Structure Formation
Lecture 2

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SLAC Summer Institute 2007

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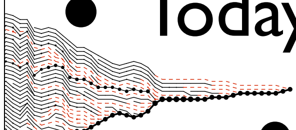
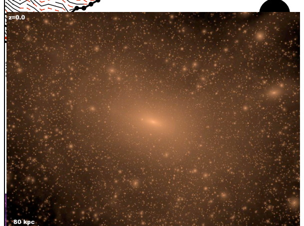
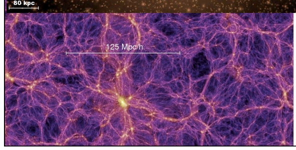
The Plan

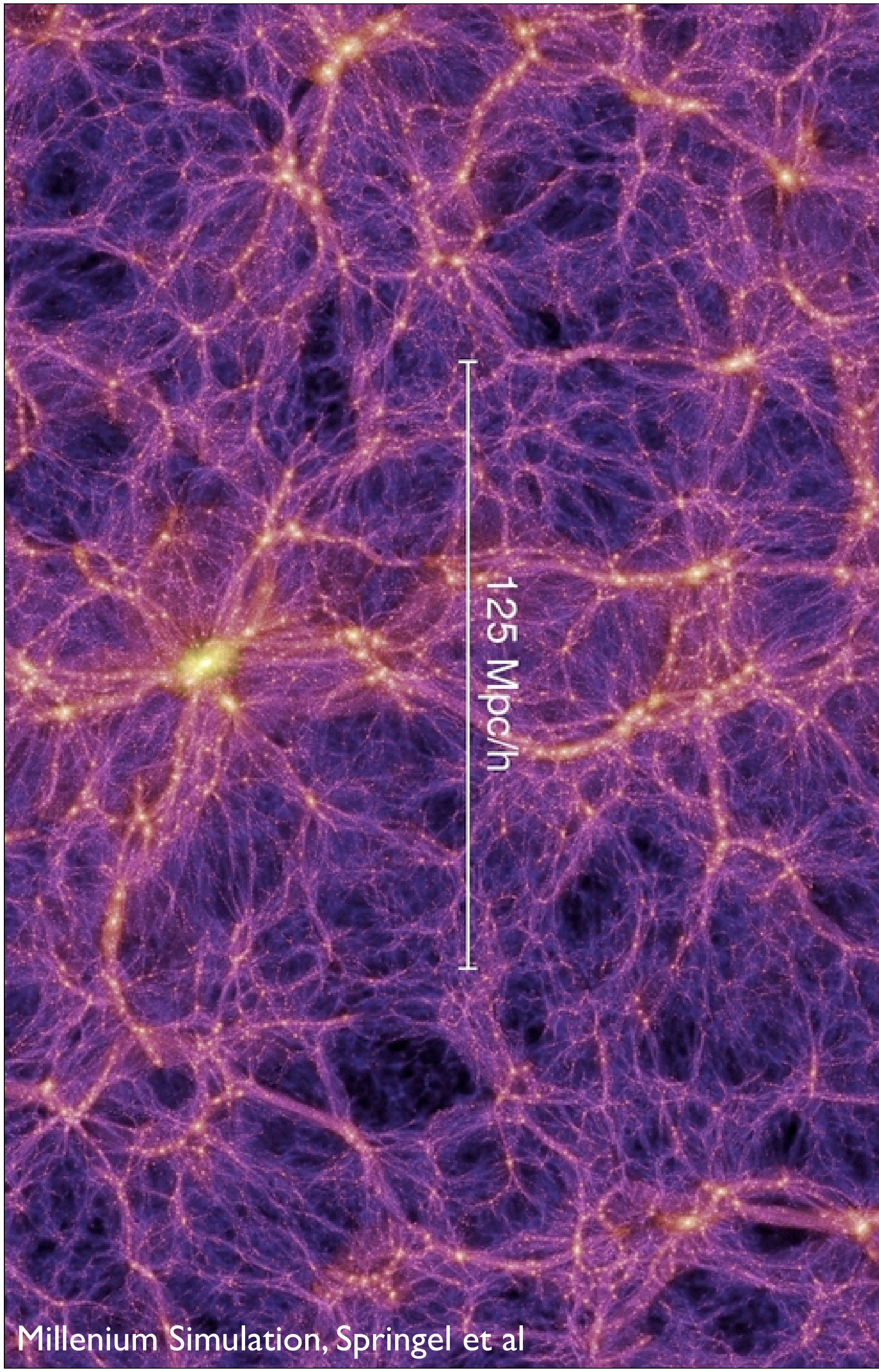
- Yesterday:
  - ☆ Quick review of the Linear Power Spectrum and Growth of Fluctuations in the Linear Regime
  - ☆ Basics of Non-Linear Structure Formation; Spherical Collapse
  - ☆ Abundance of Dark Matter Halos (The "Mass Function")
- Today:



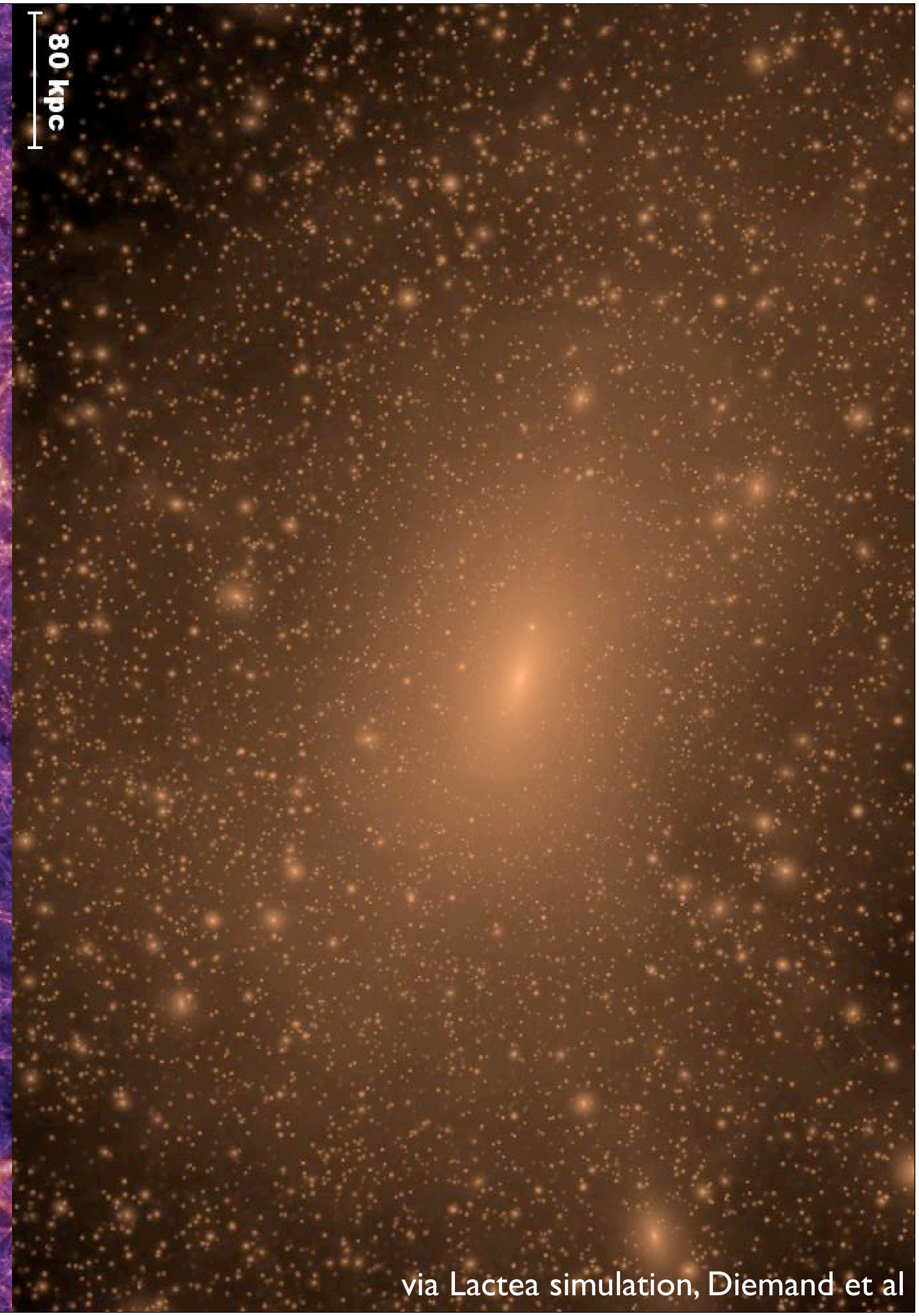




  - Growth, and Structure of Dark Matter Halos
  - Dark Matter Substructure
  - Clustering of Dark Matter, Halos, & Galaxies



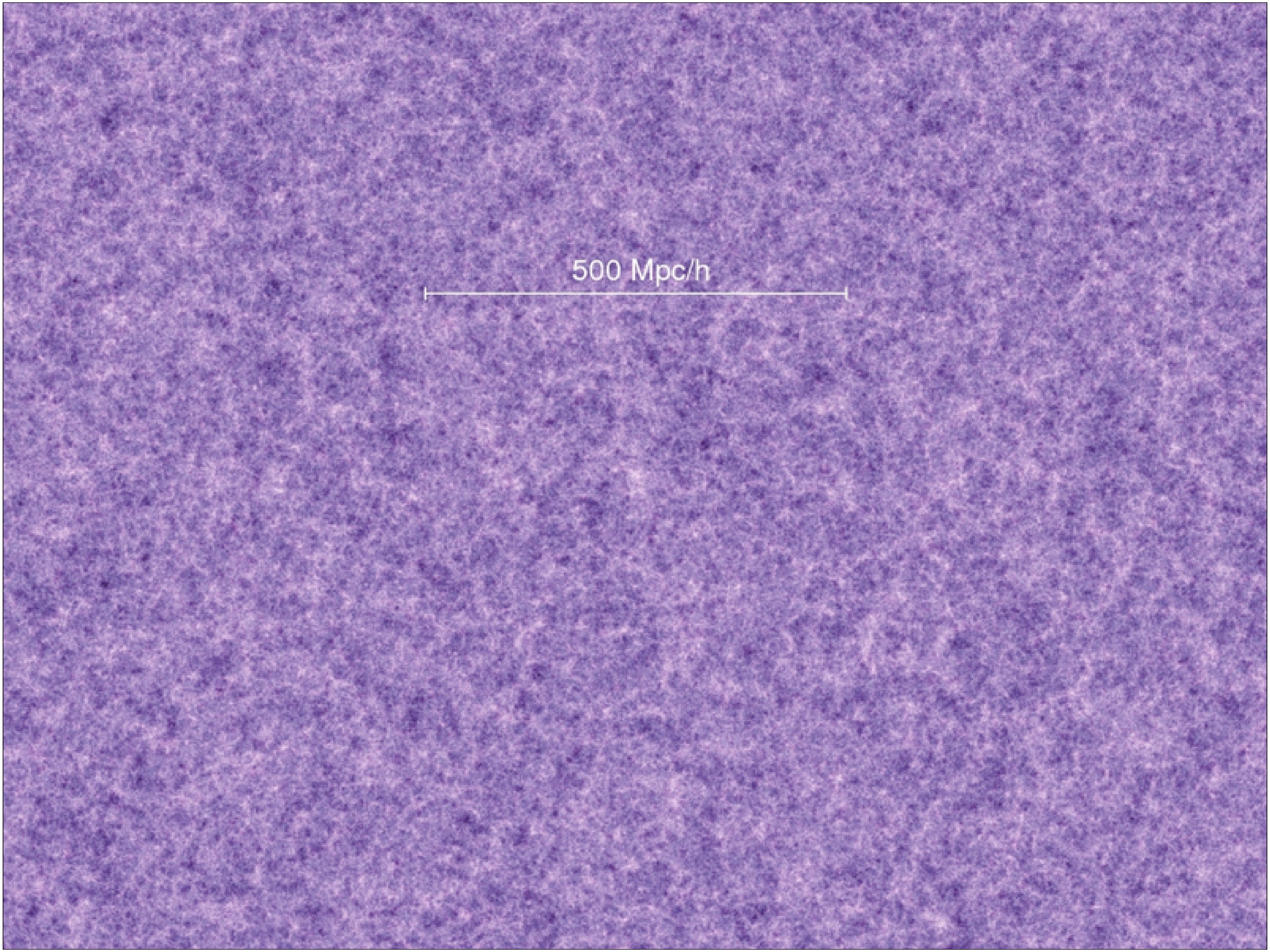
Millenium Simulation, Springel et al

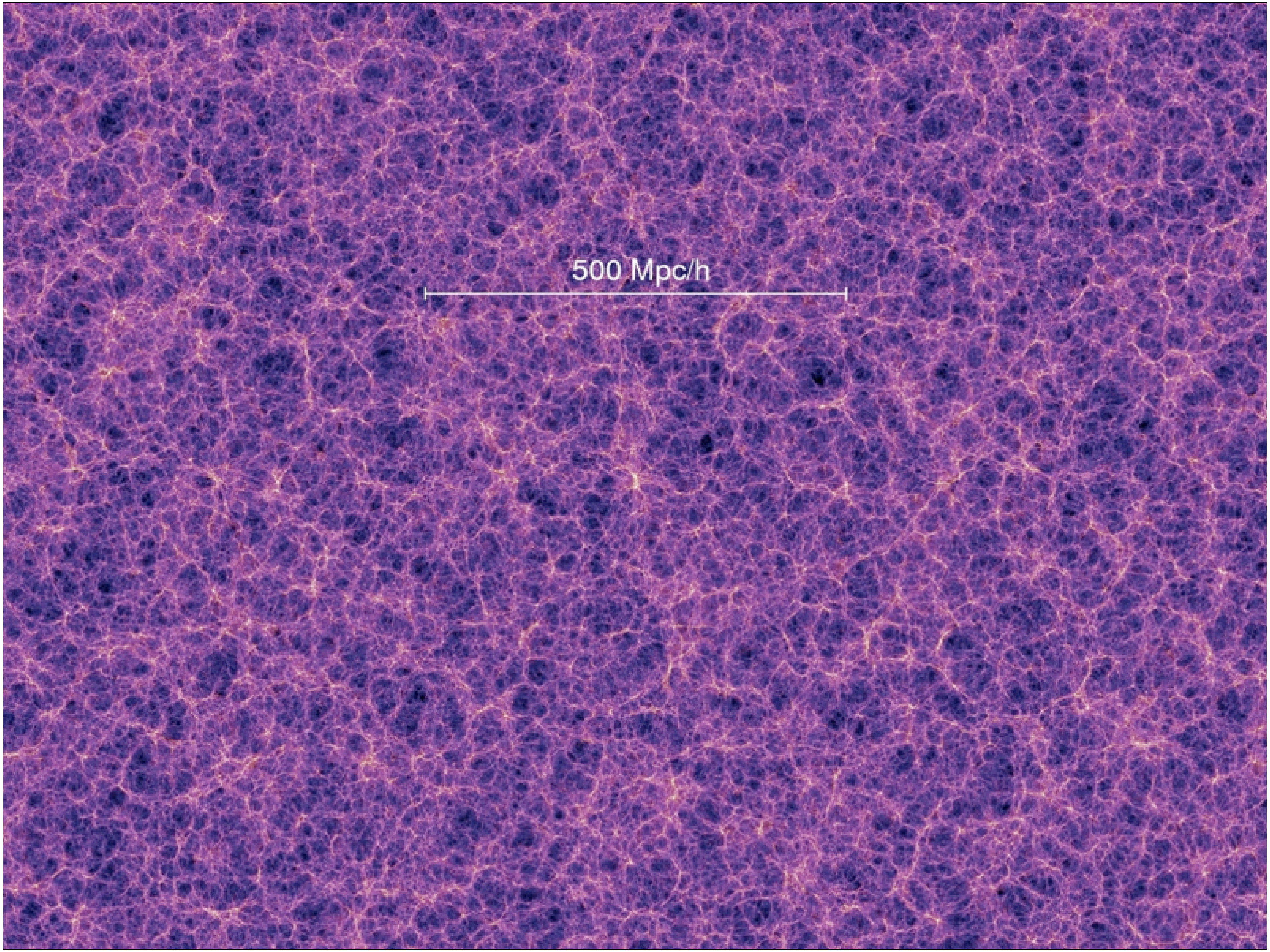


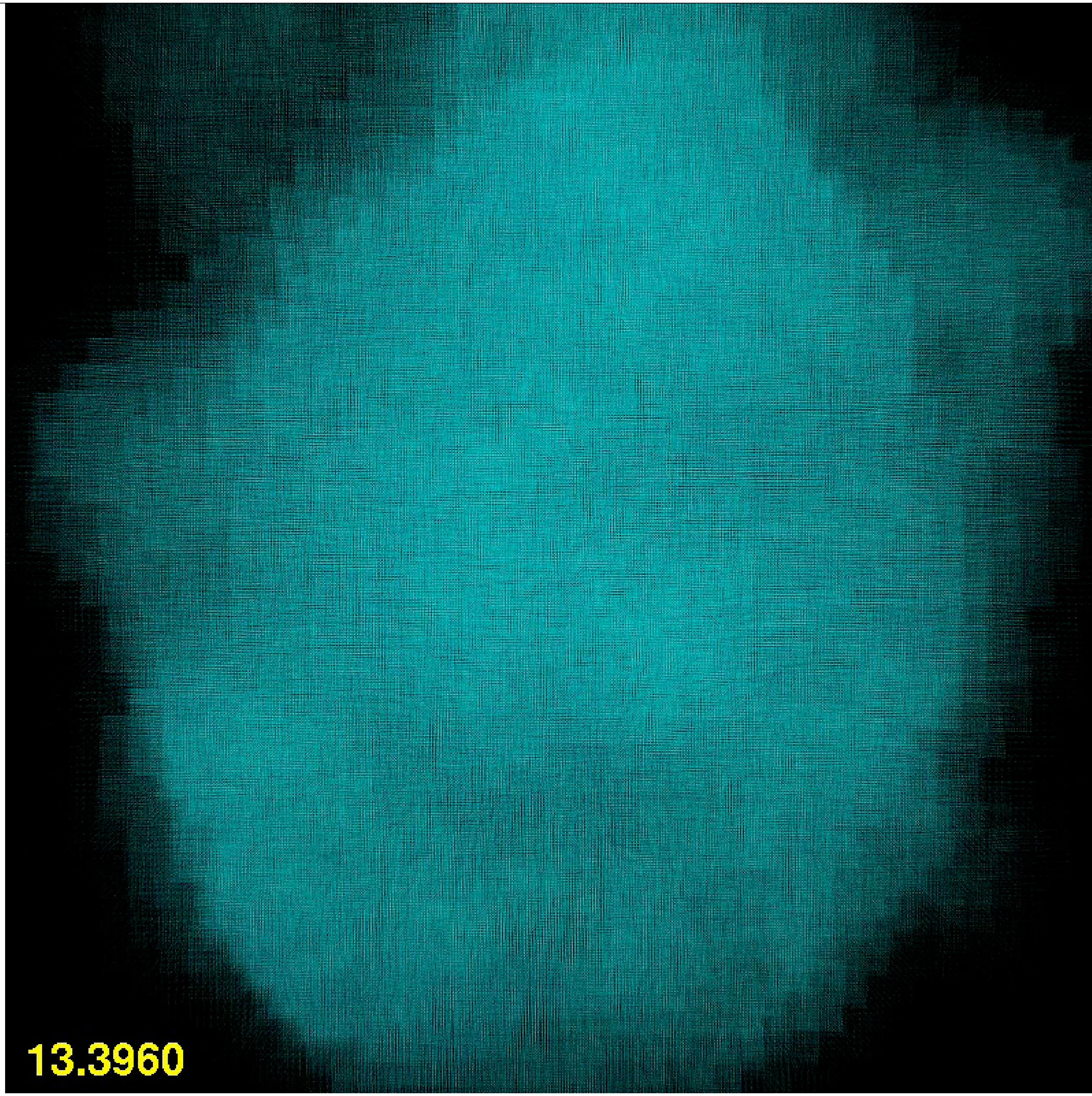
via Lactea simulation, Diemand et al

simulating the Universe

- choose a cosmological model ($\Omega_m, \Omega_\Lambda, \Omega_b, h, n$, dark matter, etc)
- choose a computational set up (box size, dynamic range, what physics to include)
- find the linear P(k)
- set up a random or constrained realization of P(k) in the linear regime (200<z<30) in the chosen box
- find yourself a computer. the bigger the better!
- follow the evolution of dark matter using particle N-body methods
- optionally, follow the evolution of the gas by numerically solving hydrodynamic equations
- optionally, add sink and source terms to hydro equations, modeling heating and cooling of the gas, star formation, etc.. ("subgrid physics")
- evolve to the redshift of interest


500 Mpc/h


500 Mpc/h

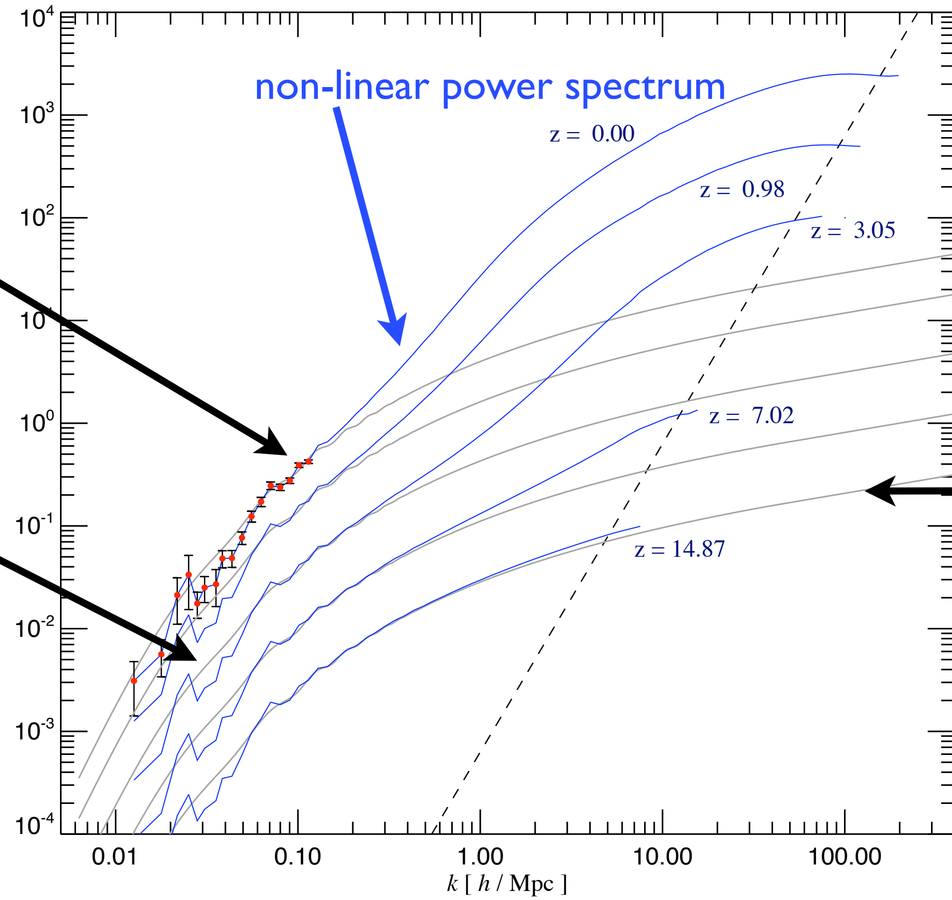

13.3960

evolution of the matter power spectrum

$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$$

largest scales are still in the linear regime

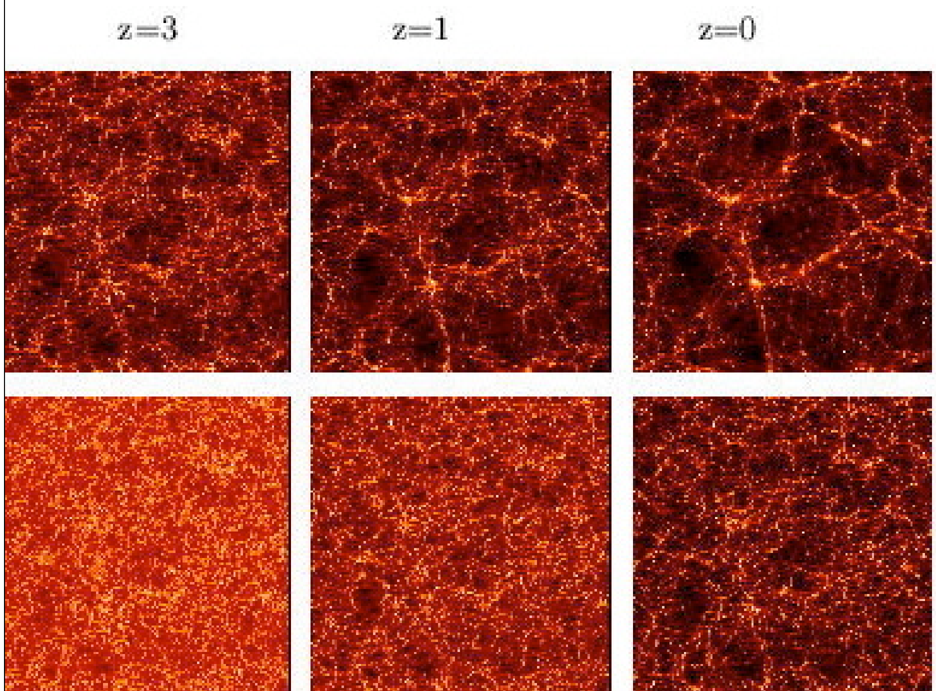
finite volume box; large modes have noise

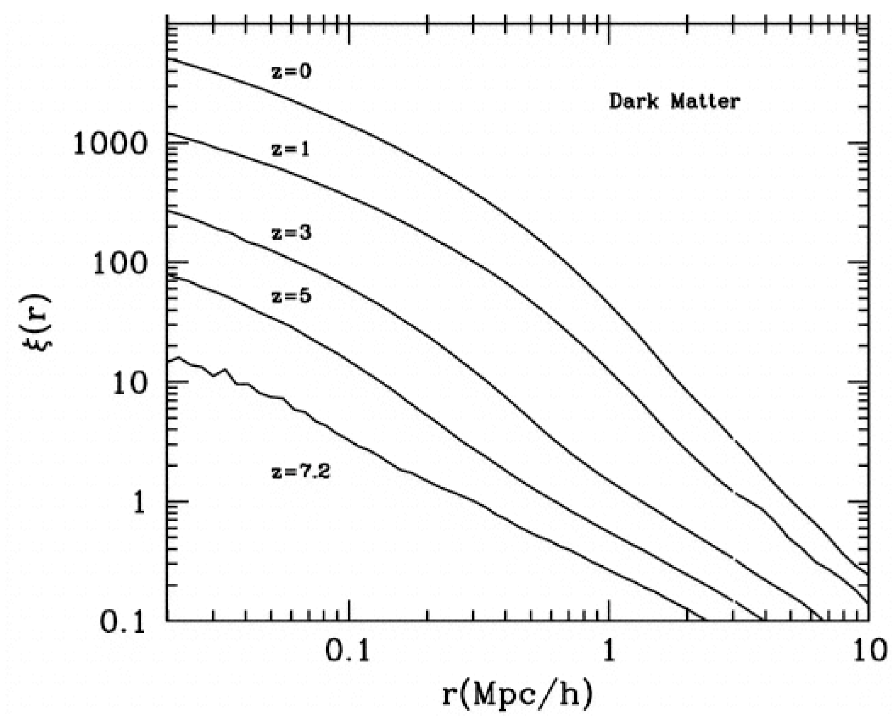


linear power spectrum

Springel et al 2005

evolution of dark matter clustering



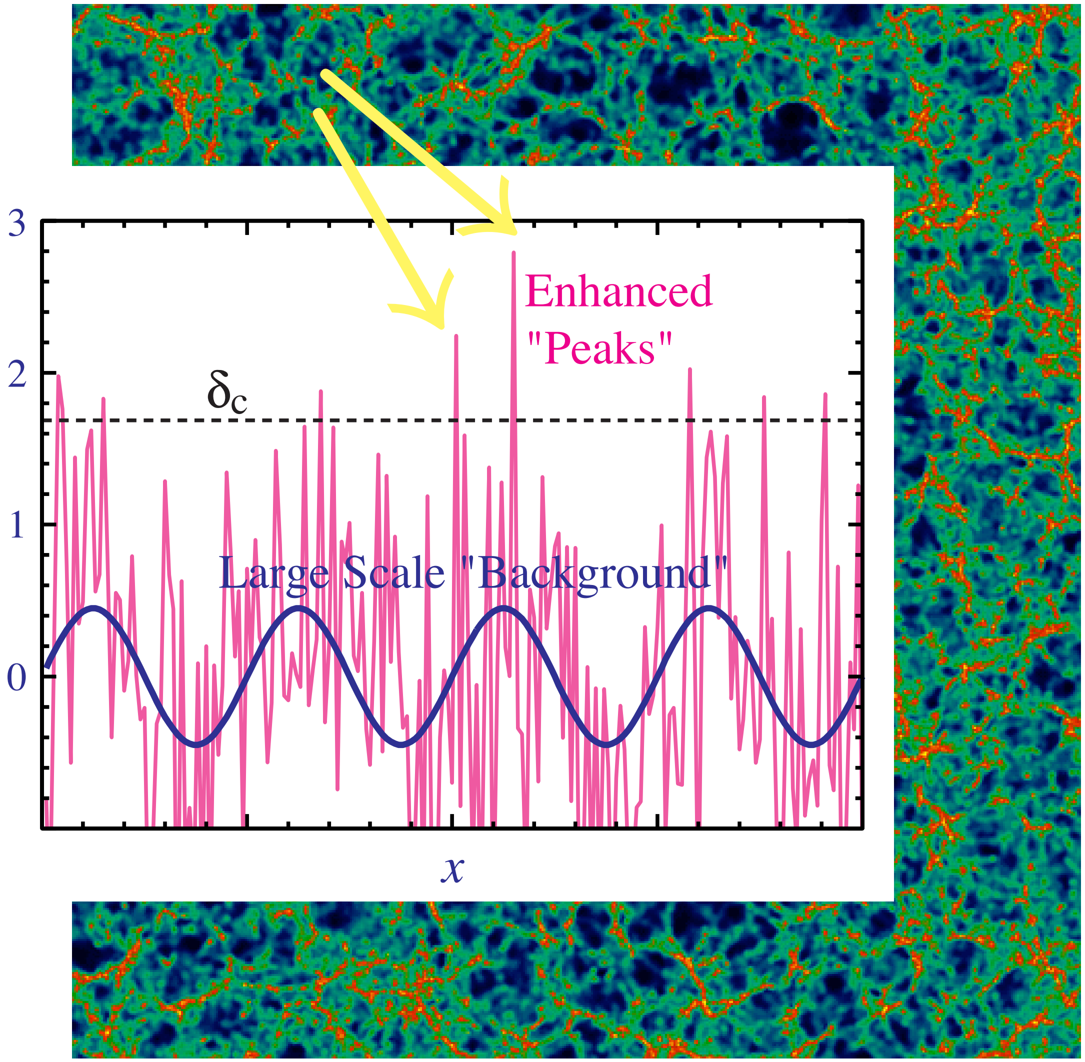


- evolves rapidly with redshift
- 2PCF not a power law; has a feature at the scale of halos
- evolution is a strong function of matter density and dark energy

Colin et al 1999

halo formation in peaks

first sites of halo formation



first sites of snowfall

Gaussian fluctuations on various scales

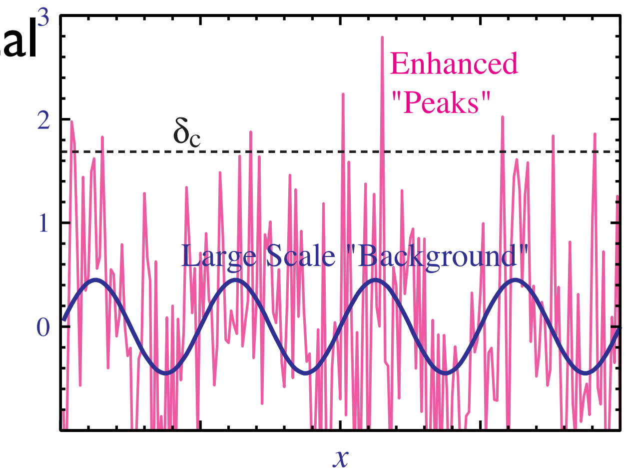
halo bias

- if halos are formed without regard to the underlying density, then

$$\frac{\delta n_h}{n_h} = \frac{\delta \rho}{\rho}$$

- but spherical collapse model indicates that the probability of forming a halo depends on the initial density field: large scale density acts as a background enhancement
- halos are "biased" tracers of the background dark matter field. bias can be calculated from spherical collapse and the form of the mass function

$$\frac{\delta n_M}{n_M} = [1 + b(M)]\, \delta$$



halo bias

eg. Mo & White 1996
Sheth, Mo & Tormen 2001
Zentner 2007

- the relative abundance of halos in dense regions compared to halos in the background is

$$\delta^{\rm L}_{\rm halo} = \frac{\mathcal{N}(M|\delta_0, S_0)}{({\rm d}n(M)/{\rm d}M)V_0} - 1$$

$$\nu \equiv \delta_{\rm c}/\sigma(M)$$

- to first order,

$$\delta^{\rm L}_{\rm halo} = \frac{\nu^2 - 1}{\delta_{\rm c}}\delta_0,$$

$$\delta_{\rm halo} = \left(1 + \frac{\nu^2 - 1}{\delta_{\rm c}}\right)\delta \equiv b_{\rm h}\delta.$$

- for Press-Schechter mass function,

$$n_M \propto \nu \exp(-\nu^2/2)$$

$$b(M) = 1 + \frac{\nu^2 - 1}{\delta_c}$$

- improved by Sheth-Torman mass function

a, p fit to sims

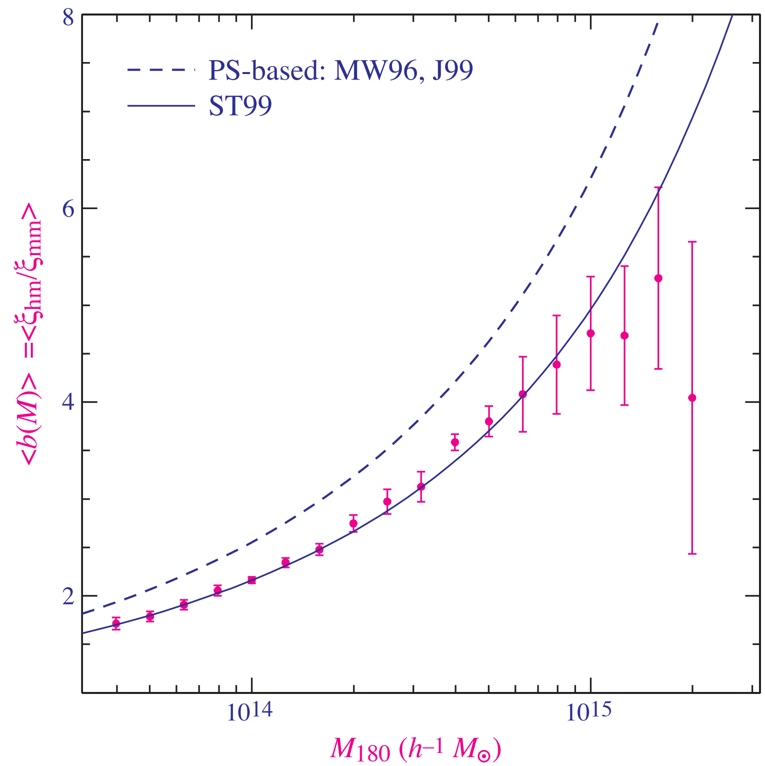
$$\nu f(\nu) = A\left(1 + \frac{1}{\nu'^p}\right)\left(\frac{\nu'}{2}\right)^{1/2}\frac{{\rm e}^{-\nu'/2}}{\sqrt{\pi}},$$

$$b(M) = 1 + \frac{a\nu^2 - 1}{\delta_c} + \frac{2p}{\delta_c[1 + (a\nu^2)^p]}$$

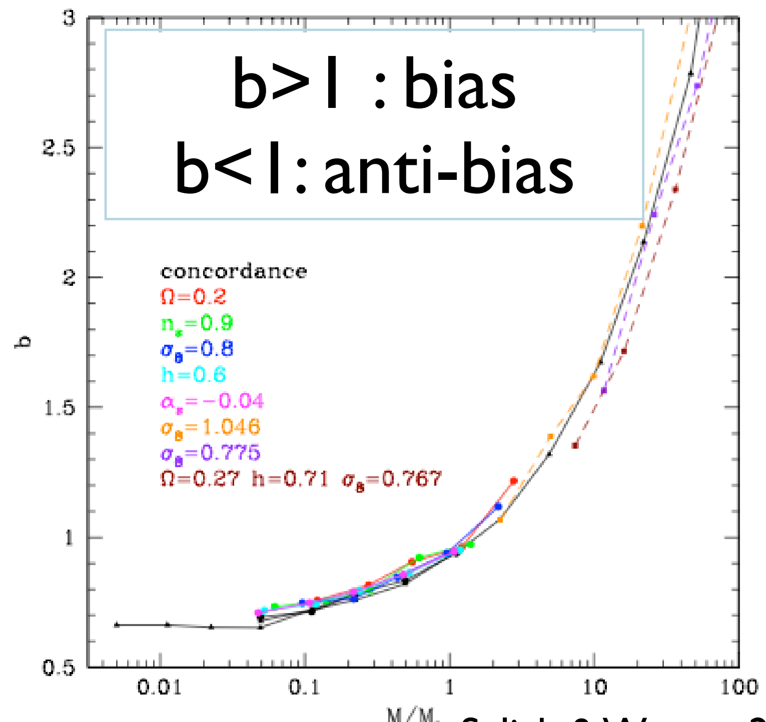
in general a given model should simultaneously give b(M) and n(M)

halo bias in simulations

$\xi_h = b^2\,\xi_{DM}$



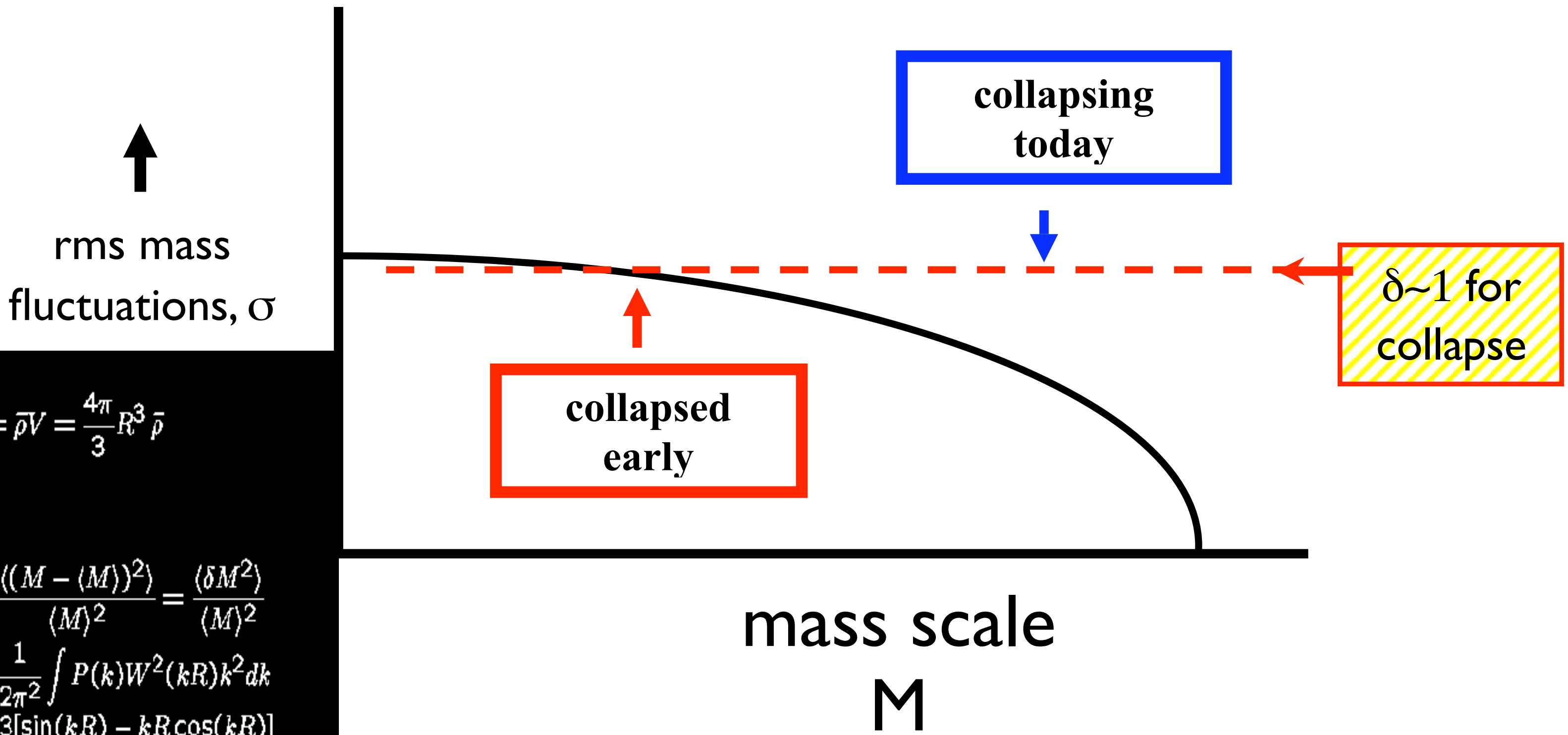
Hu & Kravtsov 2002



Seljak & Warren 2004

see also
Mo & White 1996; Sheth & Tormen 1999,
Sheth, Mo & Tormen 2001, etc.

$$\nu \equiv \delta_c / \sigma(M) \qquad \sigma(M_\star) = \delta_c$$



$$\langle M \rangle = \bar{\rho} V = \frac{4\pi}{3} R^3 \bar{\rho}$$

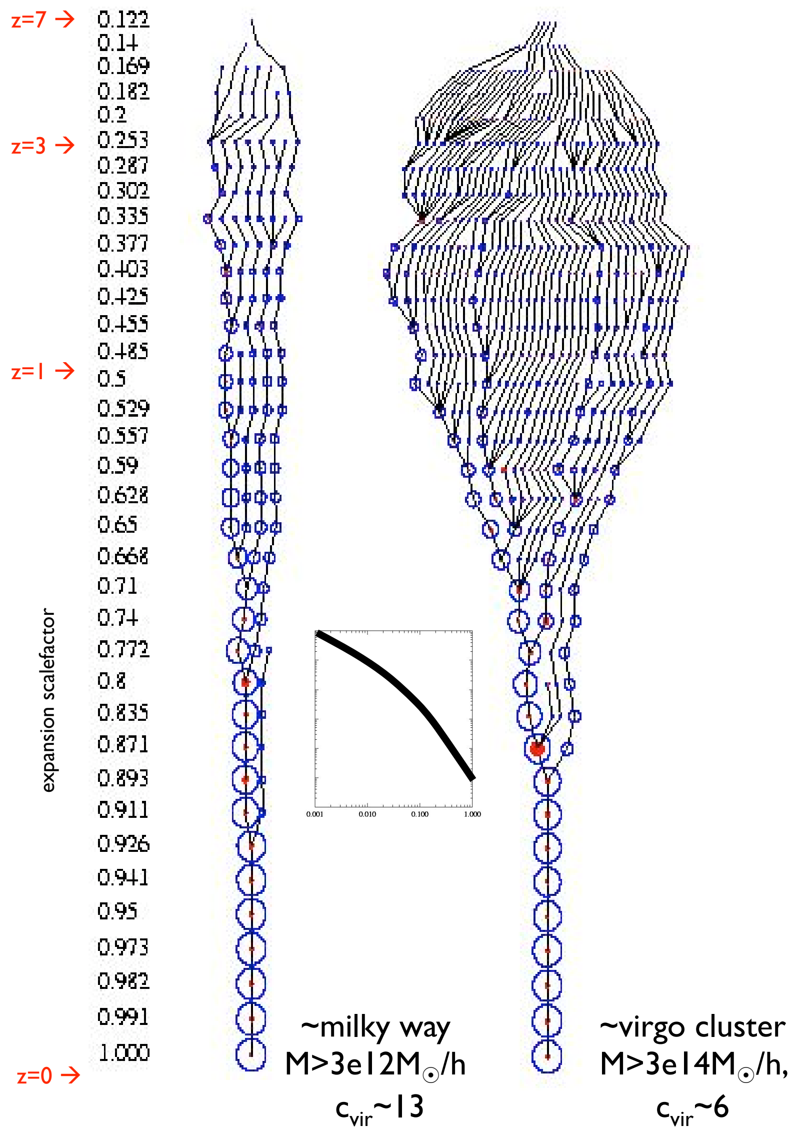
$$\sigma^2(M) = \frac{\langle (M - \langle M \rangle)^2 \rangle}{\langle M \rangle^2} = \frac{\langle \delta M^2 \rangle}{\langle M \rangle^2}$$

$$= \frac{1}{2\pi^2} \int P(k) W^2(kR) k^2 dk$$

$$W(kR) = \frac{3[\sin(kR) - kR\cos(kR)]}{(kR)^3}$$

high peaks are rarer and more clustered

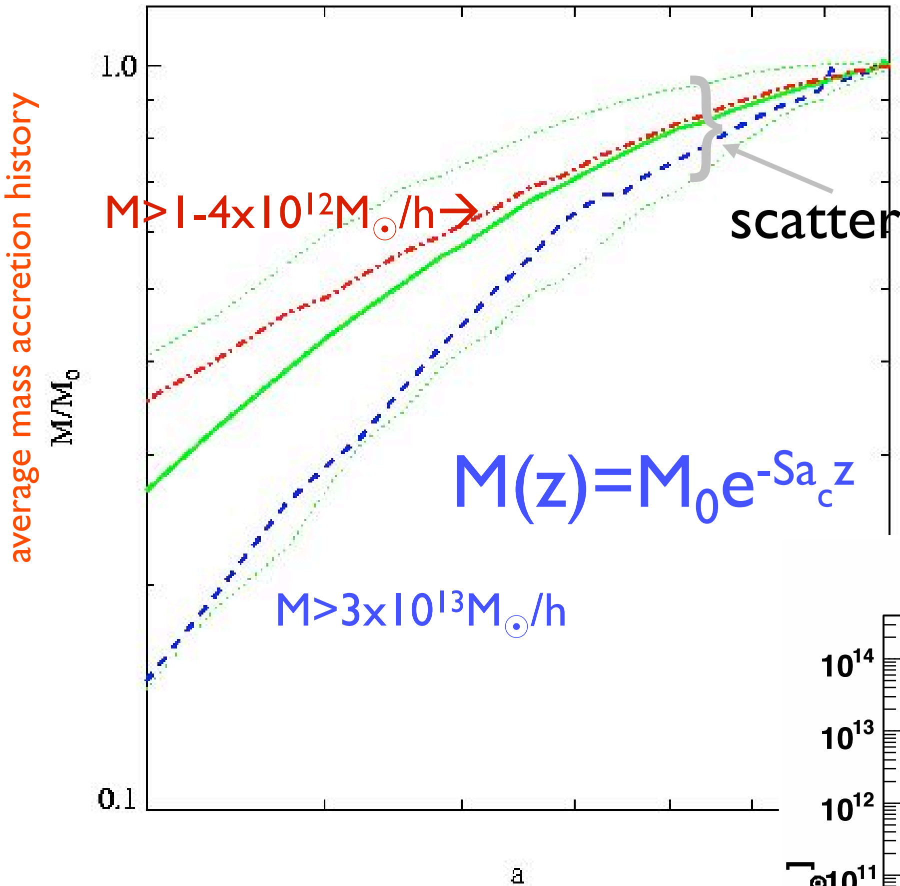
halo merger histories



Wechsler et al 2002

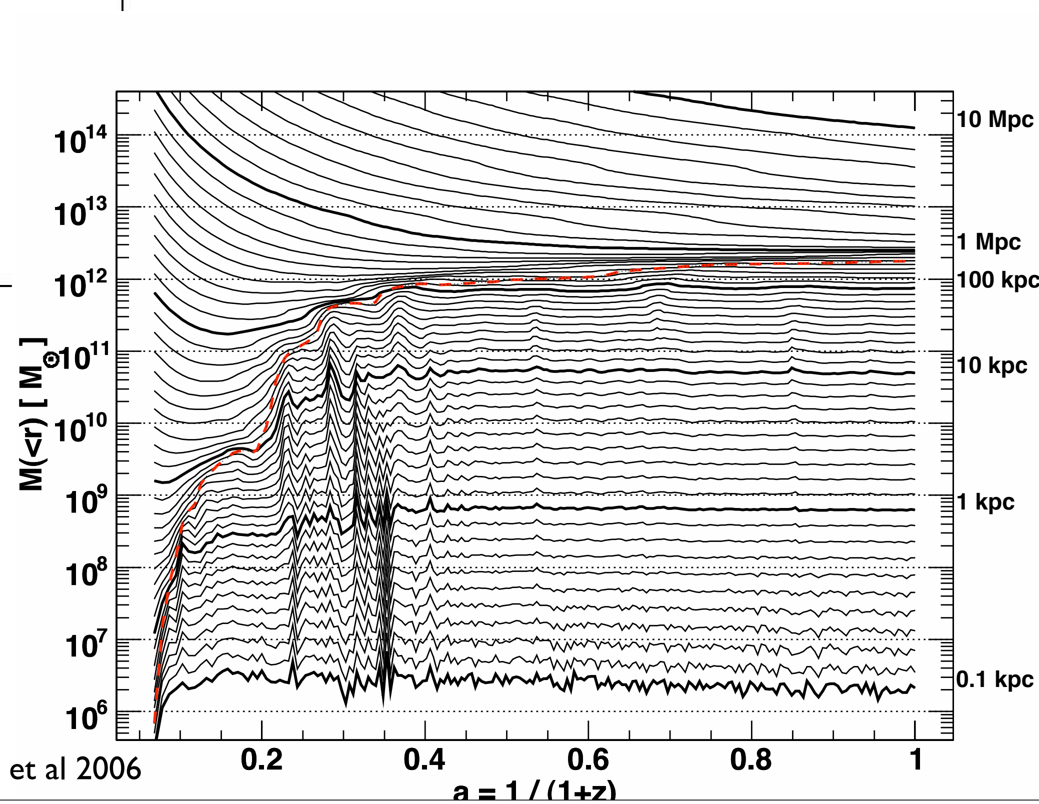
- can extract merger histories from simulations
- or get merger histories analytically from "Extended Press-Schechter"
  - PS: mass functions
  - EPS: predicts the probability of having a halo of mass M with progenitor M1.

Wechsler et al 2002



average MAH

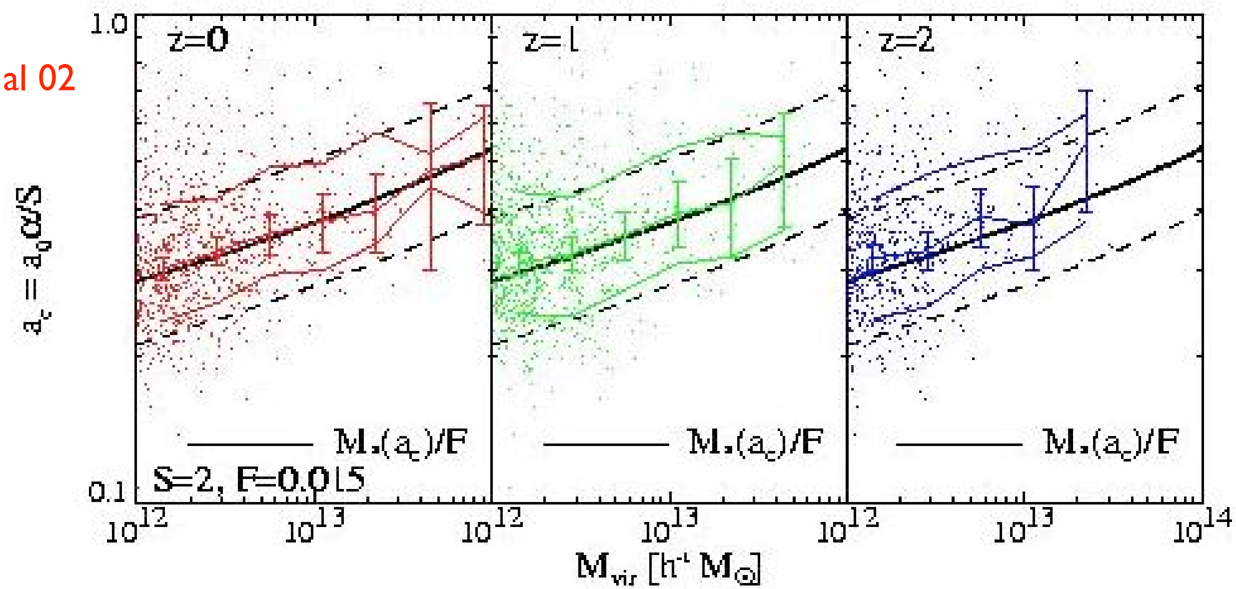
individual MAH



Diemand et al 2006

$a_c(M,z)$

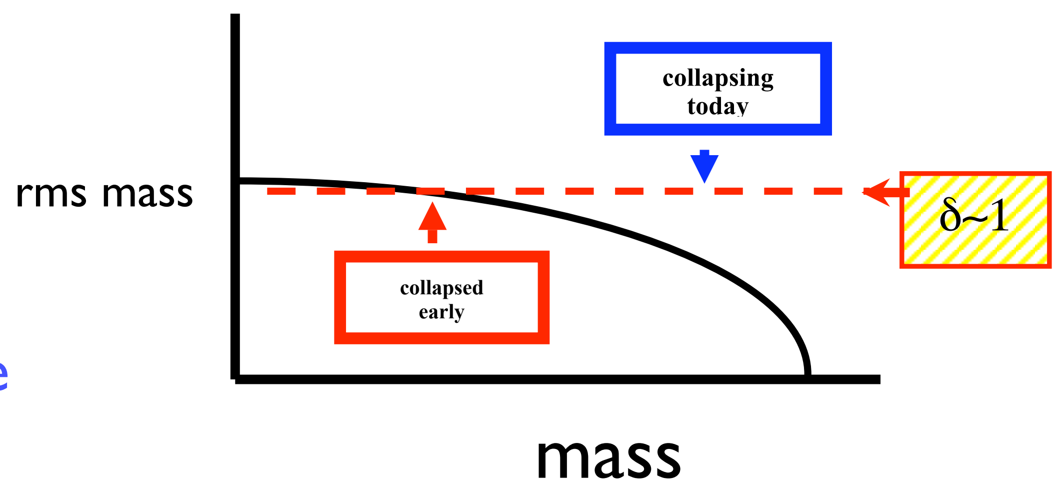
RW et al 02



scatter is 0.13 in $\log_{10} a_c$

- formation time is related to the mass fluctuation spectrum.
- formation time is measured here as the time when it had a given accretion rate, but this is equivalent to a formation time defined as the time when FM = M*

$$\sigma[M\star(a)] = \delta_c$$



how is the mass accreted?

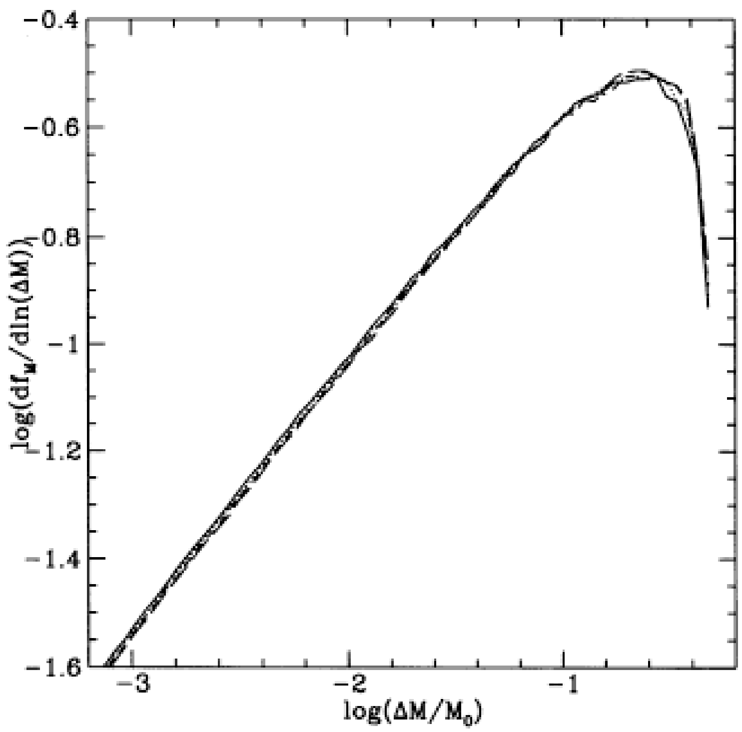
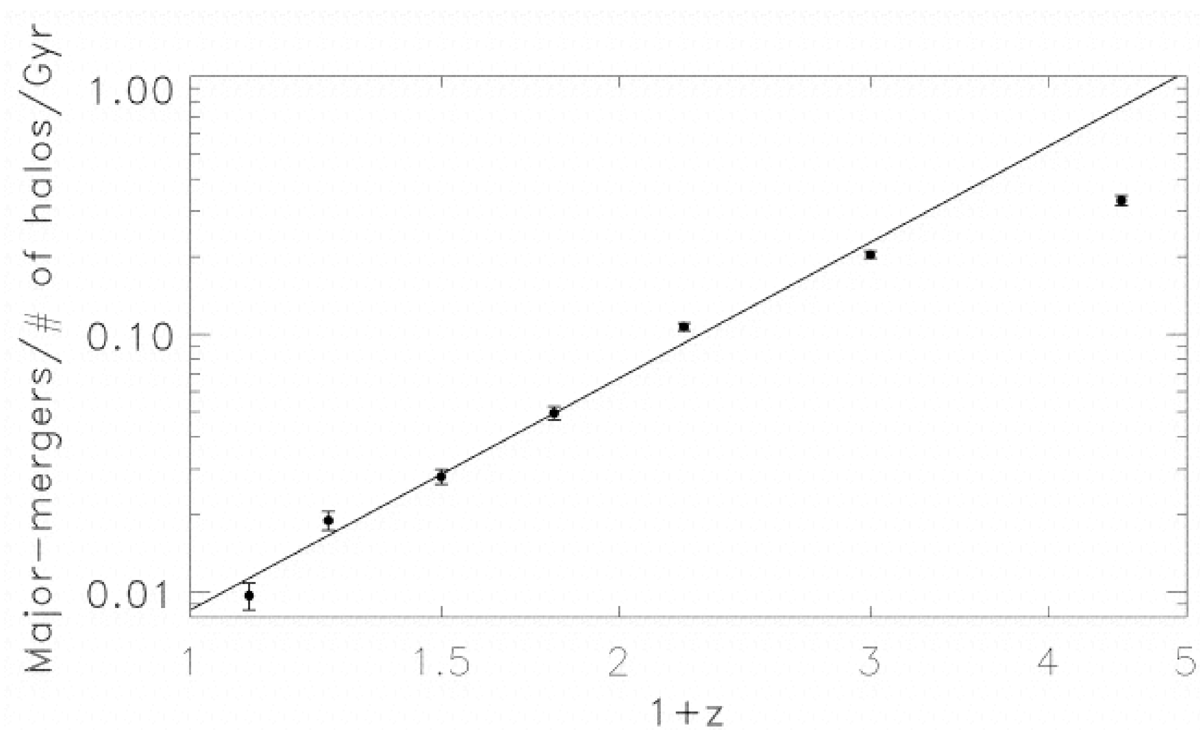


Figure 12. The fraction of mass accreted in haloes of mass ΔM by haloes of given current mass M_0, since time t_f, for power-law power spectra with $n = -2$ (solid curve), $n = -1$ (dotted), $n = 0$ (short-dashed) and $n = 1$ (long-dashed). Curves are obtained from Monte Carlo simulations of halo merger histories, with the parameters the same as in Fig. 9.

most mass accreted in halos ~ 10-30% of the host

Lacey & Cole 1993

Merger rate of DM halos



merger rate goes as $(1+z)^3$

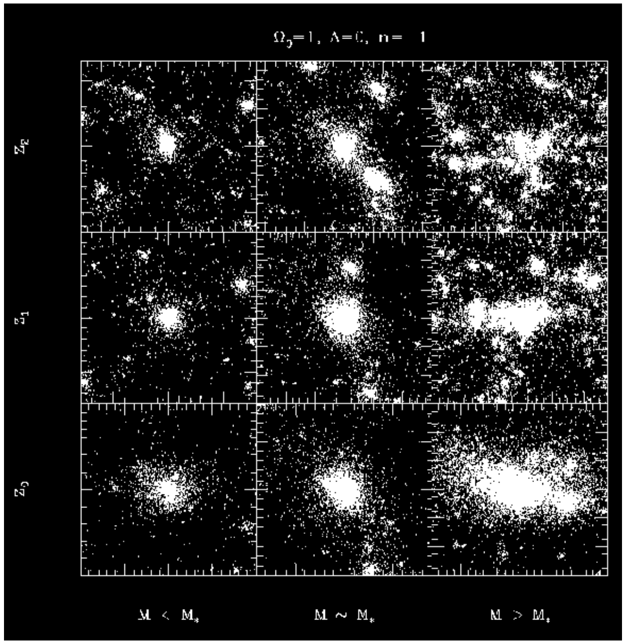
Gottloeber et al 2000

formation history summary

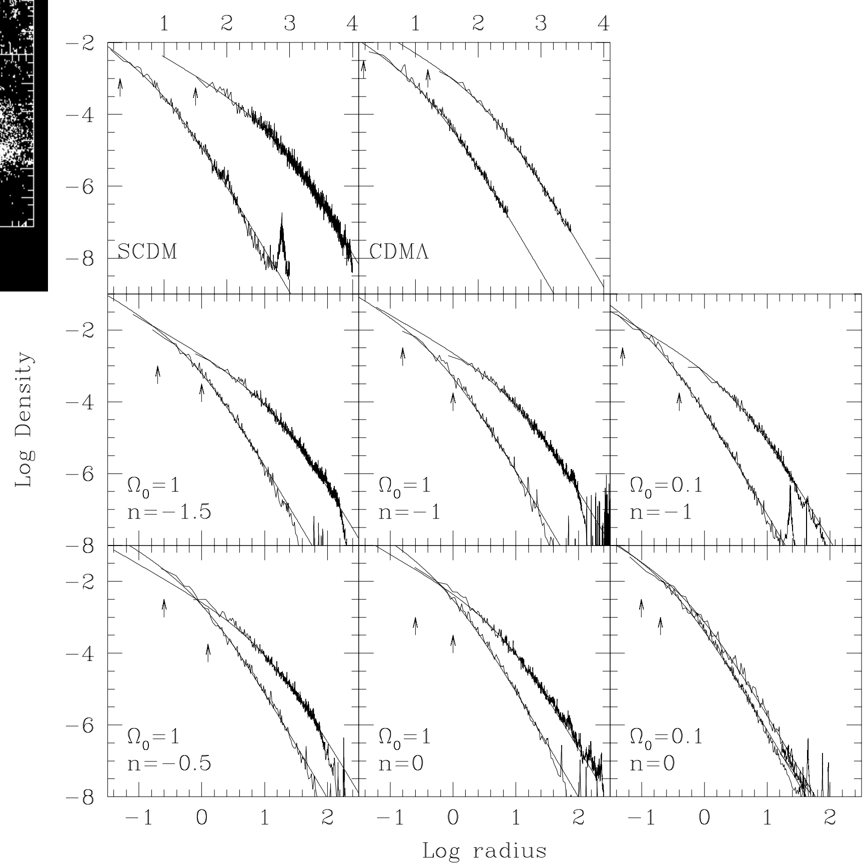
- HSF implies small halos form first and merge into bigger halos
- halo merger rate declines with z.

halo density profiles

Navarro, Frenk & White 1996, 1997



Also:
Dubinsk 1991,
Moore et al 1999
Fukushige & Makino 1999
Klypin 2001
Bullock 2001
Jing & Suto 2001
Power et al 2003
Navarro et al 2004
Maccio et al 2006
Neto et al 2007...



dark matter halos in N-body simulations found to have a roughly 'Universal' density profile

halo density profiles

- roughly self-similar form:

$$M_{\rm vir} \equiv \frac{4\pi}{3}\Delta_{\rm vir}\rho_b R_{\rm vir}^3$$

small radius

$$\rho(r) \sim \frac{1}{r^3}$$

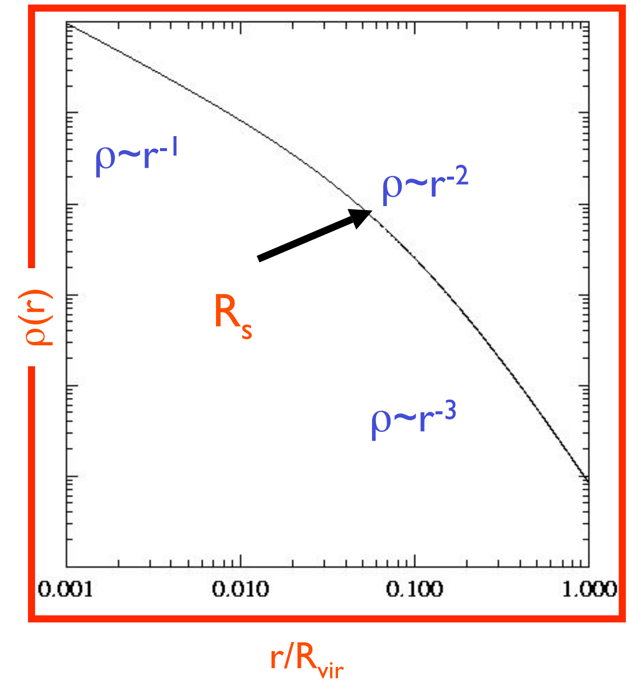
large radius

$$\rho(r) \sim \frac{1}{r^\alpha} \quad \alpha \sim 0.7 - 1.5$$

- convenient parameterization:

$$\frac{\rho(r)}{\rho_{\rm crit}} = \frac{\delta_c}{(r/R_s)(1+r/R_s)^2}$$

Navarro, Frenk & White 1996, 1997(NFW)



- concentration parameter:

$$c_{\rm vir} \equiv \frac{R_{\rm vir}^*}{R_s}$$

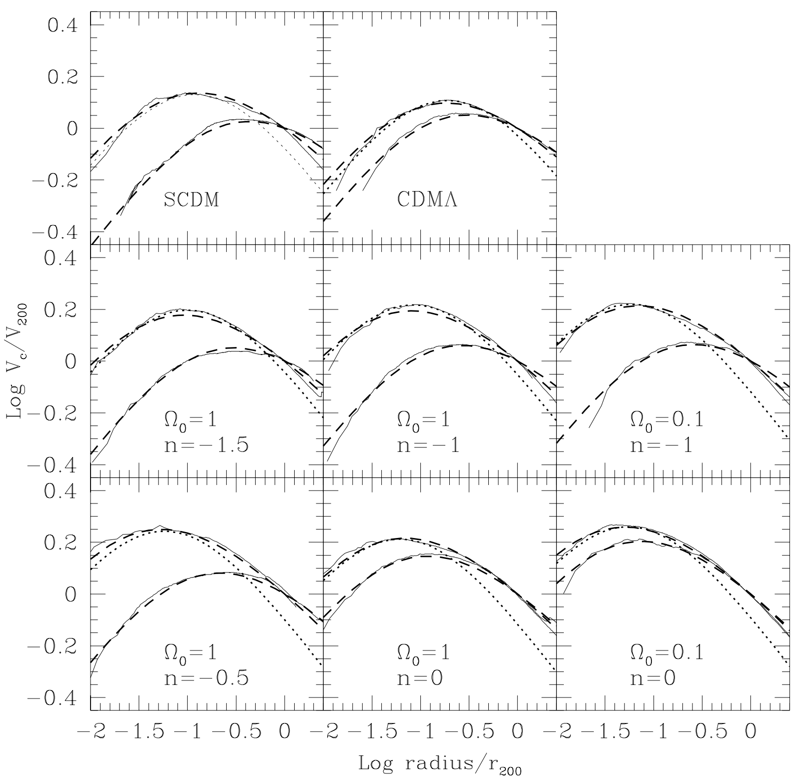
* virial radius with respect to the background density; Δ_{vir}=337 at z=0

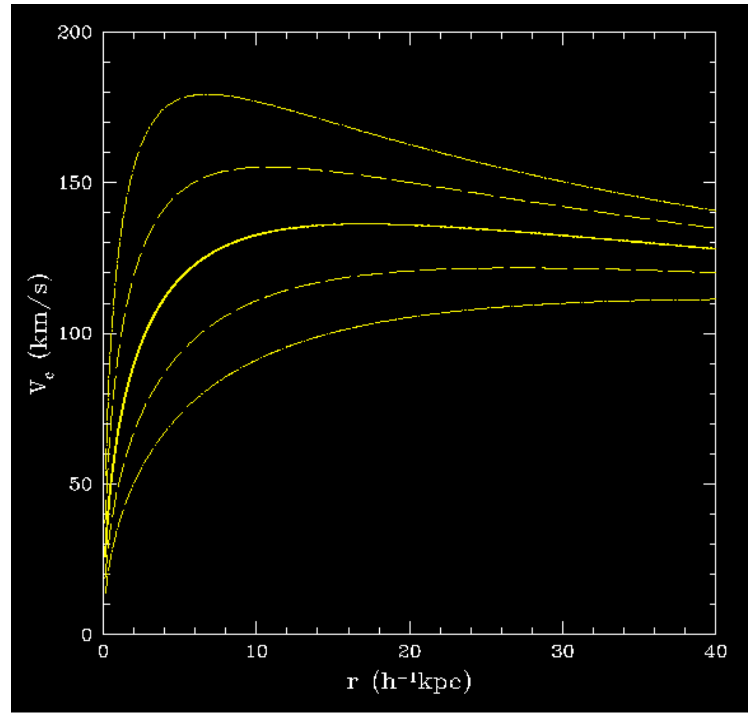
note: concentrations depend on definition of the density threshold used to define halos

halo rotation velocities

$$r_{v_{\max}} = 2.163 r_s$$

log rotation velocity





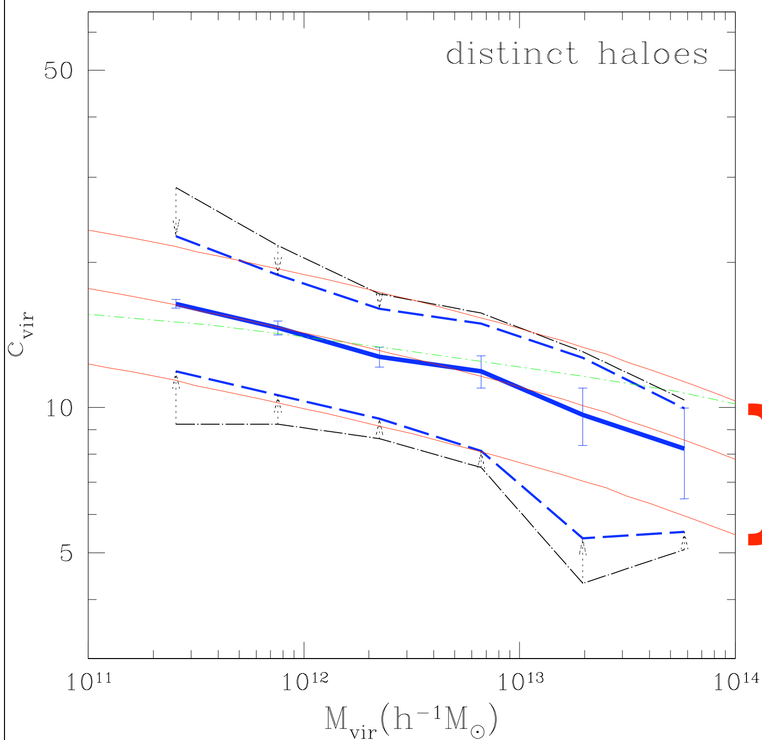
$$V_c(r) = 4\pi G \rho_s r_s^3 \frac{f(r)}{r} \qquad f(r) = ln(1 + r/r_s) - \frac{r/r_s}{1 + r/r_s}$$

log radius

Navarro, Frenk & White 1996, 1997

halo concentrations

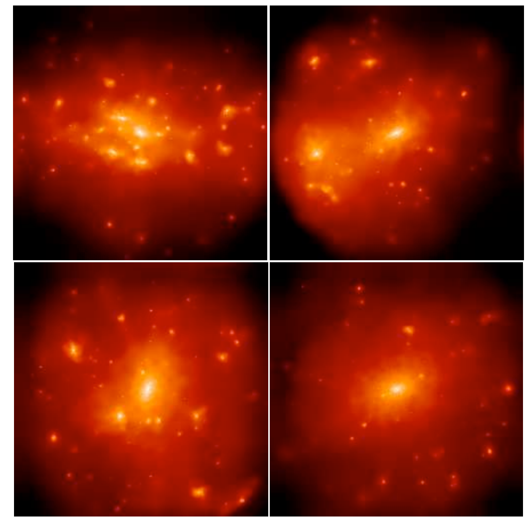
Bullock et al 2001

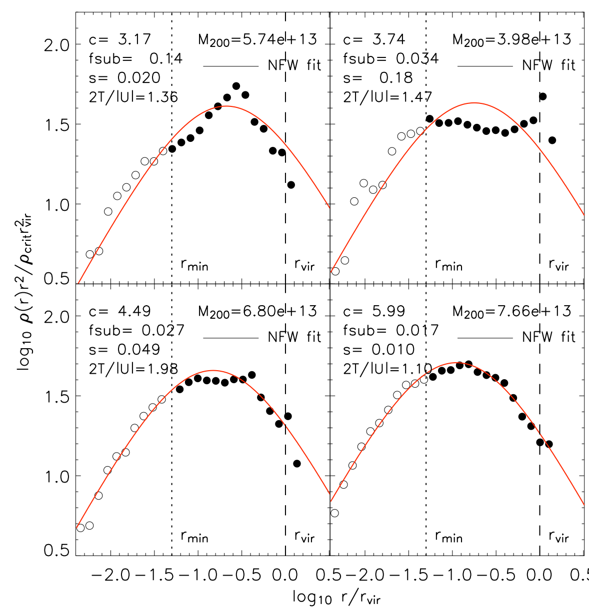


68% intrinsic scatter in halo population

$c_{vir} \sim 1/(1+z)$

Neto et al 2007





concentration vs. formation time

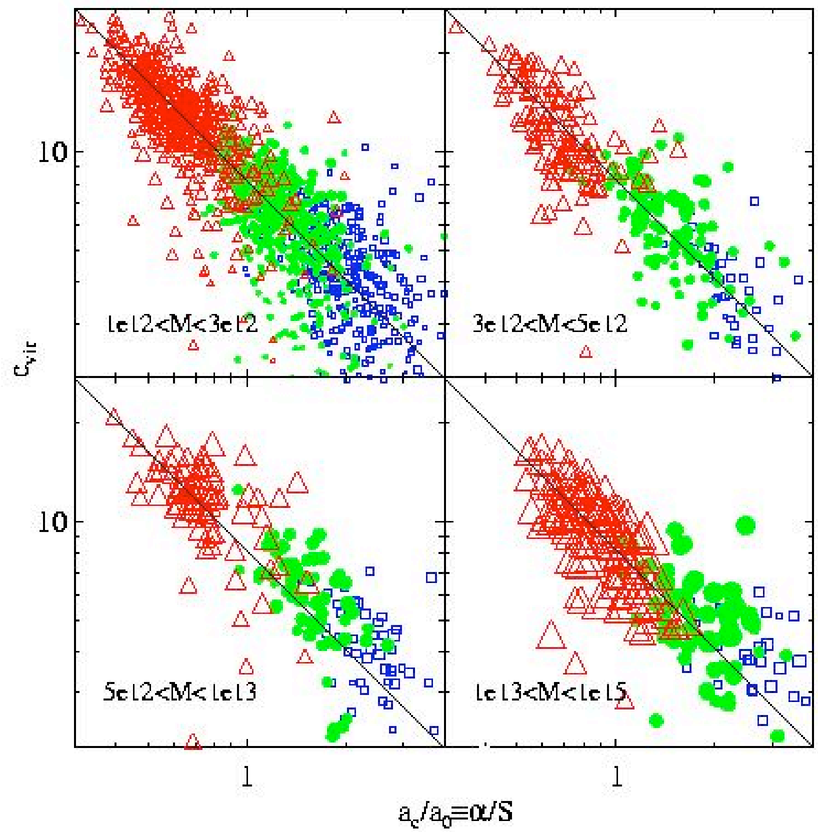
$c_{vir}=c_1 a_{obs}/a_c$

for all masses and redshifts

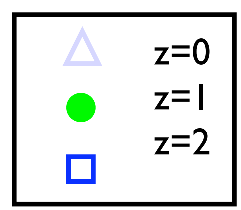
scatter at a given mass
and redshift caused by scatter
in mass accretion histories

→ correlated with galaxy type?

concentration



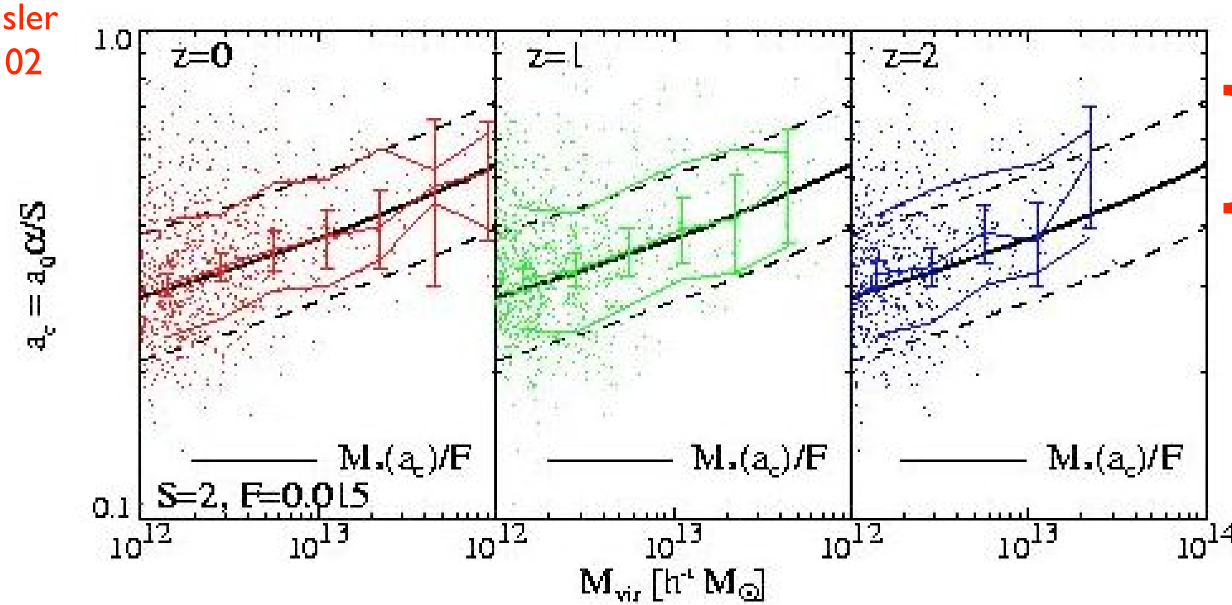
scaled formation scalefactor



RW et al 02

$a_c(M,z)$

Wechsler et al 02



} scatter in c_{vir}

simply inverting this plot gives you c(M,z) + scatter.

the same model for formation time based on M* applies.

this means that we can understand how concentrations depend on the power spectrum

$$c_{\rm vir} = K\frac{a}{a_c} \sim 9(M/M_*)^{-0.13}$$

Bullock et al 2001

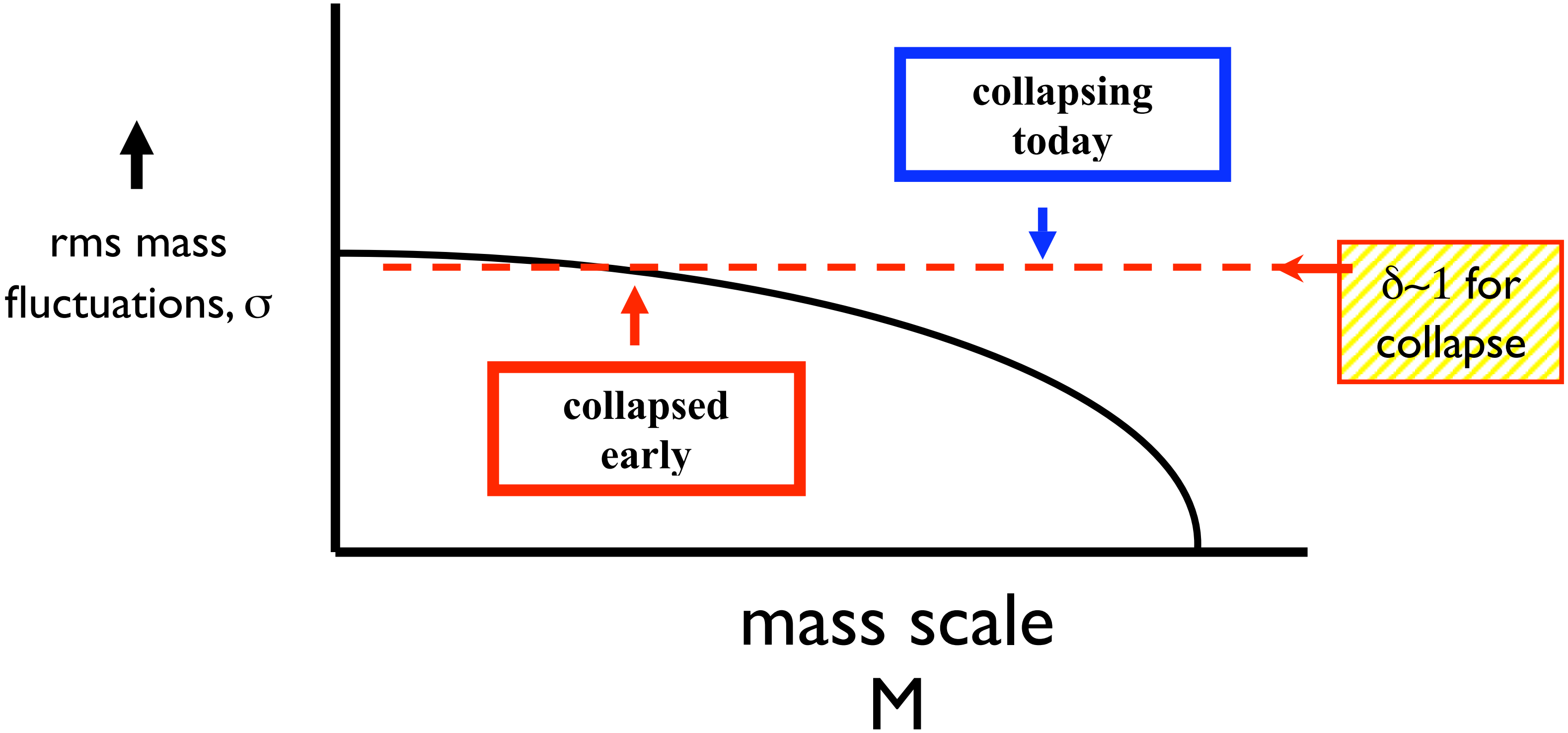
$$c_{200} = 5.26\ \left(M_{200}/10^{14}h^{-1}\ M_\odot\right)^{-0.10},$$

for *relaxed* haloes, and

$$c_{200} = 4.67\ \left(M_{200}/10^{14}h^{-1}\ M_\odot\right)^{-0.11}$$

for the complete halo sample.

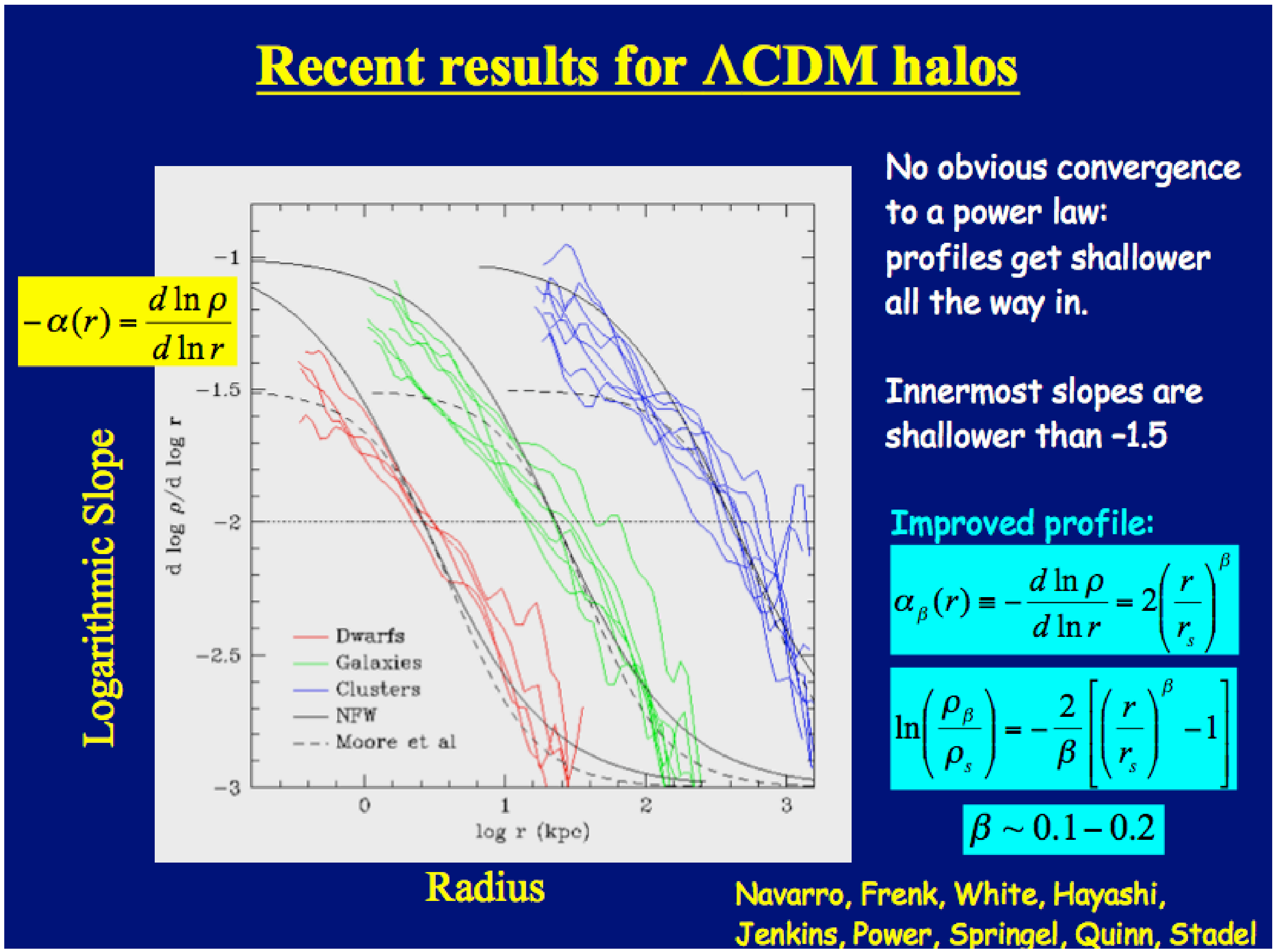
Neto et al 2007

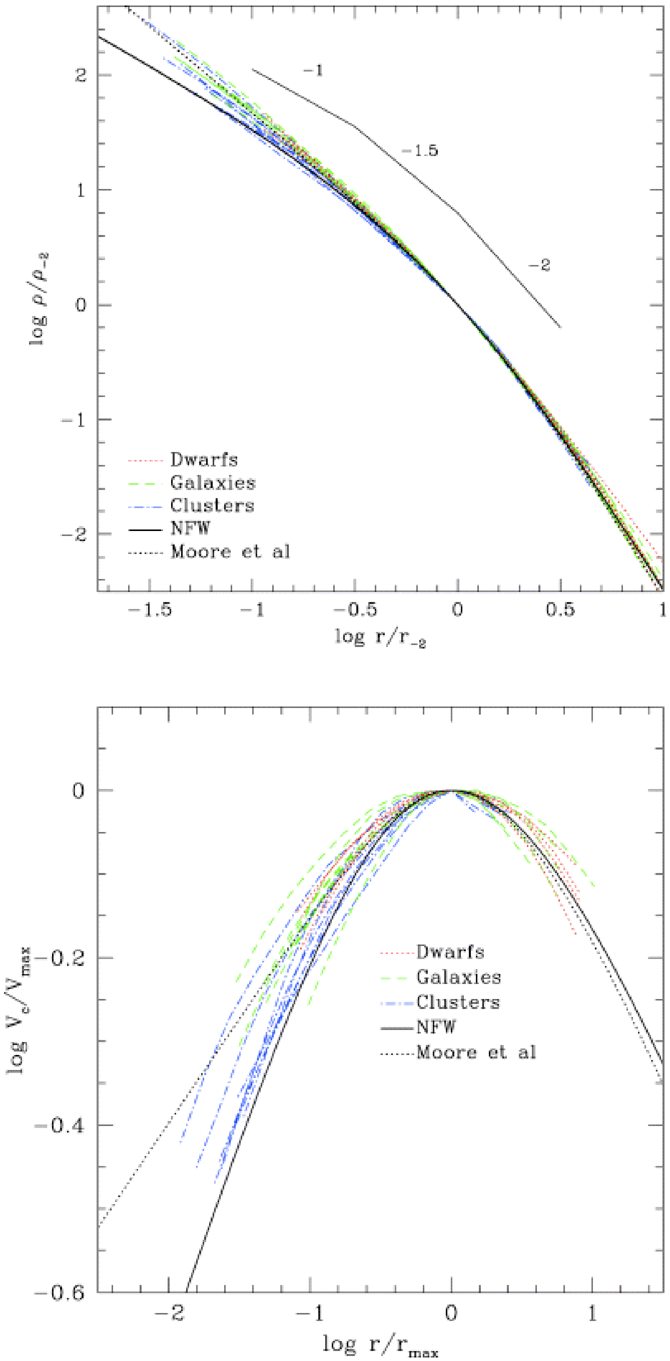


- low mass halos form early, when the universe was denser
- reducing mass fluctuations on galaxy scales (low σ_8, tilt) reduces concentrations

the inner slope

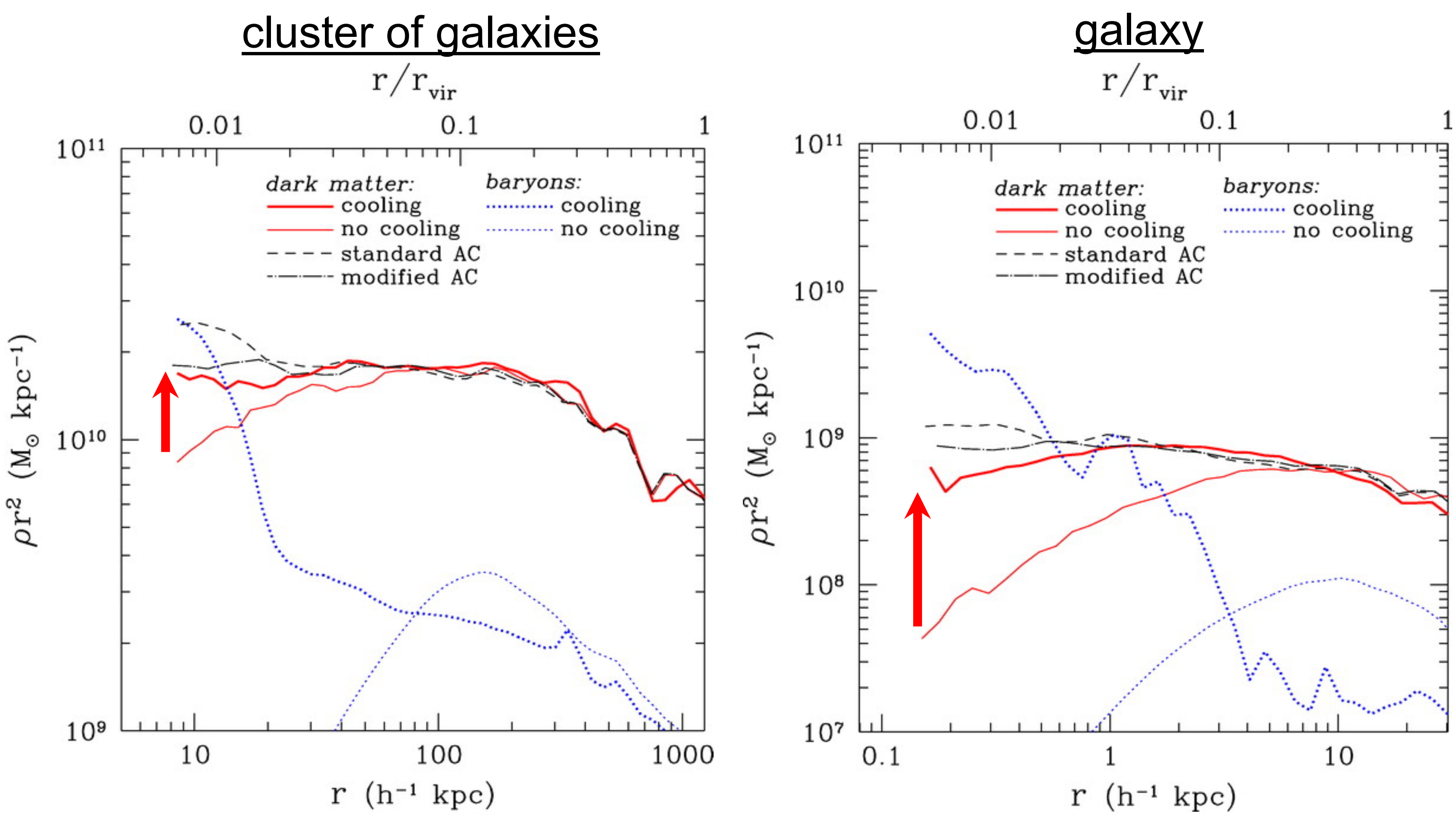
(“cuspy halo crisis”)





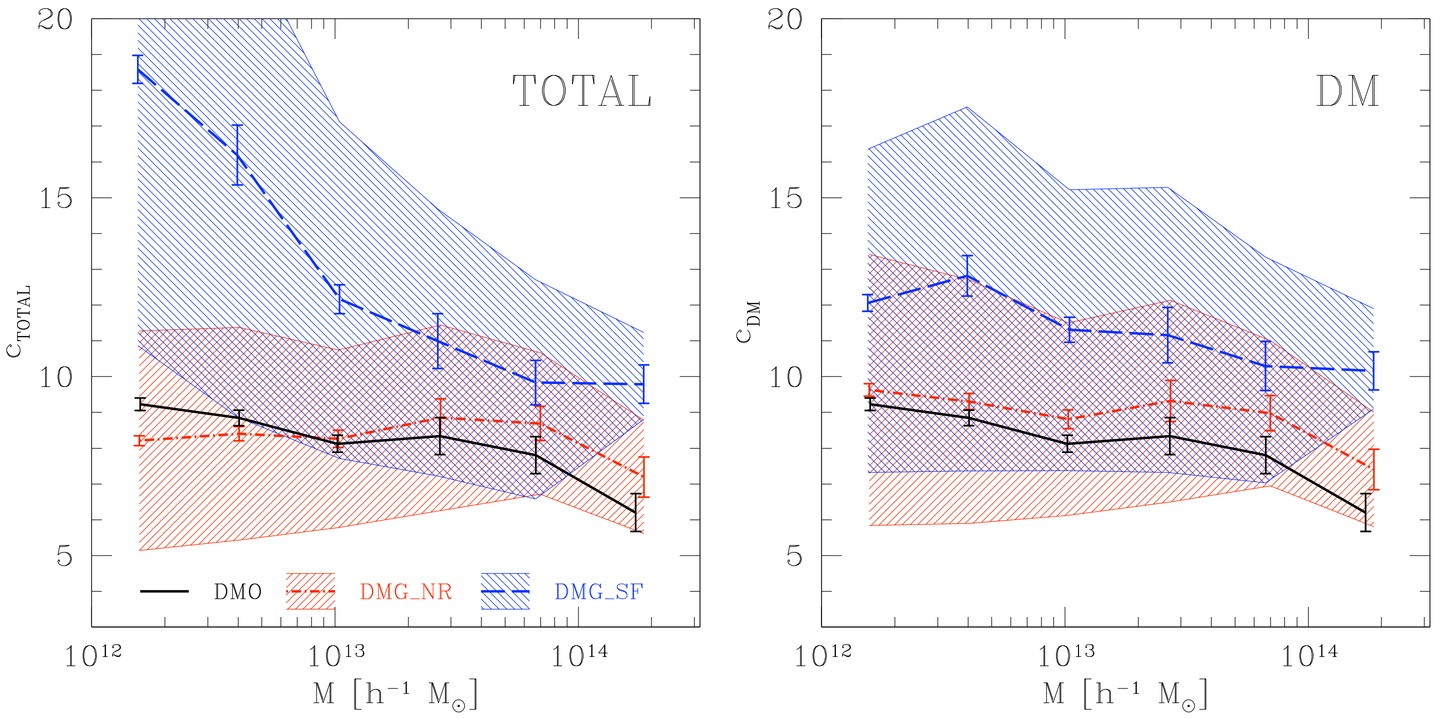
a diversity of inner slopes?

do baryons matter for profiles? adiabatic contraction



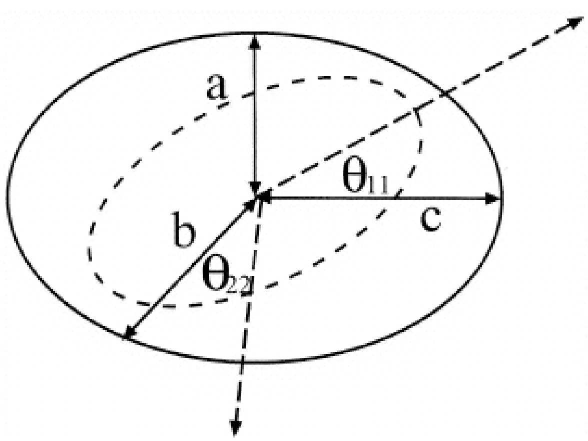
Gnedin et al 2004

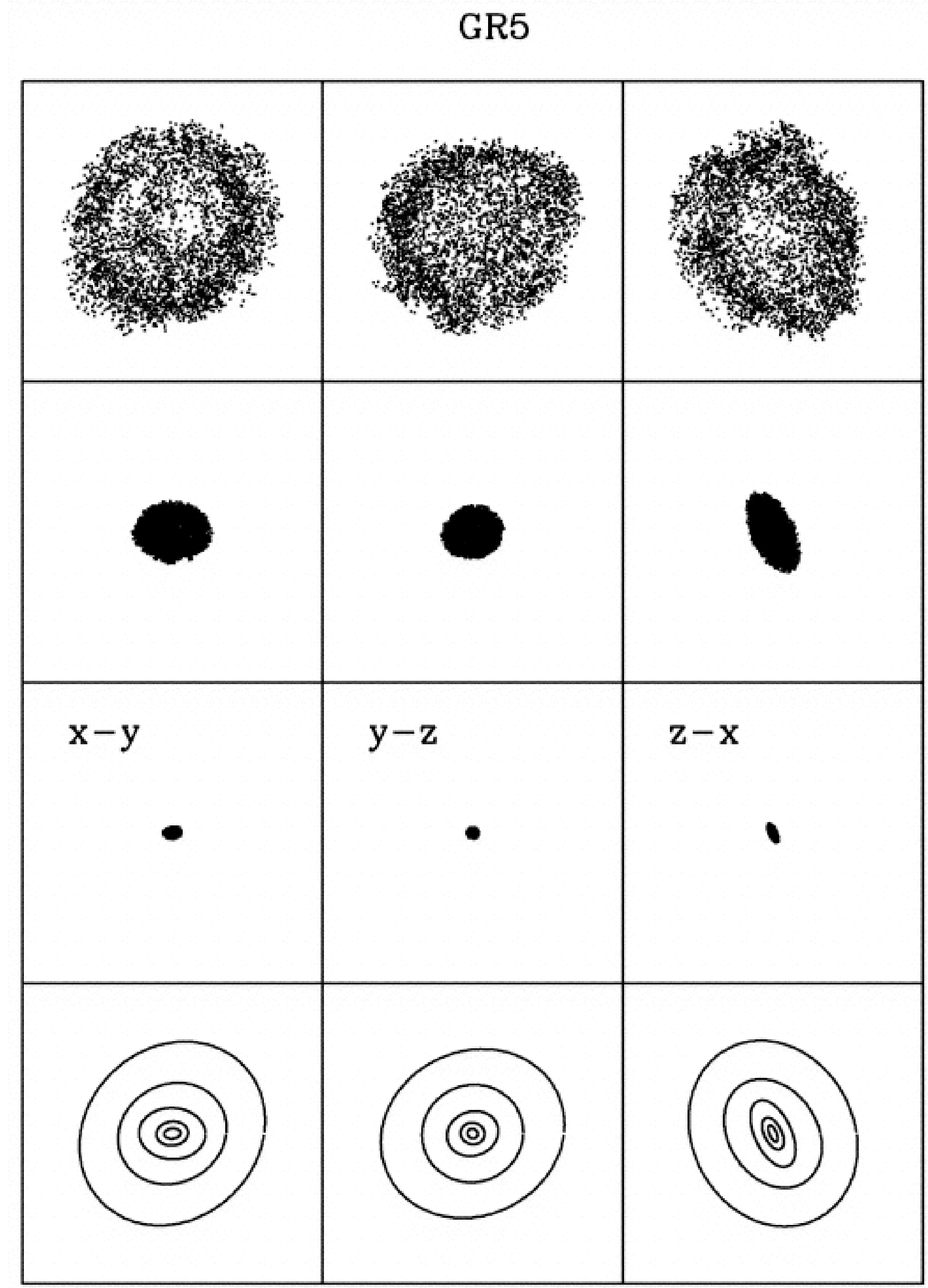
effect of baryons on concentrations



Rudd et al 2007

triaxial halos

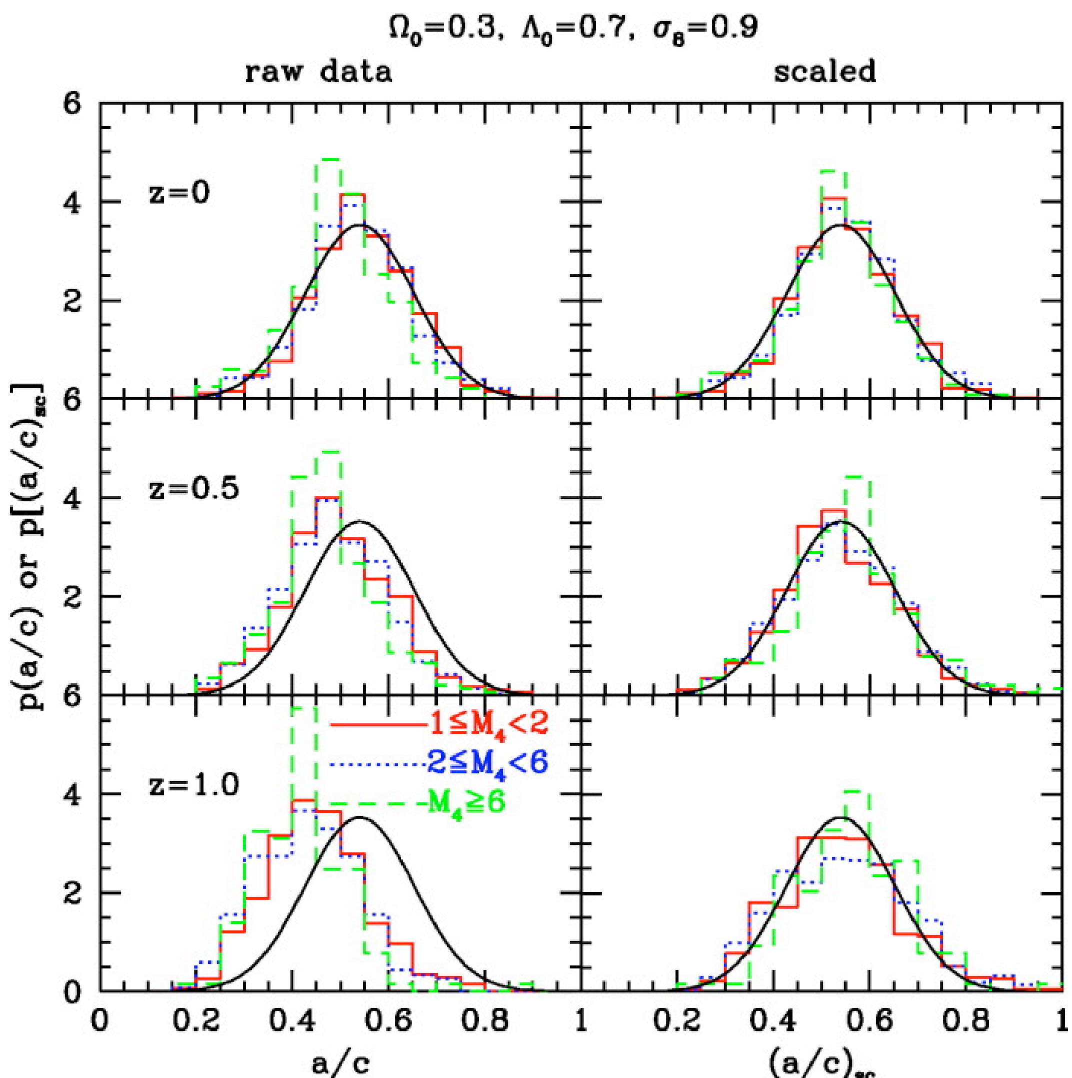




Jing & Suto 2002

triaxial halos

- wide distribution of shape parameters, but all halos fairly triaxial and prolate

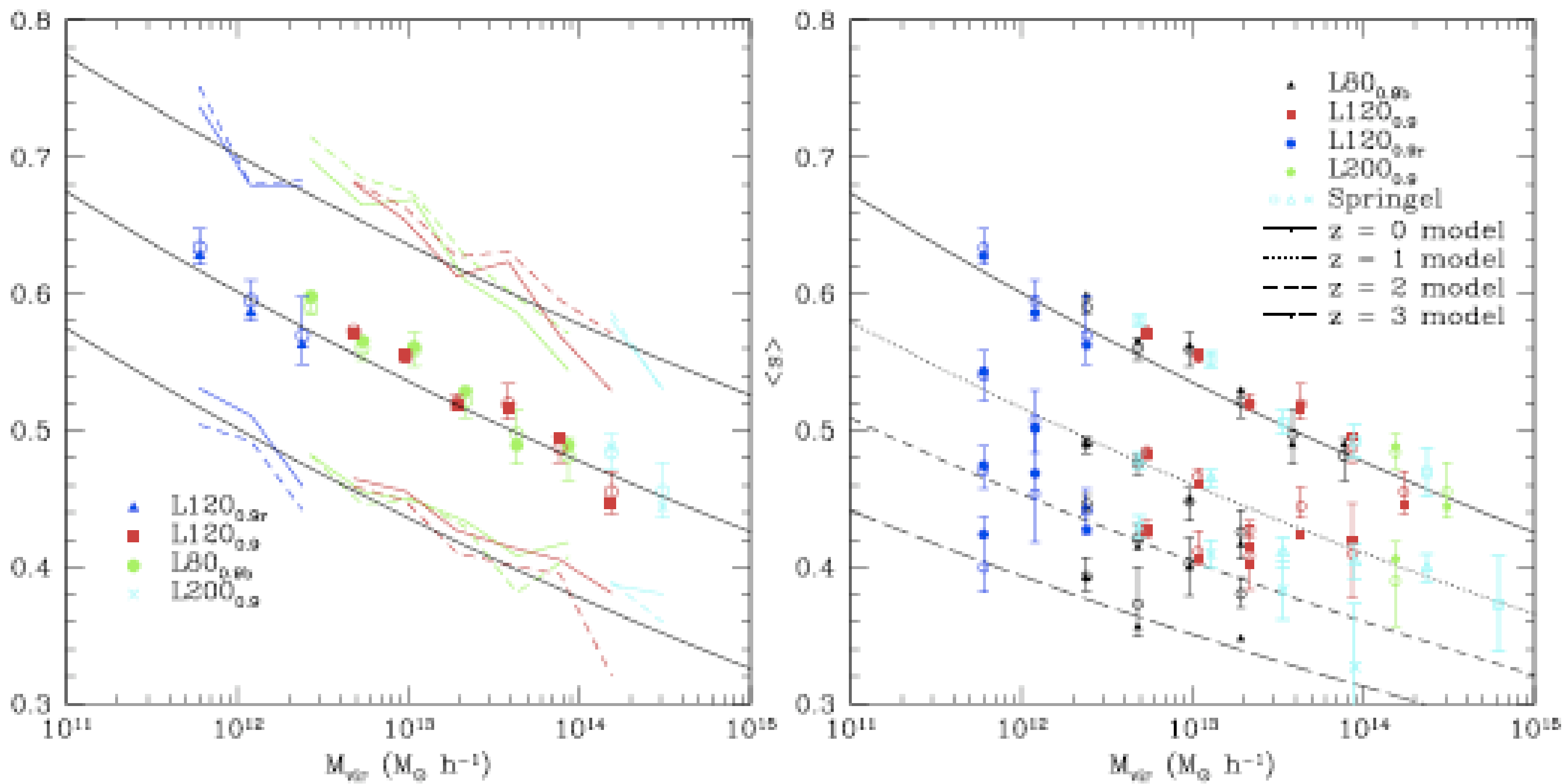


Jing & Suto 2002

halo shapes

$s = 0.54 M_{vir}/M_*^{-0.05}$

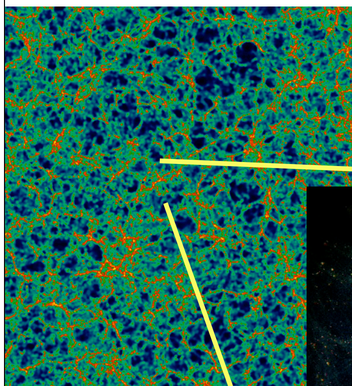
shape =
shortest axis/longest axis



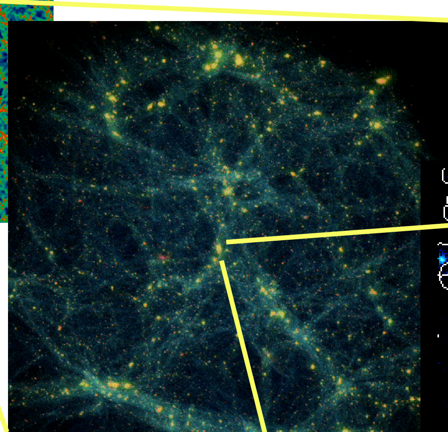
- halos get rounder with time & further out in radius
- low mass halos are the most spherical
- early forming halos more spherical

Allgood et al 2006

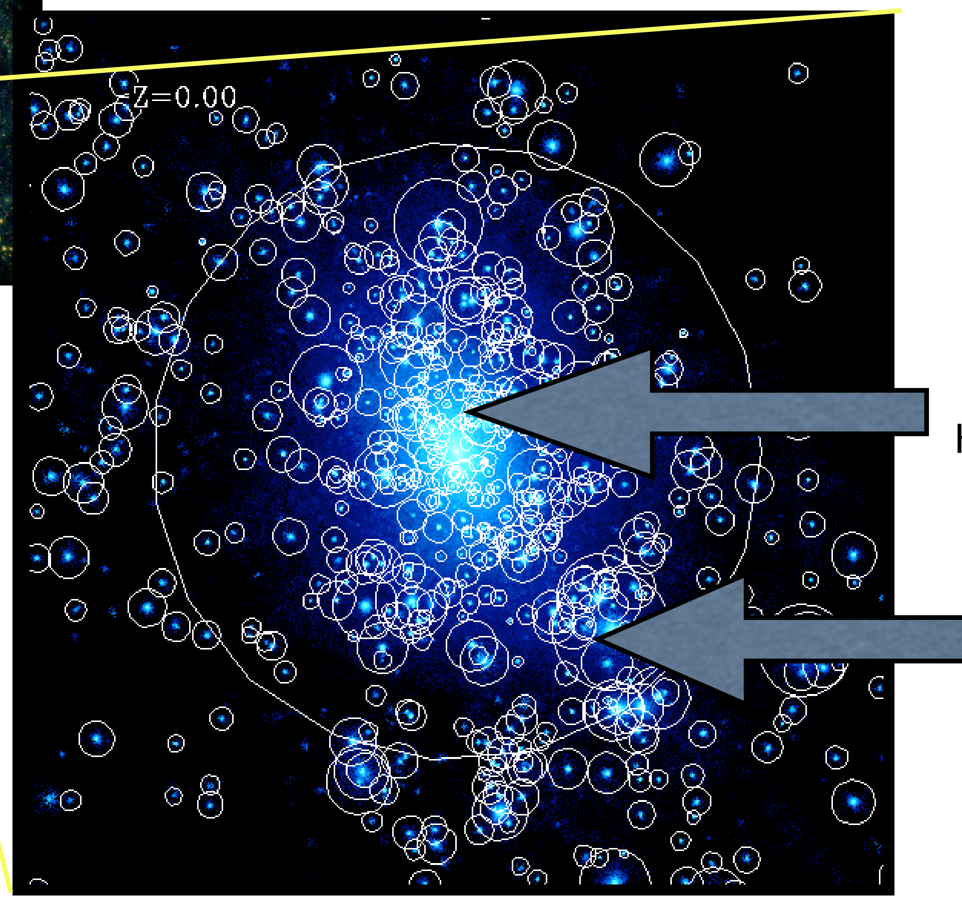
dark matter substructure



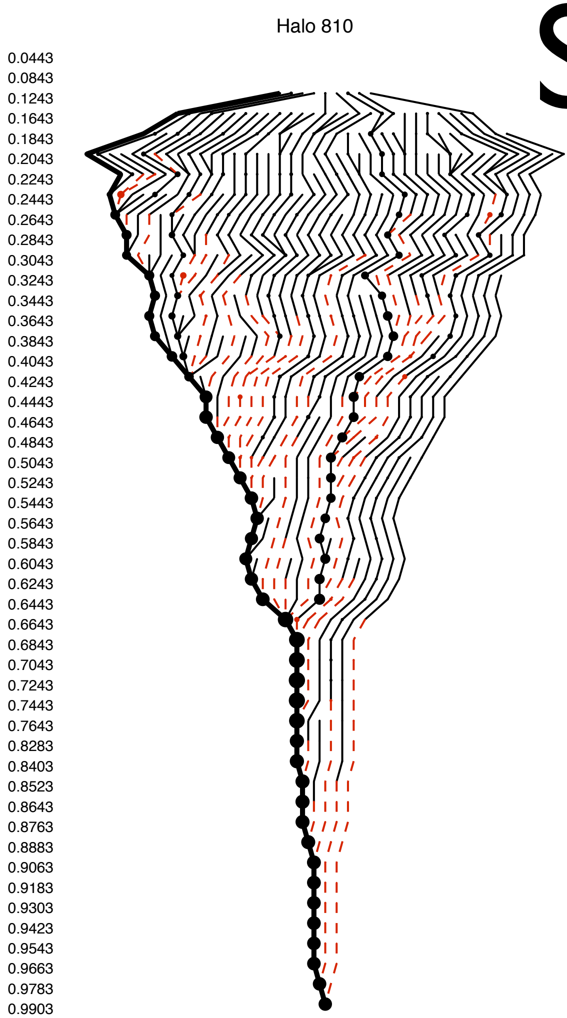
simulation by VIRGO consortium



simulation by B. Allgood



simulation by A. Kravtsov

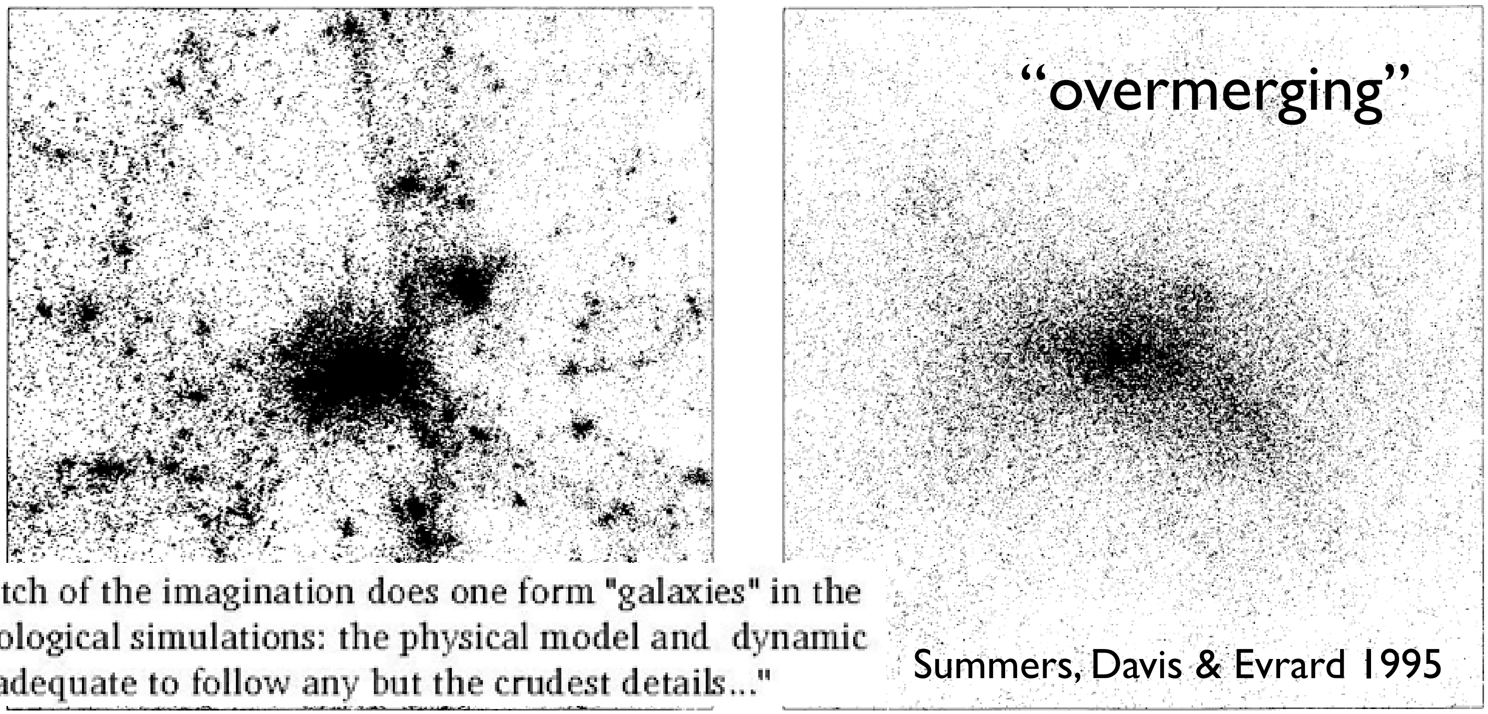


Stewart, Bullock, Wechsler
in prep

Substructure

- The best evidence for a hierarchical structure formation
- The distribution and properties of substructure contains information about the entire hierarchy and history of merging galaxies
- This includes information about the properties and nature of dark matter
- Substructures in large halos are likely the host for galaxies

substructure studies only a decade old



"... by no stretch of the imagination does one form "galaxies" in the current cosmological simulations: the physical model and dynamic range are inadequate to follow any but the crudest details..."

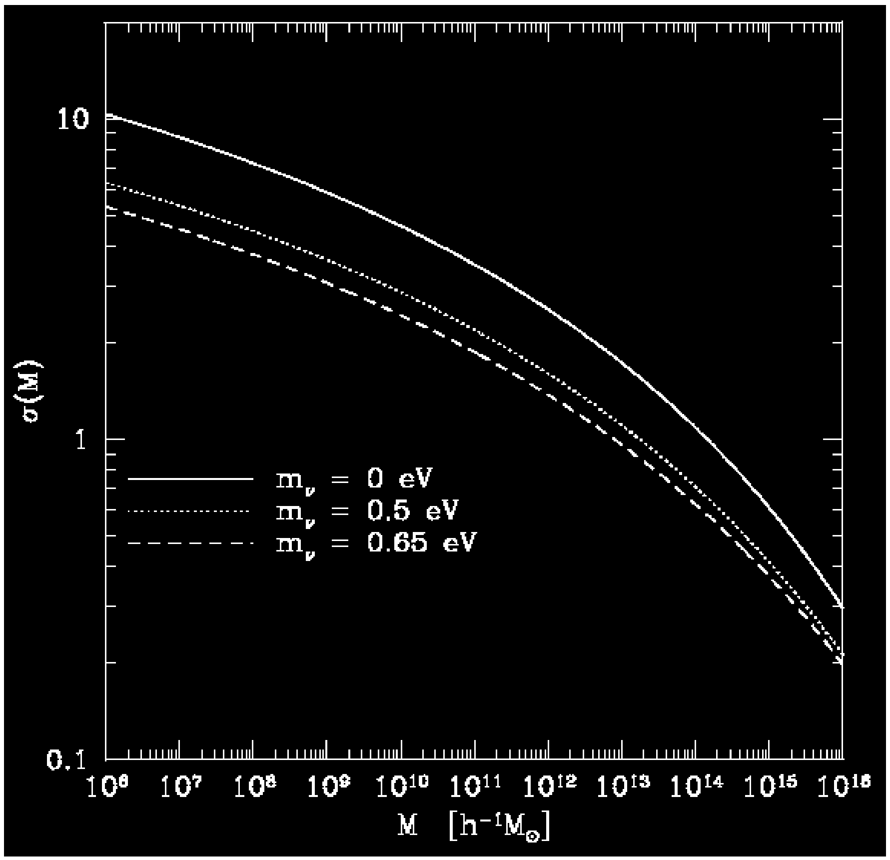
Full Box – 7 Mpc   Central Region – 700 kpc

FIG. 1.—Two plots which characterize the dark matter distribution in the simulation. Length scales here and throughout are given in physical units at $z = 1$. On the left is the entire simulation volume (a 7 Mpc cube) showing the central group, a second group above and right of center, and various filamentary structures. For clarity, only one-fourth of the particles are plotted. The right-hand side details the central region, 1/10 the box length on a side, and shows all of the particles.

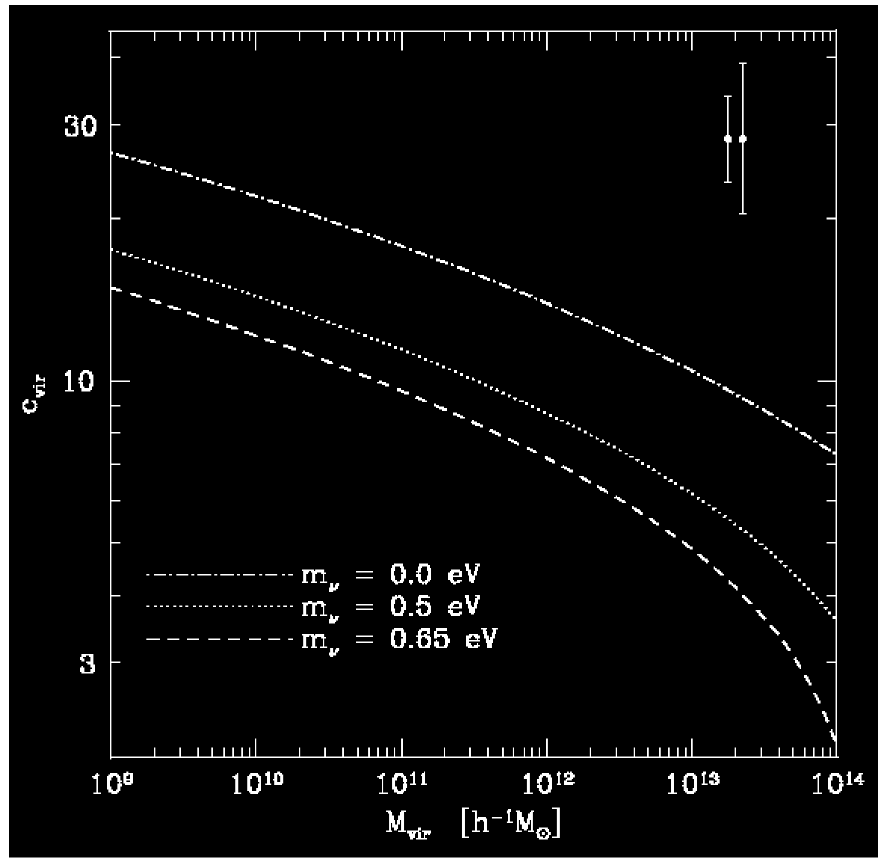
first serious substructure studies in 1998:
Klypin et al 1998, Moore et al 1998, Ghigna et al 1998

Warm Dark Matter

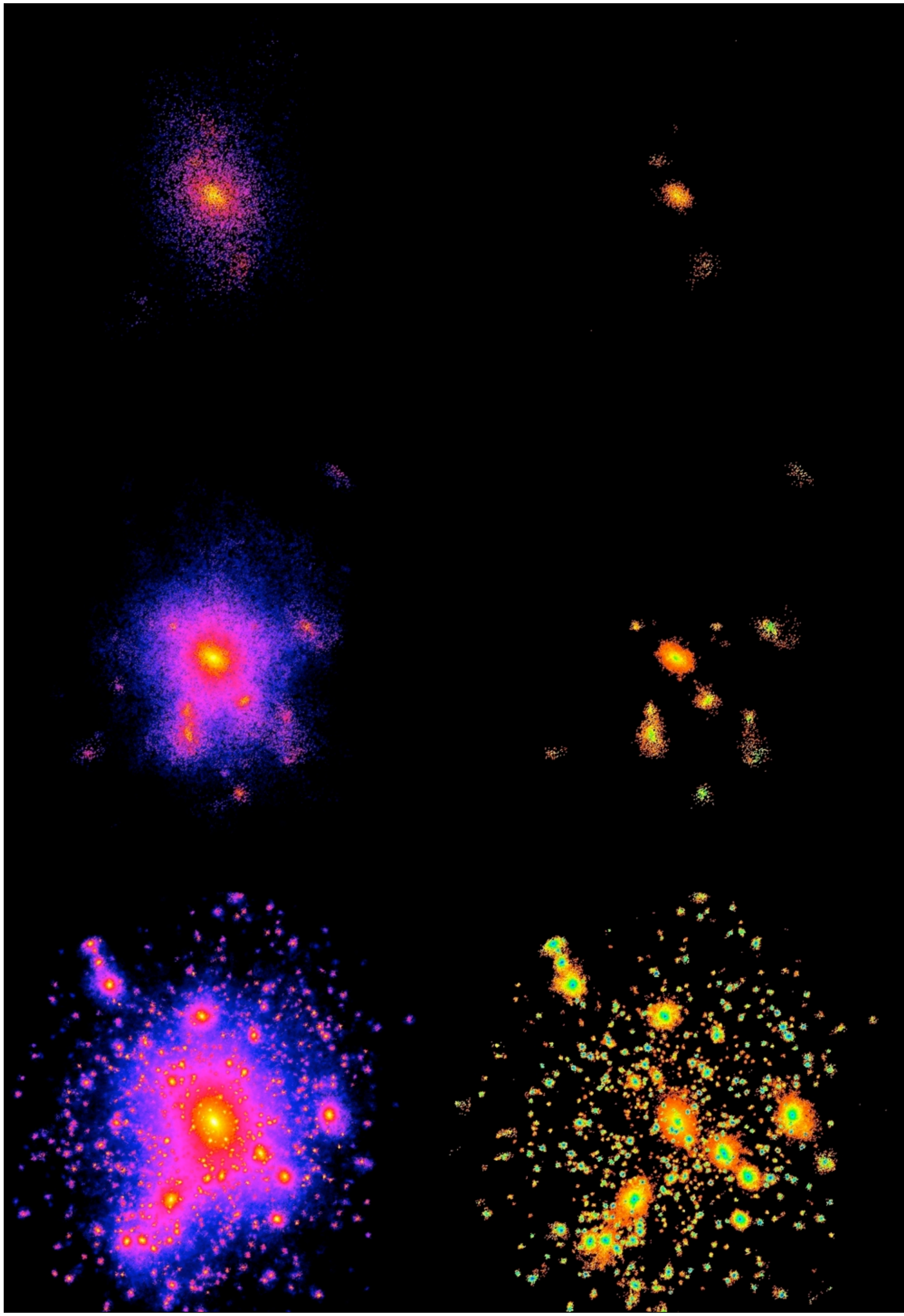
mass variance



halo concentration

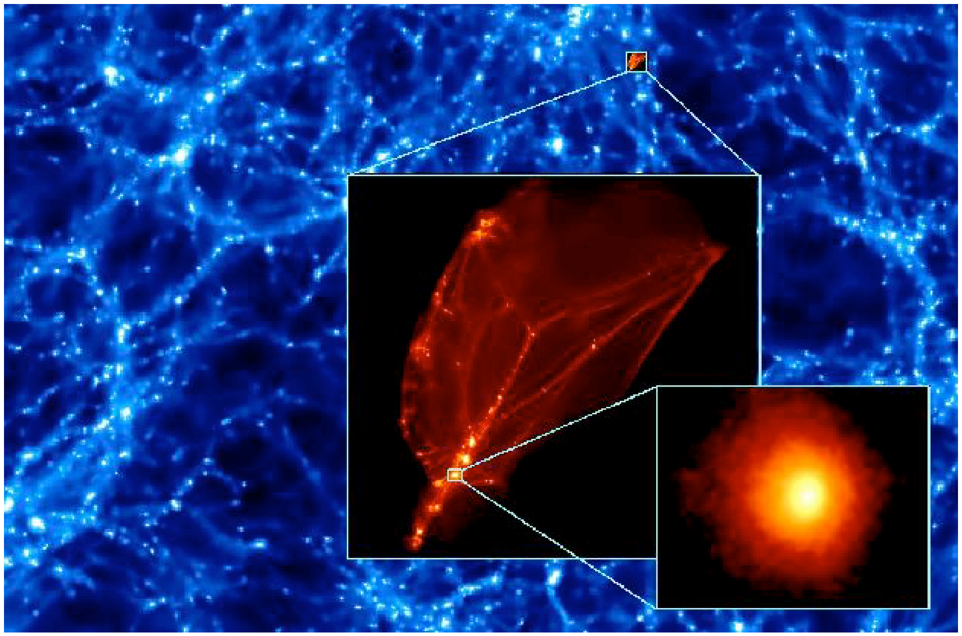


Zentner & Bullock 2002; 2003

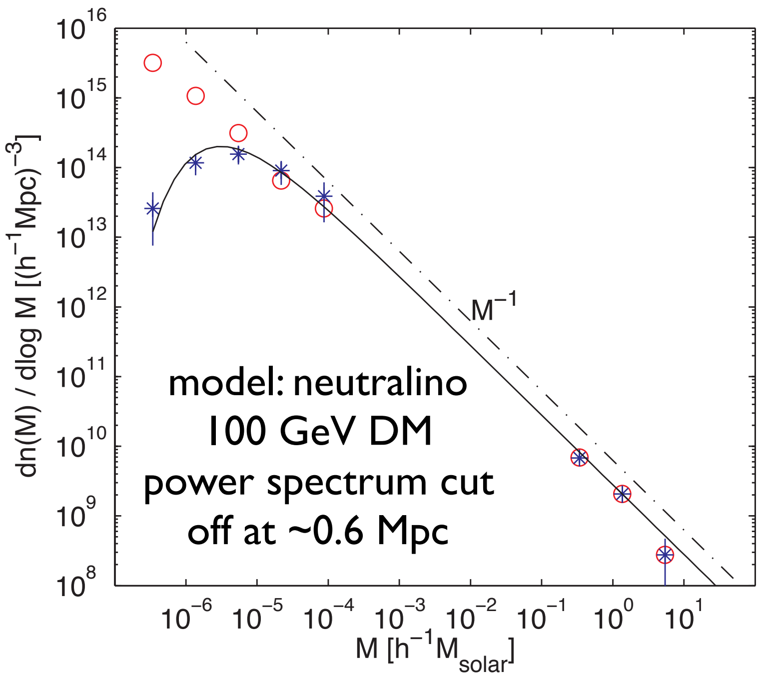


resolving substructure in simulations

CDM substructure may extend over 18-20 orders of magnitude in mass



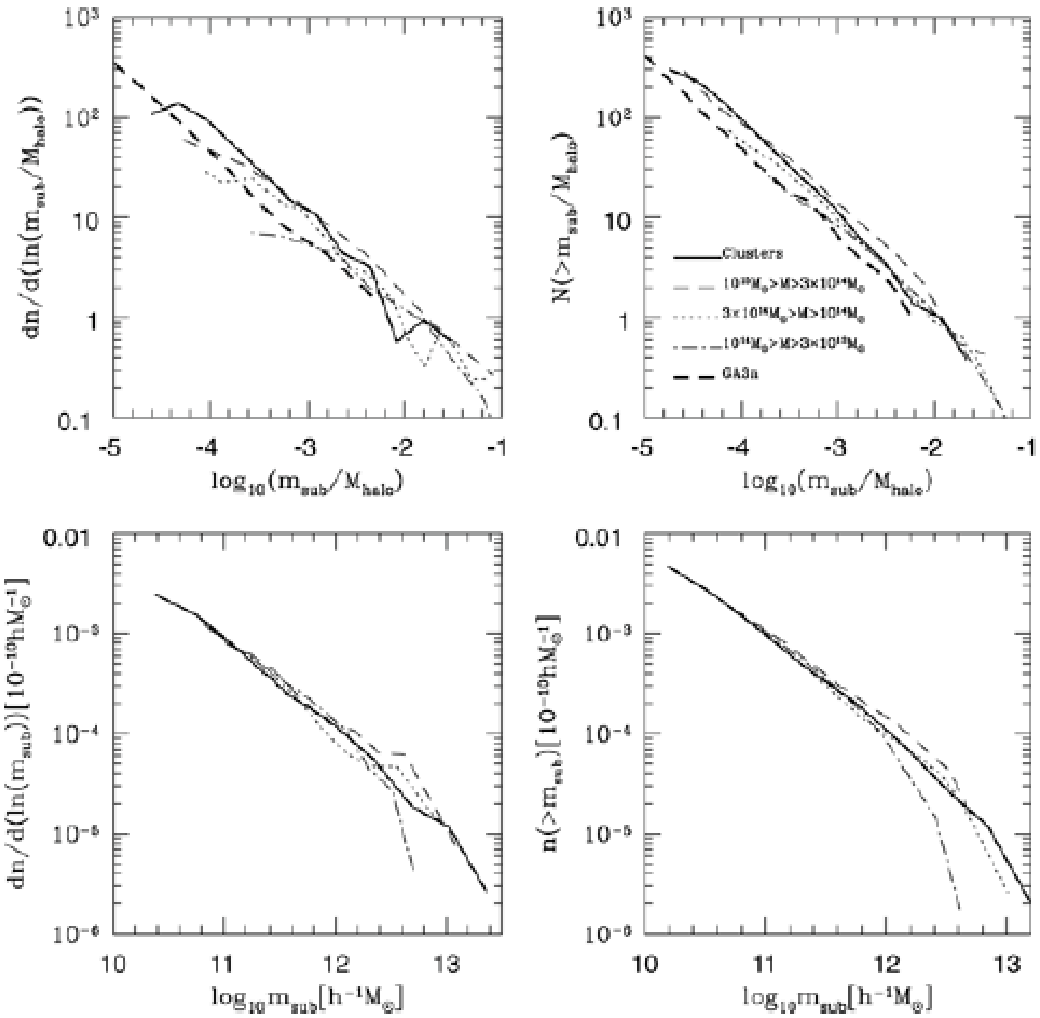
one of the first objects to form, at z~60.
smooth halo with a cuspy density profile.
earth (10^{-6}) mass substructures, size of the solar system.
10^{15} of these inside the galactic halo



Diemand et al 2006

axion DM: no cut off here(10^{-13})

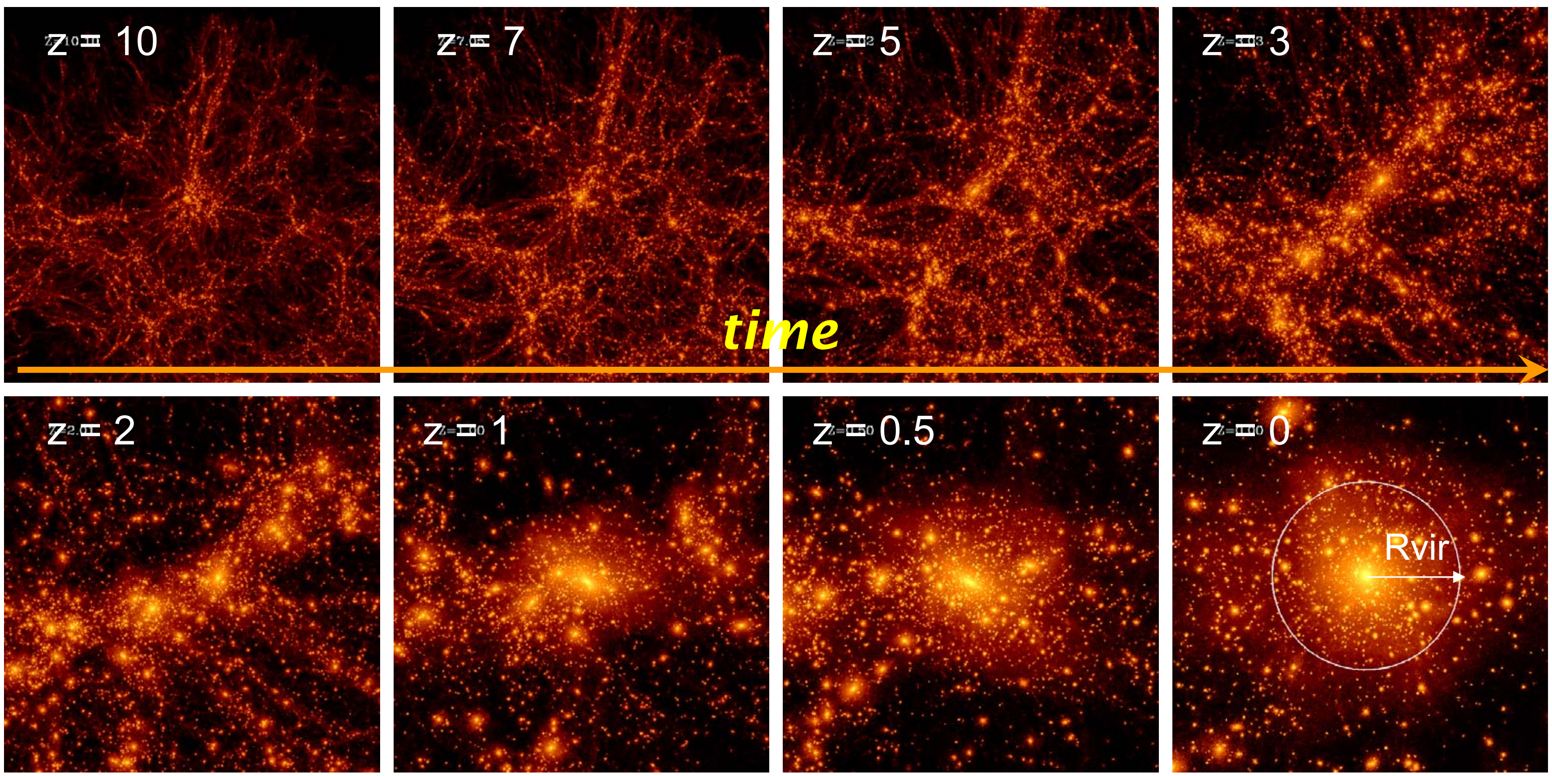
(approximately)
self-similar substructure



Abundance of subhalos in a given halo

is determined by competition between accretion of new subhalos and disruption of old subhalos

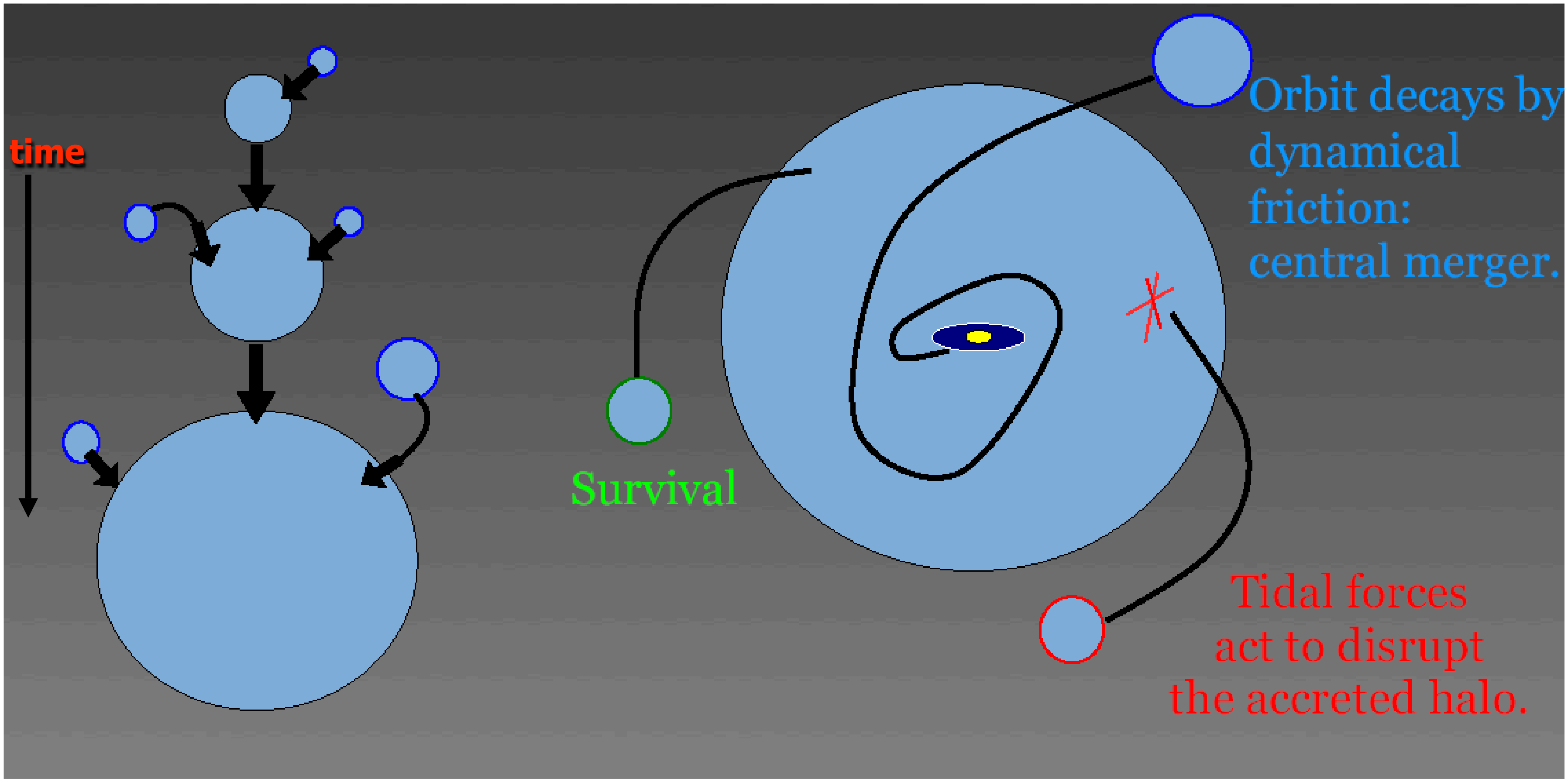
disruption = loss of identity via merging with other halos or significant mass loss due to tidal stripping



Formation of a galaxy-sized halo in LCDM, Mvir=3x10$^{12}h^{-1}$ Msun; Rvir=293h^{-1} kpc;

slide credit: Kravtsov

what processes affect substructure?



slide credit:Zentner

Gnedin & Ostriker 1999; Gnedin, Ostriker, & Hernquist 2000; Taffoni et al. 2002; Taylor & Babul 2002; Zentner & Bullock 2003; Zentner et al. 2005a,2005b

dynamical friction

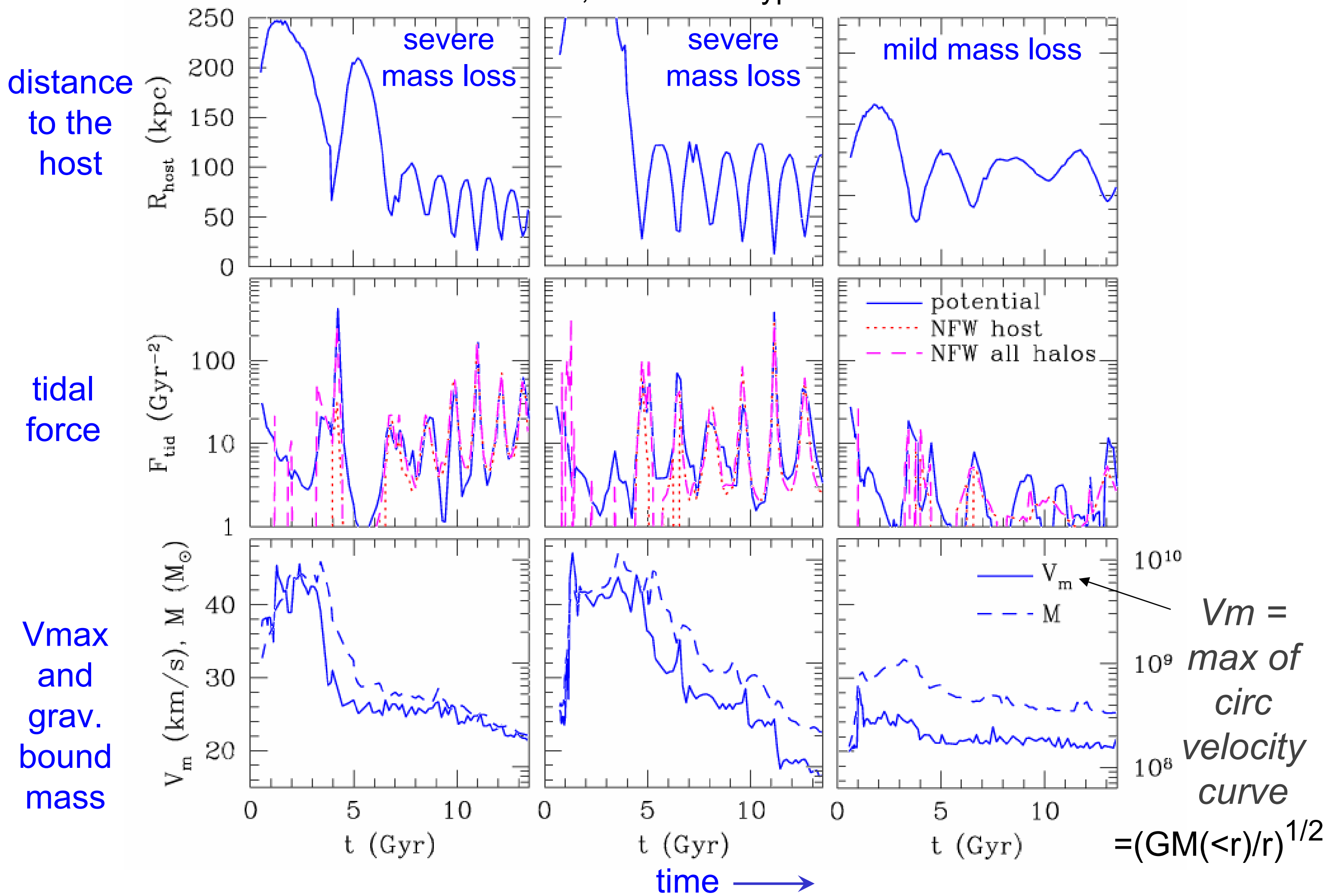
- galaxies can loose orbital energy due to a gravitational drag force

$$F_{DF} = \frac{4\pi ln(\Lambda) G^2 M_{\rm sat}^2 \rho(r)}{V_{\rm orb}^2} \left[{\rm erf}(x) - \frac{2X}{\sqrt{\pi}} {\rm exp}(-X^2) \right]$$

Chandrasekar 1943

Tidal stripping of subhalos: three examples

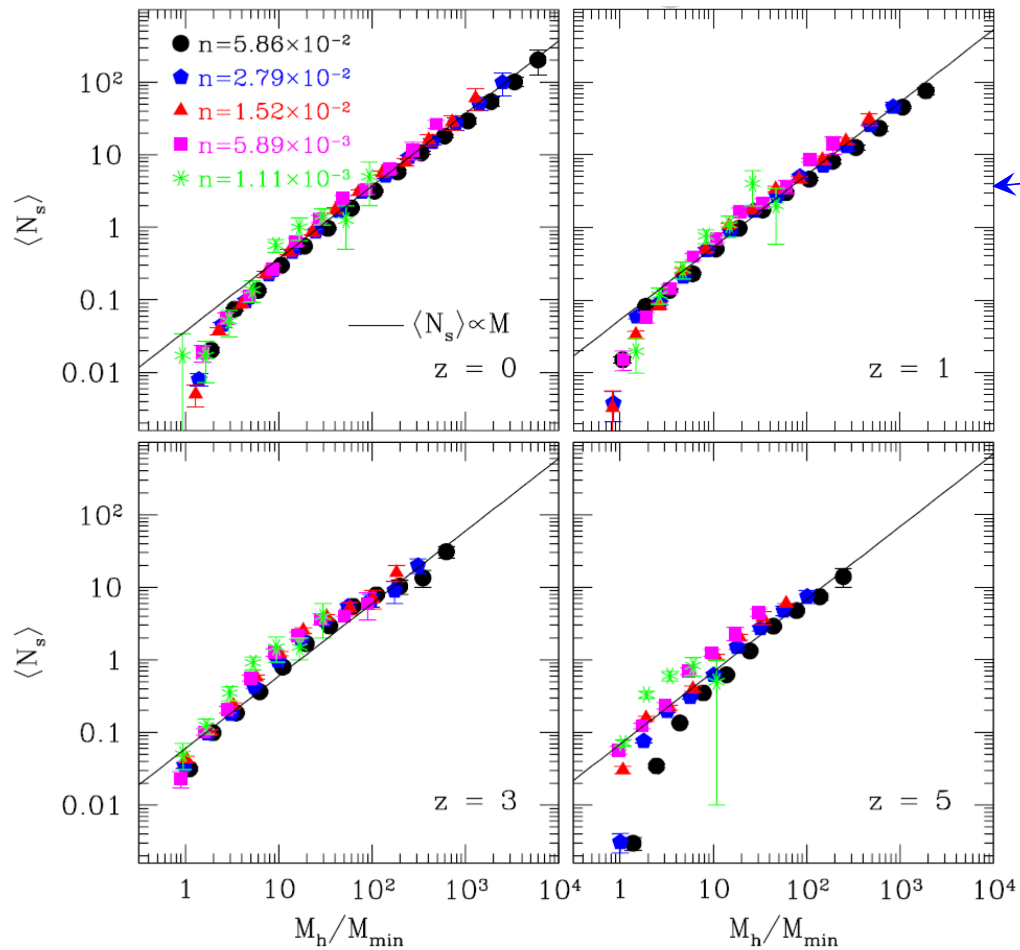
Kravtsov, Gnedin & Klypin 2004



slide credit: Kravtsov

subhalo abundance:

~ linear with halo mass



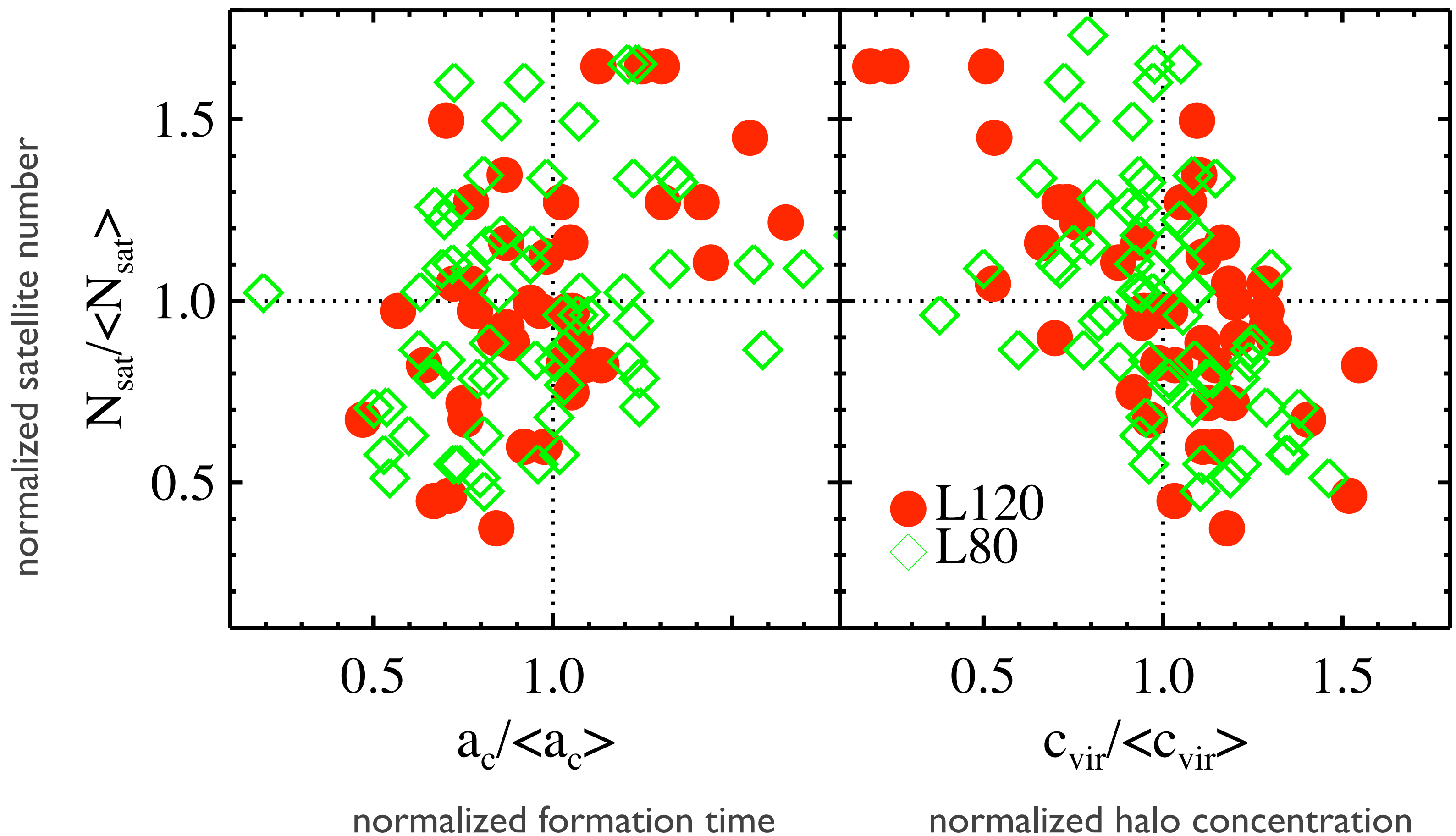
Kravtsov et al. 2004

$$\langle N(M)\rangle_{\rm sat} = \frac{M}{M_1}\exp\left(-\frac{M_{\rm cut}}{M}\right)$$

$$\log\left(M_{\rm cut}\right) = 0.76\log\left(M_1\right) + 2.3$$

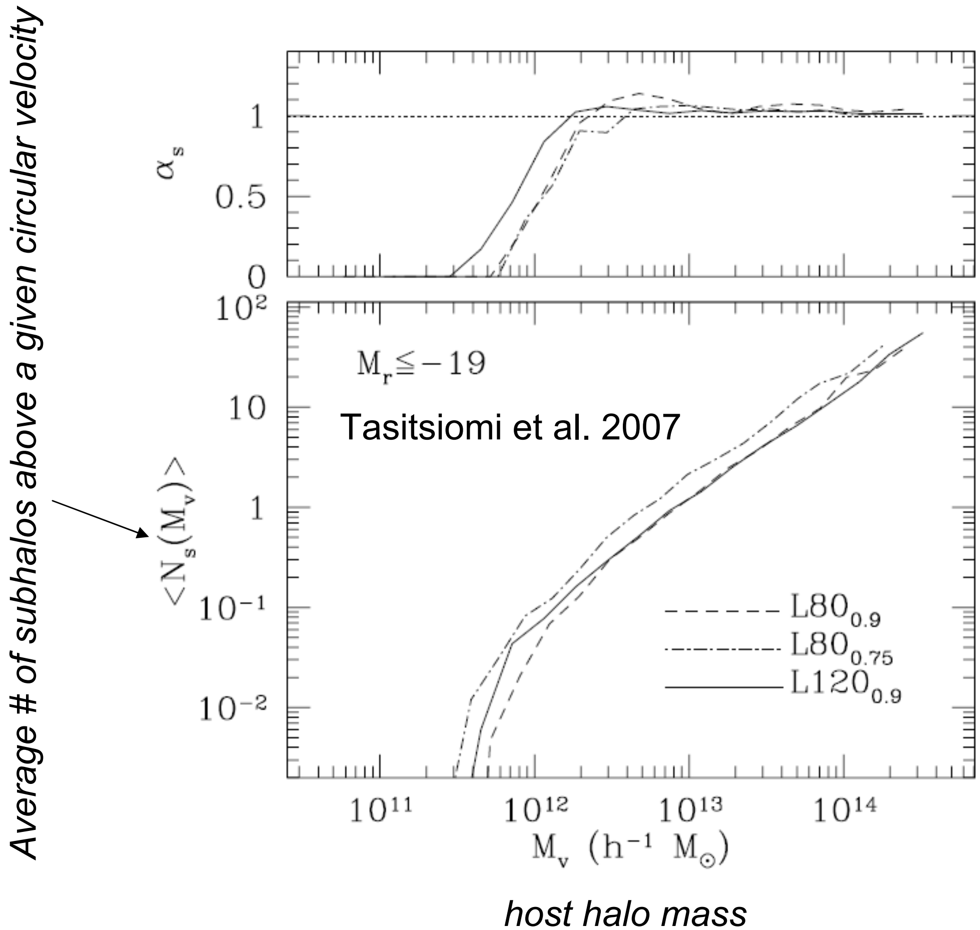
Tinker et al. 2005; Conroy et al. 2006

satellite number correlates with formation time and halo concentration

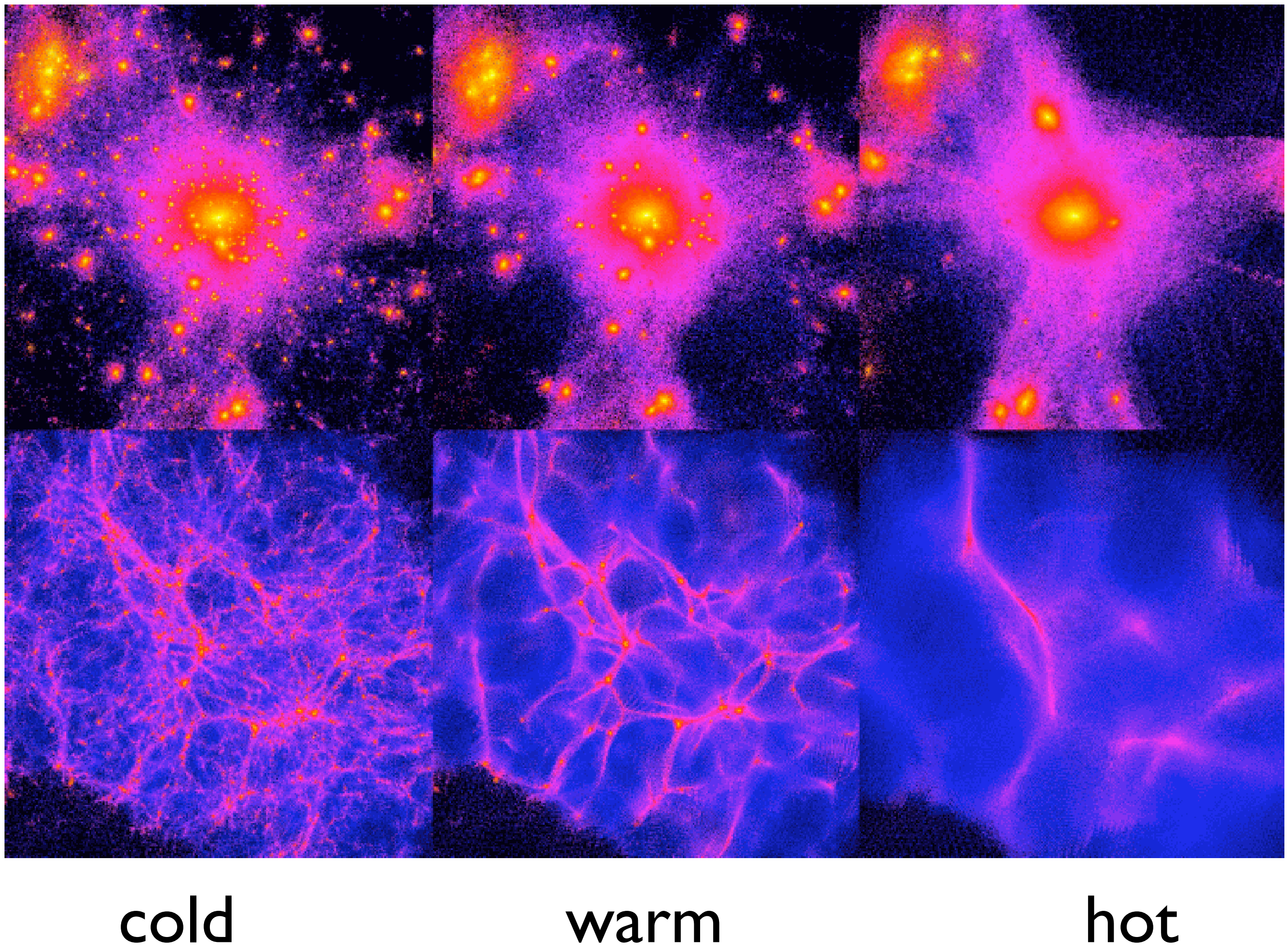


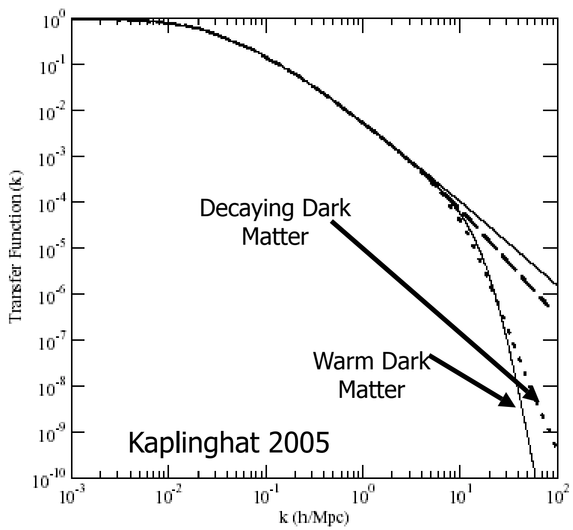
Zentner et al 2005; Wechsler et al 2006

abundance depends on power spectrum

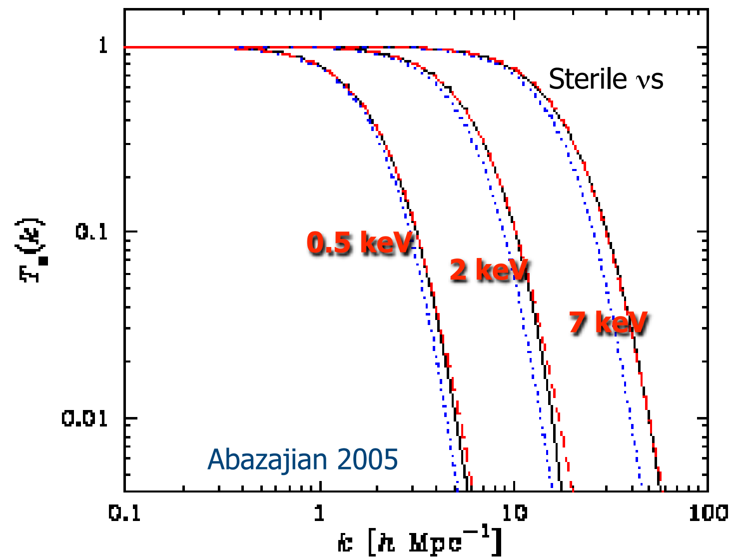


Zentner & Bullock 2002, 2003


cold
warm
hot



Suppression of small-scale power from dark matter produced through decays (Feng et al. 2003; Cembranos et al. 2005).



Suppression of small-scale power via models of non-thermal, sterile neutrino dark matter (Abazajian 2005; Asaka et al. 2005).

Warm Dark Matter has two effects:

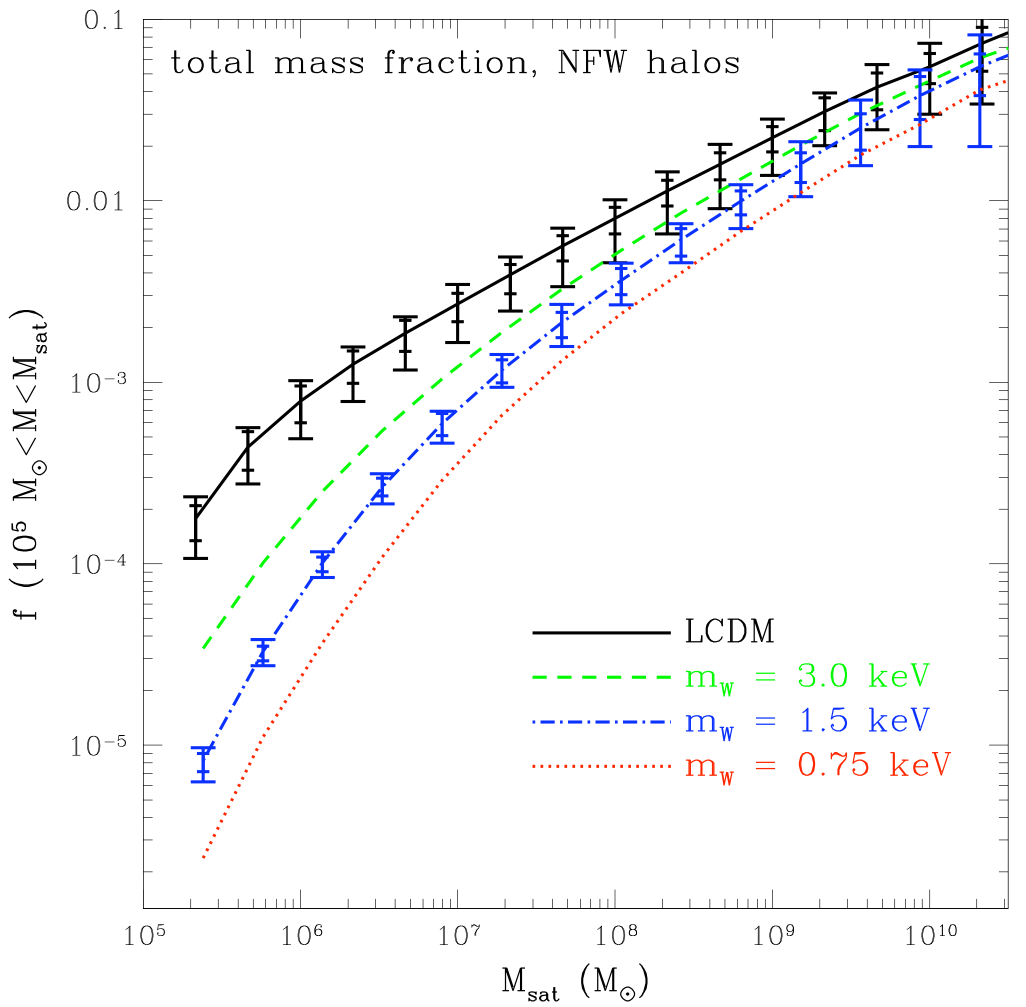
Suppression of Linear Power Spectrum due to free-streaming on scales with $k\eta_{DM} \gg 1$. For WDM models $\lambda_{FS} \sim 3\ hMpc^{-1}(m/keV)^{-4/3}$

High densities limited by finite "Phase-Packing" because initial phase-space density is relatively low, $Q \equiv \rho/\langle v^2\rangle^{3/2} \sim 10^{-4}\ (Msun/pc^3/km^3s^{-3}) \sim 10^{-24}\ Q_\chi$

- The amount of primordial power on scales relevant for galaxy formation is not well constrained λ < Mpc and scale-invariance is almost always assumed

slide credit:Zentner

fraction of mass in substructure depends on DM properties.



tidal streams in M31

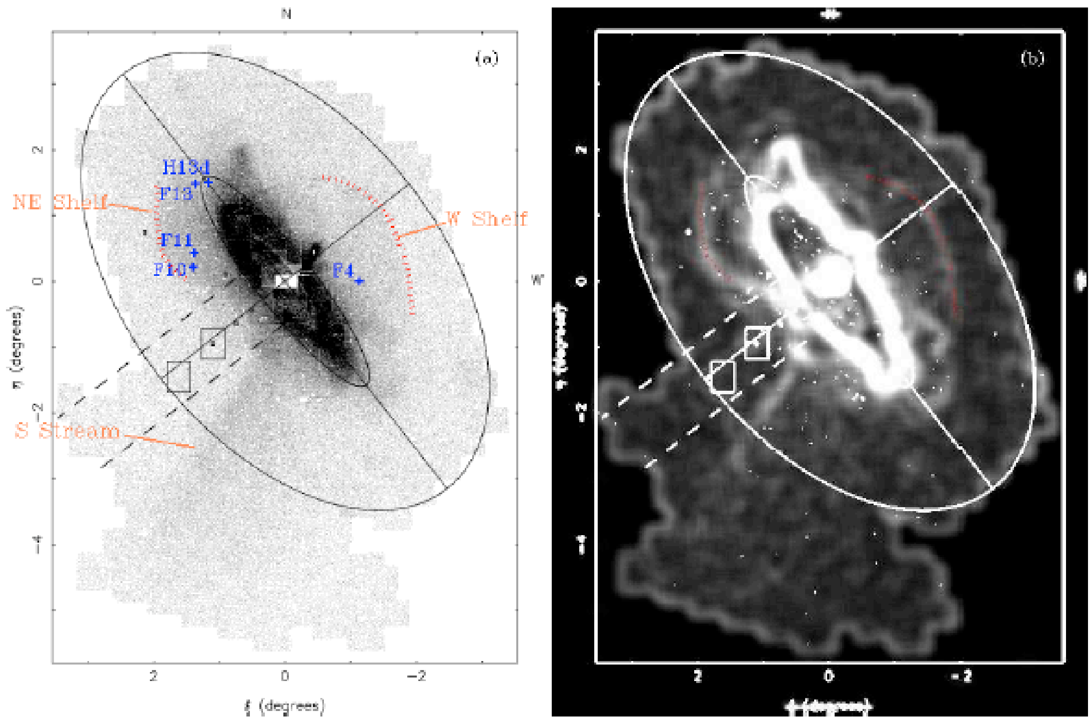


Figure 1. *Panel a:* Map of RGB count density, from Irwin et al. (2005). The edges of the NE and W shelves that are the focus of this paper are marked with red dotted lines. The H13d field of Reitzel et al. (2006) and four fields from Ibata et al. (2005) discussed below are marked with crosses. *Panel b:* Sobel-filtered version of Panel a, which detects sharp edges in the count map. To create this map, we fill in sharp features such as plot annotations and noise spikes with the surrounding pixel values, smooth the map, and apply the Sobel operator. We then fit a smooth curve to the shelf edges, which again are shown by dotted lines.

observational signatures of DM substructure

- substructure mass function in clusters from lensing
- multiple-image strong lenses of quasars
- abundance of dwarf satellite galaxies
- dynamical effects on galactic disks
- (future) annihilation signal from DM subhalos in the local group
- galaxy abundance in groups & clusters

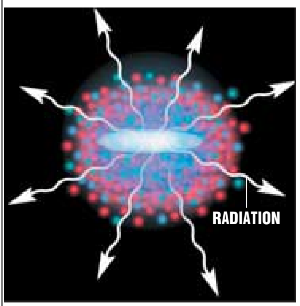
slide credit: Kravtsov

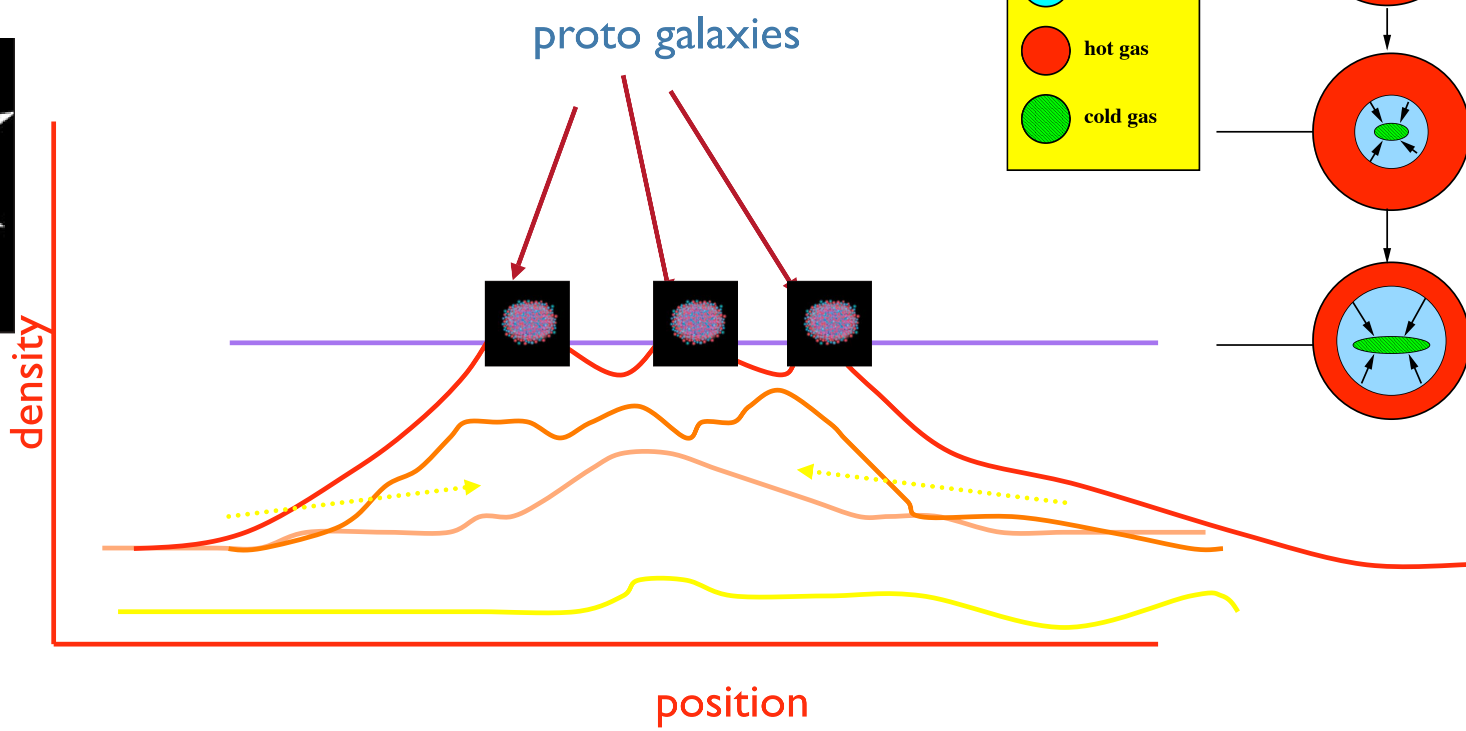
Substructure Summary

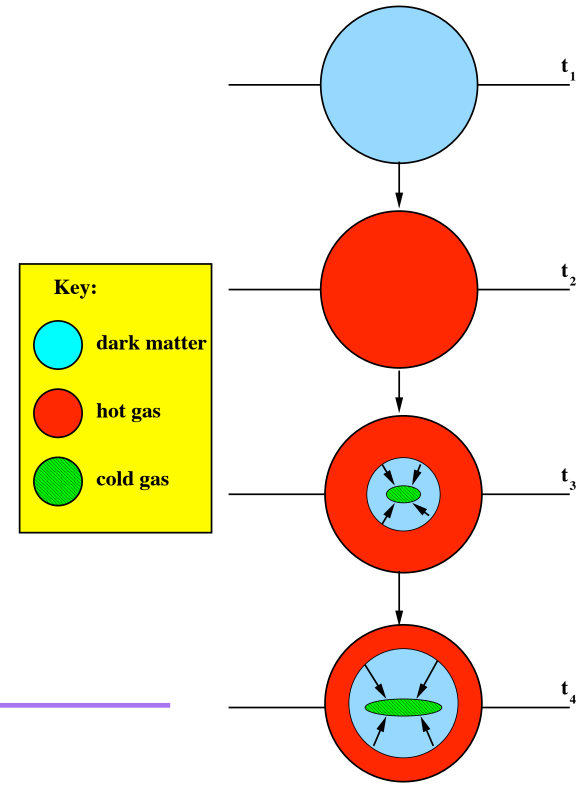
- substructures are ubiquitous in CDM
- mass function of substructure is roughly self-similar.
- the number of substructures is a trade-off between accretion (the halo merging rate) and distruction (dynamical friction; tidal stripping)
- abundance & properties may constrain small scale power spectrum (inflation; DM physics), but much work is still needed.
- lensing & stellar structure may be ways of probing dark and disrupted substructures.

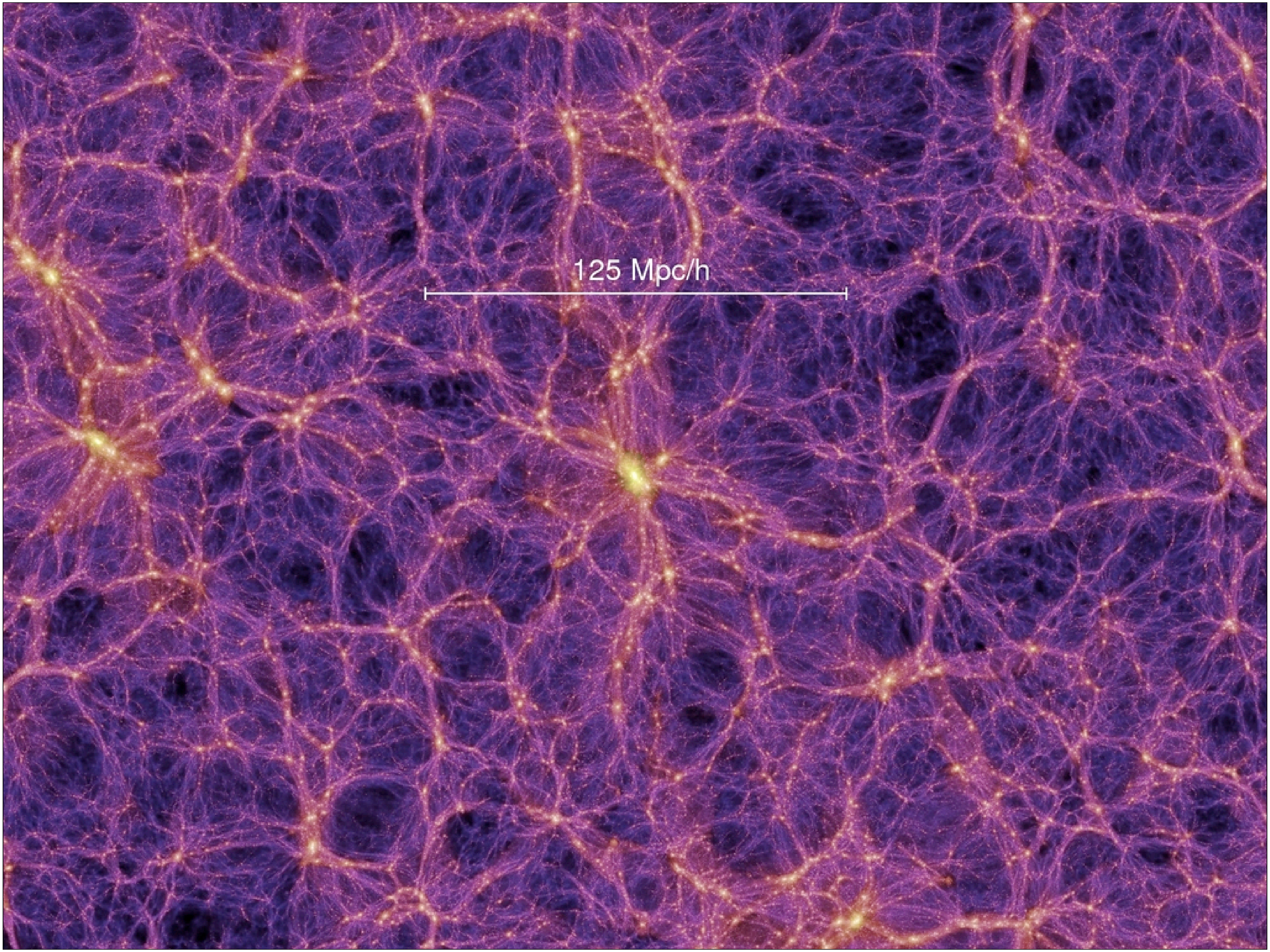
one minute galaxy formation

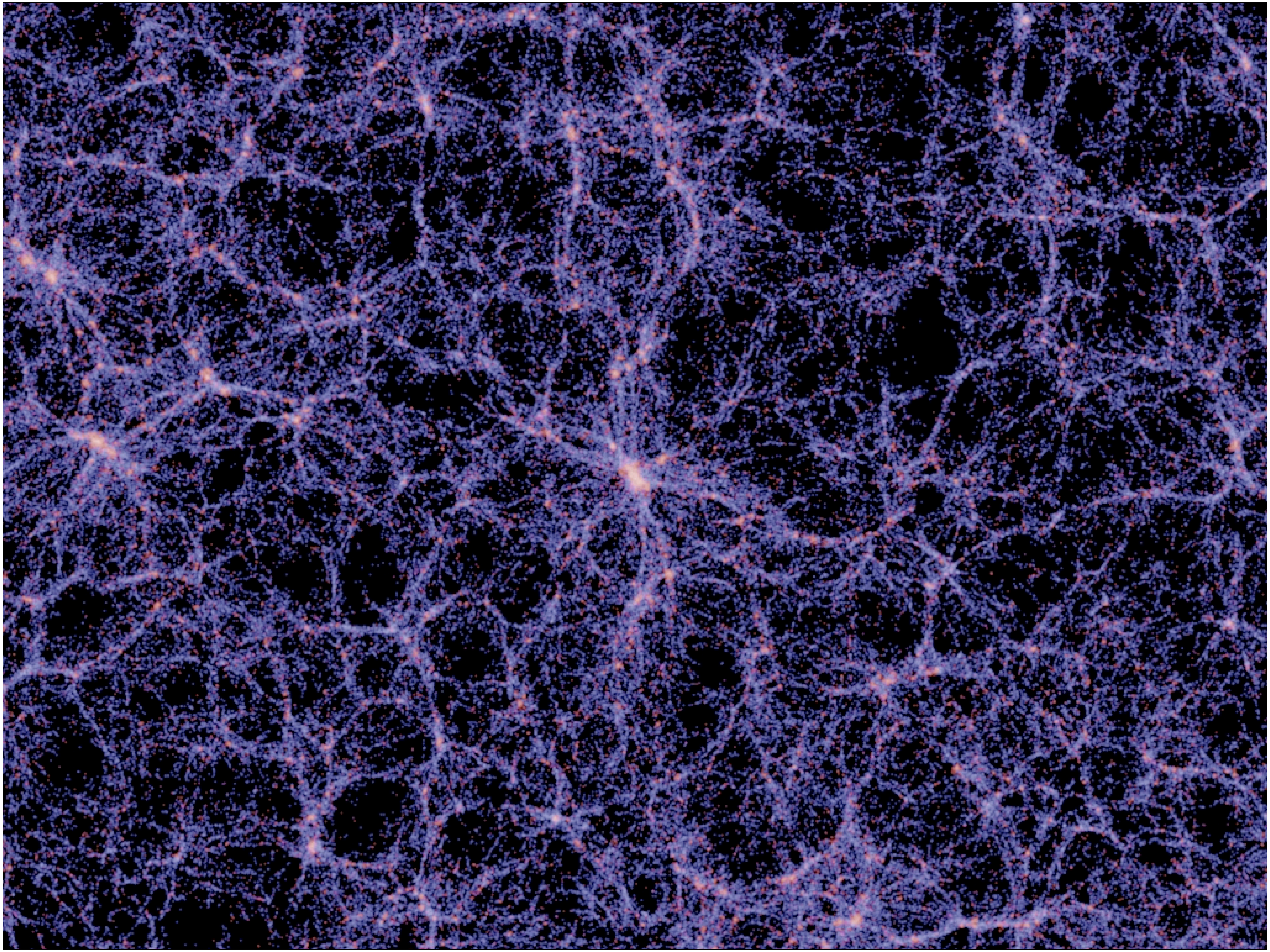
gas in density peaks shock heats to virial temperature, and then radiates and cools. expect to have a galaxy at the center of every density peak (halo) that is massive enough to allow cooling (~10^4 K).



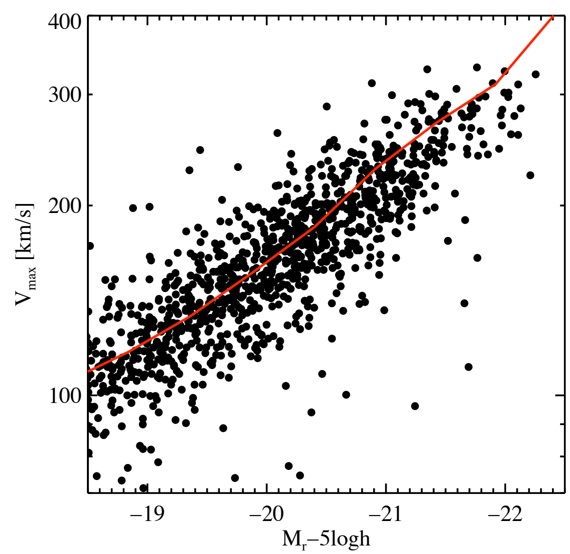





125 Mpc/h



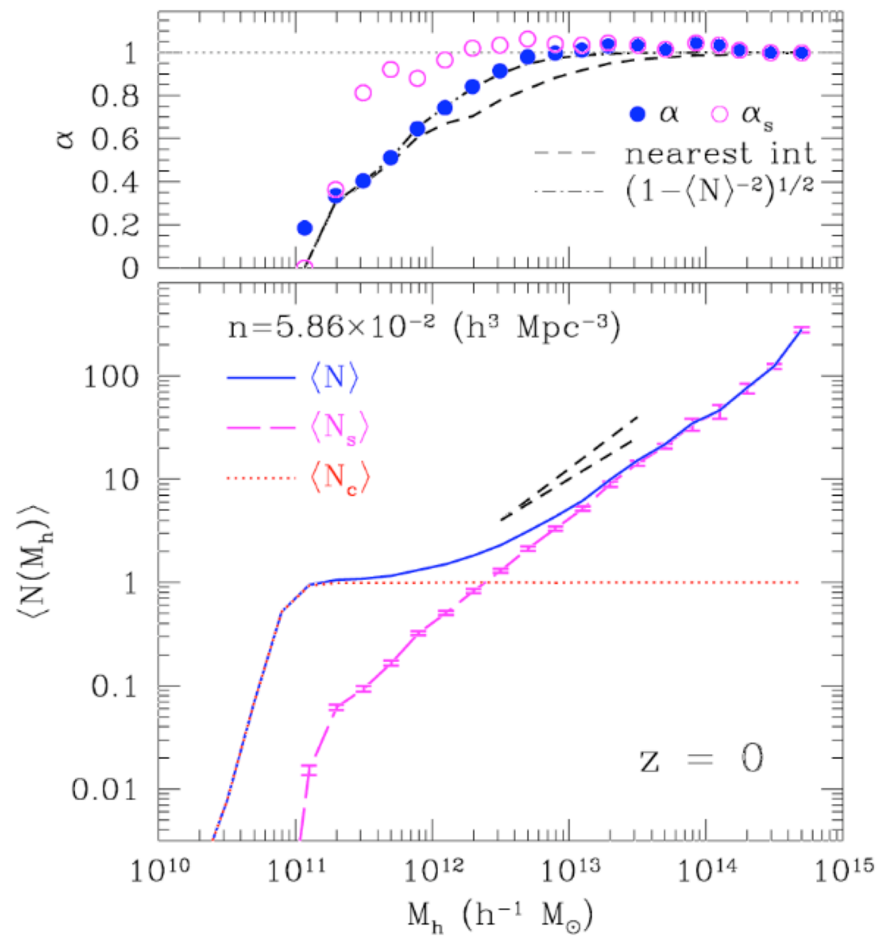
galaxy properties are tightly correlated with halo properties.



Tully-Fisher relation

halo occupation of galactic halos

Average number of galactic (sub)halos



Host halo mass

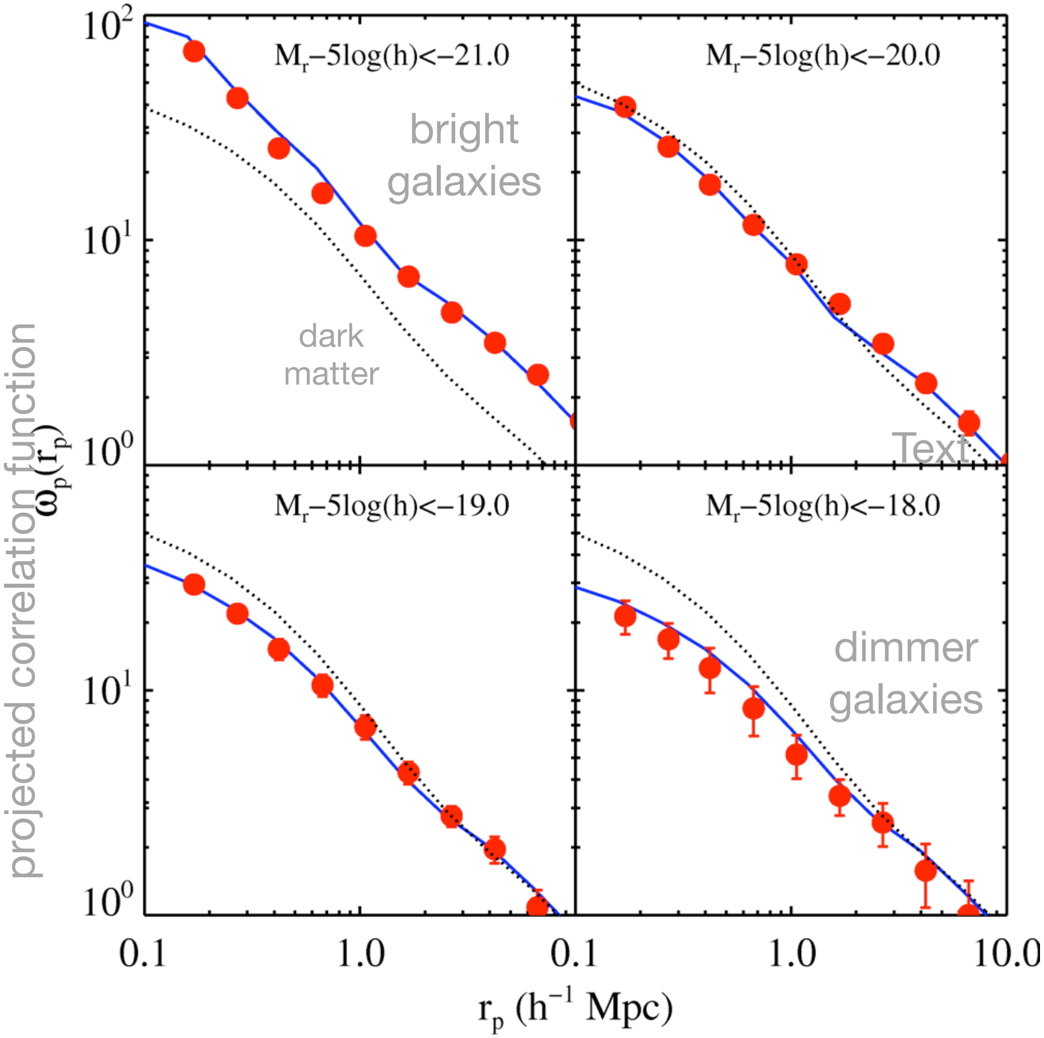
$$\alpha^2 \equiv \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}$$

$$N_{\rm sub} \sim M$$

- a physically motivated way of characterizing non-linear bias of galaxies/subhalos
- no smoothing scale; naturally incorporates stochasticity, naturally brings about scale dependence

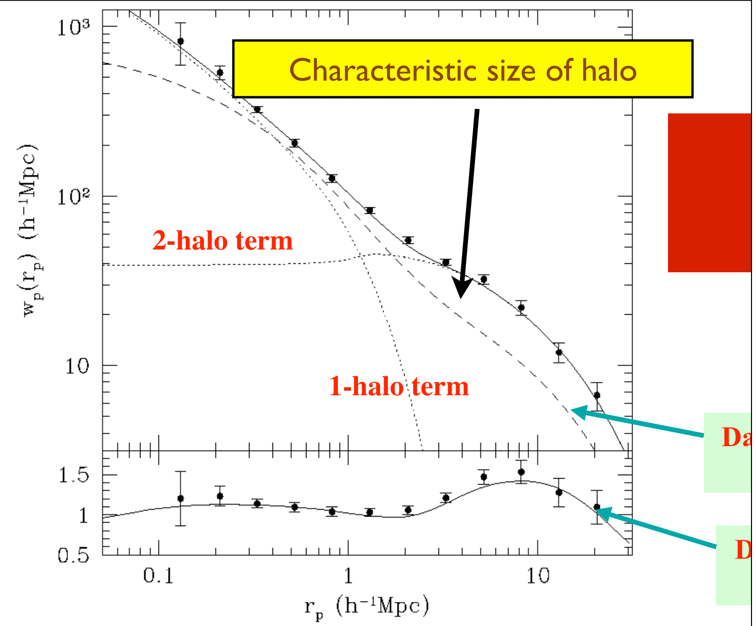
Kravtsov, Berlind, Wechsler, et al 2004

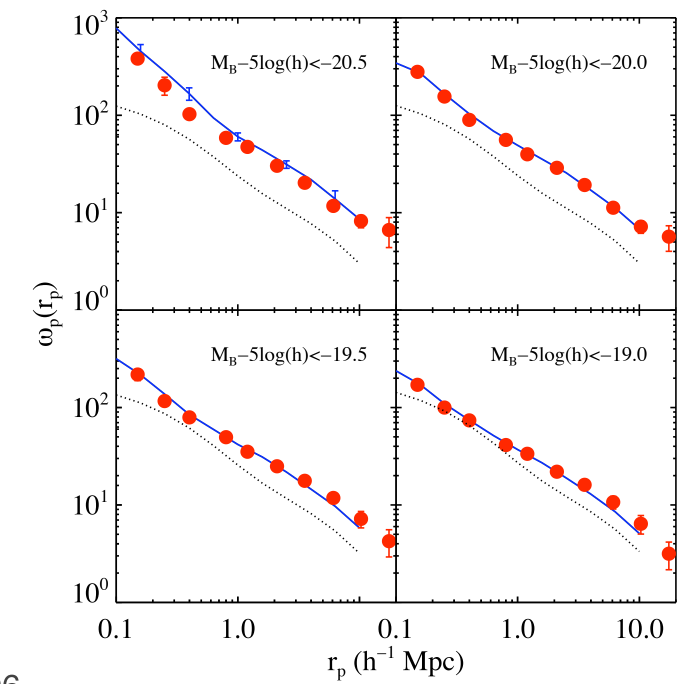
assume every galaxy lives in a subhalo.
how do they cluster?



SDSS, z=0
data: Zehavi et al 2004

Conroy, Wechsler & Kravtsov 2006





DEEP, z=1
data: Coil et al 2006

The Halo Model

- Basic idea:
  - assume that stuff (e.g.: mass, galaxies, quasars, gas) lives in dark matter halos.
  - use knowledge of dark matter halo properties + relation of stuff to halos to determine the clustering properties of the stuff (e.g., non-linear power spectrum, galaxy clustering, etc...)
  - or use clustering to constrain the relation

the halo occupation approach

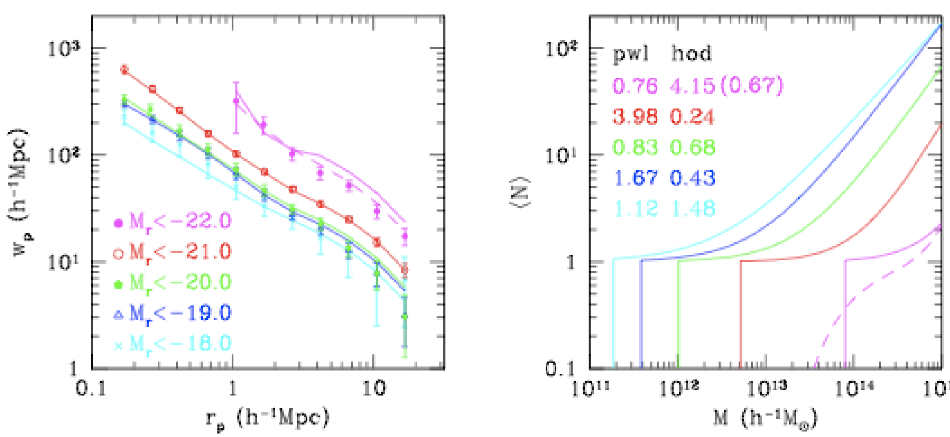
- assume galaxies live in halos
- assume P(N|M) is just a function of M

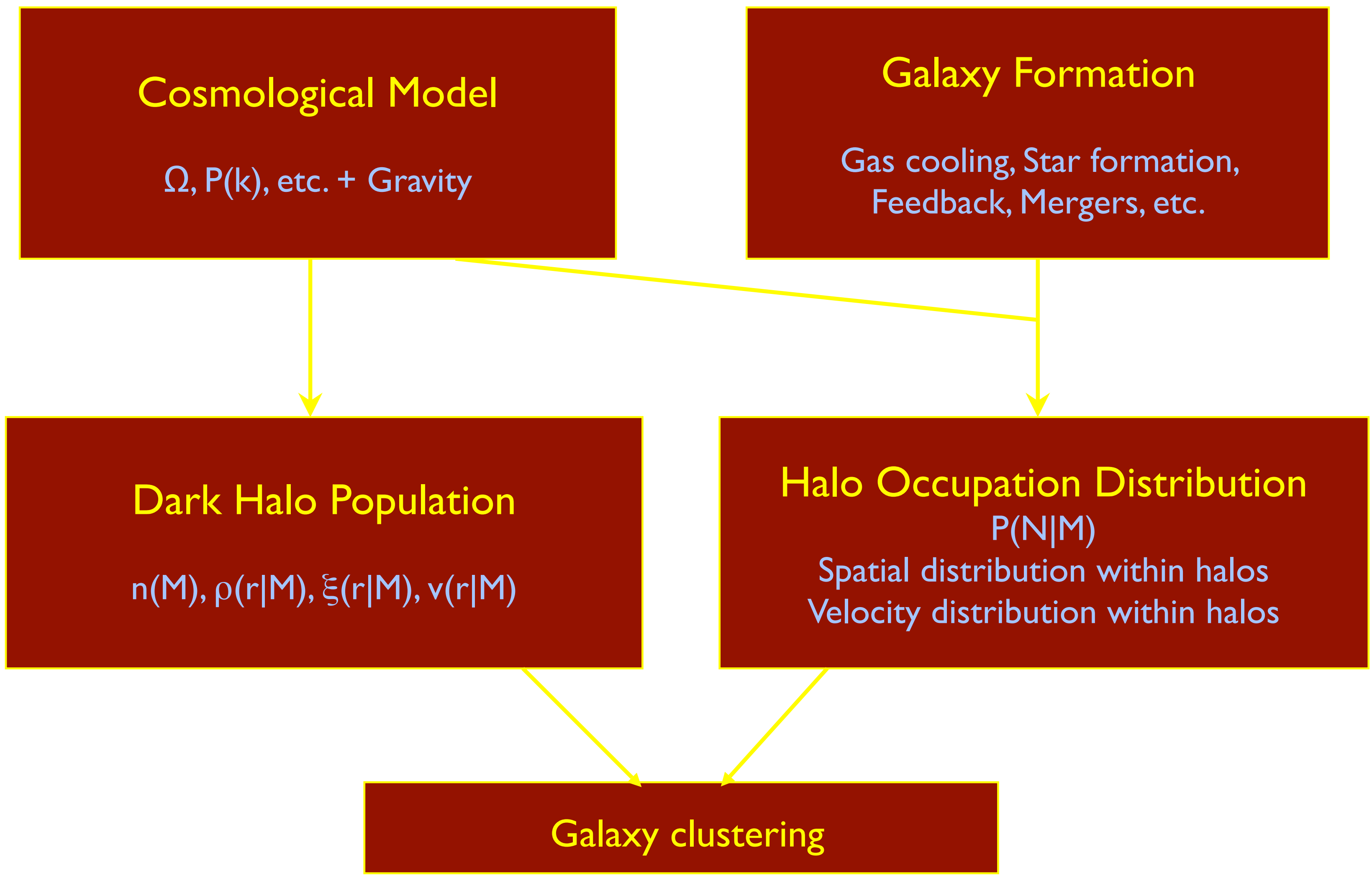
The relation between the clustering of Dark Matter and any class
of galaxies (luminosity, type, etc.) is fully defined by the
Halo Occupation Distribution (HOD):

• The probability distribution *P(N|M)* that a halo of mass *M* contains*N* galaxies of that class.

• The relation between the spatial distributions of galaxies and DM within halos.

• The relation between the velocity distributions of galaxies and DM within halos.





slide credit:Berlind

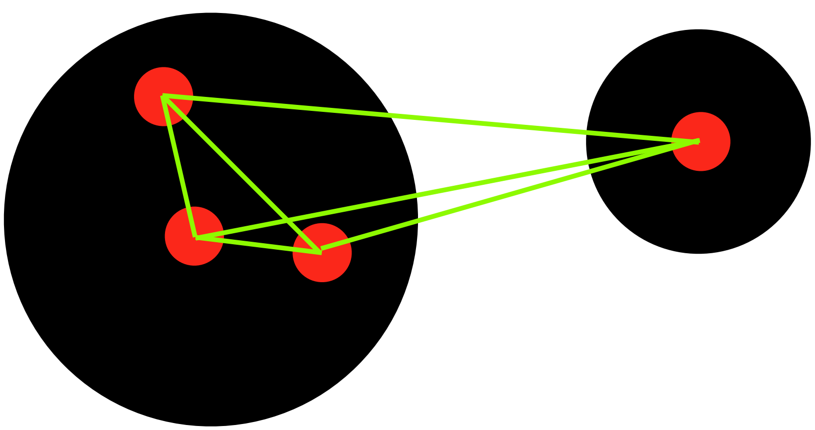
How do we compute clustering statistics?

Correlation function

Small scales: All pairs come from same halo.
1-halo term

$$1+\xi_g^{1h}(r) = \left(2\pi r^2 n_g^2\right)^{-1} \int_0^\infty dM \frac{dn}{dM} \frac{\langle N(N-1)\rangle_M}{2} \lambda(r|M)$$

Large scales: Pairs come from separate halos.



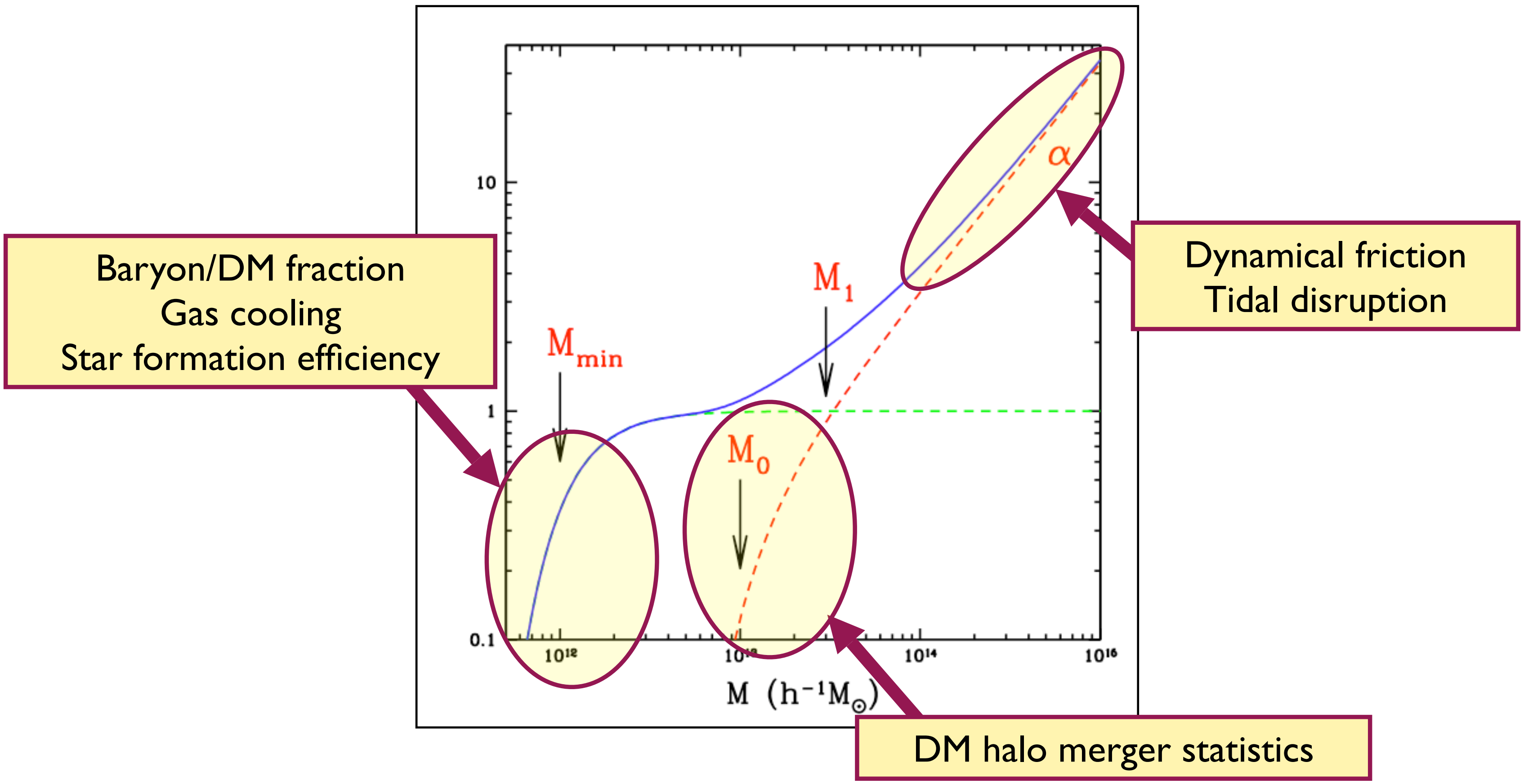
$$\xi_g(r) = b_g{}^2 \xi_m(r)$$

$$b_g = n_g^{-1} \int_0^\infty dM \frac{dn}{dM} \langle N\rangle_M b_h(M)$$

Berlind & Weinberg (2002)

slide credit:Berlind

The HOD contains information about galaxy formation physics



can also use the halo model to calculate the

Non-Linear Power Spectrum

- Non-linear power spectrum is composed of dark matter halos that are clustered according to the halo bias + the clustering due to the halo density profile

$$P_{\rm nl}(k,z) = I_2^2(k,z)P(k,z) + I_1(k,z)$$

where

$$I_2(k,z) = \int d\ln M \left(\frac{M}{\rho_m(z=0)}\right) \frac{dn}{d\ln M} b(M) y(k,M)$$

$$I_1(k,z) = \int d\ln M \left(\frac{M}{\rho_m(z=0)}\right)^2 \frac{dn}{d\ln M} y^2(k,M)$$

and y is the Fourier transform of the halo profile with $y(0,M)=1$

$$y(k,M) = \frac{1}{M}\int_0^{r_h} dr 4\pi r^2 \rho(r,M) \frac{\sin(kr)}{kr}$$

slide credit:Hu

Summary