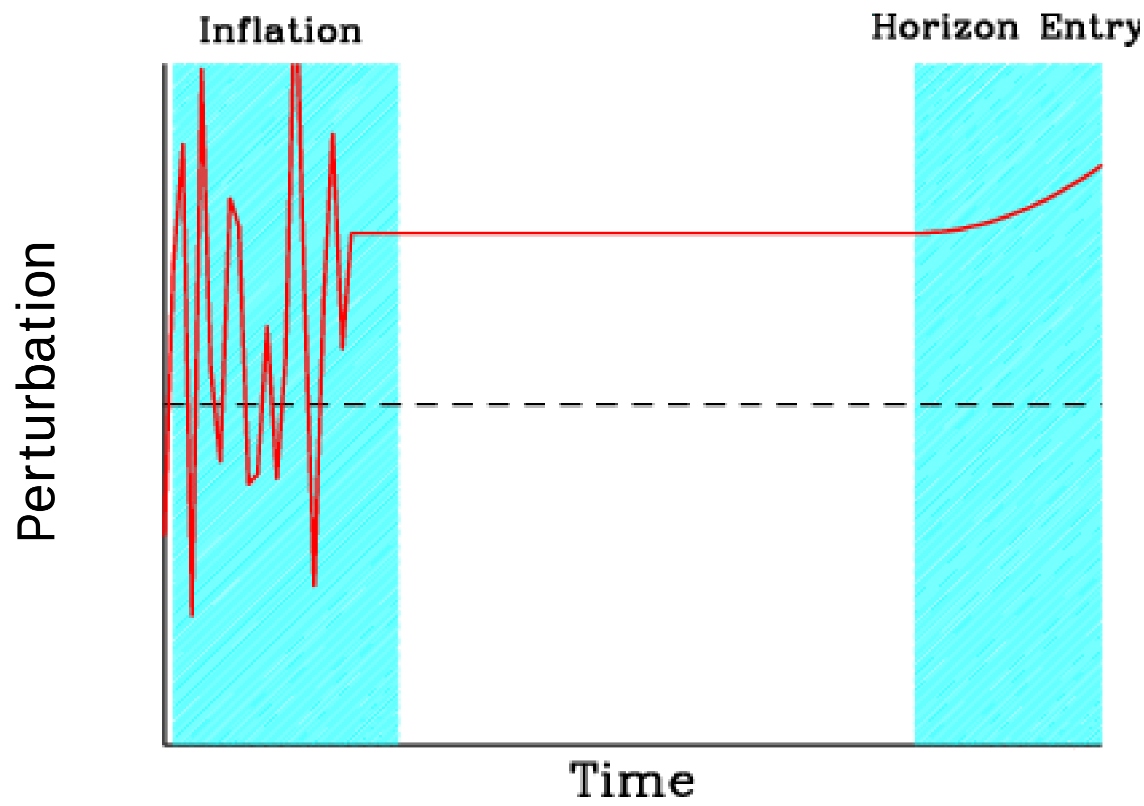


In this simple example, all modes have same wavelength/frequency



More generally, at each wavelength/frequency, need to average over many modes to get spectrum

Perturbations start as quantum mechanical fluctuations and turn classical



Perturbations to the FRW metric

$$g_{00} = -1 - 2\Psi(\vec{x},t) \; ; \; g_{ij} = \delta_{ij} a^2(t)\left(1 + 2\Phi(\vec{x},t)\right)$$

The scalar potentials typically satisfy $\Phi = -\Psi$ (but beware sign conventions) so we will use them interchangably.

During inflation, Ψ fluctuates quantum mechanically around a smooth background

The mean value of Ψ is zero, but its variance is

$$\langle \Psi^2(\vec{x}) \rangle = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} e^{i\vec{k}'\cdot\vec{x}} \langle \tilde{\Psi}(\vec{k}) \tilde{\Psi}(\vec{k}') \rangle$$

$$= \int \frac{dk}{k} \frac{k^3 P_\Psi(k)}{2\pi^2}$$

$$\left\langle \tilde{\Psi}(\vec{k}) \tilde{\Psi}(\vec{k}') \right\rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') P_\Psi(k)$$

Get contributions from all scales equally if

$P_\Psi \propto k^{-4+n}$ with *n=1* (scale-invariant spectrum)

Inflation predicts ...

Two regions with the scalar field taking the same value at slightly different times have relative potential

$$\Psi \sim \frac{\delta a}{a} \sim \frac{\dot{a}}{a}\delta t = H\delta t$$

But

$$\delta t = \frac{\delta\phi}{\dot{\phi}} \sim \frac{\delta\phi}{V'/H}$$

The last equality following from the equations of motion

Leading to ...

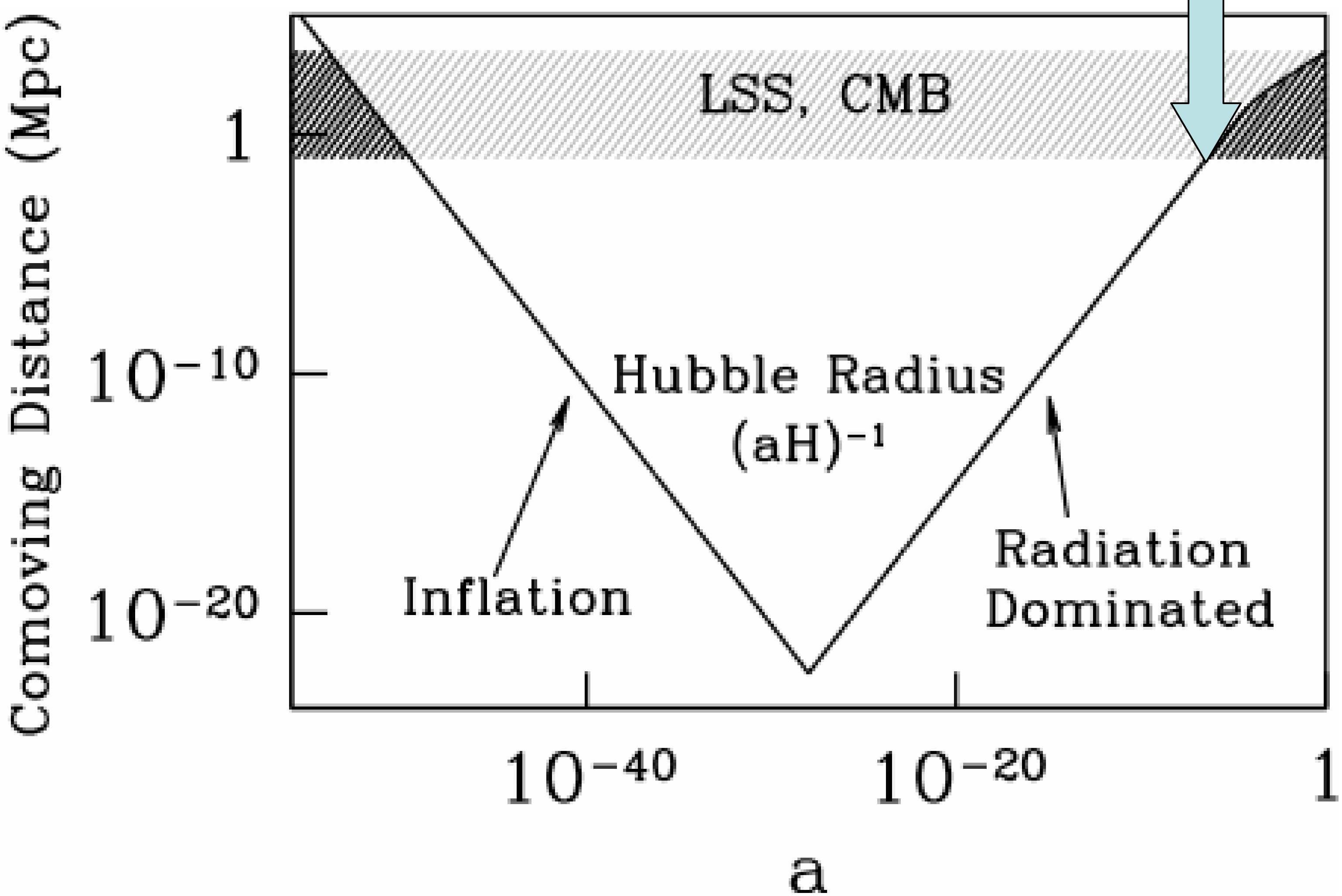
$$\Psi \simeq \frac{\delta\phi H^2}{V'}$$

RMS fluctuations in the scalar field are roughly equal to the Hubble rate, and the Friedmann equation tells us that $H^2 \sim GV$

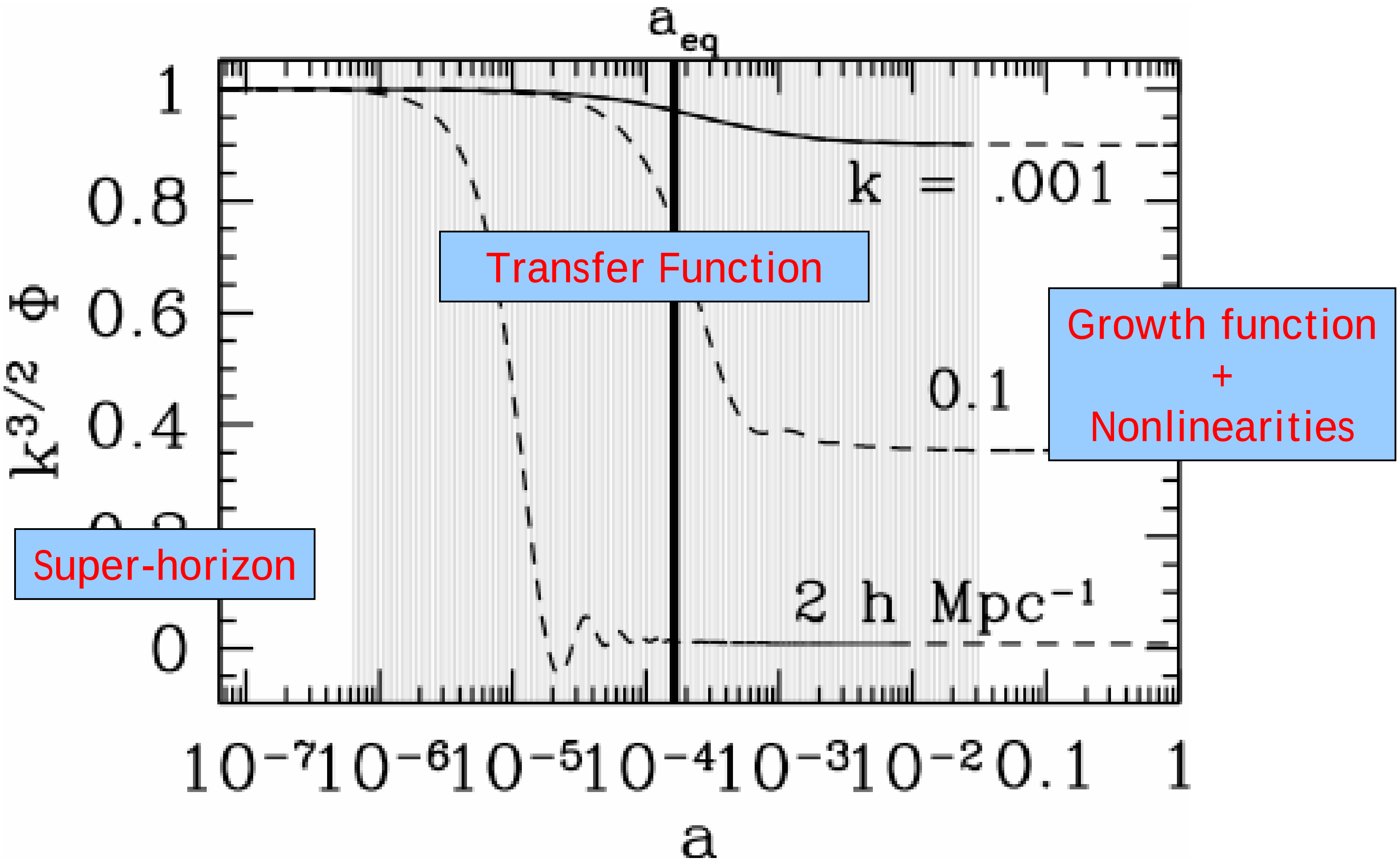
$$\Psi_{\rm RMS} \sim \left[k^3 P_\Psi\right]^{1/2} \sim \frac{(GV)^{3/2}}{V'}$$

Each value of k is associated with φ [the value of φ when k exits the horizon]. Since the RHS here is near constant for a slowly rolling field, n≈1.

Now determine the evolution of perturbations when they re-enter the horizon.



Earlier than a_{EQ}, most of the energy density is relativistic; afterwards non-relativistic



Growth of Structure: Gravitational Instability

Define overdensity:

$$\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t)-\overline{\rho}(t)}{\overline{\rho}(t)}$$

Fundamental equation governing overdensity in a matter-dominated universe when scales are within horizon:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\overline{\rho}_m\delta = 0$$

Growth of Structure: Gravitational Instability

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m \delta = 0$$

Example 1: No expansion (H=0,energy density constant)

$$\delta \propto e^{\pm t\sqrt{4\pi G\bar{\rho}_m}}$$

- Two modes: growing and decaying
- Growing mode is exponential (the more matter there is, the stronger is the gravitational force)

Gravitational Instability in an Expanding Universe

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m\delta = 0$$

<u>Example 2:</u> Matter density equal to the critical density in an expanding universe.

The coefficient of the 3rd term is then $3H^2/2$, so

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta = 0$$

In this universe $a=(t/t_0)^{2/3}$ so $H=2/(3t)$

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

Gravitational Instability in an Expanding Universe

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

Insert solution of the form: $\delta \sim t^p$

Growing mode: $\delta \sim a$. Dilution due to expansion counters attraction due to overdensity. Result: power law growth instead of exponential growth

$$p = \frac{\ }{6} \pm \frac{\ }{2}\sqrt{\frac{\ }{9} + \frac{8}{3}} = \begin{cases} 2/3 \\ -1 \end{cases}$$

Gravitational Potential

Poisson's Equation: $$\nabla^2\Phi = 4\pi G\bar{\rho}\delta$$

In Fourier space, this becomes: $$-\frac{k^2}{a^2}\tilde{\Phi} \propto \frac{\tilde{\delta}}{a^3}$$

So the gravitational potential remains constant! Delicate balance between attraction due to gravitational instability and dilution due to expansion.

Only holds if all the energy is in non-relativistic matter. Dark energy or massive neutrinos lead to potential decay.

Matter Power Spectrum

Poisson says: $$k^2\tilde{\Phi} \propto \tilde{\delta}$$

So the power spectrum of matter (which measures the density *squared)* scales as:

$$P_\delta \propto k^4 P_\Phi \propto k^n$$

Valid on large scales which entered the horizon at late times when the universe was matter dominated.

Sub-horizon modes oscillate and decay in the radiation-dominated era

Newton's equations - with radiation as the source - reduce to

Here using η as time variable

$$\ddot{\Phi} + \frac{4}{\eta}\dot{\Phi} + \frac{k^2}{3}\Phi = 0$$

with analytic solution

$$\Phi(\eta) = 3\Phi(0)\frac{\sin(k\eta/\sqrt{3}) - (k\eta/\sqrt{3})\cos(k\eta/\sqrt{3})}{(k\eta/\sqrt{3})^3}$$

Expect less power on small scales

For scales that enter the horizon well before equality,

$$\Phi(\eta_{\rm EQ}) \rightarrow \Phi(0) \frac{\cos(k\eta_{\rm EQ}/\sqrt{3})}{(k\eta_{\rm EQ}/3)^2}$$

So, we expect the transfer function to fall off as

$$\lim_{k\to\infty} T(k) \equiv \lim_{k\to\infty} \frac{\Phi_{today}(k)}{\Phi_{initial}(k)} \propto k^{-2}$$

Shape of the Matter Power Spectrum

$$P(k) \propto k^n T^2(k) \propto \begin{cases} k^n & \text{Large scales} \\ k^{n-3}\ln^2(k) & \text{Small scales} \end{cases}$$

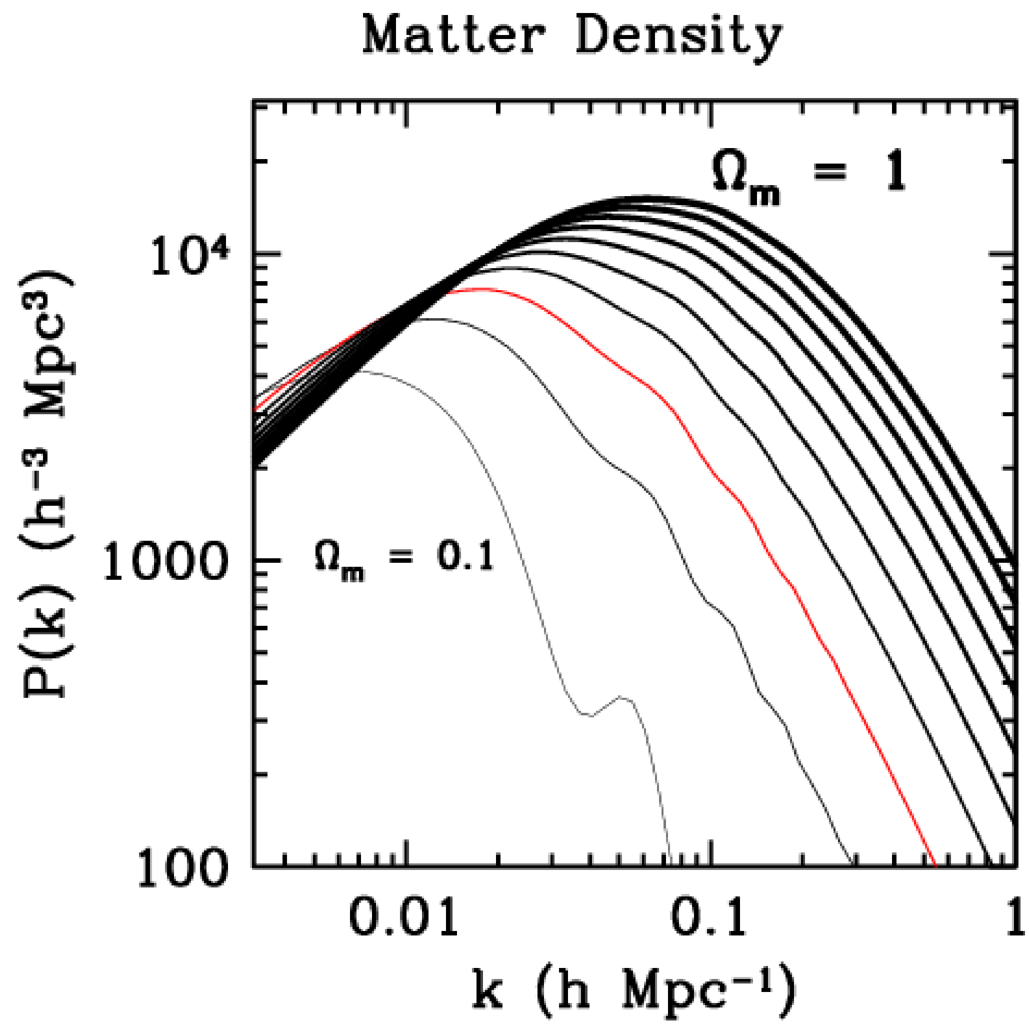
Log since structure grows slightly during radiation era when potential decays

The *turnover* scale is the one that enters the horizon at the epoch of matter-radiation equality:

$$k_{EQ} = 0.073\Omega_m h^2 \text{Mpc}^{-1}$$

Therefore, measuring the shape of the power spectrum will give a precise estimate of Ω_m

Turnover scale sensitive to the matter density



Neutrinos affect large scale structure

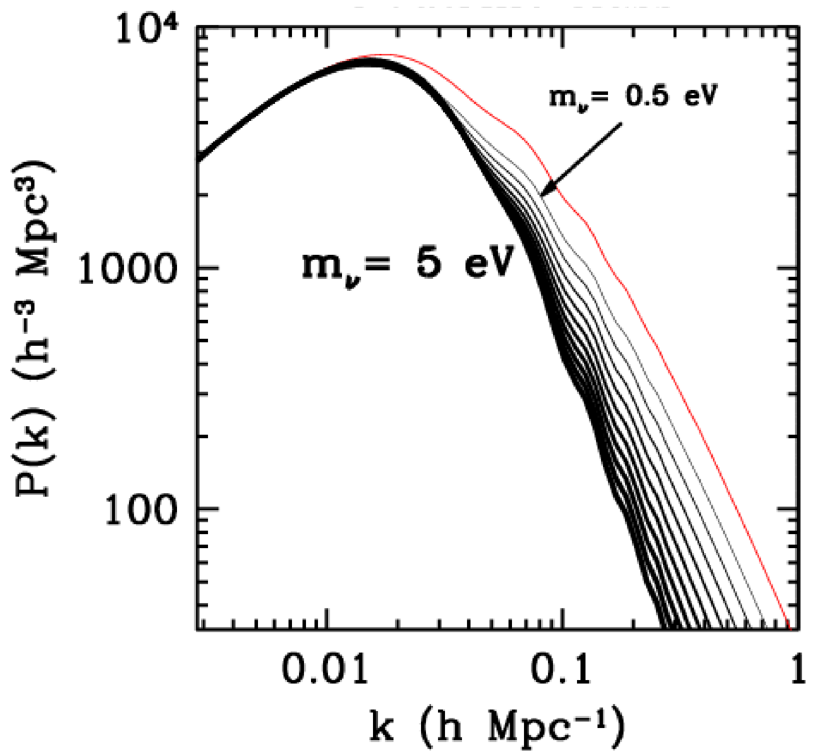
Recall $$\Omega_\nu = 0.02 \frac{m_\nu}{1\,\mathrm{eV}}$$

This fraction of the total density does *not* participate in collapse on scales smaller than the freestreaming scale

$$k_{\rm fs}^{-1} \simeq \frac{vt}{a} \simeq \frac{(T/m)H^{-1}}{a}$$

At the relevant time, this scale is 0.02 Mpc^{-1} for a 1eV ν; power on scales small than this is suppressed.

Neutrino mass suppresses the power spectrum on small scales



Even for a small neutrino mass, get large impact on structure: power spectrum is excellent probe of neutrino mass

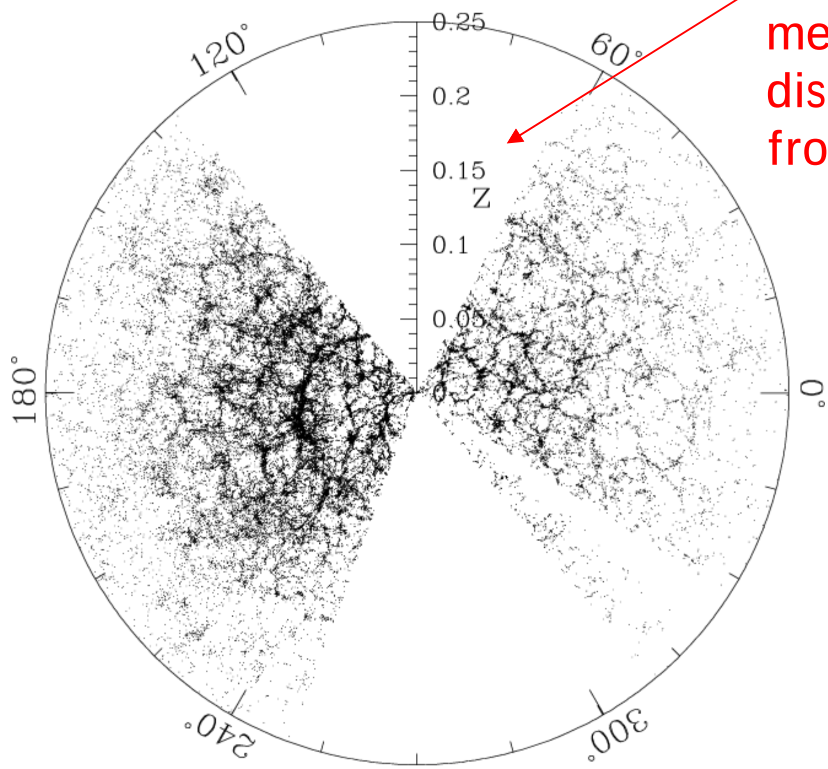
How do we probe the matter distribution?

Most of the matter in the universe is dark, i.e. does not emit light. How can we probe its distribution?

Several Large Galaxy Surveys

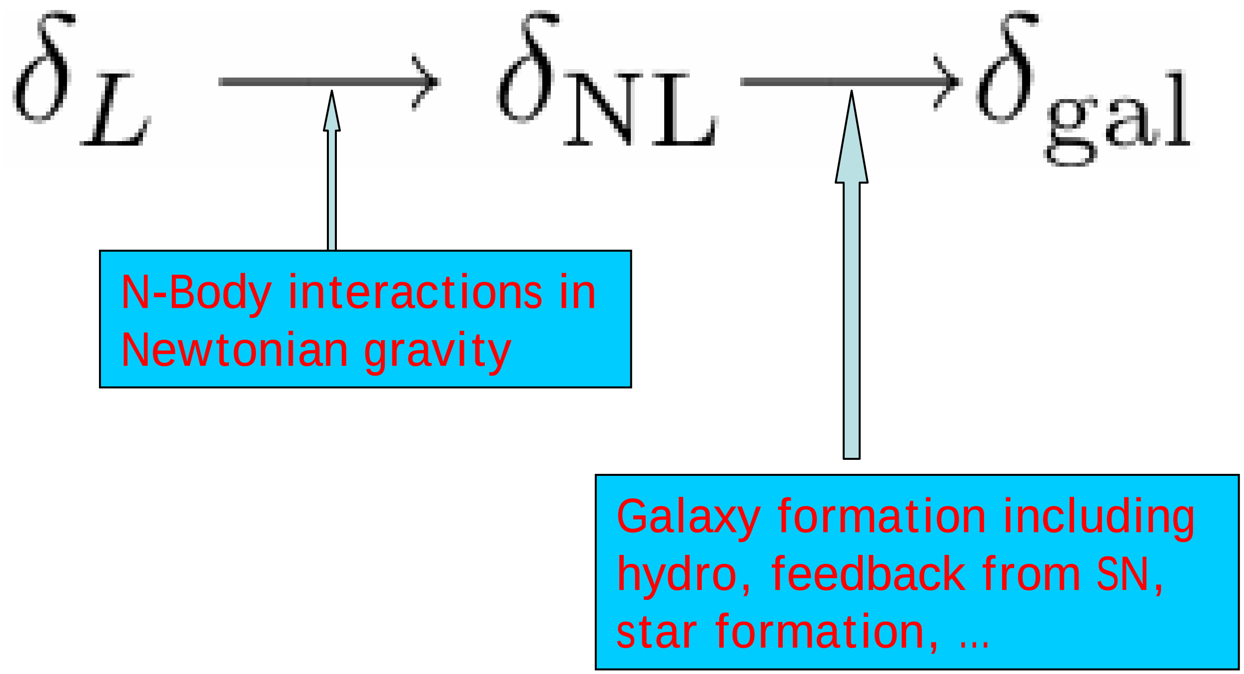
The *Sloan Digital Sky Survey (SDSS)* and the *Two Degree Field (2dF)* both have measured positions and redshifts (which are related to distances) of hundreds of thousands of galaxies

Blanton et al. (2003) (astro-ph/0210215)



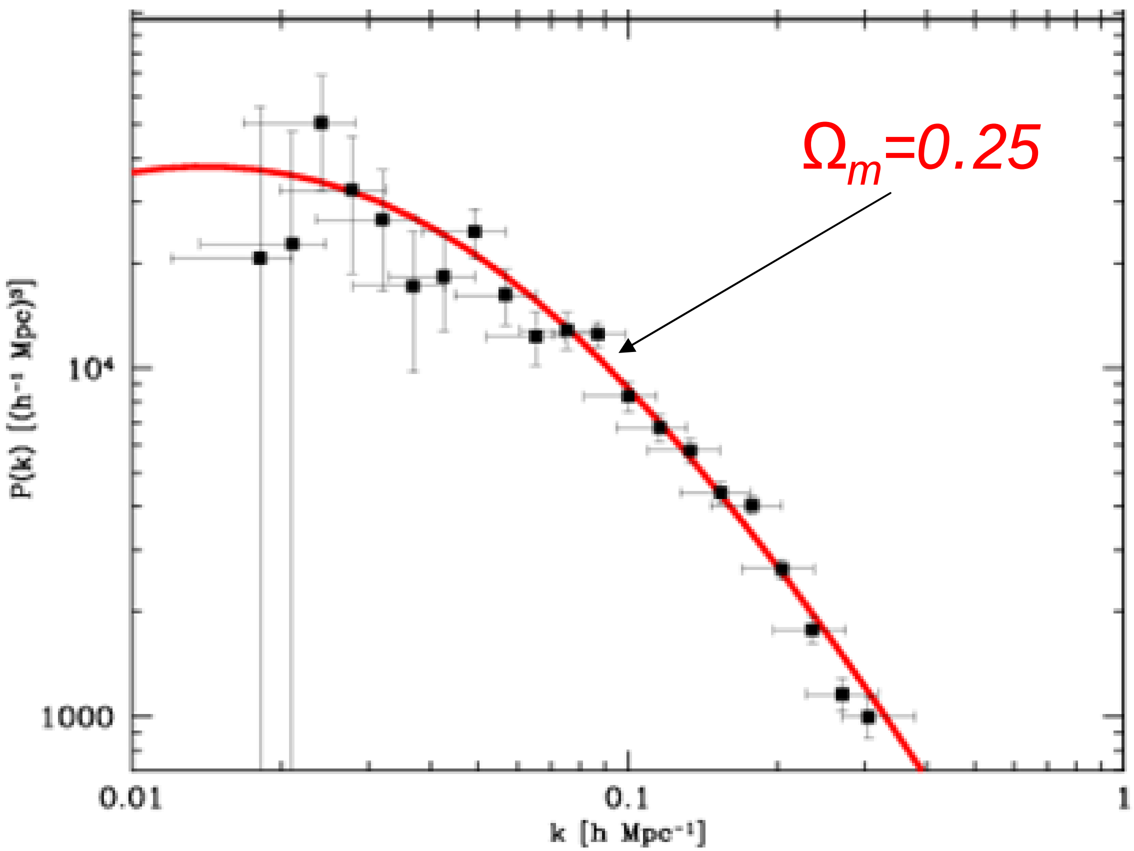
Non-trivial to compare observation to theory

The observables, δ_{gal}, are complicated *functionals* of the easy-to-predict linear matter density field, δ_L.



SDSS Galaxy Power Spectrum

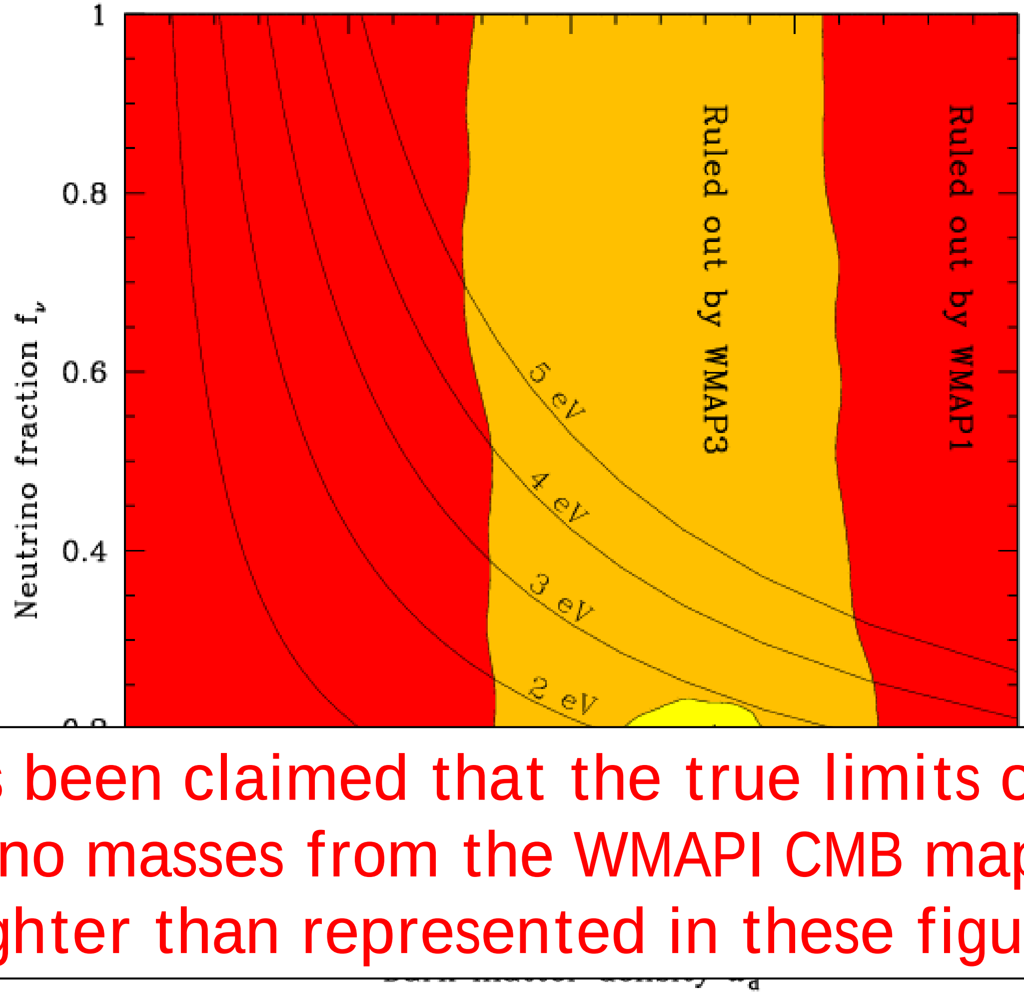
SDSS analysis of >200,000 galaxies



Tegmark et al. 2004

Results

Peaks and troughs in CMB sensitive to matter density: need both CMB and large scale structure to tighten constraints on neutrino mas

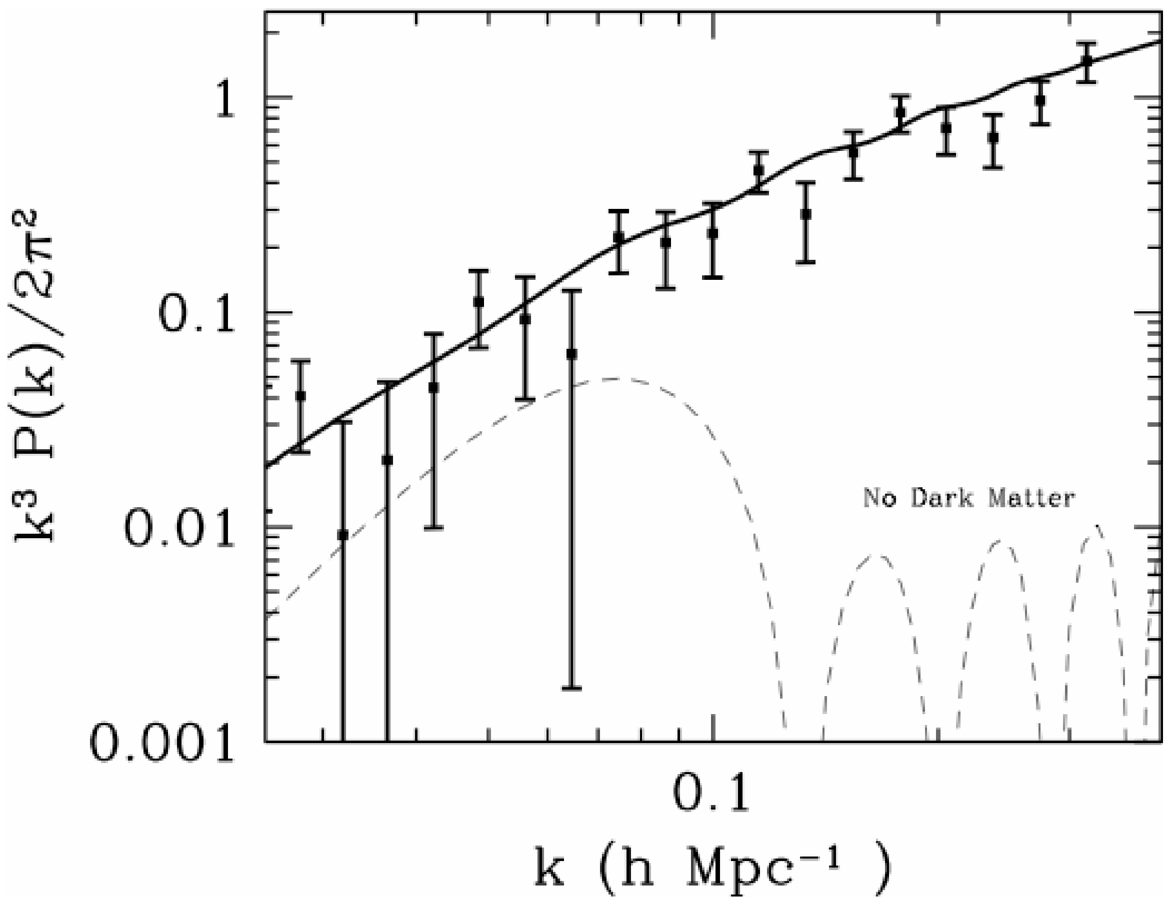


*It has been claimed that the true limits on neutrino masses from the WMAPI CMB maps are tighter than represented in these figures.

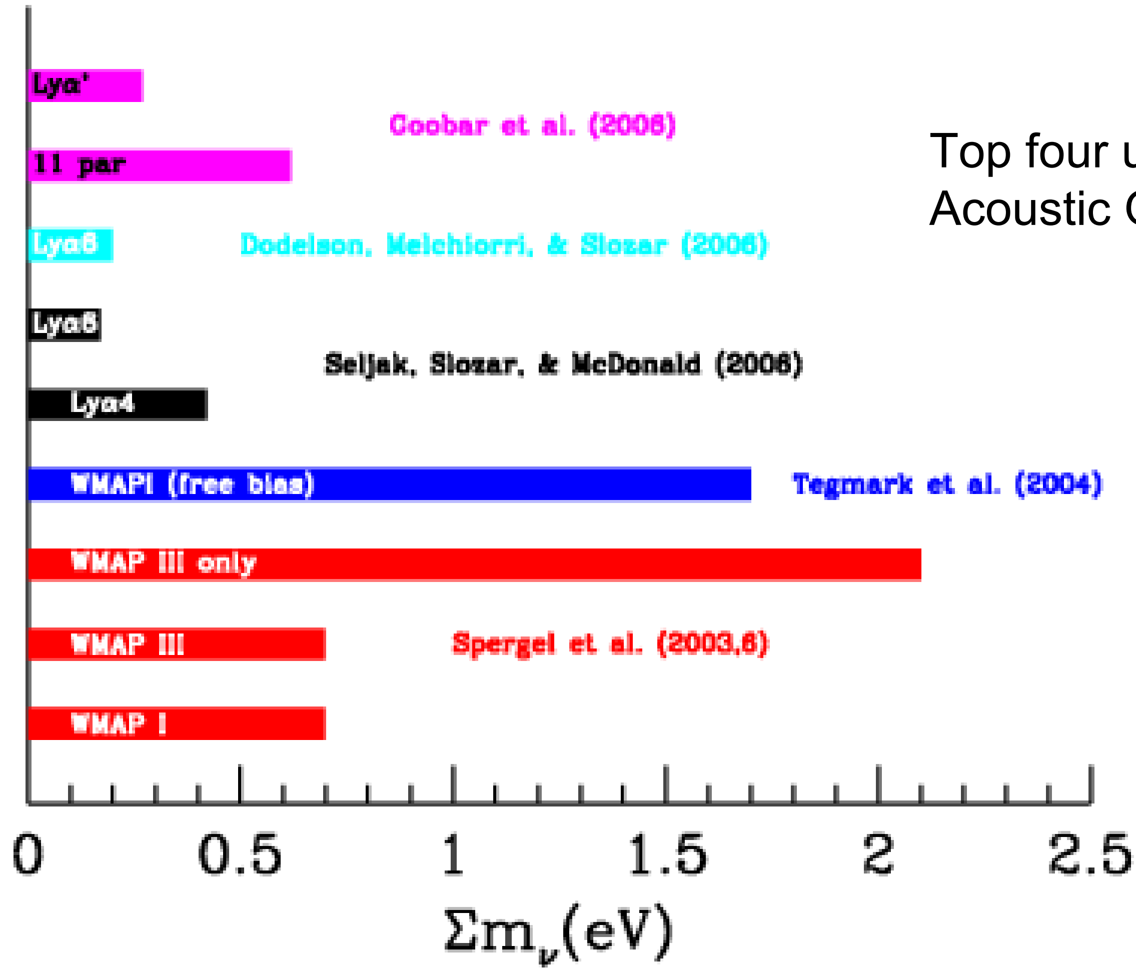
Tegmark, et al. 2006

No-Dark-Matter is strongly ruled out

Y-axis shows dimensionless measure of clumpiness; if it stays below one (as it would if there were no DM), no nonlinear structures form in the universe



Neutrino Mass constraints are typically sub-eV

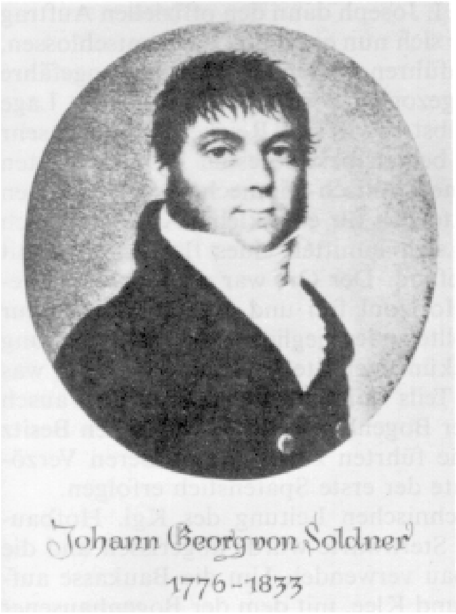


Top four use Baryon Acoustic Oscillations

Can we do better?

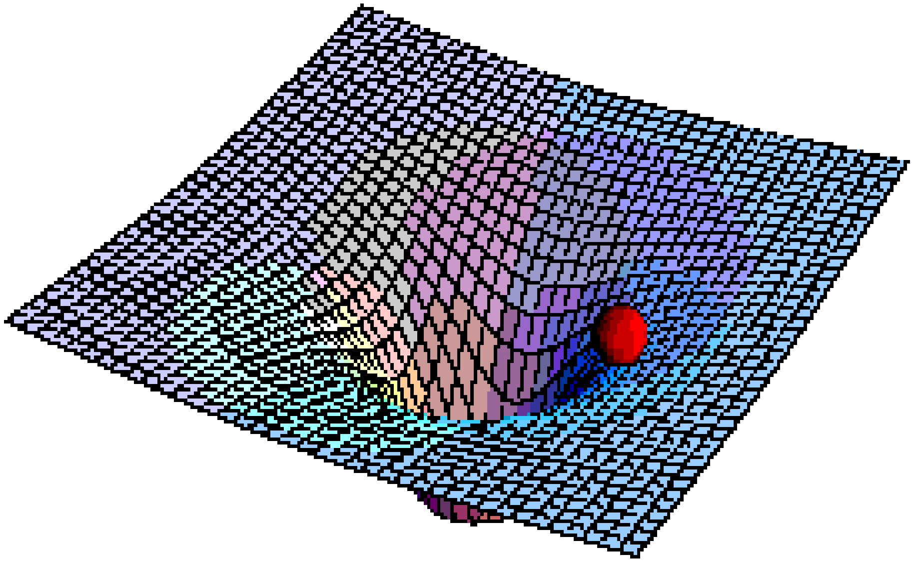
Currently running up against systematics, induced largely because we are depending on mass *tracers.*

1804: Astronomer Johann Soldner computes deflection of light due to Sun



Straightforward exercise in Newtonian gravity to show that particle passing within distance d of point mass M gets deflected by angle $2GM/d$

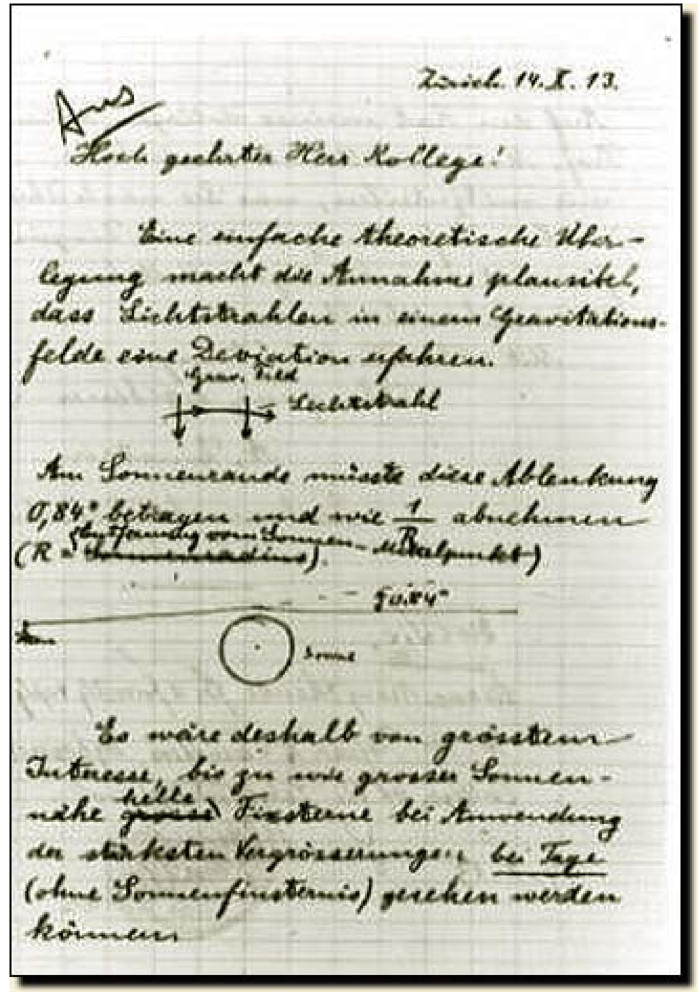
Over a hundred years later Einstein posits that mass distorts space: even light paths would be affected



Could this effect be detected?

Einstein writes to George Hale (Director of Mount Wilson Observatory) in 1913. He mentions the 0.84'' ($2GM_{\odot}/R_{\odot}c^2$) deflection expected from the Sun.

Wambsganss 1998


Zürich. 14.X.13.

Aus

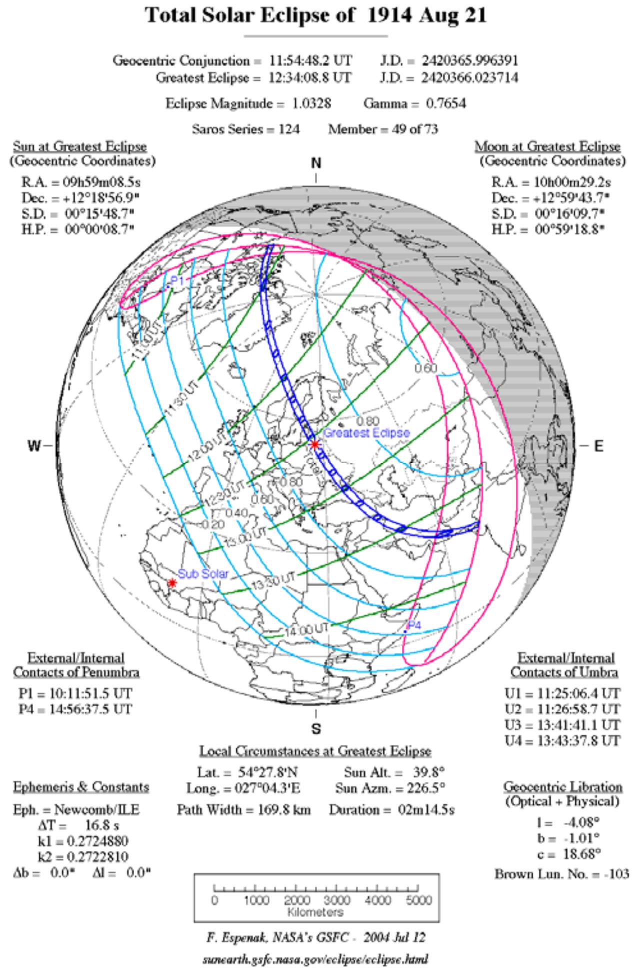
Hoch geehrter Herr Kollege!

Eine einfache theoretische Überlegung macht die Annahme plausibel, dass Lichtstrahlen in einem Gravitationsfelde eine Deviation erfahren.

Am Sonnenrande müsste diese Ablenkung 0,84" betragen und wie 1/R abnehmen (R = Entfernung vom Sonnen-Mittelpunkt).

Es wäre deshalb von grösstem Interesse, bis zu wie grosser Sonnennähe helle Fixsterne bei Anwendung der stärksten Vergrösserungen bei Tage (ohne Sonnenfinsternis) gesehen werden können.

The next total solar eclipse was August 21, 1914. An expedition was sent to observe in the region of greatest eclipse ...



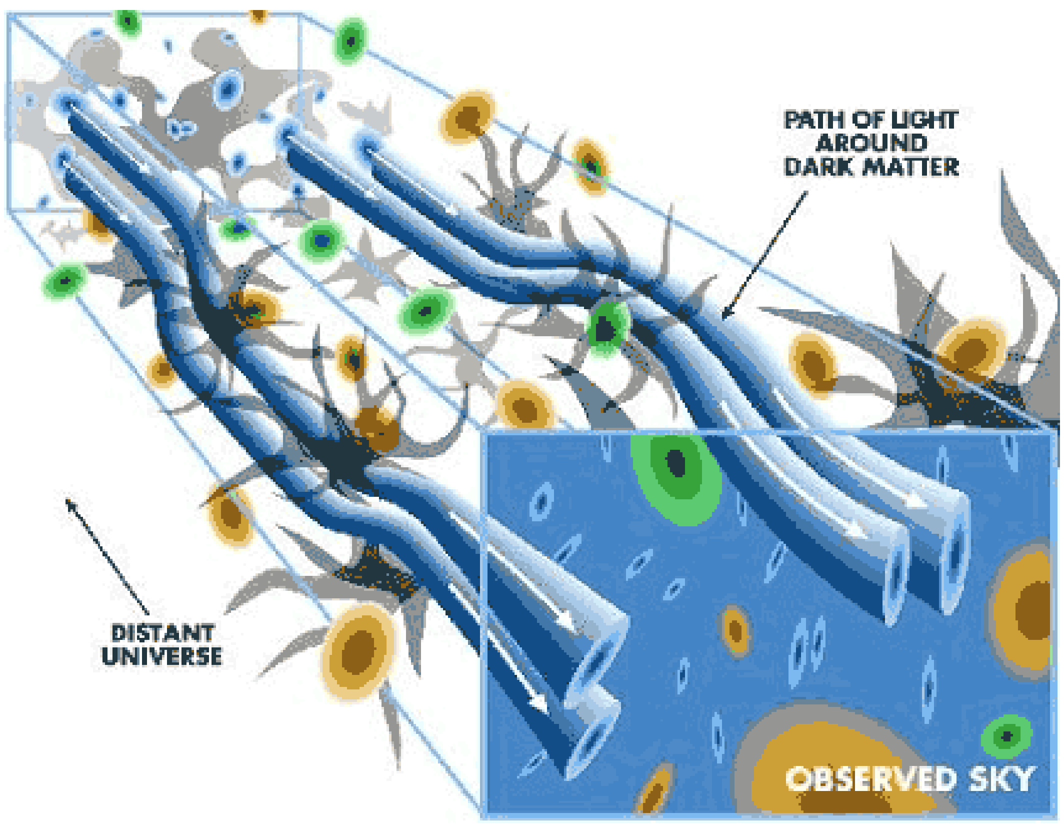
Russian Crimean Peninsula

1914 was not a good time to start a scientific expedition in Europe

The astronomers were captured by Russian soldiers and released a month later … with no data

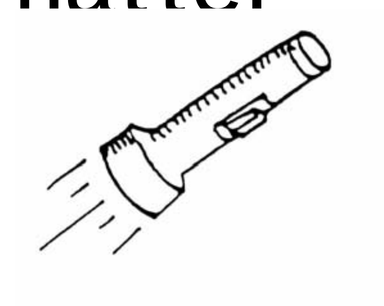
… which in retrospect is a good thing. Einstein improved his theory over the next several years. He eventually concluded that the deflection should be twice as large as the Newtonian result … And this was confirmed by the famous expeditions in 1919.

Exploit this: Gravitational Lensing



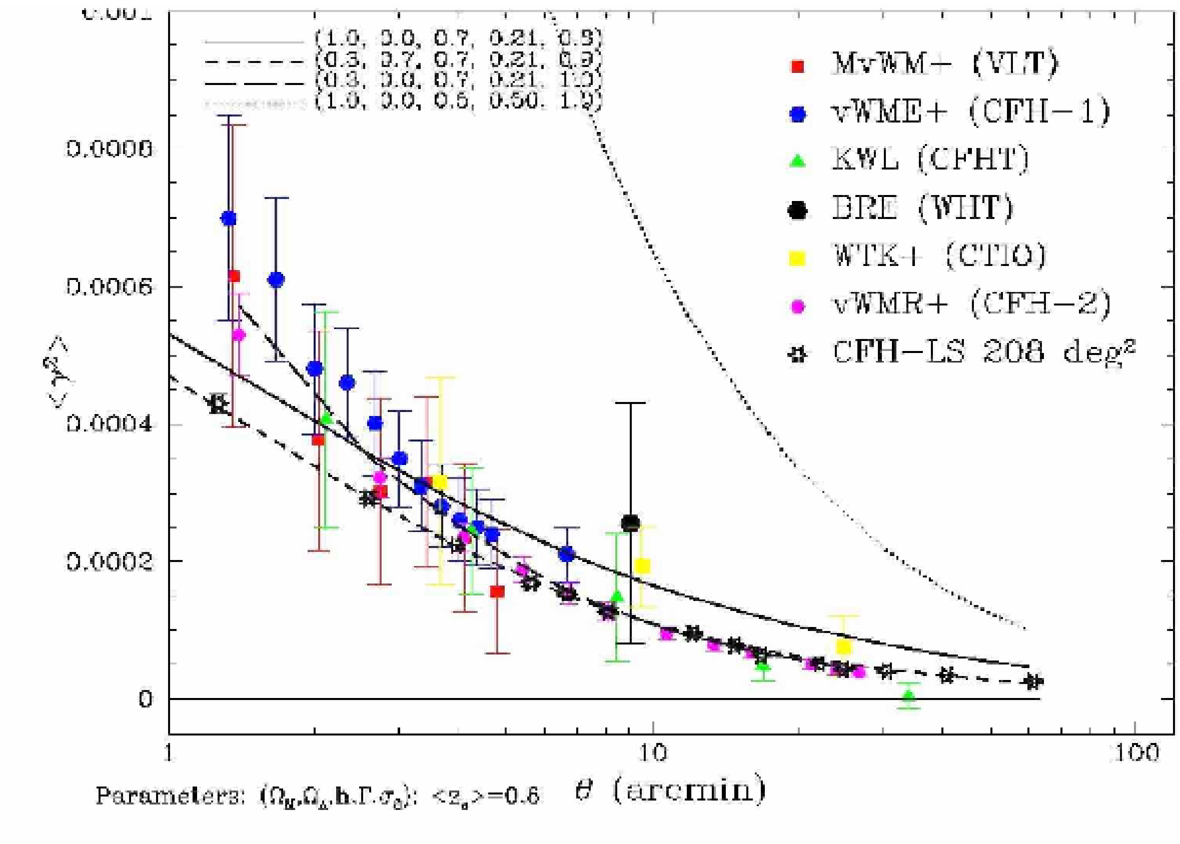
Learn about the intervening mass distribution

Big Advantage of Lensing: Sensitive to potential wells due to **all** matter



No longer need to use tracers

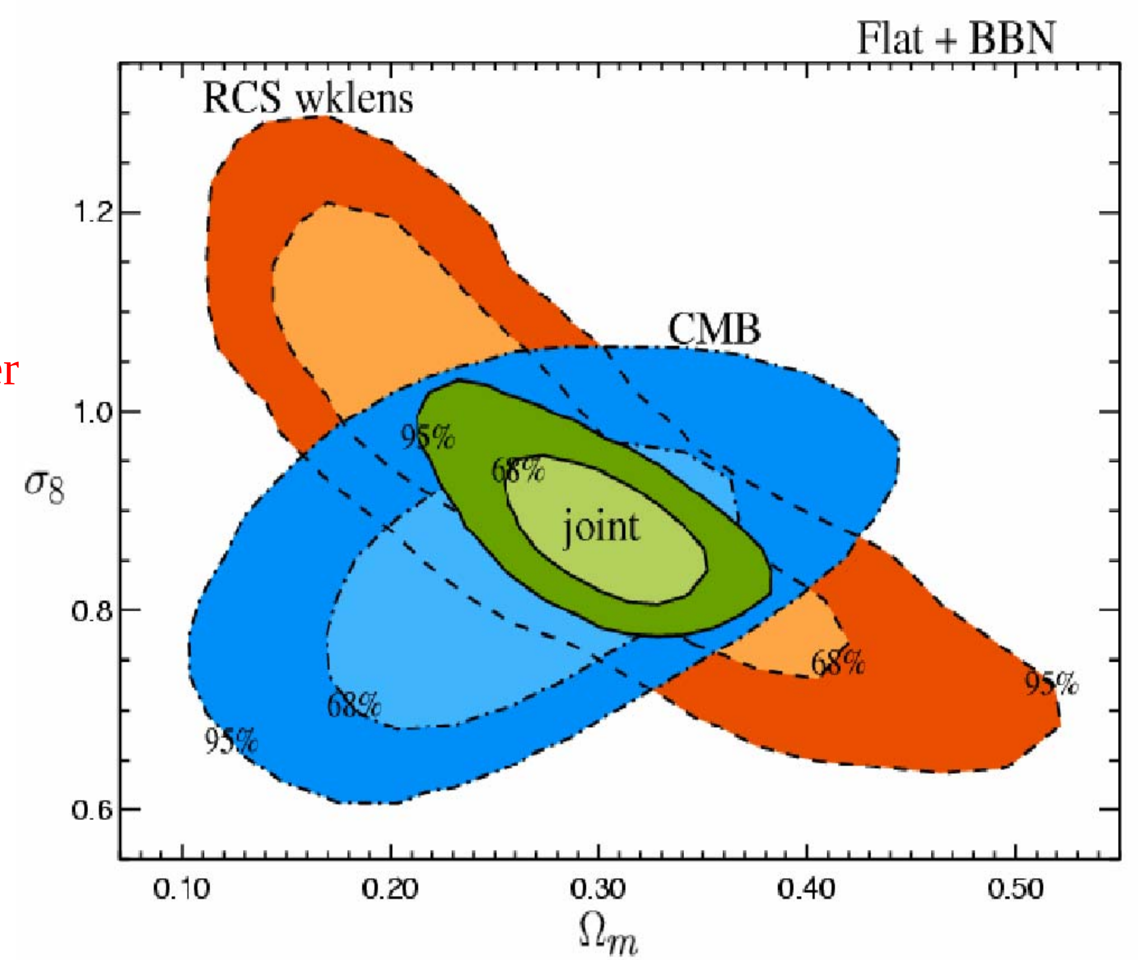
Four seminal measurements in 2000; ~dozen since then



Van Waerbeke & Mellier 2003

Constraints on parameters

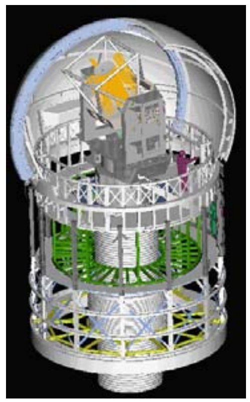
Amplitude of Matter
fluctuations



Contaldi, Hoekstra, Lewis: astro-ph/0302435

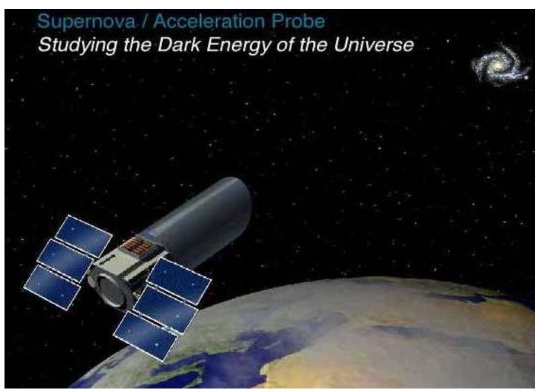
Several Upcoming Surveys

Panstarrs

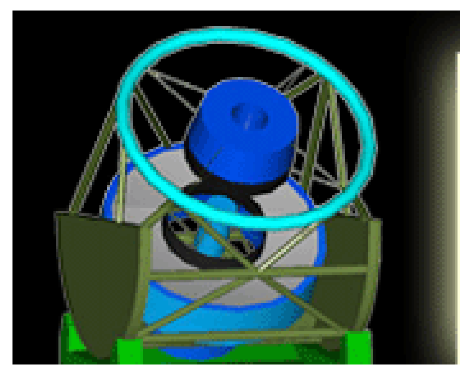


DARK ENERGY
Survey

SNAP



LSST



Final Slide

If you want to get your hands dirty check out ...

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http://www.mpa-garching.mpg.de/gadget/
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Meet me at the bar tonight if you have a good idea about ...

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why different analyses [sometimes
of the same data!] get different
limits on the neutrino mass.
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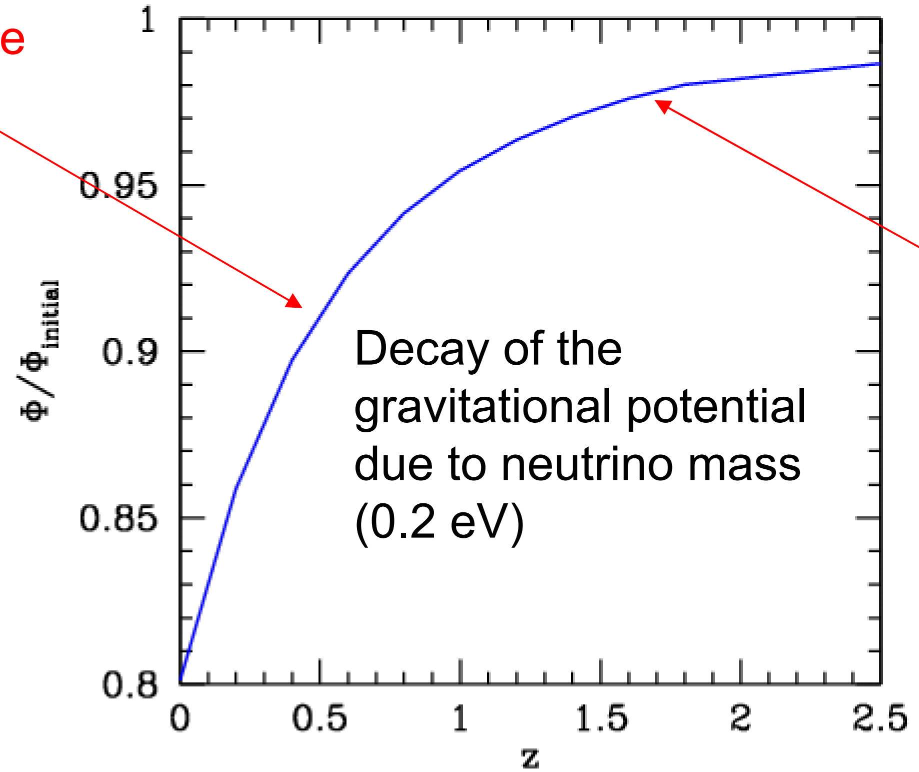
Tomography: Divide Background (Tracer) Galaxies into High and Low Redshift Bins

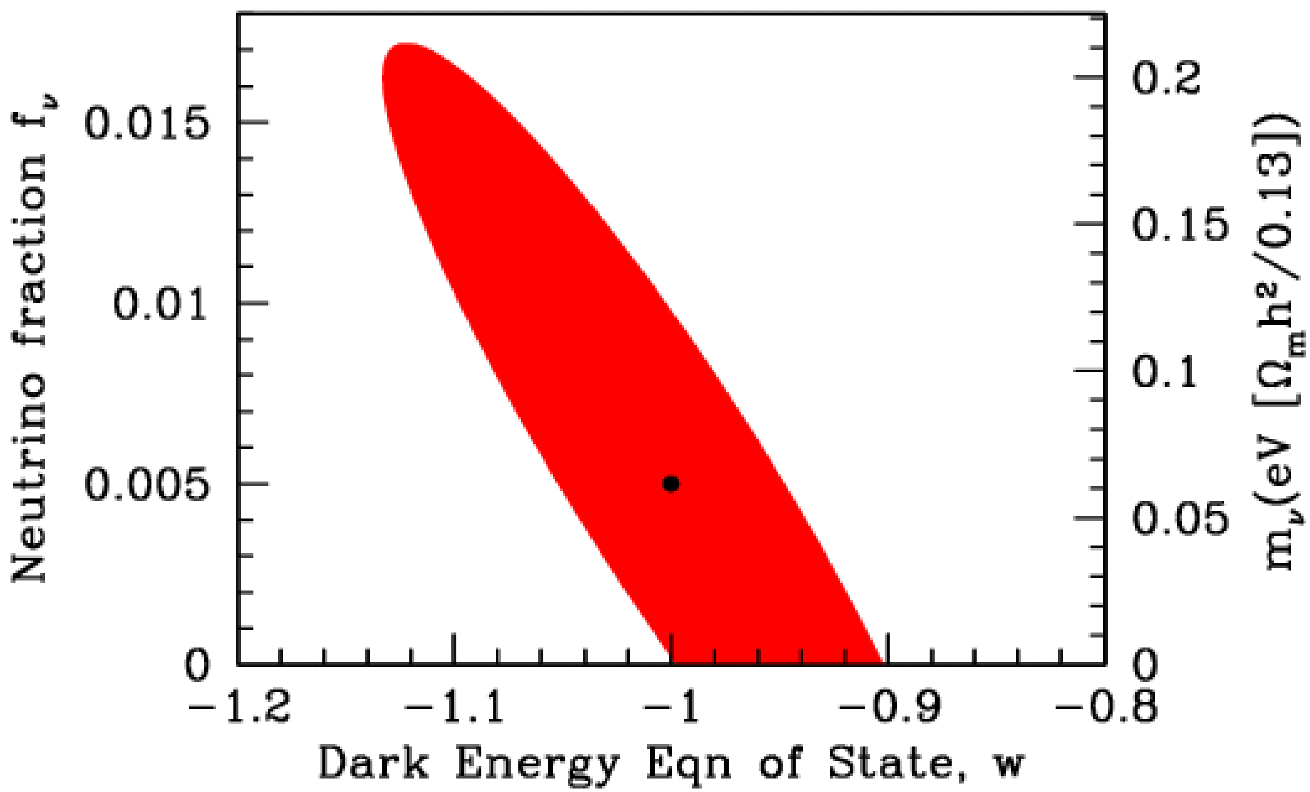
Low redshift galaxies sensitive to this



Very high redshift galaxies sensitive to this

Even if you're here only to learn about neutrinos, you need to understand dark energy

Projection for deep survey over 1/10 of the sky



Abazjian & Dodelson 2003

Clumping on Scale *k*

- Dimensionless quantity akin to $l^2 C_l$

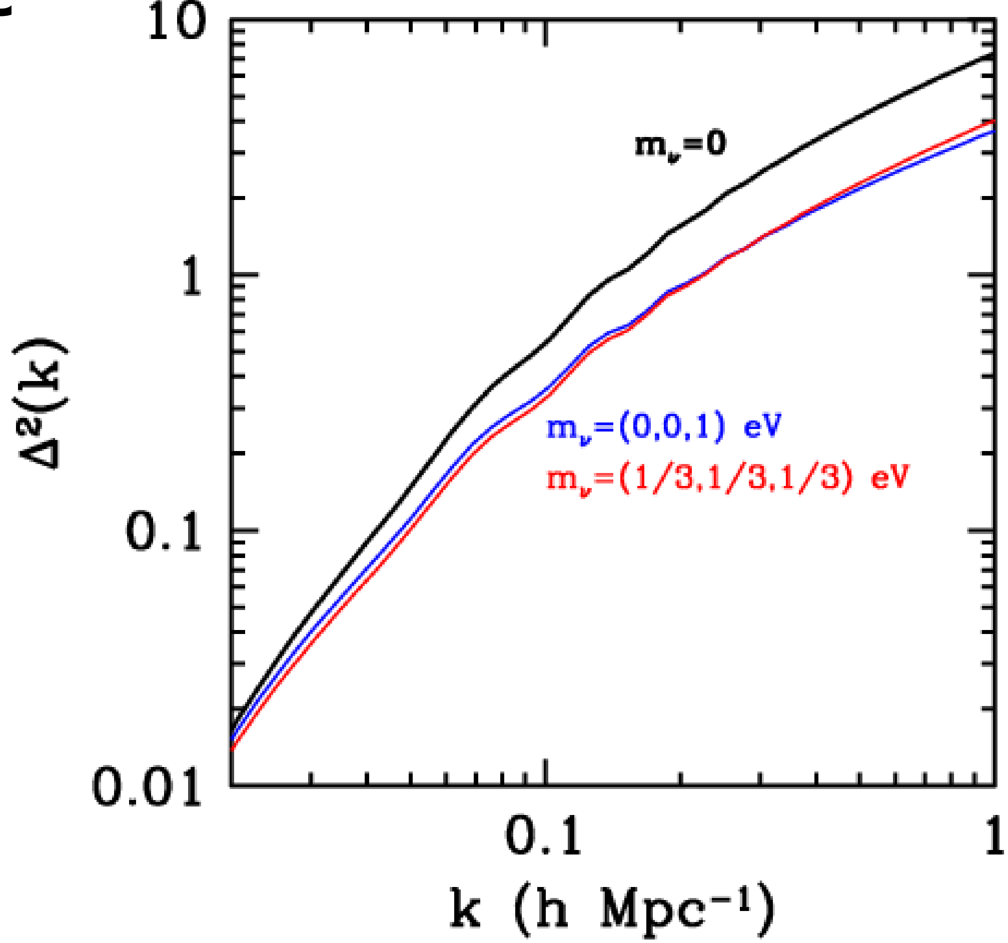
$$\Delta^2 \equiv \frac{k^3 P(k)}{2\pi^2}$$

- Variance of density:

$$\left\langle \left(\frac{\delta\rho}{\rho} \right)^2 \right\rangle = \int \frac{dk}{k} \Delta^2(k)$$

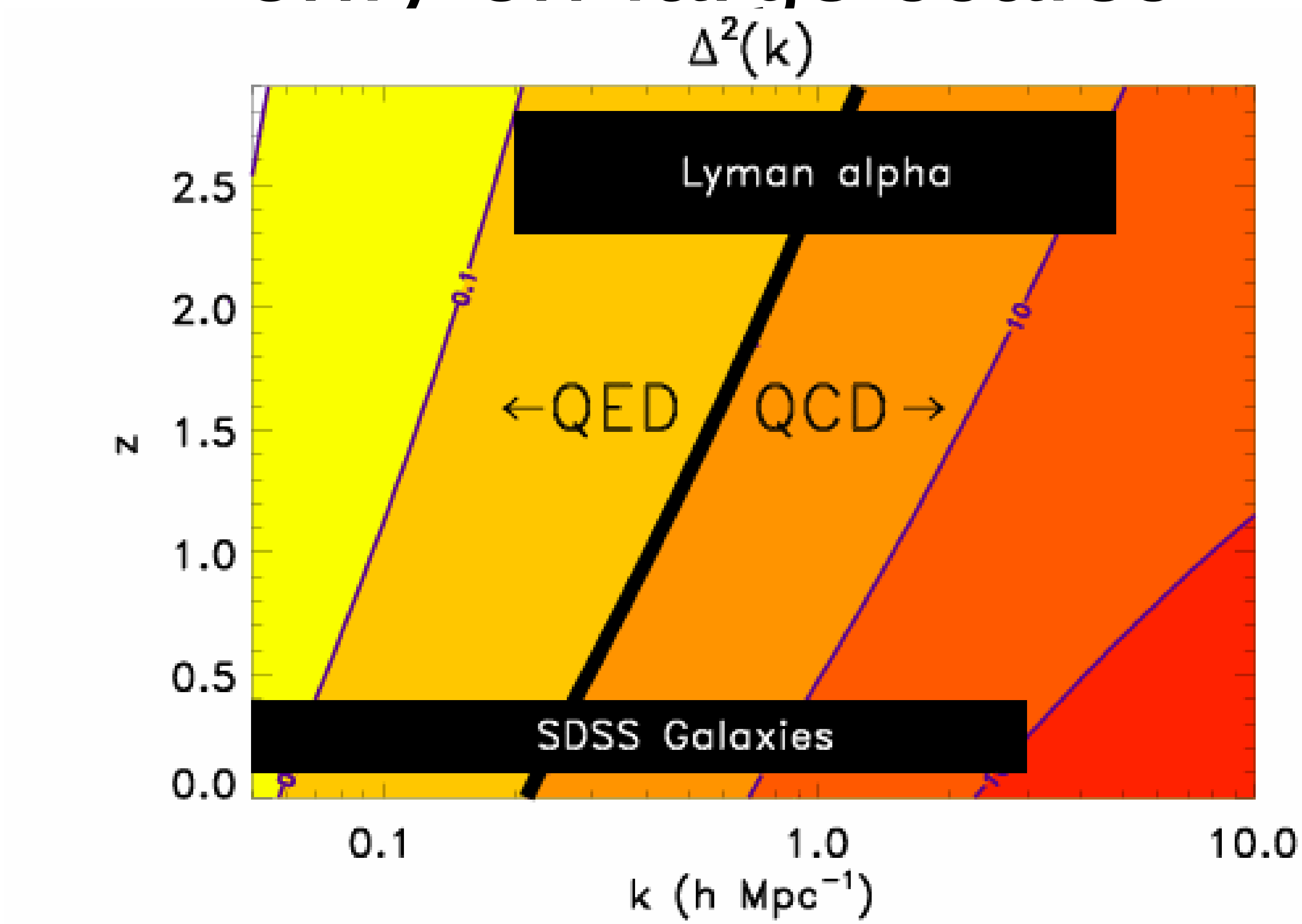
- Onset on nonlinearity: $\Delta^2 > 1$

Power spectrum depends only on massive neutrino energy density



Large Scale Structure probes Σm_ν

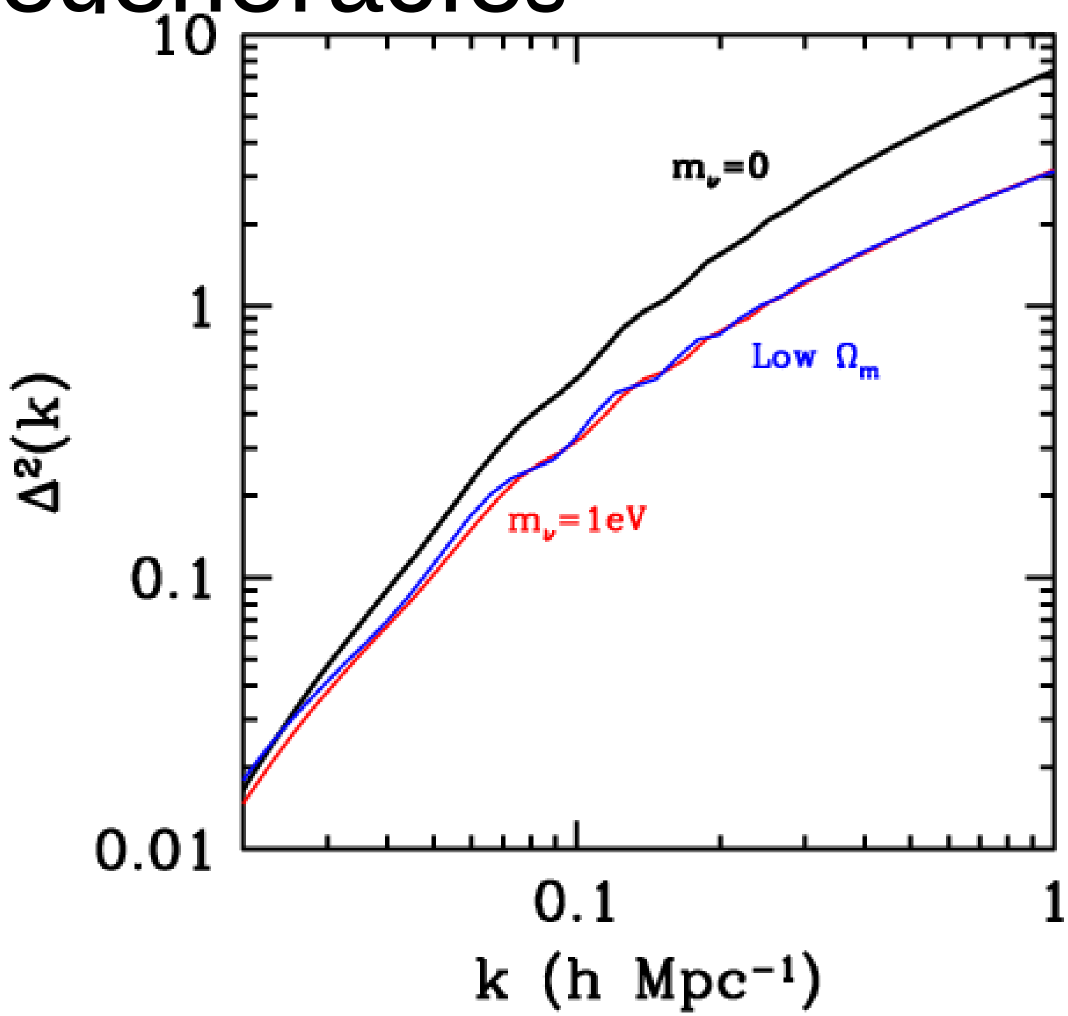
Comparing to predictions is easy only on large scales



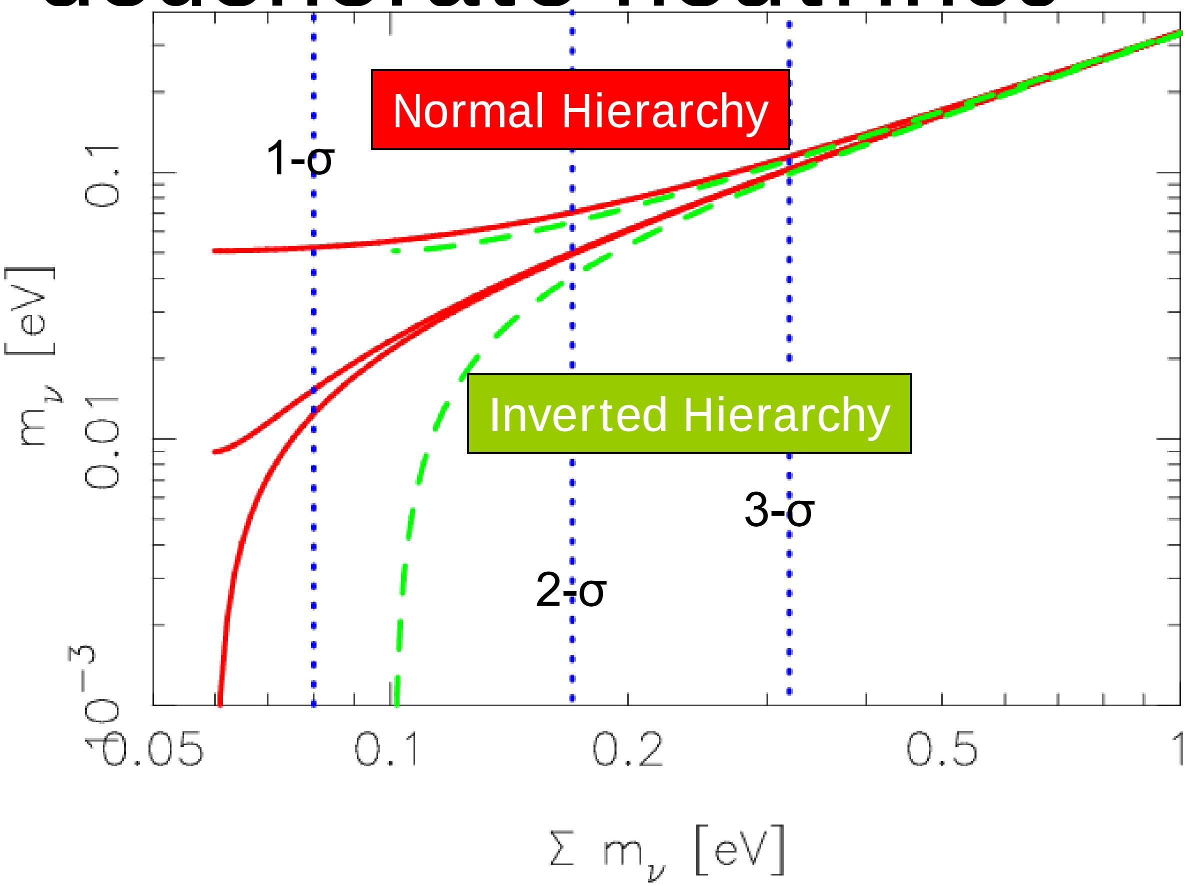
Degeneracies

- Lowering the matter density suppresses the power spectrum

- Close to degenerate with non-zero neutrino mass



Most aggressive limit disfavors 3 degenerate neutrinos



Seljak, Slosar, & McDonald 2006