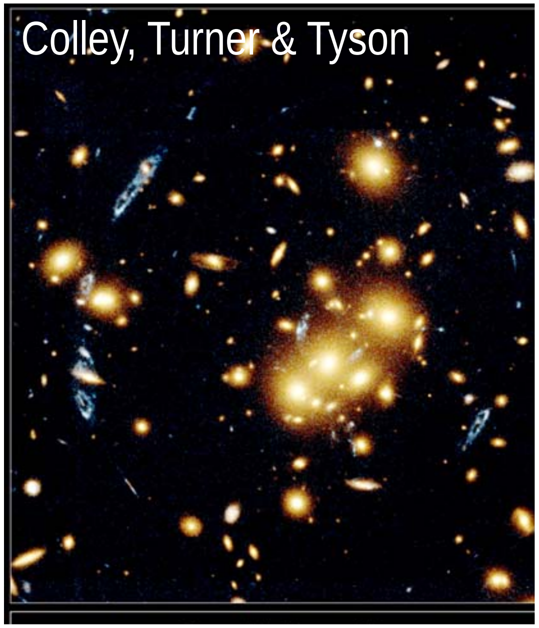
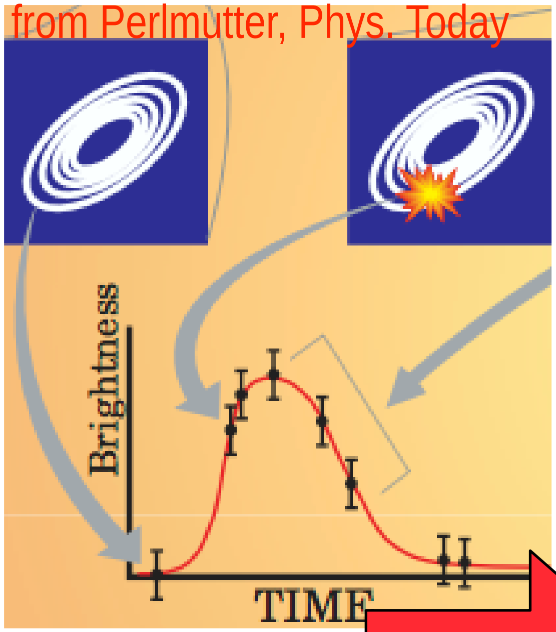
Direct Detection of WIMP Dark Matter: Part 1 - a primer

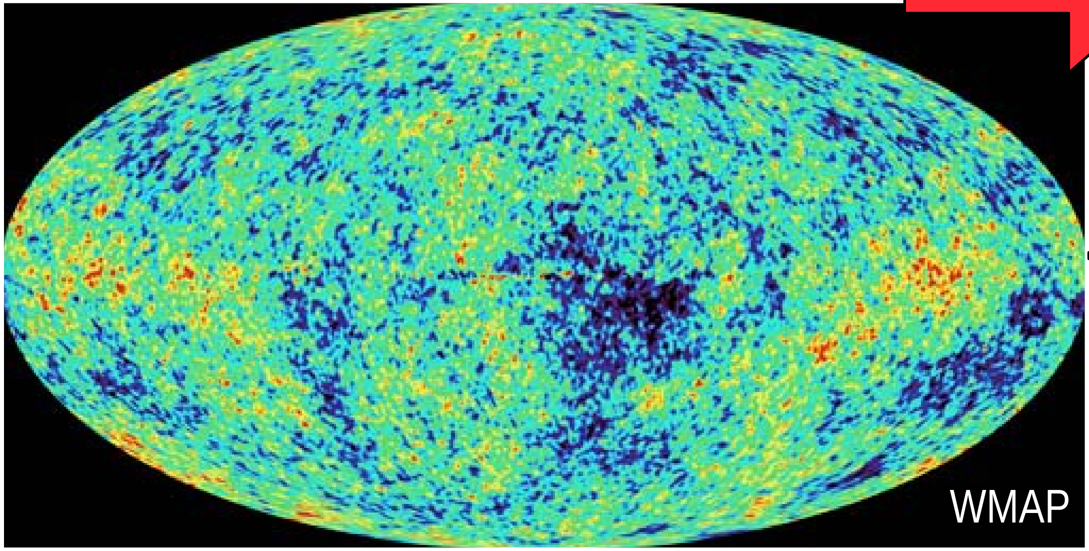
Dan Akerib
Case Western Reserve University
CDMS Collaboration

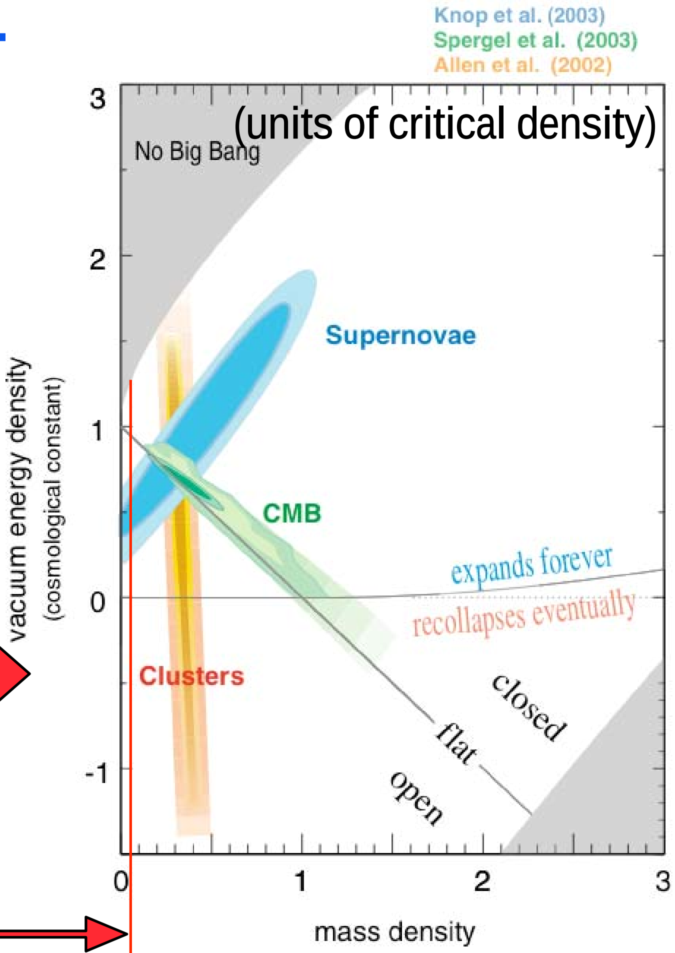
SLAC Summer Institute
Aug 2, 2007

Standard Cosmology

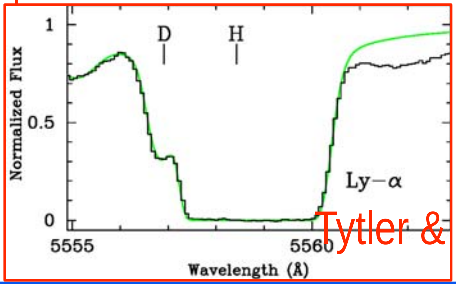

Colley, Turner & Tyson


from Perlmutter, Phys. Today


WMAP



(units of critical density)



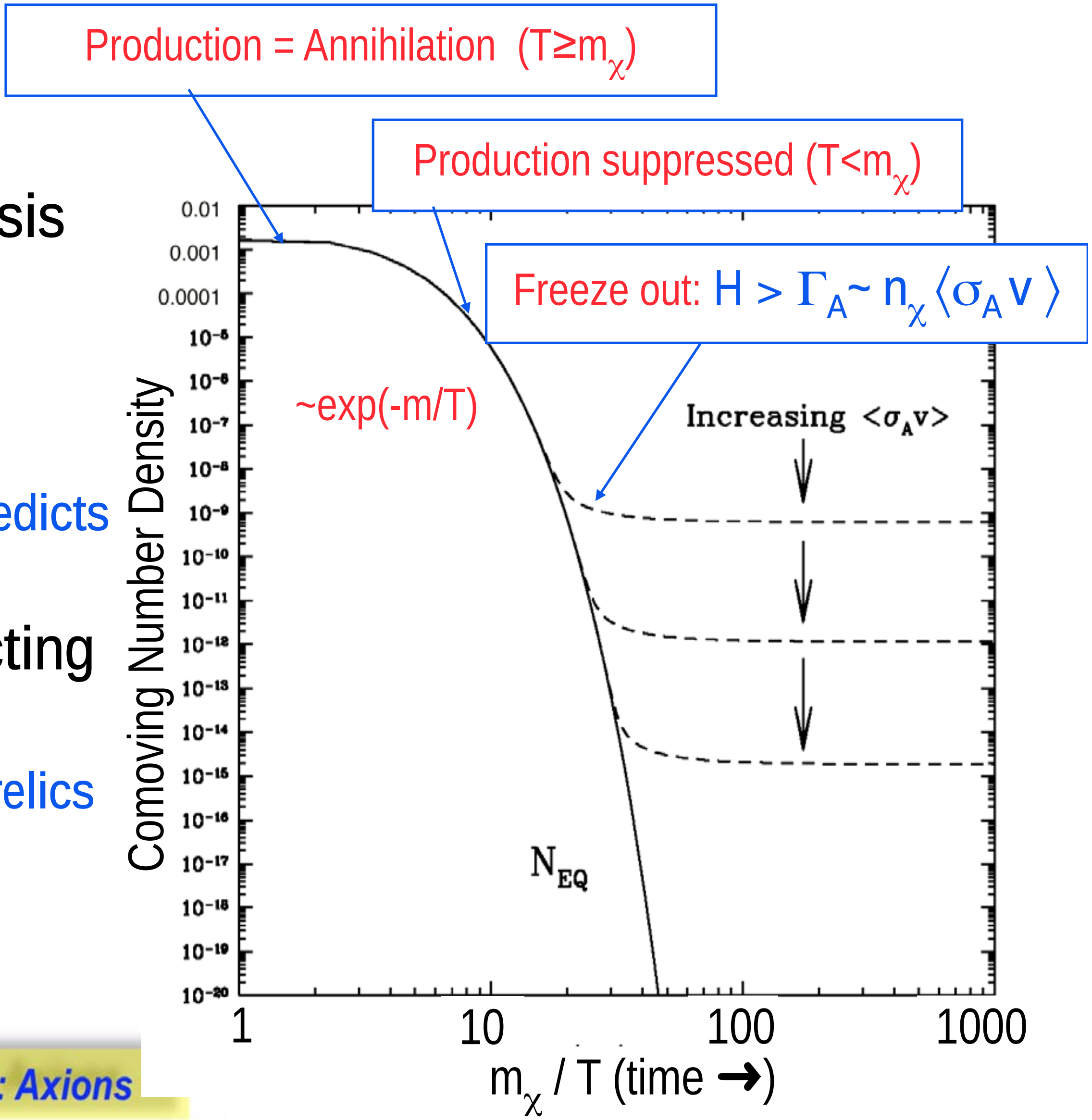
Tytler & Burles

Non-Baryonic Dark Matter

- Matter density
  - $\Omega_{\text{Matter}} = 0.30 \pm 0.04$
- Big Bang Nucleosynthesis
  - $\Omega_{\text{Baryons}} = 0.05 \pm 0.005$
- Nature of dark matter
  - Non-baryonic
  - Large scale structure predicts DM is 'cold'
- WIMPs – Weakly Interacting Massive Particle
  - ~10--1000 GeV Thermal relics
  - T_{FO} ~ m/20
  - $\sigma_A \sim$ electroweak scale

SUSY/LSP

Next week: Axions

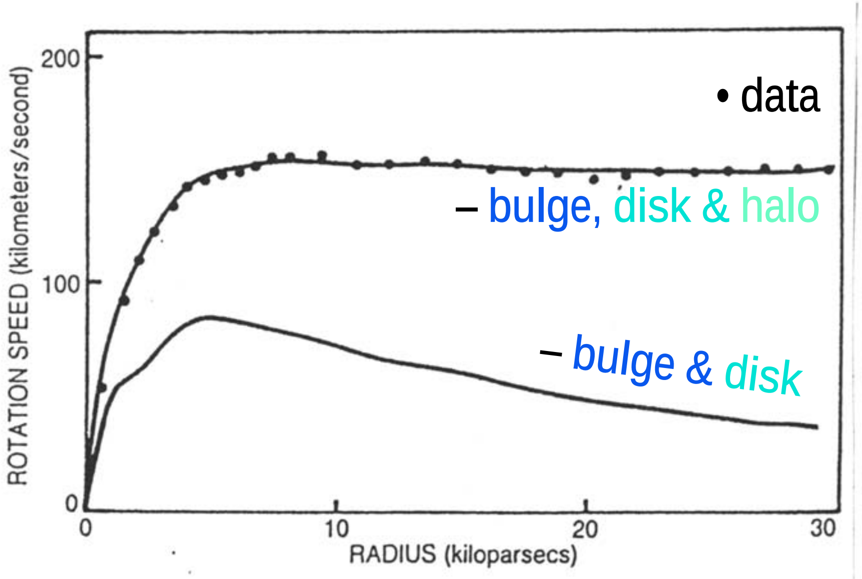

Production = Annihilation $(T \geq m_\chi)$

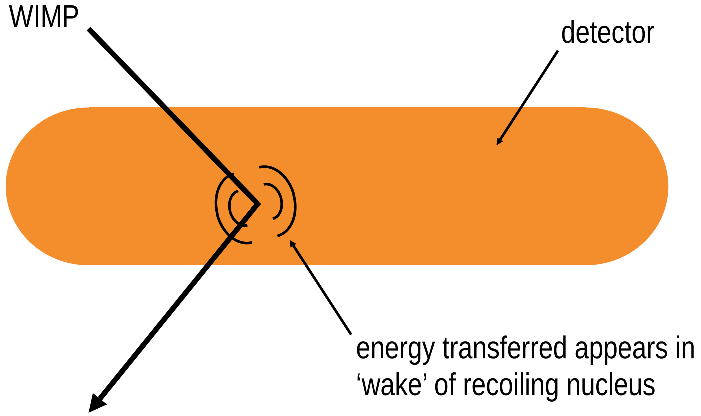
Production suppressed $(T < m_\chi)$

Freeze out: $H > \Gamma_A \sim n_\chi \langle \sigma_A v \rangle$

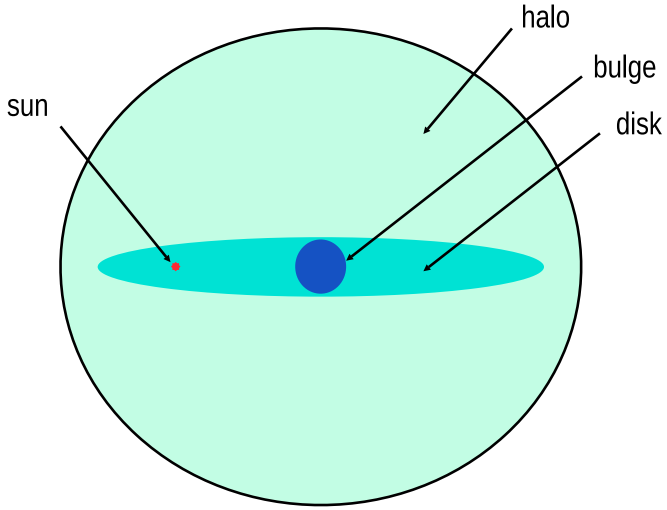
~exp(-m/T)

WIMPs in the Galactic Halo



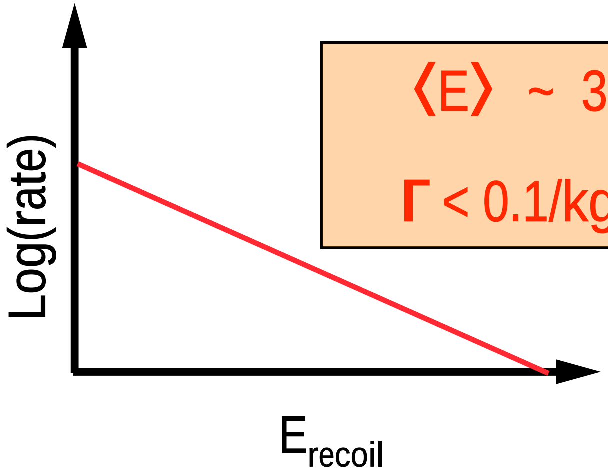


WIMP-Nucleus Scattering



The Milky Way

Scatter from a Nucleus in a Terrestrial Particle Detector



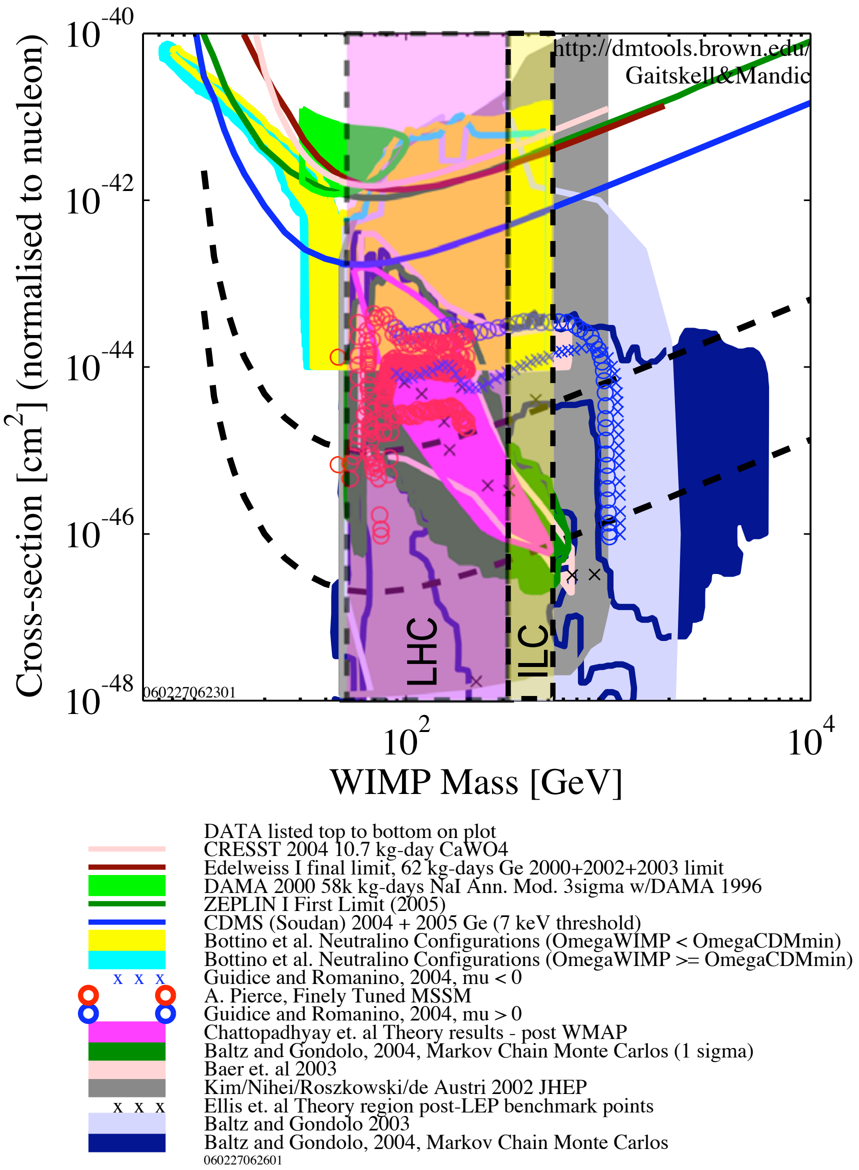
$\langle E \rangle \sim$ 30 keV

$\Gamma < 0.1$/kg/day

SUSY Dark Matter: elastic scattering cross section

- The 'standard' progress plot
  - Direct-search experimental bounds
- Theory
  - Sample SUSY parameter space
  - Apply accelerator and model-specific particle physics constraints
  - Apply cosmological bound on relic density

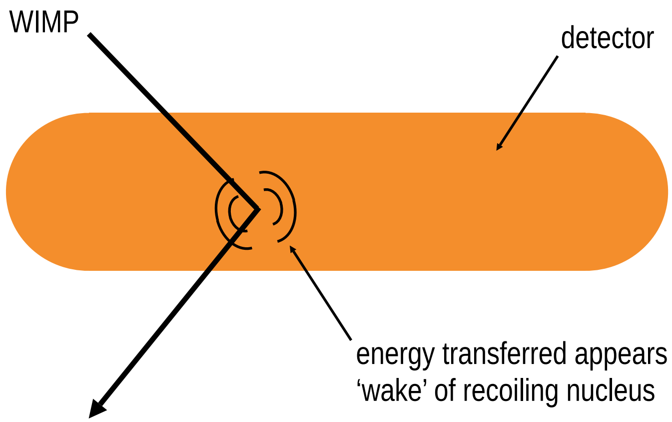
⇒ Extract allowed region for WIMP-nucleon cross-section versus WIMP mass



Broad theoretical landscape: much of it testable with next and next-next generation DM searches and/or next and next-next generation accelerators

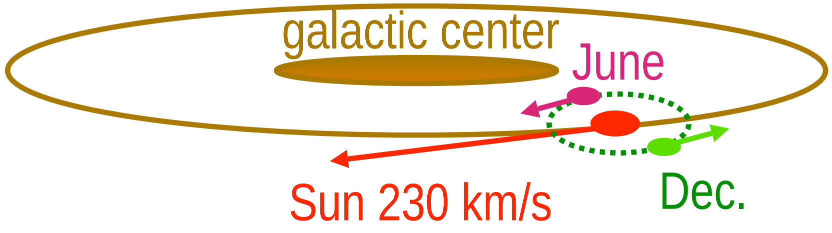
WIMPs in the Galactic Halo

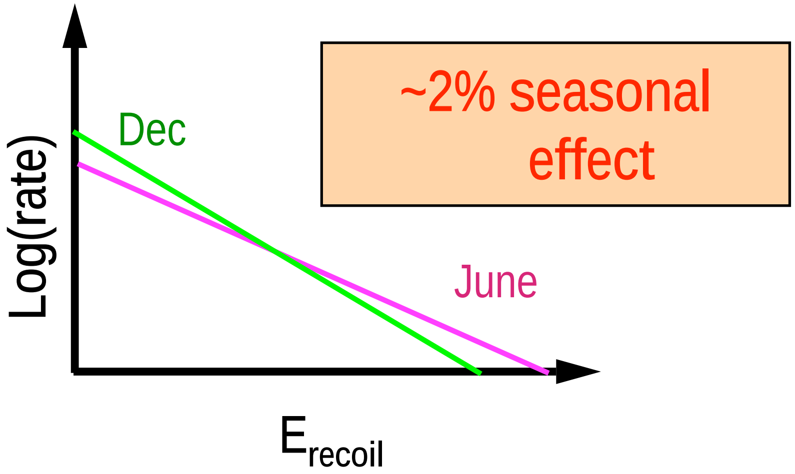
- Exploit movements of Earth/ Sun through WIMP halo
  - Direction of recoil -- most events should be opposite Earth/Sun direction (Spergel 1988)
  - Annual modulation -- harder spectrum when Earth travels with sun (Drukier, Freese, & Spergel 1986)



WIMP-Nucleus Scattering

Scatter from a Nucleus in a Terrestrial Particle Detector

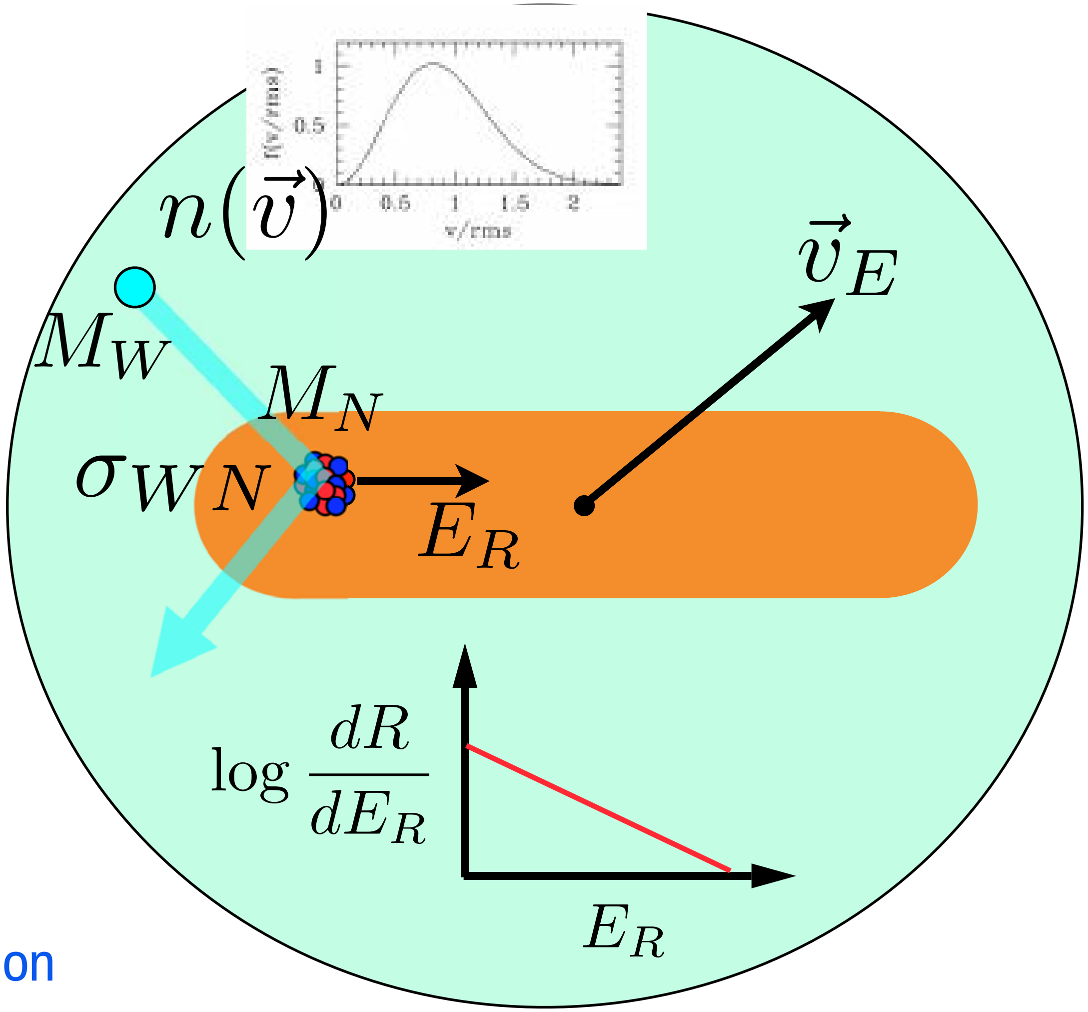




Defining the Signal

- Kinematics
  - halo potential
  - WIMP mass
  - target mass & velocity
- Rate
  - halo density
  - cross section
    - SD/SI
    - coherence & form factors
- Primary signal
- Secondary features
  - annual modulation of rate
  - diurnal modulation of direction
- Backgrounds
- Experimental methods & results

} mostly tomorrow

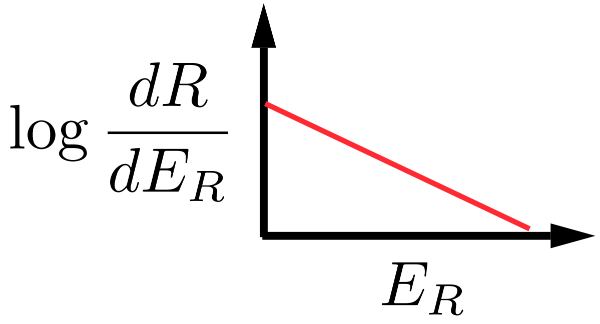


References and further reading

- References and notation - generally following the treatment of two key review articles:
  - J.D. Lewin and P.F. Smith, Astroparticle Physics 6 (1996)
  - G. Jungman, M. Kamionkowski and K. Griest, Physics Reports 267 (1996)
- A very nice pedagogic reference
  - S. Golwala, Ph.D. thesis, UC Berkeley (2000) (http://cdms.berkeley.edu)
    - 10th CDMS dissertation - first astrophysics results!
    - winner of first APS Tanaka dissertation prize!
- See also
  - R.J. Gaitskell (experiment review) in Ann. Rev. Nucl. Part. Sci. 54 (2004)
  - G. Heusser (low background techniques) in Ann. Rev. Nucl. Part. Sci. 45 (1995)

Differential energy spectrum (simplified)

$$\frac{dR}{dE_R} = \frac{R_0}{E_0 r} e^{-E_R/E_0 r}$$



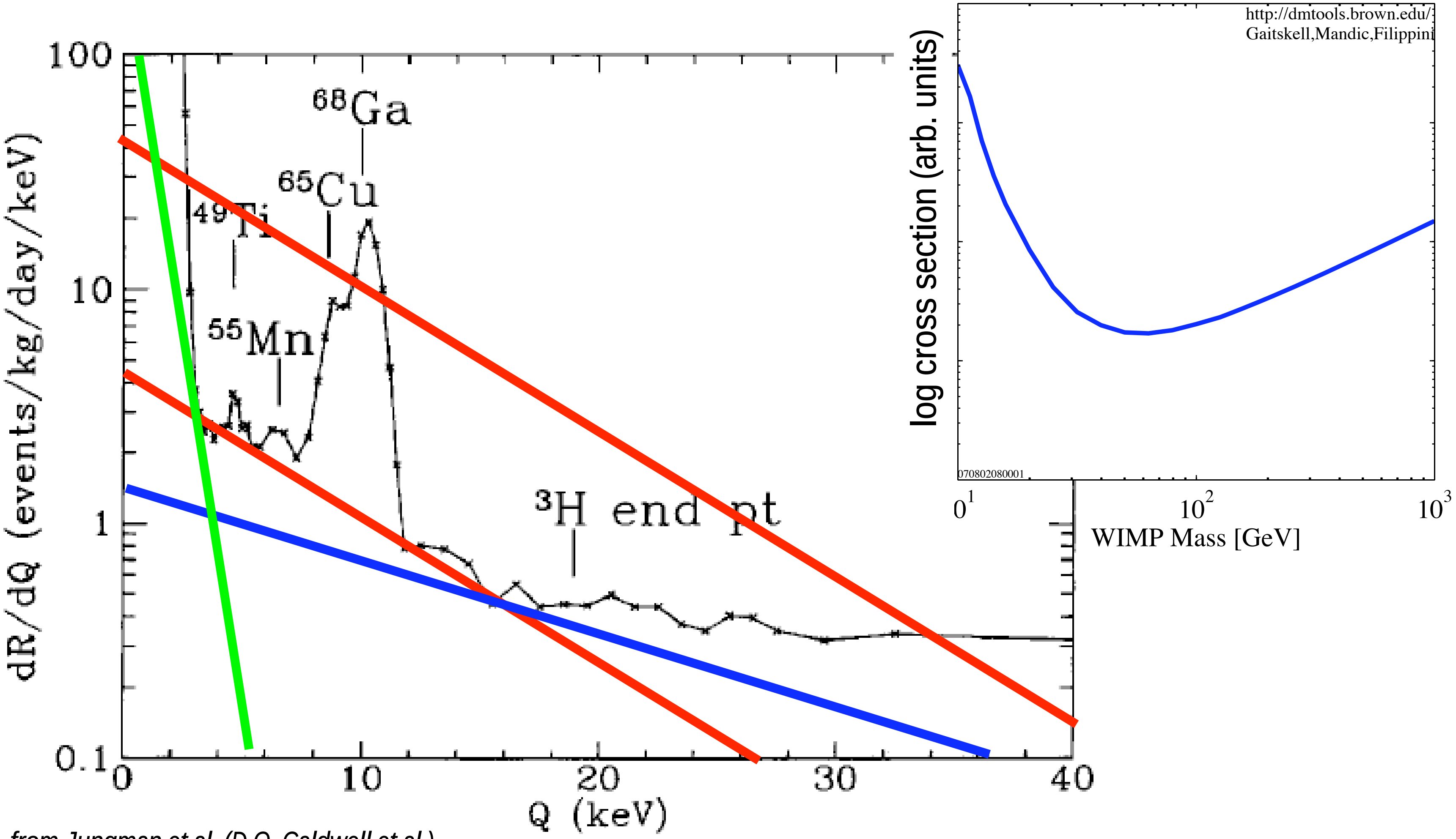
$$\begin{aligned}
R &= \text{event rate per unit mass} \\
E_R &= \text{recoil energy} \\
R_0 &= \text{total event rate} \\
E_0 &= \text{most probable incident} \\
& \text{energy (Maxwellian)} \\
r &= \frac{4M_W M_N}{(M_W + M_N)^2} \\
M_W &= \text{mass of WIMP} \\
M_N &= \text{mass of target nucleus}
\end{aligned}$$

$$\int_0^\infty \frac{dR}{dE_R} dE_R = R_0$$

$$\begin{aligned}
\langle E_R \rangle &= \int_0^\infty E_R \frac{dR}{dE_R} dE_R \\
&= E_0 r
\end{aligned}$$

WIMP search - c.1988

- Germanium ionization detector (UCSB/UCB/LBL)



from Jungman et al. (D.O. Caldwell et al.)

Typical numbers

For:

$$\begin{aligned} M_W &= M_N = 100\,\mathrm{GeV/c^2} \\ &\Rightarrow r = 1 \end{aligned}$$

$$\beta \sim 0.75 \times 10^{-3} = 220\,\mathrm{km/s}$$

$$\begin{aligned} \langle E_R \rangle = E_0 &= \tfrac{1}{2}\, M_W \beta_0^2 \mathrm{c}^2 \\ &= \tfrac{1}{2}\, 100\, \frac{\mathrm{GeV}}{\mathrm{c}^2}\, (0.75 \times 10^{-3})^2\, \mathrm{c}^2 \\ &= 30\,\mathrm{keV} \end{aligned}$$

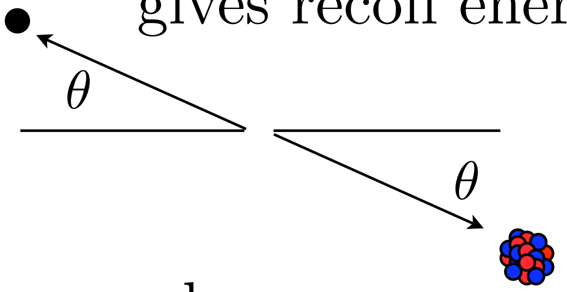
Refinements

$$\left.\frac{dR}{dE_R}\right|_{OBS} = R_0\, S(E_R)\, F^2(E_R)\, I$$

$$\begin{aligned} S(E_R) &= \text{spectral function — masses and kinematics} \\ F^2(E_R) &= \text{form factor correction, with } E_R = q^2/2M_W \\ I &= \text{interaction type} \end{aligned}$$

Kinematics

DM particle with velocity v and incident KE $E_i = \frac{1}{2} M_W v^2$
scattered at angle θ in CM frame
gives recoil energy in lab frame



$$E_R = E_i \, r \, \frac{(1 - \cos\theta)}{2}$$

where

$$r = \frac{4 m_r^2}{M_W \, M_N} = \frac{4 \, M_W \, M_N}{(M_W + M_N)^2}$$

and

$$m_r = \frac{M_W \, M_N}{M_W + M_N}$$

is the reduced mass

Kinematics

Isotropic scattering: uniform in $\cos\theta$

Incident WIMP with energy E_i gives recoil energies uniformly in

$$E_R = 0 \longrightarrow E_i\, r$$

Recall "familiar" case for equal masses ($r = 1$), target at rest, head-on collision

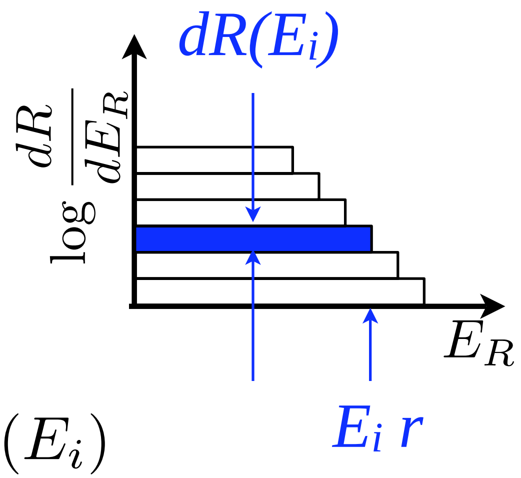
$$E_R = E_i$$

Overall spectrum? —sample incident spectrum

In each interval $E_i \rightarrow E_i + dE_i$

contribution to spectrum in $E_R \rightarrow E_R + dE_R$

at rate $dR(E_i)$ of



$$d\left(\frac{dR}{dE_R}(E_R)\right) = \frac{dR(E_i)}{E_i\, r}$$

Kinematics

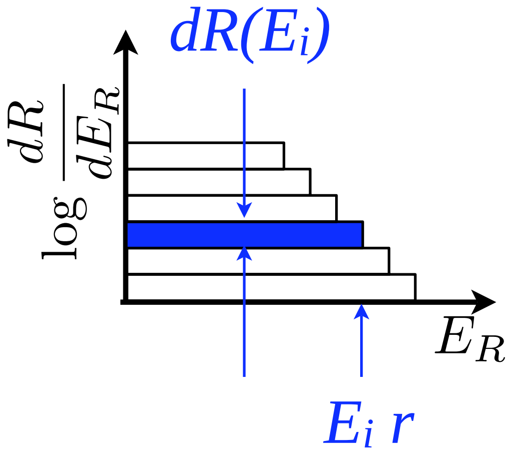
Need to integrate over range of incident energies

$$\frac{dR}{dE_R}(E_R) = \int_{E_{min}}^{E_{max}} \frac{dR(E_i)}{E_i\, r}$$

For E_{max} use ∞ or v_{esc} (later. . .)

For E_{min}, to get recoil of energy E_R need incident energy

$$E_i \geq \frac{E_R}{r} \equiv E_{min}$$



and also need differential rate. . .

Differential rate

In a kilogram of detector of nuclear mass number A

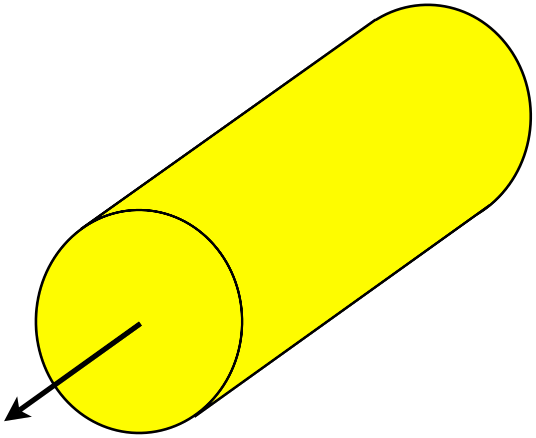
$$dR = \frac{N_0}{A}\,\sigma\, v\, dn$$

where the differential density dn is taken as a function v

$$dn = \frac{n_0}{k}\, f(\vec{v}, \vec{v}_E)\, d^3\vec{v}$$

with $n_0 = \rho_{DM}/M_W$ and normalization

$$k = \int f\, d^3\vec{v}$$



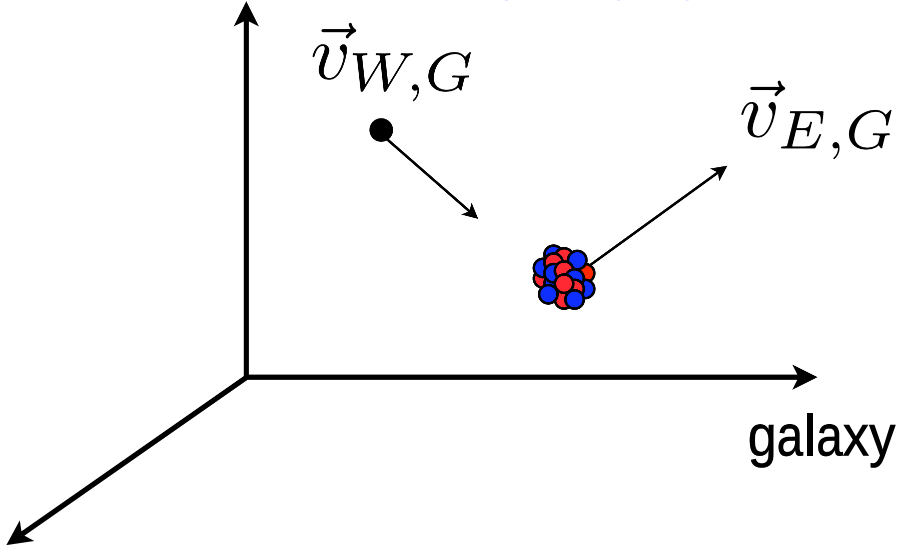
volume **σ**v swept per unit time contains dn(v) particles with velocity v

Coordinate system

collision kinematics

$$\begin{aligned} \vec{v} \quad = \quad \vec{v}_{W,E} &= \text{WIMP velocity in the target/Earth frame} \\ \vec{v}_{E,G} &= \text{Earth velocity in the Galaxy frame} \\ \vec{v}_{W,G} &= \text{WIMP velocity in the Galaxy frame} \end{aligned}$$

galaxy dynamics



$$\begin{aligned} \vec{v}_{W,G} &= \vec{v}_{W,E} + \vec{v}_{E,G} \\ &= \vec{v} \qquad + \vec{v}_E \end{aligned}$$

$$f(\vec{v}, \vec{v}_E) = e^{-(\vec{v}+\vec{v}_E)^2/v_0^2}$$

Maxwellian velocity distribution

Differential rate

For simplified case of $v_E = 0$ and $v_{esc} = \infty$

$$dR = R_0 \frac{1}{2\pi v_0^4}\, v\, f(v,0)\, d^3v$$

with $R_0 = 2\pi^{-\frac{1}{2}} \frac{N_0}{A} \frac{\rho_{DM}}{M_W} \sigma_0 v_0$

For Maxwellian $f(v,0) = e^{-v^2/v_0^2}$,
isotropic $d^3v \to 4\pi v^2 dv$,
$E_i = \frac{1}{2} M_W v^2$ and $E_0 = \frac{1}{2} M_W v_0^2$:

$$\begin{aligned} \frac{dR}{dE_R}(E_R) &= \int_{\frac{E_R}{r}}^{\infty} \frac{dR(E_i)}{E_i\, r} = \frac{R_0}{r(\frac{1}{2} M_W v_0^2)^2} \int_{v_{min}}^{\infty} e^{-v^2/v_0^2} v\, dv \\ &= \frac{R_0}{E_0 r} e^{-E_R/E_0\, r} \end{aligned}$$

$$v_{min} = \sqrt{2E_R/rM_W}$$

Corrections: escape velocity

For finite v_{esc}

$$\frac{dR}{dE_R} = \frac{k_0}{k_1(v_{esc},0)} \frac{R_0}{E_0 r} \left(e^{-E_R/E_0\, r} - e^{-v_{esc}^2/v_0^2}\right)$$

but $\frac{k_0}{k_1} = 0.9965$ for $v_{esc} = 600\,\mathrm{km/s}$,

and for $M_W = M_N = 100\,\mathrm{GeV/c^2}$,
maximum $E_R = 200\,\mathrm{keV}$

$$\Rightarrow \text{cutoff energy} \gg \langle E_R \rangle = 30\,\mathrm{keV}$$

Corrections: earth velocity

Clearly $\vec{v}_E \neq 0$ — but $\sim v_0 = 230\,\mathrm{km/s}$. Full calculation yields:

$$\frac{dR(v_{esc}, v_E)}{dE_R} = \frac{k_0}{k_1}\frac{R_0}{E_0 r}\left(\frac{\sqrt{\pi}}{4}\frac{v_0}{v_E}\left[\mathrm{erf}\left(\frac{v_{min}+v_E}{v_0}\right) - \mathrm{erf}\left(\frac{v_{min}-v_E}{v_0}\right)\right] - e^{-v_{esc}^2/v_0^2}\right)$$

where $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$, $v_{min} = v_0\sqrt{E_R/E_0 r}$, $k_0 = (\pi v_0^2)^{\frac{3}{2}}$
and

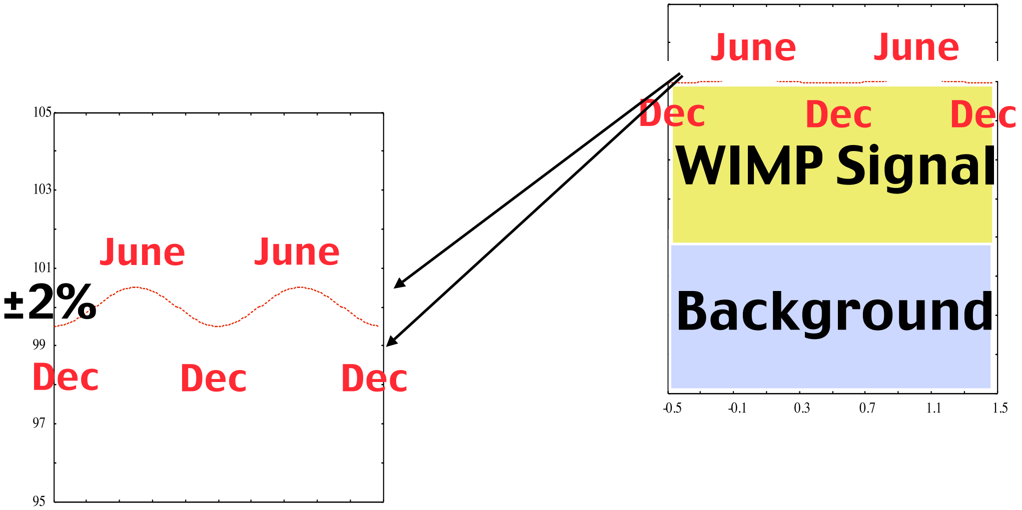
$$k_1 = k_0\left[\mathrm{erf}\left(\frac{v_{esc}}{v_0}\right) - \frac{2}{\sqrt{\pi}}\frac{v_{esc}}{v_0} - e^{-v_{esc}^2/v_0^2}\right]$$

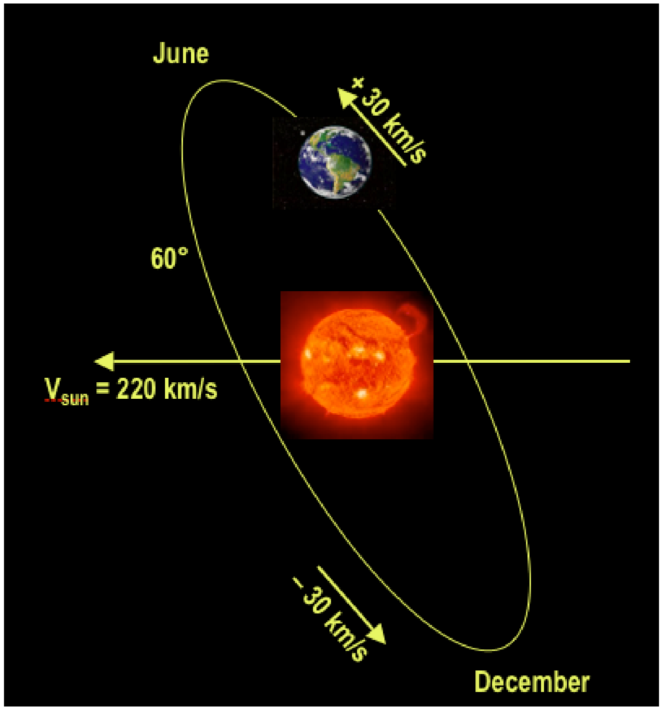
Fortunately, average value well approximated by numerical fit

$$\frac{dR(v_{esc} = \infty, v_E)}{dE_R} = c_1\frac{R_0}{E_0 r}e^{-c_2 E_R/E_0\, r}$$

~30% increase in integrated rate = c_1/c_2 = 0.751/0.561
and harder spectrum

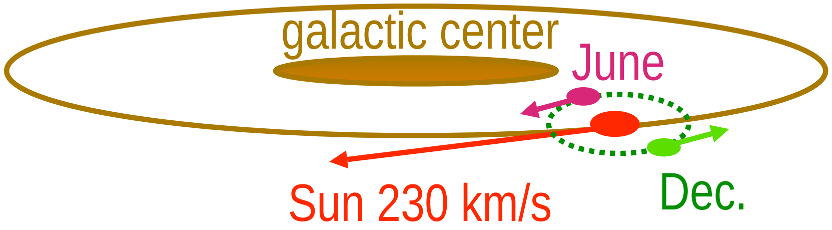
Signal modulations: annual effect

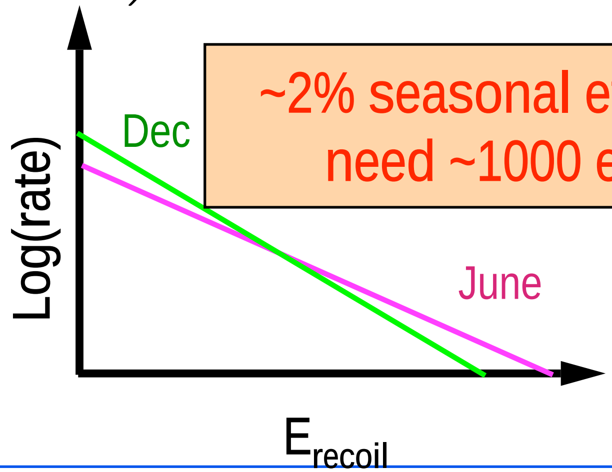




$$v_E(t)[\mathrm{km/s}] = 232 + 15\cos\left(2\pi\,\frac{t-152.5}{365.25}\right)$$

t in days after January 1





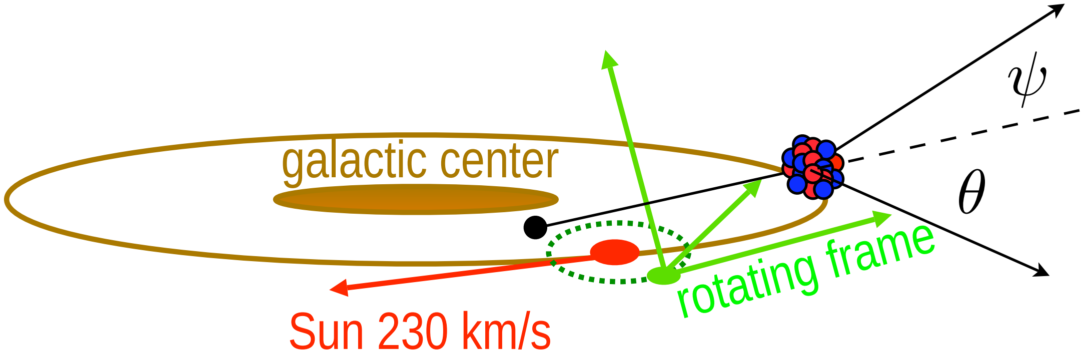
~2% seasonal effect -
need ~1000 events

Signal modulations: recoil direction

- Differential angular spectrum:

$$\frac{d^2R}{dE_R d(\cos\psi)} = \frac{1}{2}\frac{R_0}{E_0 r}e^{-(v_E\cos\psi - v_{min})^2/v_0^2}$$

- Asymmetry → more recoils in forward direction by 5x: ~10 events
- Orientation of lab frame rotates relative to forward direction
  - eg, definition of forward/backward in lab frame changes as earth rotates
  - ⊥ versus || reduces asymmetry to 20% effect: ~300 events



Refinements

$$\left.\frac{dR}{dE_R}\right|_{OBS} = R_0\, S(E_R)\, F^2(E_R)\, I$$

✔ $S(E_R) \quad = \quad$ spectral function — masses and kinematics

time dependence

$F^2(E_R) \quad = \quad$ form factor correction, with $E_R = q^2/2M_W$

$I \quad = \quad$ interaction type

in zero-velocity limit (v<<c), scalar and axial vector interactions dominate → **spin independent** and spin dependent couplings

these dominate

Nuclear form factor and Spin Ind. interactions

- Scattering amplitude: Born approximation $\vec{q} = \hbar\,(\vec{k}' - \vec{k})$
- Spin-independent scattering is coherent $\lambda = \hbar/q \sim \text{few fm}$

$$M(\vec{q}) = f_n\, A \underbrace{\int d^3x\, \rho(\vec{x})\, e^{i\,\vec{q}\cdot\vec{x}}}_{F(\vec{q})} \quad \Rightarrow \quad \sigma \propto |M|^2 \propto A^2$$

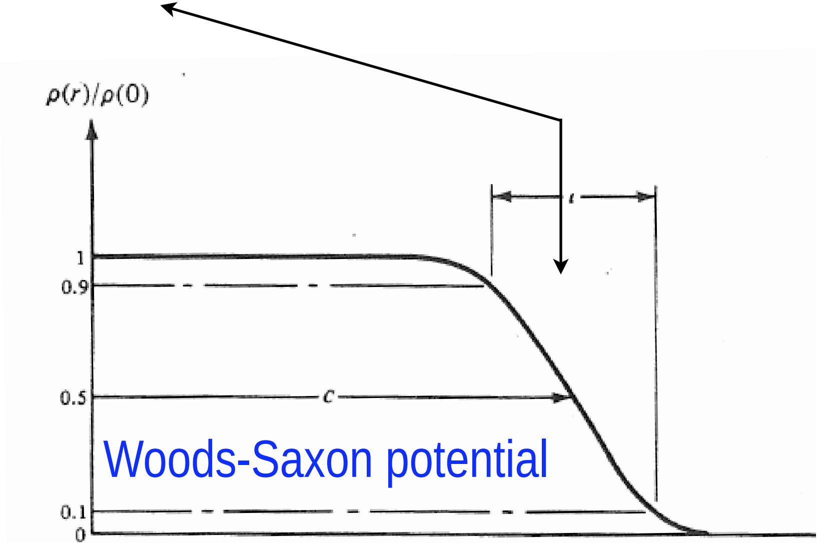
fundamental coupling to nucleon

mass number

“Helm” form factor

$$F(qr_n) = \underbrace{\frac{3[\sin(qr_n) - qr_n\cos(qr_n)]}{(qr_n)^3}}_{j_1(qr_n)}\, e^{-(qs)^2/2}$$

- j$_1$ exact for ‘sharp’ density cutoff
  - ◆ r_n nuclear radius
  - ◆ s skin thickness parameter

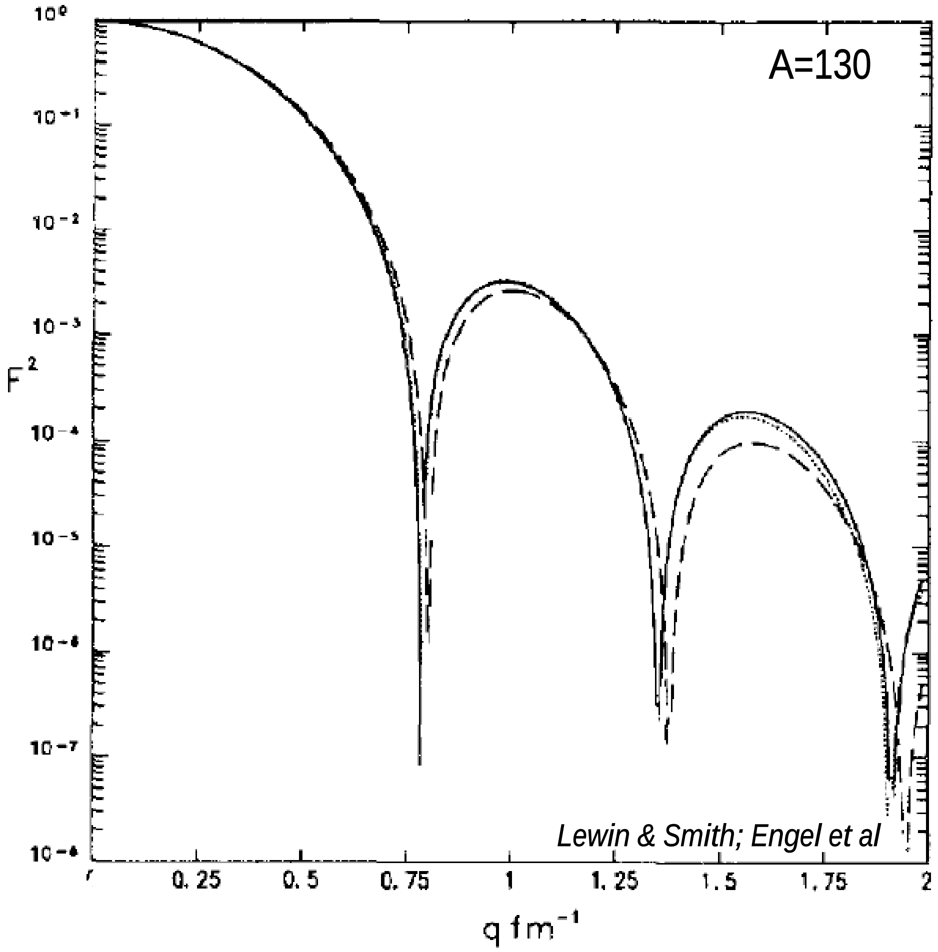


Woods-Saxon potential

swiped from Fraunfelder and Henley

Nuclear form factor and Spin Ind. interactions

- Loss of coherence as larger momentum transfer probes smaller scales



Lewin & Smith; Engel et al

SI cross section

- Now have dependence on ***q*²** and nucleus → separate out fundamental WIMP-nucleon cross section
- Differential cross section can be written

$$\frac{d\sigma_{WN}(q)}{dq^2} = \frac{\sigma_{0WN}\,F^2(q)}{4m_r^2 v^2}$$

rel. velocity in CM frame

where σ_{0WN} is total cross section for $F = 1$.
From Fermi's Golden Rule

$$\frac{d\sigma_{WN}(q)}{dq^2} = \frac{1}{\pi v^2}|M|^2 = \frac{1}{\pi v^2} f_n^2 A^2 F^2(q)$$

- Can identify "unity-form-factor" cross sections:

$$\sigma_{0WN} = \frac{4m_r^2}{\pi} f_n^2 A^2 = \underbrace{\frac{4}{\pi} m_n^2 f_n^2}_{\sigma_{Wn}} \frac{m_r^2}{m_n^2} A^2$$

nucleus nucleon

all the particle physics, here

SI cross section and differential rate

- Putting this all together

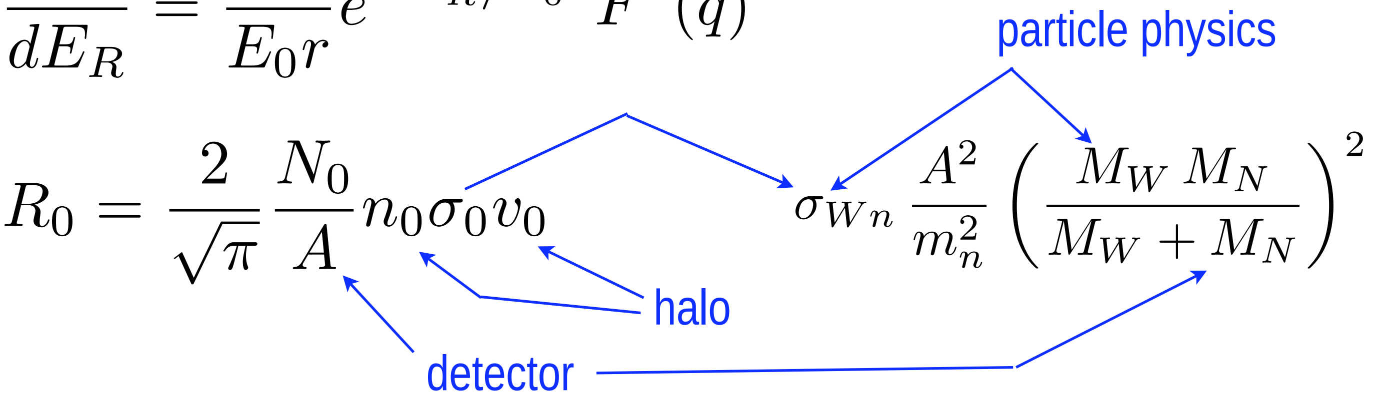
$$\frac{d\sigma_{WN}(q)}{dq^2} = \frac{1}{4m_n^2 v^2}\,\sigma_{Wn}\,A^2\,F^2(q)$$

- Recall

$$\frac{dR}{dE_R} = \int \frac{dR(E)}{Er} \qquad \text{(where } dR(E) \text{ contained } \sigma\text{)}$$

- The ***Er*** factor was from isotropic scattering - corresponds to the ***v***^{2} in the differential cross section. Including now the FF:

$$\frac{dR}{dE_R} = \frac{R_0}{E_0 r} e^{-E_R/E_0 r} F^2(q)$$



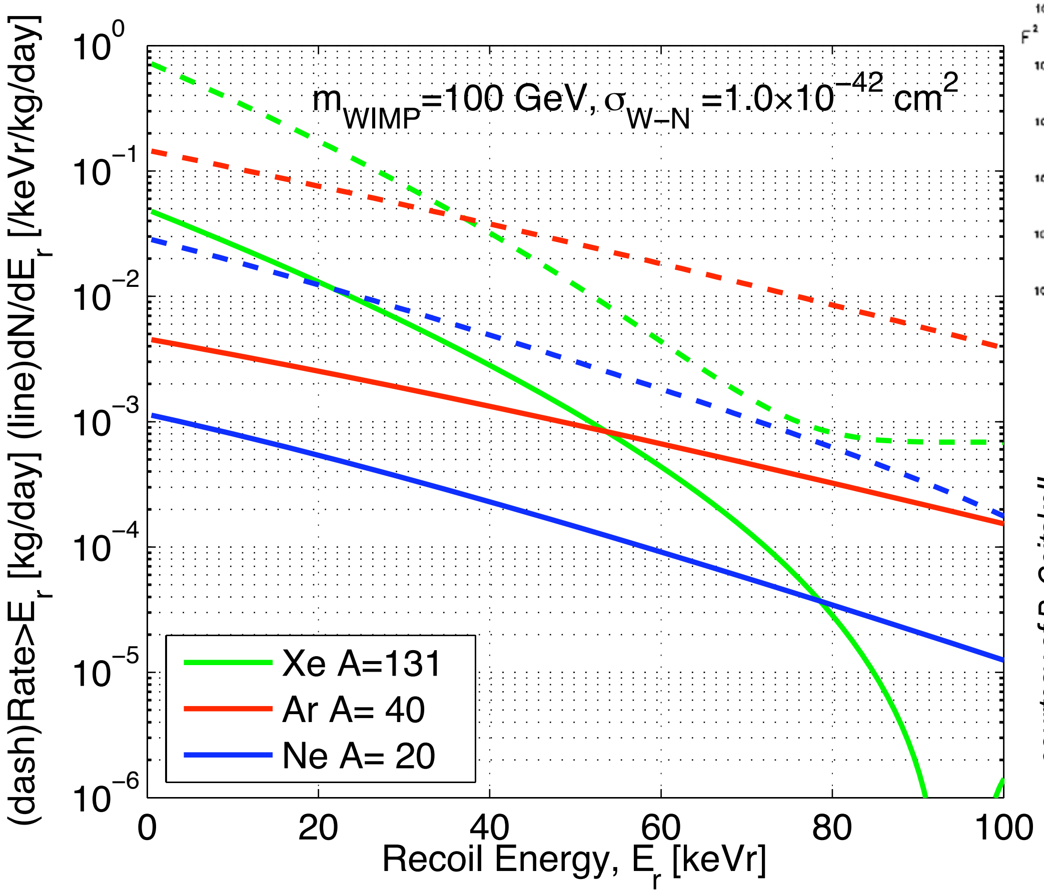
$$R_0 = \frac{2}{\sqrt{\pi}}\frac{N_0}{A}n_0\sigma_0 v_0 \qquad \sigma_{Wn}\,\frac{A^2}{m_n^2}\left(\frac{M_W\,M_N}{M_W+M_N}\right)^2$$

particle physics

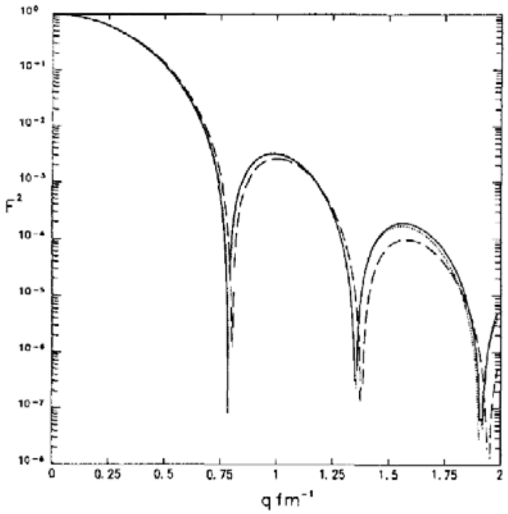
halo

detector

SI cross section and differential rate



courtesy of R. Gaitskell



Nuclear form factor and Spin Dep. interactions

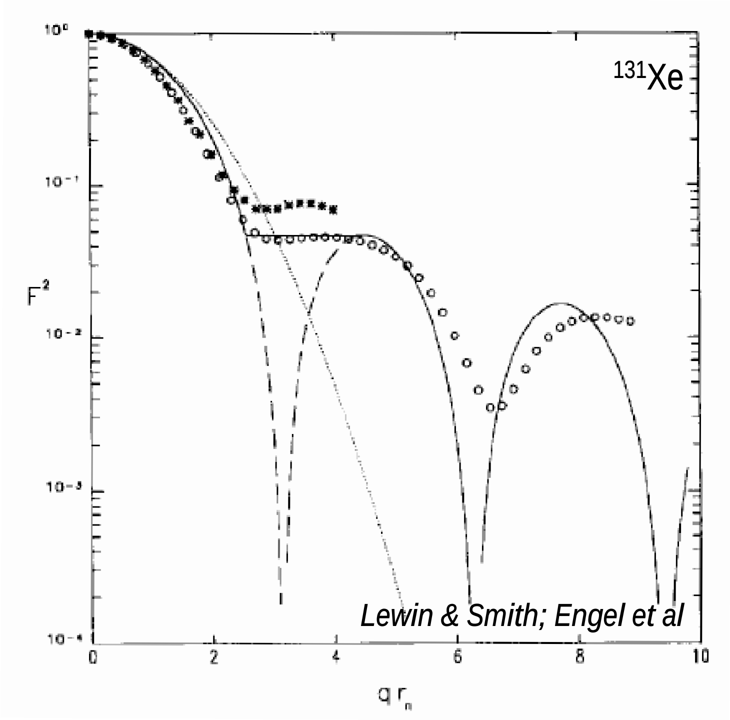
- Scattering amplitude dominated by unpaired nucleon
  - paired nucleons ↑↓ tend to cancel -- couple to net spin *J*

$$\frac{d\sigma}{dq^2} = \frac{8}{\pi v^2}\Lambda^2 G_F^2 J(J+1)F^2(q)$$

- Simplified model based on thin-shell valence nucleon

$$F(qr_n) = j_0(qr_n) = \frac{\sin(qr_n)}{qr_n}$$

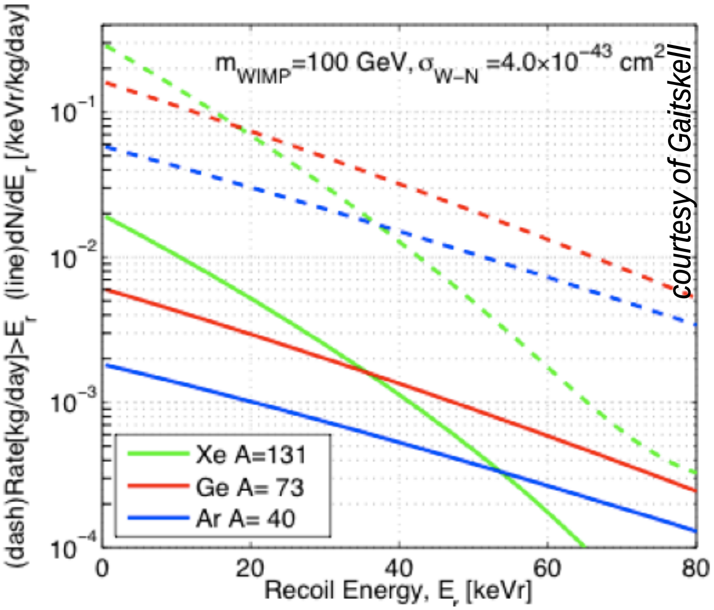
- Better: detailed nucleus specific calcs.
  - average over odd-group nucleons
  - use measured nuclear magnetic moment



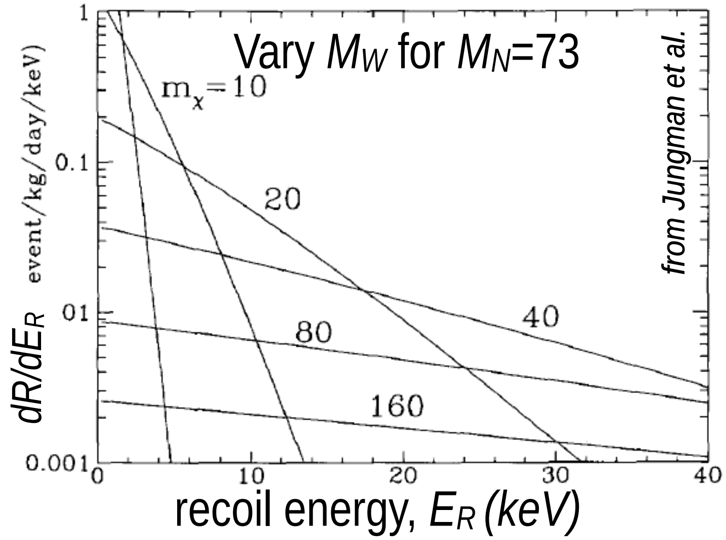
Lewin & Smith; Engel et al

signal characteristics

- A^2 dependence
  - coherence loss
  - relative rates
- M_W relative to M_N
  - large M_W - lose mass sensitivity
  - if ~100 GeV
- Present limits on rate
- Following a detection (!), many cross checks possible
  - A^2 (or J, if SD coupling)
  - WIMP mass if not too heavy
    - different targets
    - accelerator measurements
  - galactic origin
    - annual
    - diurnal/directional - WIMP astronomy



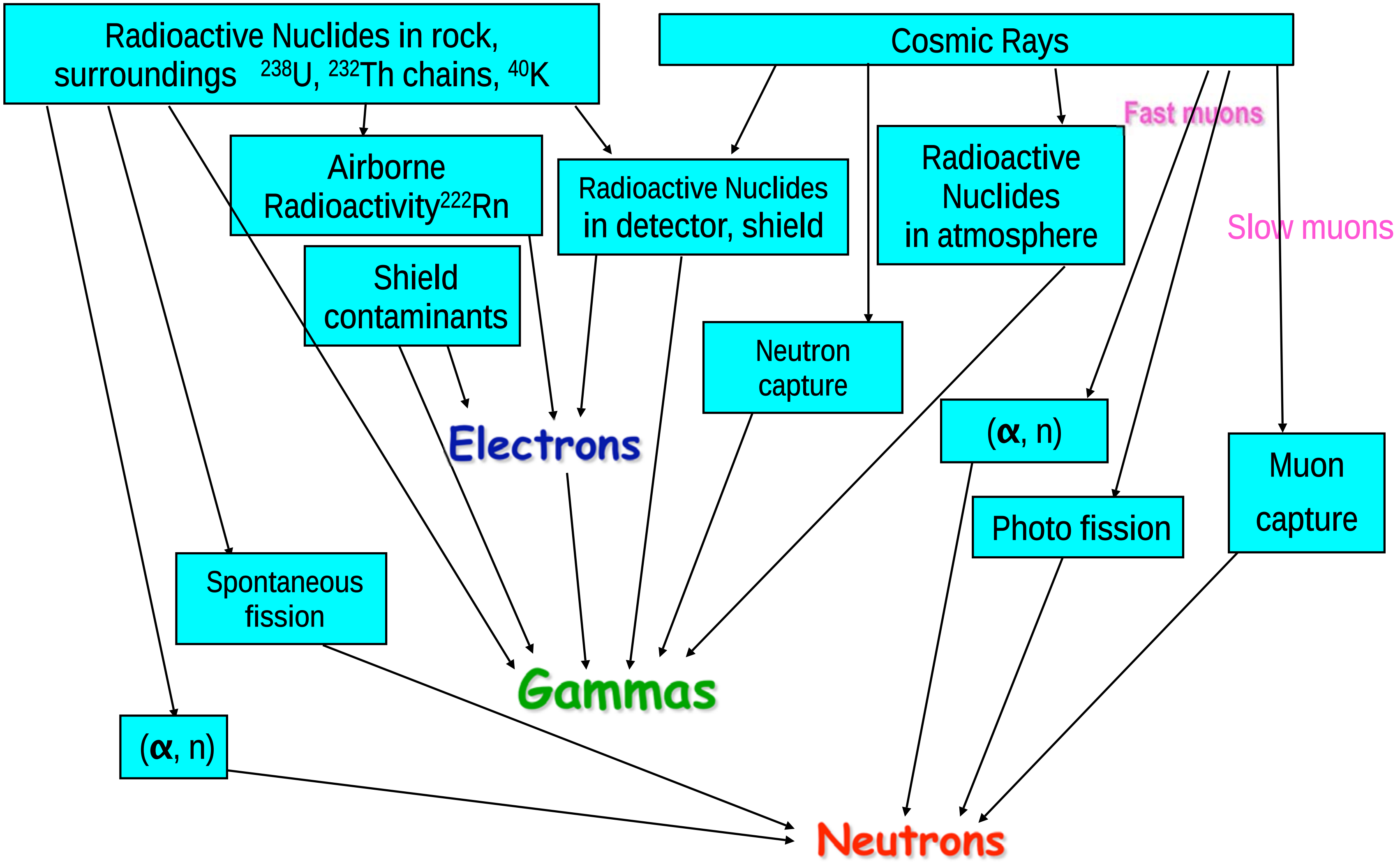
courtesy of Gaitskell



from Jungman et al.

Backgrounds: cosmic rays and natural radioactivity

WIMP scatters **(< 1 evts /10 kg/ day)** swamped by backgrounds **(> 10^{6-7} evts/kg-d)**



Radioactive Nuclides in rock, surroundings ^{238}U, ^{232}Th chains, ^{40}K

Cosmic Rays

Fast muons

Slow muons

Airborne Radioactivity ^{222}Rn

Radioactive Nuclides in detector, shield

Radioactive Nuclides in atmosphere

Shield contaminants

Neutron capture

Electrons

(**α**, n)

Muon capture

Photo fission

Spontaneous fission

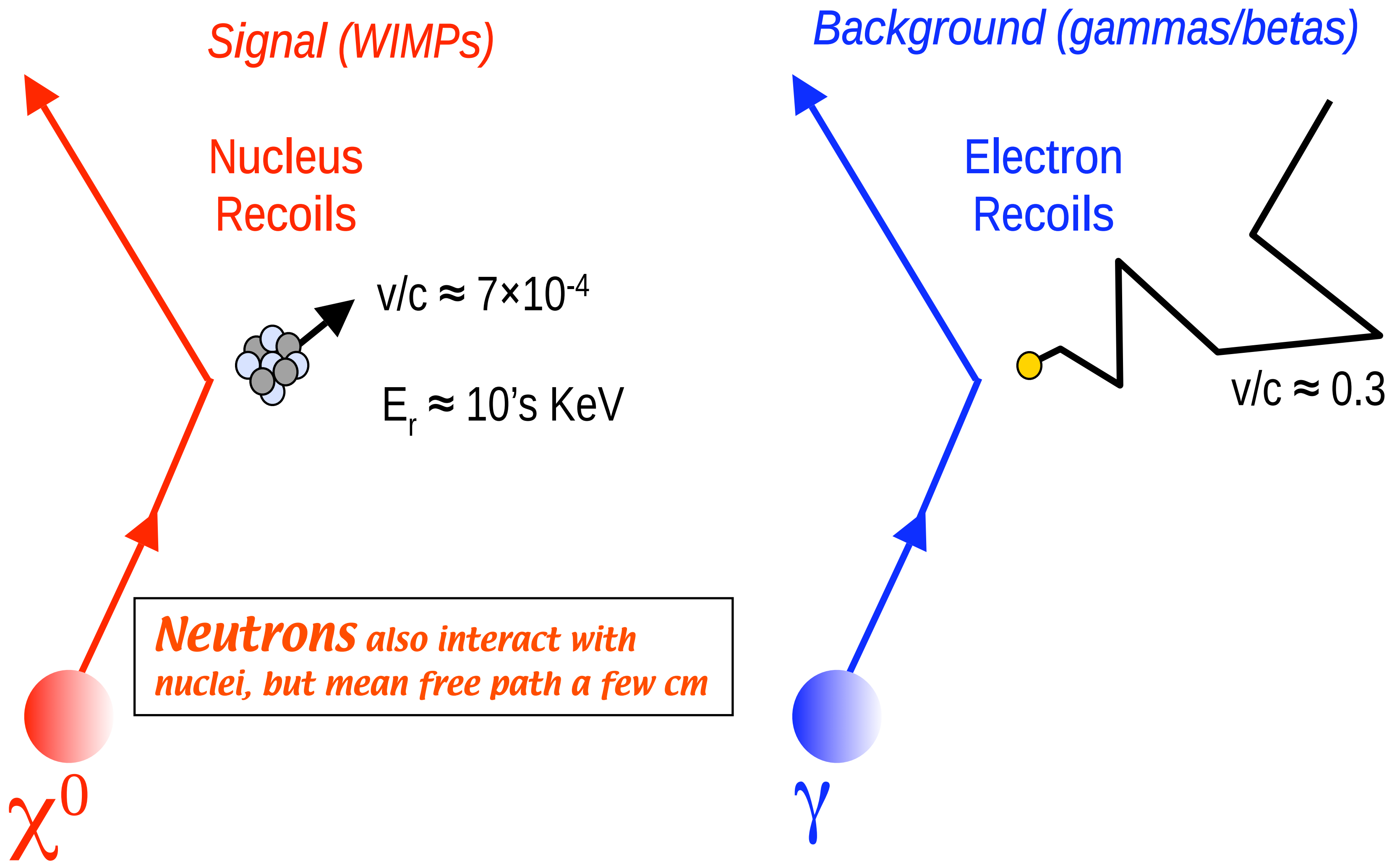
Gammas

(**α**, n)

Neutrons

courtesy of S. Kamat

The Signal and Backgrounds



Signal (WIMPs)

Nucleus Recoils

$v/c \approx 7\times10^{-4}$

$E_r \approx 10\text{'s KeV}$

***Neutrons** also interact with nuclei, but mean free path a few cm*

χ^0

Background (gammas/betas)

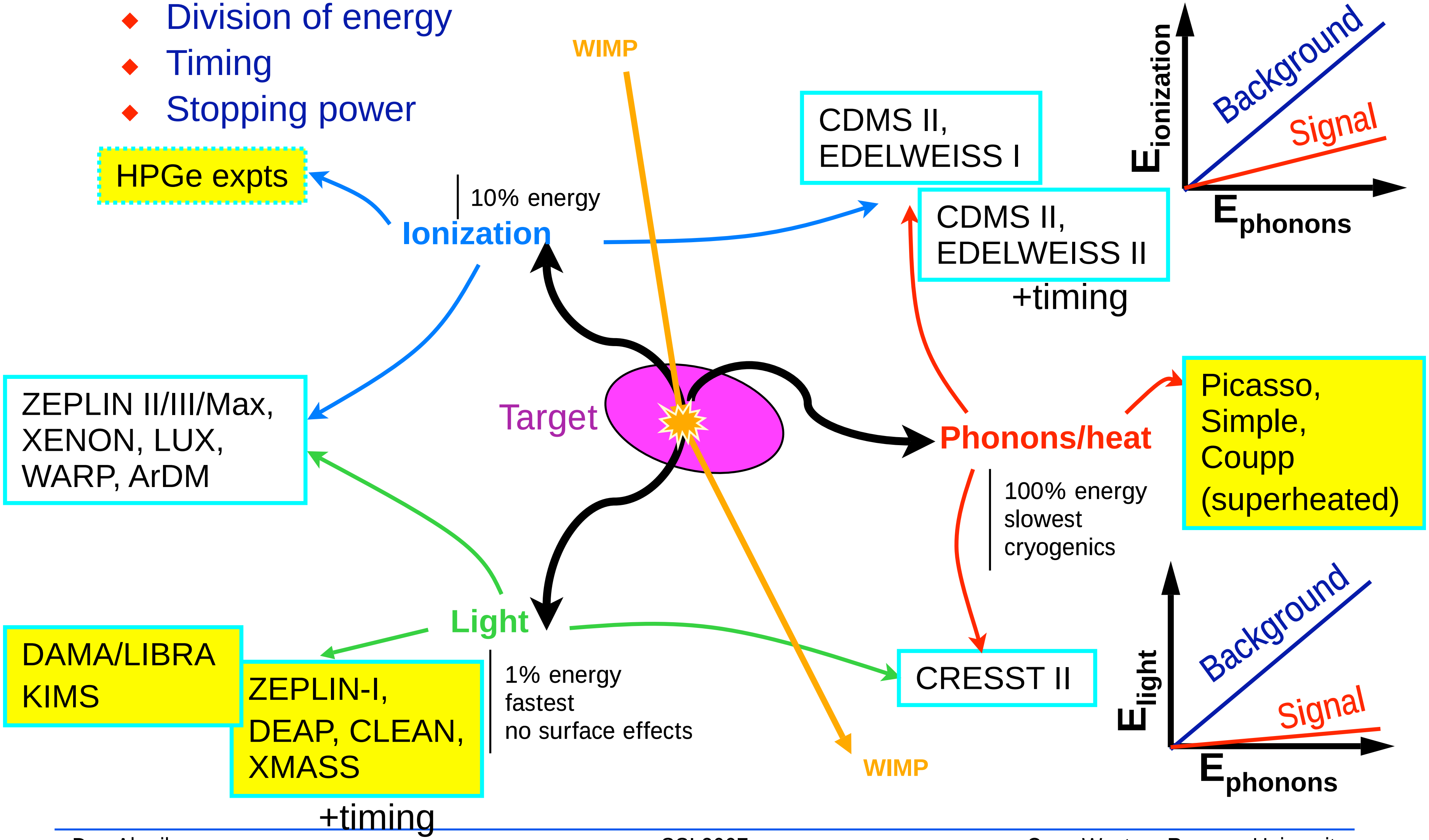
Electron Recoils

$v/c \approx 0.3$

γ

Nuclear-Recoil Discrimination

- Nuclear recoils vs. electron recoils
  - Division of energy
  - Timing
  - Stopping power



HPGe expts

Ionization

10% energy

CDMS II, EDELWEISS I

CDMS II, EDELWEISS II

+timing

ZEPLIN II/III/Max, XENON, LUX, WARP, ArDM

Phonons/heat

100% energy
slowest
cryogenics

Picasso, Simple, Coupp (superheated)

Light

1% energy
fastest
no surface effects

DAMA/LIBRA
KIMS

ZEPLIN-I, DEAP, CLEAN, XMASS

+timing

CRESST II

Next: WIMP search experiments