Axions –Theory

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Lectures from an Axion Workshop

Strong CP Problem and Axions
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Astrophysical Axion Bounds
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Axion Cosmology
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Symmetries

Symmetry $\Leftrightarrow$ Invariance of Lagrangian

Familiar cases:

Lorentz symmetries $\Leftrightarrow$
Invariance under changes of coordinates

Other symmetries: Invariances under field redefinitions

e.g (local) gauge invariance in electromagnetism:

$$A_\mu(x) \to A_\mu(x) + \delta_\mu \Omega(x)$$
$$F_{\mu\nu} = \delta_\mu A_\nu - \delta_\nu A_\mu \to F_{\mu\nu}$$

How to build an (effective) Lagrangian

Choose symmetries to impose on L:
gauge, global and discrete.

Choose representation content of matter fields

Write down every (renormalizable) term allowed by the symmetries with arbitrary couplings $(d \leq 4)$

Add Hermitian conjugate (unitarity) -

Fix renormalized coupling constants from match to data
(subtractions, usually defined perturbatively)

"Naturalness" - an artificial criterion to avoid arbitrarily "fine tuned" couplings

Symmetries of Standard Model

Gauge Symmetries

Strong interactions: $\boldsymbol{SU(3)_{\mathrm{color}}}$
unbroken but confined; quarks in triplets

Electroweak $\boldsymbol{SU(2)_{\mathrm{weak}}}$ X $\boldsymbol{U(1)_Y}$
more later:representations, "spontaneous breaking"

Discrete Symmetries
CPT –any field theory (local, Hermitian L);
C and P but not CP broken by weak couplings
CP (and thus T) breaking - to be explored below - arises from quark-Higgs couplings

Global symmetries

$\boldsymbol{U(1)_{B-L}}$ accidental; more?

Chiral symmetry

Massless four component Dirac fermion is
two independent chiral fermions

$$\psi = \frac{(1+\gamma_5)}{2}\psi + \frac{(1-\gamma_5)}{2}\psi = \psi_R + \psi_L$$

$$\gamma_5 \equiv \gamma_0\gamma_1\gamma_2\gamma_3 \qquad \gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$$

Chiral Rotations: rotate L and R independently

$$\psi \longrightarrow e^{i(\alpha+\beta\gamma_5)}\psi$$

$$\psi_R \longrightarrow e^{i(\alpha+\beta)}\psi_R; \quad \psi_L \longrightarrow e^{i(\alpha-\beta)}\psi_L$$

Kinetic and gauge coupling terms in L are chirally invariant

$$\bar{\psi}\gamma_\mu\psi = \bar{\psi}_R\gamma_\mu\psi_R + \bar{\psi}_L\gamma_\mu\psi_L \quad \textbf{since} \quad \bar{\psi} = \psi^\dagger\gamma_0$$

Chiral symmetry breaking

any fermion mass term violates chiral symmetry

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

as it must –chirality is a frame dependent concept

for a complex mass term hermiticity requires:

$$m\bar{\psi}_L\psi_R + m^*\bar{\psi}_R\psi_L = \bar{\psi}[(m + m*) + (m - m*)\gamma_5]\psi$$

can make any fermion Dirac mass real by a chiral rotation

in QED – no other consequences

Standard Model Weak SU(2)

Left handed quarks and leptons in doublets

$q^i_L = \binom{u^i_L}{d^i_L}$ and $l_L = \binom{\nu_L}{e_L}$

Right handed quarks and leptons in singlets

$\Rightarrow$ do not couple to W

Explicit violation of P and C (but not CP)

no SU(2) singlet quark or lepton mass term

$SU(N_F)_L$ X $SU(N_F)_R$ chiral symmetry

Higgs-gauge couplings

Higgs field is a (color singlet) weak SU(2) doublet:

$$\phi = \binom{\phi^0}{\phi^-} \qquad \tilde{\phi} \equiv \phi^{\dagger T} = \binom{\phi^+}{\phi^{0*}}$$

Higgs potential such that $\langle \phi^0 \rangle = v$

Spontaneously breaks SU(2) X $U(1)_Y$ to $U(1)_{EM}$

gives Massive W's and Z;
(removes 3 of 4 Higgs degrees of freedom)

$\Rightarrow$ v = 174 GeV scale set by e, W and Z masses

Z is a mixture of "B" and Y; photon is the orthogonal mixture

Higgs-Fermion couplings

Can write Yukawa couplings of the form

$Y_u^{ij}\phi\bar{u}_R^i q_L^j + Y_d^{ij}\tilde{\phi}\bar{d}_R^i q_L^j$ + hermitian conjugate

explicitly breaks chiral symmetries, distinguishes generations (i,j)

quark mass terms arise from $\langle\phi^0\rangle = v$

diagonalize quark mass matrices

W couplings not diagonal in mass eigenstate basis

coupling matrix is called CKM matrix
Cabibbo; Kobayashi and Maskawa

CP Violation?

quantum interference between two paths

(two different Feynman amplitudes)

$$\langle f|i\rangle = A_1 + A_2 = g_1 a_1 + g_2 a_2$$

where g_i product of all coupling constants, a_i all else

Then hermiticity of L gives

$$\langle f|\mathrm{CP}|i\rangle = \pm(g_1^* a_1 + g_2^* a_2)$$

CP violating rate differences

$$|\langle f|i\rangle|^2 - |\langle f|\mathrm{CP}|i\rangle|^2 = 2Im(g_1 g_2 *)Im(a_1 a_2 *)$$

only relative phases are physically meaningful!

Case of QED -Dirac Equation

CP symmetry is automatic

Gauge invariance plus hermiticity makes gauge coupling real

Fermion mass can be made real by a chiral rotation

Pauli 1933 (after positron discovery)

I do not believe in the hole theory, I would like to have the asymmetry between positive and negative electricity in the laws of nature. (It does not satisfy me to shift the empirically established asymmetry to one of the initial condition.)

When do we get complex couplings?

For couplings involving multiple different fields, field redefinitions can change coupling phases

$$\boldsymbol{Y\phi\bar{\psi}_i\psi_j + Y^*\phi^*\bar{\psi}_j\psi_i}$$

redefine any (or all) of the three fields e.g $\boldsymbol{\phi \longrightarrow e^{i\eta}\phi}$

equivalent to $\boldsymbol{Y \rightarrow Ye^{i\eta}}$; can choose $\boldsymbol{\eta = -\mathrm{Arg}Y}$

Unless there are more (non-gauge) couplings than fields
$\longrightarrow$ automatic CP symmetry

For Standard Model Electroweak theory

CP Violation $\Rightarrow$ three generations
OR multiple Higgs multiplets

Massive neutrinos $\longrightarrow$ possible lepton sector CP violation

What about QCD?

Lagrangian looks very similar to QED $F^a_{\mu\nu} F^{a\mu\nu}$

Polyakov; 't Hooft : Instantons, QCD theta vacuum

many non-trivial ground states, n-vacua ,
gauge equivalent to A=0

$|n> = \frac{g^2}{32\pi^2} \int d^3x \epsilon_{ijk} f^{abc} A^a_i A^b_j A^c_k$

$A_0 = 0$ gauge; SU(3): $[\lambda^a, \lambda^b] = f^{abc} \lambda^c$

instanton is a tunneling event $|n> \longrightarrow |n+1>$

θ Vacua

$\Rightarrow$ Vacuum must be
gauge invariant superposition of n-vacua

$$|\theta> = \Sigma_n e^{i\theta n}|n>$$

This looks like an additional coupling in L

$$<\theta|e^{i\int d^4xL}|\theta>= \Sigma_{n_f}\Sigma_{n_i} < n_f|e^{i(\theta(n_i-n_f)+\int d^4xL)}|n_i>$$

where $n_f - n_i = \int d^4x(F\tilde{F} \equiv \epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma})$

$$F^a_{\mu\nu} = \delta_\mu A^a_\nu - \delta_\nu A^a_\mu + igf^{abc}A^b_\mu A^c_\nu$$

is CP violating -an ($E.B$) type term

Physical Consequences of θ term ?

None if there are massless quarks
–remove it by a chiral rotation

$\psi \longrightarrow e^{i\alpha\gamma_5}\psi$ causes $\theta \longrightarrow \theta - \alpha$

$F\tilde{F}$ is divergence of axial current of fermions

If there are quark masses –mass matrix M –then

$\theta_{\rm eff} = \theta - \mathbf{ArgDetM}$
unchanged by chiral rotation

Again the physically relevant thing is the relative phase!

Strong CP Problem

theta term induces neutron electric dipole moment

$$d \approx e\theta m_q / M_N^2$$

experimental neutron electric dipole moment limit

$$d \leq 6 10^{-26} \mathrm{e\ cm} \qquad \Rightarrow \theta_{\mathrm{eff}} \leq 10^{-10}$$

Why so small?

Peccei and Quinn 1977

Asked ourselves how to get rid of strong CP violation

Cosmological Clue:

Quarks are massless in the early Universe

How can theta parameter be irrelevant there but not later?

Its not – chiral rotation changes Yukawa couplings to Higgs

What fixes the phase of the $< \phi^0 >$?

Can we "tip" the Higgs potential so that $\theta_{\mathrm{eff}} = 0$?

Theta dependence of Higgs Potential

In Standard Model?

Treat instantons as dilute gas (an interaction vertex) – couples to all chiral fermions

Integrate out fermions $\rightarrow$ a multi-Higgs interaction term

$\Delta V \propto e^{i\theta}\phi^{N(up)}\phi^{*N(down)}$ +hermitian conjugate

$\propto (\phi\phi^*)^{N_f}$; no phase dependence

$\longrightarrow \theta_{\rm eff}$ arbitrary

Can we change this?

Ways out

1. Keep one quark massless
2. Impose $\theta_{\rm eff} = 0$ by requiring exact CP
3. Add additional Higgs-type fields;
   allows new $U(1)_{PQ}$ symmetry

gives Higgs potential proportional to $(1 - cos(\theta_{eff}))$

Zero Quark mass?

Pion masses and current algebra says no for physical up-quark mass
(the lightest quark)

Could bare up-quark mass be zero?

Lattice calculations say no to that as well

Seems this answer is ruled out

Imposing CP Symmetry

Must have spontaneous CP violation to get observed CP breaking

Spontaneous Weak CPV induces small strong CPV

May be small enough—in SM its a multi-loop effect

Data on weak CP violation $\rightarrow$ very contrived models

CKM matrix has explicit CP violation and fits data beautifully

$U(1)_{PQ}$

Add additional Higgs multiplets

arrange V to be invariant under U(1)

fields type j redefined by a phase $e^{i\alpha q^j_{PQ}}$)

But theta term breaks this symmetry

V_{eff} is θ dependent $\propto (1 - cos(\theta - c\alpha))$
c depends on PQ charges of fermions but

V minimizes at $\theta_{\mathrm{eff}} = 0$

U(1) excitations – a not-quite Goldstone boson—axion

Roberto and I missed this

Wilczek and Weinberg (separately) pointed it out

Our (wrong) parenthetical remark

"This same lack of U(1) symmetry was noted by 't Hooft and explains the lack of a Goldstone boson in this theory even though one might naively have expected to find one. In a more physical model it explains the absence of a ninth light pseudoscalar meson."

There is mixing between η and axion
both get there masses from the chiral anomaly, $F\tilde{F}$
but there are still two particles, with quite different properties.

Isospin violation in Higgs couplings ($m_u \neq m_d$)
$\rightarrow \pi^0$ mixes too

Many possible versions of $U(1)_{PQ}$

different additional Higgs multiplets and q^i_{PQ}

PQ version (straw man): add a second doublet
$\langle \phi^0_i = v_i e^{i\alpha q^i_{PQ}} \rangle$

doublet vacuum value contributes to W and Z mass, this restricts values of v_i

$\Sigma_i v_i^2 = v^2 = (174\mathrm{GeV})^2$

Only free parameter is $\frac{v_1}{v_2}$

thus axion mass and couplings constrained

PQ model ruled out eg by limits on $K \to \pi a$,

Invisible axion models

Add singlet scalar (Higgs-type) multiplet ϕ_S

Kim; Shifman, Vainshtein, Zakharov (KSVZ)
add just 1 singlet

Dine, Fischler Srednicki; Zhitnitsky (DFSZ)
add doublet and singlet

vacuum value of singlet $< \phi_S >= f_a e^{i\alpha q^S_{PQ}}$
phenomenology defined by f_a
and three parameters λ_i (all order 1, or maybe N_f)

$$m_a = \lambda_m \left(\frac{m_\pi f_\pi}{f_a}\right) = \lambda_m \left(\frac{174\text{GeV}}{f_a}\right) 25\text{keV} \; ;$$

Mixing $\Leftrightarrow$ Couplings to matter

$$\xi_{a\pi} = \lambda_3 \frac{f_\pi}{f_a}; \quad \xi_{a\eta} = \lambda_0 \frac{f_\pi}{f_a}$$

Axions in Astrophysics

Cooling mechanism for stellar interiors

Astrophysical constraints
—e.g. lifetime of red giant stars, supernovae

$$f_a \geq 10^9 \text{ GeV}$$

$$\Rightarrow m_a \leq 10^{-5}\text{eV}$$

A possible dark matter particle –very light and very weakly coupled

Axion Cosmology

Axion is so light – How come it gives COLD dark matter?

Dominant component of axion vacuum density is coherent –not thermal

axion mass is irrelevant until universe cools enough to see theta dependence (GeV scale)

– initial value of axion field α_0 is random

uniform over large regions
–due to expansion after T of order f_a

Coherent axions

Case 1 $f_a > T_{\text{reheating}}$ axion field forms before inflation
– only a zero mode $\longrightarrow$ only zero momentum axions

energy density in axions $\propto f_a^2 \alpha_0^2$

dark matter density
$\longrightarrow$ either α_0 restricted or $f^a \leq 10^{12} GeV$

Case 2 $f_a \leq T_{\text{reheating}}$ a bit more complicated,
domain walls, axion strings...but similar bound on f_a

There are early Universe axion fluctuations as well as gravitational fluctuations –further model constraints

In conclusion

Dominant axions today are still non-relativistic

Axion dark matter is viable, but not WIMP-like

SUSY is more popular but not more motivated!

Axion searches –tomorrow – van Bibber