Cosmic Neutrinos

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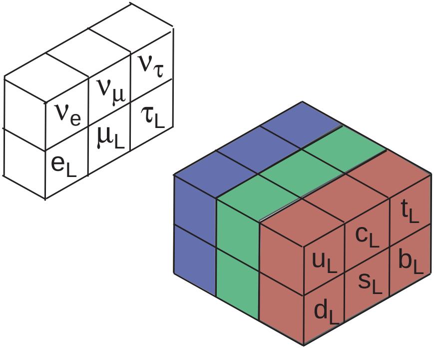
Neutrinos are abundant!

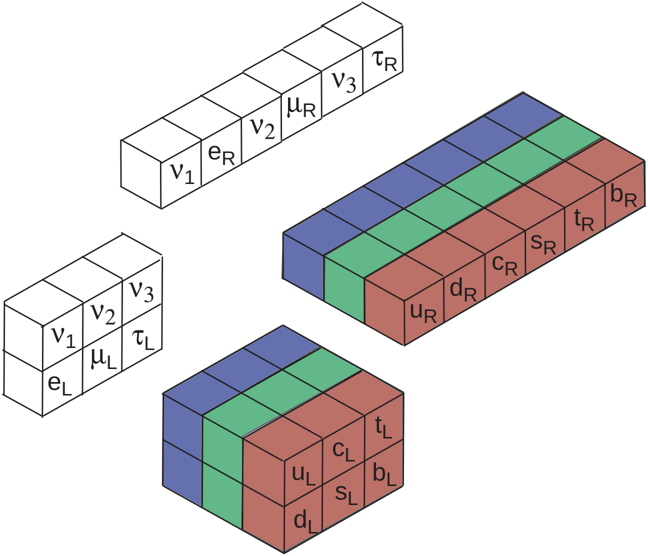
- Each second, some 10^{14} neutrinos made in the Sun pass through your body.
- Each second, about 10^{3} neutrinos made in Earth's atmosphere by cosmic rays pass through your body.
- Other neutrinos reach us from natural (radioactive decays of elements inside the Earth) and artificial (nuclear reactors, particle accelerators) sources.
- Inside your body are more than 10^{7} neutrino relics from the early universe.

ν_i, $\bar{\nu}_i$ number density now: 56 cm^{-3}, $\propto (1+z)^3$

Neutrinos in the electroweak theory



Neutrinos in the electroweak theory



Leptons are observed as free particles

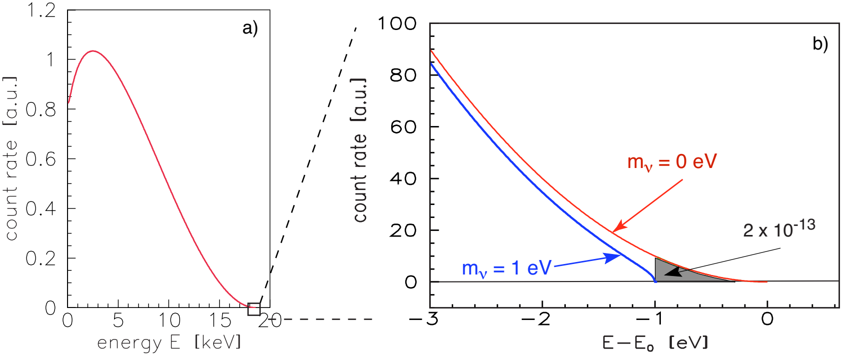
Lepton	Mass	Lifetime
ν_e	< 2 eV	
e^-	$0.510\,998\,918(44)$ MeV	$> 4.6 \times 10^{26}$ y (90% CL)
ν_μ	< 0.19 MeV (90% CL)	
μ^-	$105.658\,369\,2(94)$ MeV	$2.197\,03(4) \times 10^{-6}$ s
ν_τ	< 18.2 MeV (95% CL)	
τ^-	1776.90 ± 0.20 MeV	$290.6 \pm 1.0 \times 10^{-15}$ s

All spin-$\frac{1}{2}$, pointlike ($\lesssim$ few $\times 10^{-17}$ cm)

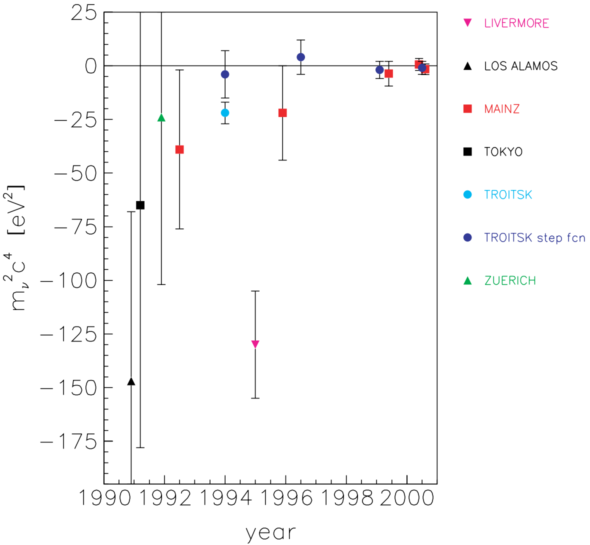
kinematically determined ν masses consistent with 0
(ν oscillations $\Rightarrow$ nonzero, unequal masses)

Tritium β-decay spectrum

$$^3\mathrm{H} \to\ ^3\mathrm{He}\ e^-\ \bar{\nu}_e\text{:}\ \ Q \approx 18.57\ \mathrm{keV}$$

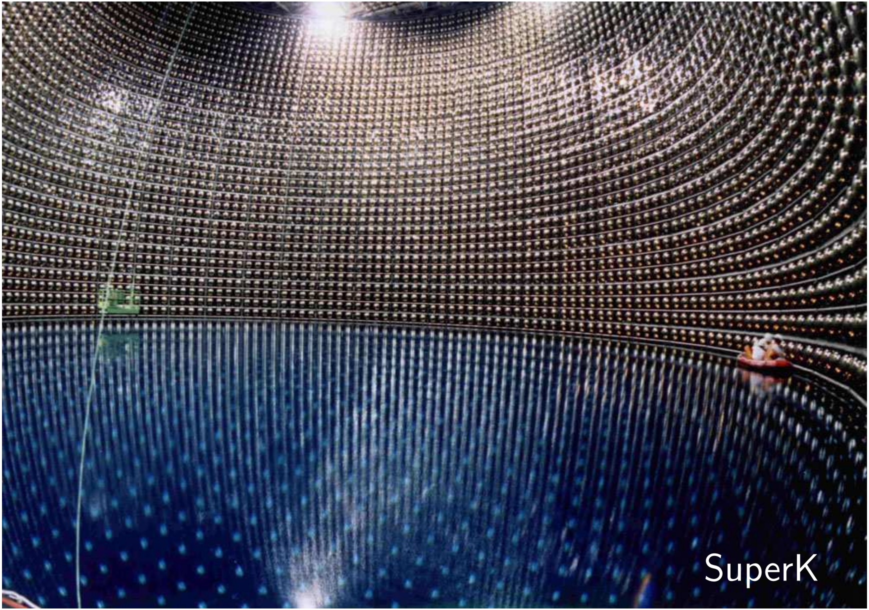


ν mass limits (e-associated)



KATRIN aims for $m_\nu \lesssim 0.2$ eV

Neutrino flavor change $\rightsquigarrow$ neutrinos have mass



The essence of neutrino oscillations

If neutrinos of definite flavor $(\nu_e, \nu_\mu, \nu_\tau)$ are superpositions of different mass eigenstates (ν_1, ν_2, ν_3)
the mass eigenstates evolve in time with different frequencies . . . and so the superposition changes in time

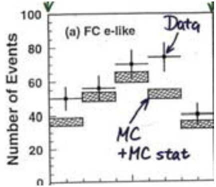
$$\boxed{\text{Beam created as flavor } \nu_\alpha \text{ evolves into a flavor mixture}}$$

Suppose $\nu_\alpha = \nu_1 \cos\theta + \nu_2 \sin\theta$; $\nu_\beta = -\nu_1 \sin\theta + \nu_2 \cos\theta$:

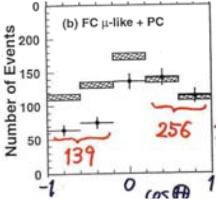
After distance L, beam created as ν_α with energy E has

$$P_{\alpha\to\beta} = \sin^2 2\theta \sin^2\left(\Delta m^2 L/4E\right)$$

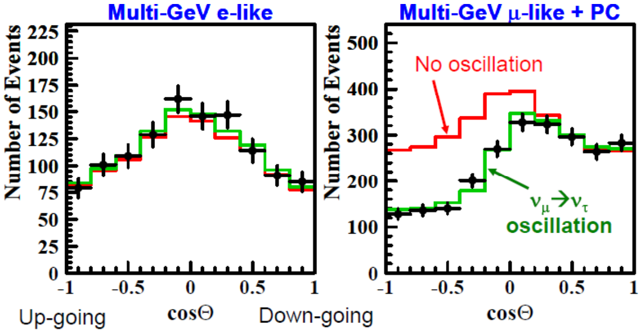
SuperK Atmospheric Neutrinos



@Neutrino98
(535 day)

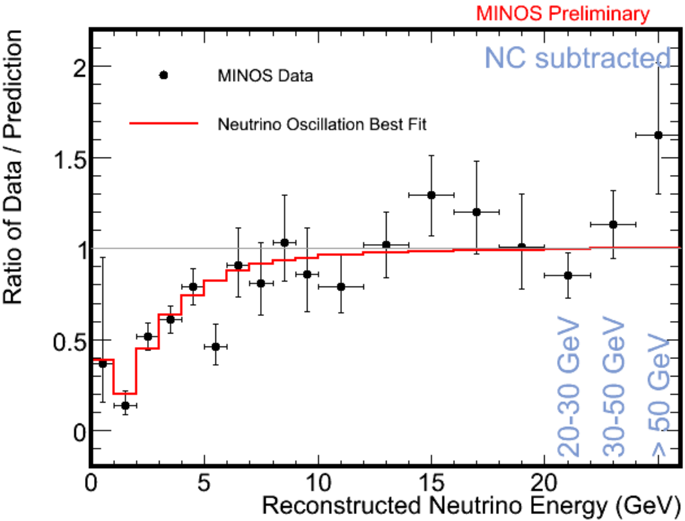


Now
(2293 day)

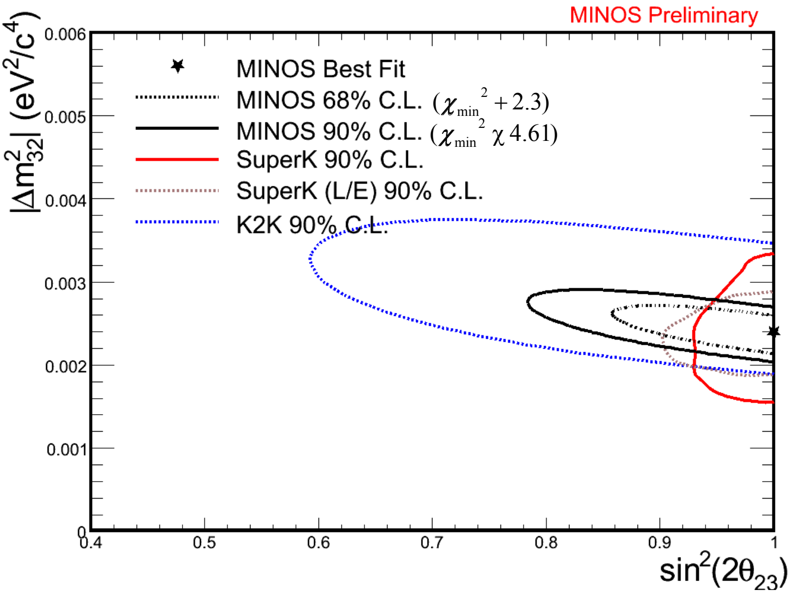


Downward ν travel 15 km, upward ν up to 13 000 km

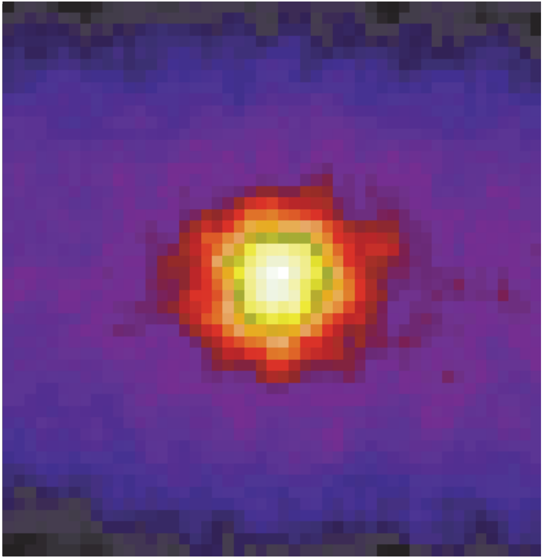
MINOS ν_μ Deficit



Atmospheric Neutrino Summary



SuperK All-Sky Map in Neutrinos ($\nu e \to \nu e$)



$$4p \to {}^4\text{He} + 2e^+ + 2\nu_e + 25\ \text{MeV}$$

Solar neutrinos: measure three reaction rates

[CC] Charged-current dissociation: sensitive to ν_e flux

$$\nu_e \mathrm{d} \to e\ p\ p \quad W\text{-exchange}$$

[NC] Neutral-current dissociation: sensitive to total ν flux

$$\nu_\ell \mathrm{d} \to \nu_\ell\ p\ p \quad Z\text{-exchange}$$

[ES] Elastic scattering: sensitive to $\approx \nu_e + \frac{1}{7}(\nu_\mu + \nu_\tau)$ flux

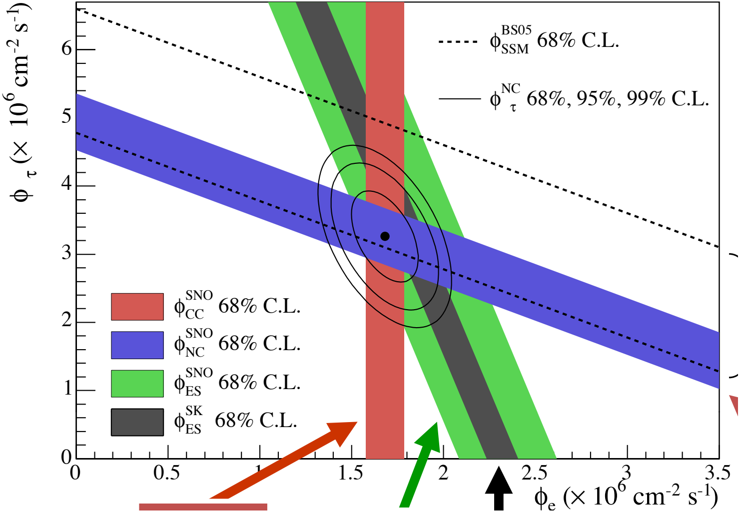
$$\begin{aligned} \sigma(\nu_{\mu,\tau} e \to \nu_{\mu,\tau} e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \quad Z\text{-exchange} \\ \sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right] \quad W + Z\text{-exchange} \end{aligned}$$

$$L_\ell = 2\sin^2\theta_W - 1 \approx -\tfrac{1}{2};\ R_\ell = 2\sin^2\theta_W \approx \tfrac{1}{2}$$

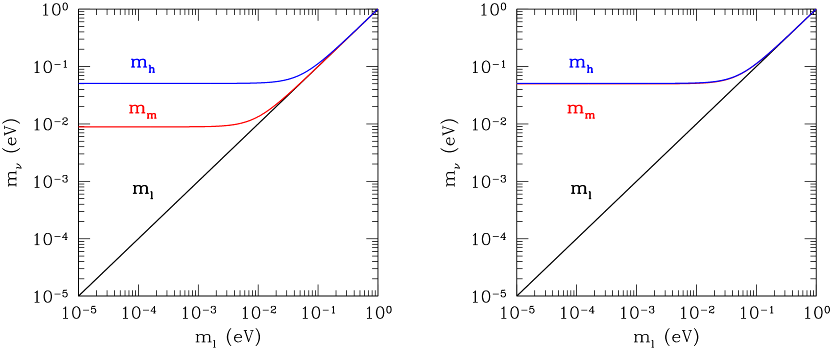
Solar neutrino observations: SuperK & SNO

Total flux agrees with solar model, but only 30% arrive as ν_e

Solar ν emerge as ν_2



Absolute scale of neutrino masses is not yet known



Normal spectrum

Inverted spectrum

$m_2^2 - m_1^2 = \Delta m_\odot^2 = 7.9 \times 10^{-5}\ \text{eV}^2$ $\quad |m_3^2 - m_1^2| = \Delta m^2_{\text{atm}} = 2.5 \times 10^{-3}\ \text{eV}^2$

$$\rho_c \equiv 3H_0^2/8\pi G_N = 1.05h^2 \times 10^3\ \text{eV cm}^{-3} = 5.6 \times 10^3\ \text{eV cm}^{-3}$$

$$\boxed{(56\nu_i + 56\bar{\nu}_i)\ \text{cm}^{-3} \rightsquigarrow \sum_i m_{\nu_i} \lesssim 50\ \text{eV}}$$

Expectations for relic neutrinos

Relate to Cosmic Microwave Background:

$$\frac{dn_\gamma(T)}{d^3p} = \frac{1}{(2\pi)^3}\,\frac{1}{\exp(p/T) - 1}\,,$$

p: relic momentum, T: photon temperature

$$n_\gamma(T) = \frac{1}{(2\pi)^3}\int d^3p\,\frac{1}{\exp(p/T) - 1} = \frac{2\zeta(3)}{\pi^2}\,T^3,$$

$T_0 = (2.725 \pm 0.002)$ K: $n_{\gamma 0} \equiv n_\gamma(T_0) \approx 410\ \text{cm}^{-3}$

Expectations for relic neutrinos

ν decoupled around 1 MeV (0.1 s), were not reheated by e^+e^- annihilation: $T_\nu/T = \left(\frac{4}{11}\right)^{1/3}$ (below m_e).

$$T_{\nu 0} = \left(\tfrac{4}{11}\right)^{1/3} T_0 = 1.945 \text{ K} \rightsquigarrow 1.697 \times 10^{-4} \text{ eV} .$$

$$\frac{dn_{\nu_i}(T_\nu)}{d^3p} = \frac{1}{(2\pi)^3} \frac{1}{\exp\left(p/T_\nu\right) + 1} .$$

$$\begin{aligned} n_{\nu_i}(T_\nu) &= \frac{1}{(2\pi)^3}\int d^3p \, \frac{1}{\exp\left(p/T_\nu\right) + 1} \\ &= \frac{3\zeta(3)}{4\pi^2} T_\nu^3 = \tfrac{3}{22} n_\gamma(T) . \end{aligned}$$

Expectations for relic neutrinos

In the present Universe,

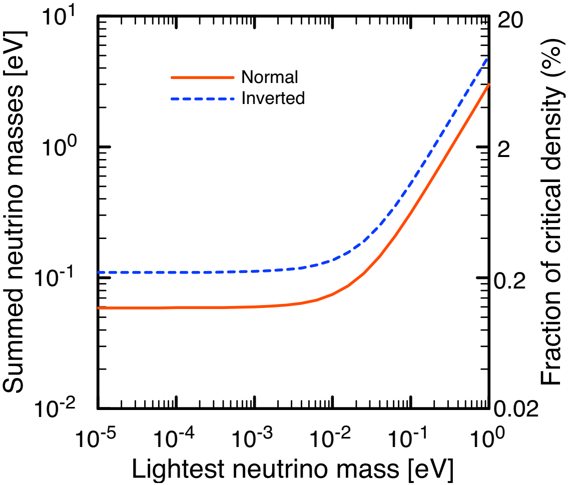
$$n_{\nu_i 0} = n_{\nu_i^c 0} \equiv n_{\nu_i}(T_{\nu 0}) \approx 56 \text{ cm}^{-3}$$

rms neutrino momentum:

$$\langle p_{\nu 0}^2 \rangle^{\frac{1}{2}} \approx 3.597\, T_{\nu 0} \approx 6.044 \times 10^{-4} \text{ eV}$$

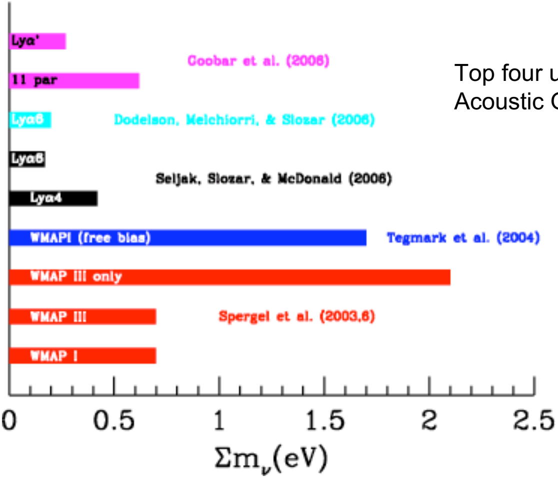
not necessarily small on scale of m_ν

$\sum_i m_{\nu_i}$



$\Omega_\nu \gtrsim (1.2, 2.2) \times 10^{-3}$ for (normal, inverted) spectrum

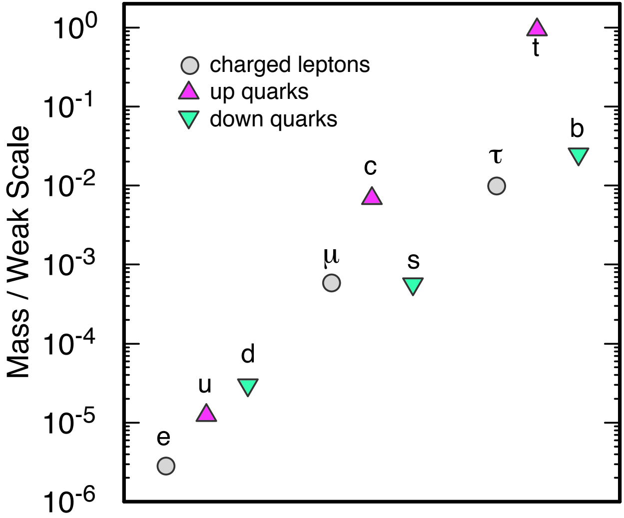
$\sum_i m_{\nu_i}$: cosmological constraints



Top four use Baryon Acoustic Oscillations

S. Dodelson, Fermilab–KEK ν School: http://nuss.fnal.gov

Quark & charged-lepton masses



If neutrinos have Dirac masses, ν Yukawa couplings $\lesssim 10^{-11}$

Summary of ν mixing parameters

$$\text{Define } \boldsymbol{\nu} = (\nu_1, \nu_2, \nu_3) \qquad \boldsymbol{\ell}_\mathsf{L} = (e_\mathsf{L}, \mu_\mathsf{L}, \tau_\mathsf{L})$$

$$\mathcal{L}_{\mathsf{CC}}^{(q)} = -\frac{g}{\sqrt{2}} \bar{\boldsymbol{\nu}} \, \gamma^\mu \mathcal{V}^\dagger \boldsymbol{\ell}_\mathsf{L} W_\mu^+ + \text{ h.c. },$$

$\mathcal{V}$: ν mixing matrix

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_{e1} & \mathcal{V}_{e2} & \mathcal{V}_{e3} \\ \mathcal{V}_{\mu 1} & \mathcal{V}_{\mu 2} & \mathcal{V}_{\mu 3} \\ \mathcal{V}_{\tau 1} & \mathcal{V}_{\tau 2} & \mathcal{V}_{\tau 3} \end{pmatrix}$$

Convention: ν_1, ν_2: solar pair, $m_1 < m_2$

ν_3 separated by Δm^2_{atm}; above or below?

Experiment tells us . . .

$$|\mathcal{V}| = \begin{pmatrix} 0.79 - 0.88 & 0.47 - 0.61 & < 0.20 \\ 0.19 - 0.52 & 0.42 - 0.73 & 0.58 - 0.82 \\ 0.20 - 0.53 & 0.44 - 0.74 & 0.56 - 0.81 \end{pmatrix}$$

$$\mathcal{V} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$s_{ij} = \sin\theta_{ij},\ c_{ij} = \cos\theta_{ij}$

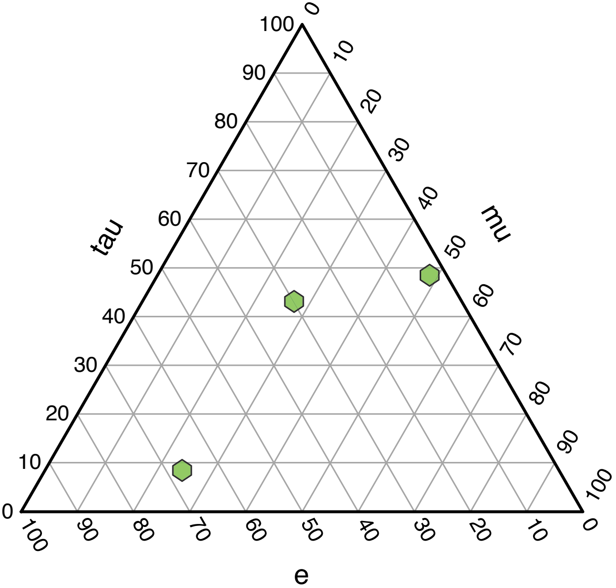
$30^\circ \lesssim \theta_{12} \lesssim 38^\circ$: solar

$35^\circ \lesssim \theta_{23} \lesssim 55^\circ$: atm

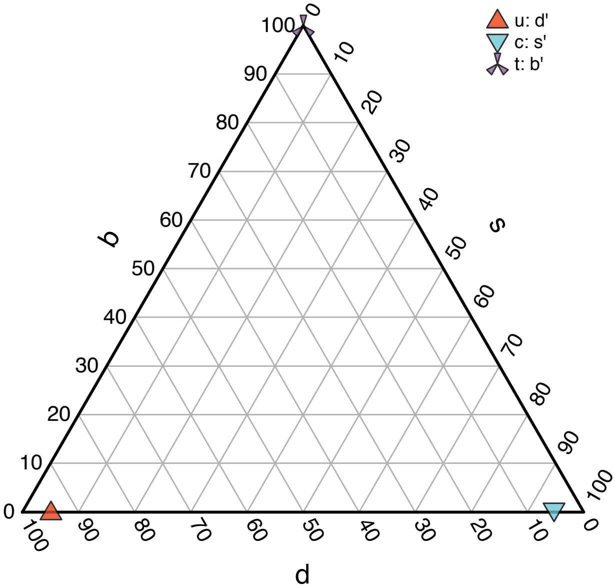
$\theta_{13} \lesssim 10^\circ$

CP phase δ unconstrained

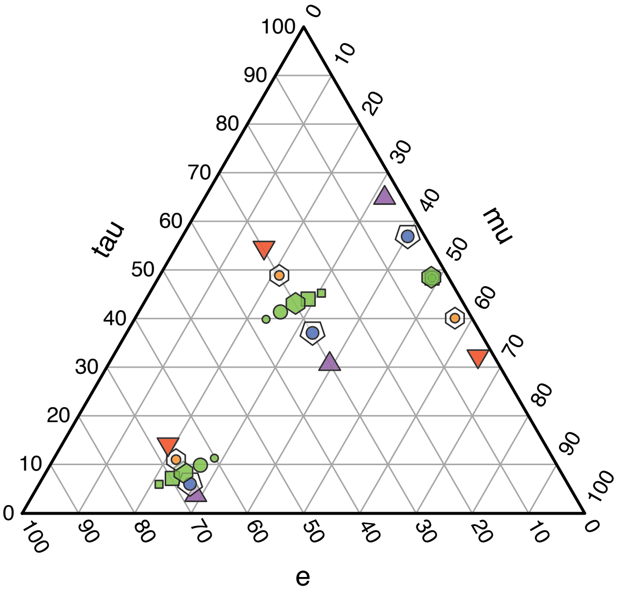
Neutrino Mixing (representative θ_{12}, θ_{23} values, $\theta_{13} = 10^\circ$)



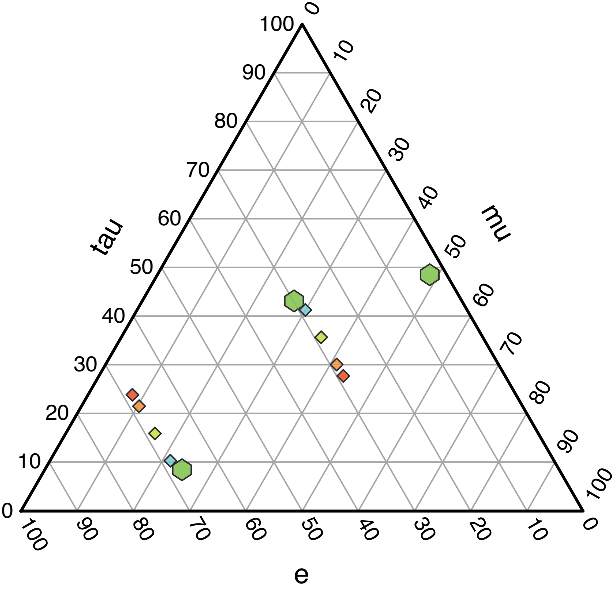
Quark Mixing (Components: $|V_{u\alpha}|^2$, etc.)



Neutrino Mixing (θ_{12}, θ_{23} variations, $\theta_{13} = 10^\circ$)



Neutrino Mixing (θ_{12}, θ_{23} fixed, δ variations, $\theta_{13} = 10^\circ$)



Varieties of neutrino mass: Dirac mass

Chiral decomposition of Dirac spinor:

$$\psi = \tfrac{1}{2}(1-\gamma_5)\psi + \tfrac{1}{2}(1+\gamma_5)\psi \equiv \psi_\mathsf{L} + \psi_\mathsf{R}$$

Dirac mass connects LH, RH components of *same field*

$$\mathcal{L}_D = -D(\bar{\psi}_\mathsf{L}\psi_\mathsf{R} + \bar{\psi}_\mathsf{R}\psi_\mathsf{L}) = -D\bar{\psi}\psi$$

$\Longrightarrow$ mass eigenstate $\psi = \psi_\mathsf{L} + \psi_\mathsf{R}$

(invariant under global phase rotation $\nu \to e^{i\theta}\nu$*,*
$\ell \to e^{i\theta}\ell$*, so that lepton number is conserved)*

Varieties of neutrino mass: Dirac mass

Add RH neutrino N_{R} to the standard-model spectrum

N_{R}: $\mathrm{SU(2)_L}$ singlet with $Y = 0$, so sterile

$$\mathcal{L}_{\mathrm{D}}^{(\nu)} = -\zeta_\nu \left[(\bar{\mathsf{L}}_\ell \bar{\phi}) N_{\mathrm{R}} + \bar{N}_{\mathrm{R}} (\bar{\phi}^\dagger \mathsf{L}_\ell) \right] \rightarrow -m_{\mathrm{D}} \left(\bar{\nu}_{\mathrm{L}} N_{\mathrm{R}} + \bar{N}_{\mathrm{R}} \nu_{\mathrm{L}} \right)$$

$$m_{\mathrm{D}} = \zeta_\nu v / \sqrt{2}$$

Some argue that $\zeta_\nu \lesssim 10^{-11}$ *is unnatural, while the range* $\zeta_t \approx 1$ *to* $\zeta_e \approx$ *few* $\times\, 10^{-6}$ *is only puzzling.* *But all Dirac masses involve physics beyond the standard model.*

Varieties of neutrino mass: Majorana mass

Neutrinos: no color, no Q: their own antiparticles?

Majorana fermions

Charge conjugate of RH field is LH: $\psi_\mathsf{L}^c \equiv (\psi^c)_\mathsf{L} = (\psi_\mathsf{R})^c$

Majorana joins LH, RH components of *conjugate fields*

$$\begin{aligned} -\mathcal{L}_{\mathsf{MA}} &= A(\bar{\nu}_\mathsf{R}^c \nu_\mathsf{L} + \bar{\nu}_\mathsf{L} \nu_\mathsf{R}^c) = A\bar{\chi}\chi \\ -\mathcal{L}_{\mathsf{MB}} &= B(\bar{\nu}_\mathsf{L}^c \nu_\mathsf{R} + \bar{\nu}_\mathsf{R} \nu_\mathsf{L}^c) = B\bar{\omega}\omega \end{aligned}$$

for which the mass eigenstates are

$$\begin{aligned} \chi &\equiv \nu_\mathsf{L} + \nu_\mathsf{R}^c = \chi^c = \nu_\mathsf{L} + (\nu_\mathsf{L})^c \\ \omega &\equiv \nu_\mathsf{R} + \nu_\mathsf{L}^c = \omega^c = \nu_\mathsf{R} + (\nu_\mathsf{R})^c \end{aligned}$$

Lepton number violation

Majorana ν: no conserved additive quantum number

$\mathcal{L}_{\mathsf{M}}$ violates lepton number by two units

$\Rightarrow$ Majorana ν can mediate $\beta\beta_{0\nu}$ decays

$$(Z, A) \rightarrow (Z+2, A) + e^{-} + e^{-}$$

$$\boxed{\text{Detecting } \beta\beta_{0\nu} \text{ would offer decisive evidence for Majorana nature of } \nu}$$

Active ν_{L} mass generated by $I = 1$ Higgs with vev or effective operator containing two $I = \frac{1}{2}$ Higgs combined to transform as $I = 1$.

Neutrino observatories: expectations

Cosmic ν flux may exceed atmospheric background at $E_\nu \approx$ few TeV

prospect for sources · characterize sources · study ν properties

Sources include AGN (at $\sim 10^2$ Mpc) 1 Mpc $\approx 3.1 \times 10^{22}$ m

$$pp \text{ or } p\gamma \Rightarrow \approx \text{numbers of } \pi^+\ \pi^0\ \pi^-$$

$$\pi^+ + \pi^0 + \pi^- \Rightarrow 2\gamma + 2\nu_\mu + 2\bar{\nu}_\mu + 1\nu_e + 1\bar{\nu}_e$$

$$\boxed{\Phi^0_{\text{std}} = \{\varphi^0_e = \tfrac{1}{3}, \varphi^0_\mu = \tfrac{2}{3}, \varphi^0_\tau = 0\} \quad (\nu = \bar{\nu})}$$

Detection (in volumes → 1 km^3)

$$\boxed{(\nu_\mu, \bar{\nu}_\mu)N \to (\mu^-, \mu^+) + \text{anything}}$$

Can we achieve efficient, calibrated $(\nu_e, \bar{\nu}_e)$ detection?
Good $(\nu_\tau, \bar{\nu}_\tau)$ detection, NC capability desirable

$\nu_\mu N \to \mu^- +$ anything

$$\frac{d^2\sigma}{dxdy} = \frac{2G_F^2 M E_\nu}{\pi} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 \left[xq(x,Q^2) + x\bar{q}(x,Q^2)(1-y)^2 \right]$$

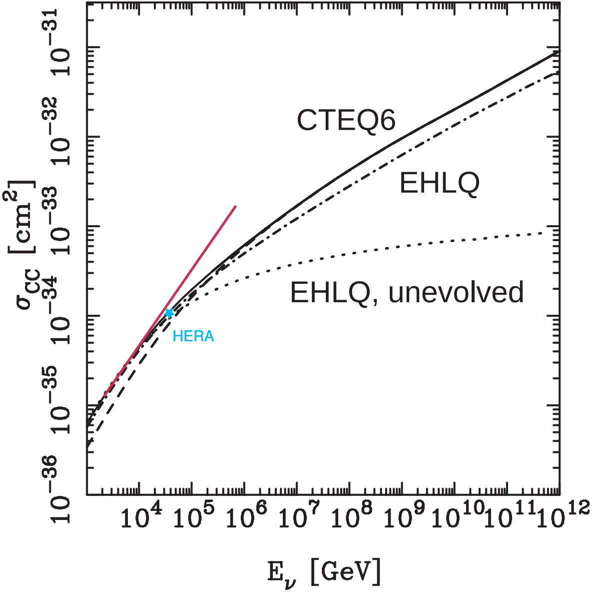
$$\begin{aligned} q(x,Q^2) &= \frac{u_v(x,Q^2) + d_v(x,Q^2)}{2} + \frac{u_s(x,Q^2) + d_s(x,Q^2)}{2} \\ &\quad + s_s(x,Q^2) + b_s(x,Q^2) \end{aligned}$$

$$\bar{q}(x,Q^2) = \frac{u_s(x,Q^2) + d_s(x,Q^2)}{2} + c_s(x,Q^2) + t_s(x,Q^2),$$

. . . isoscalar nucleon

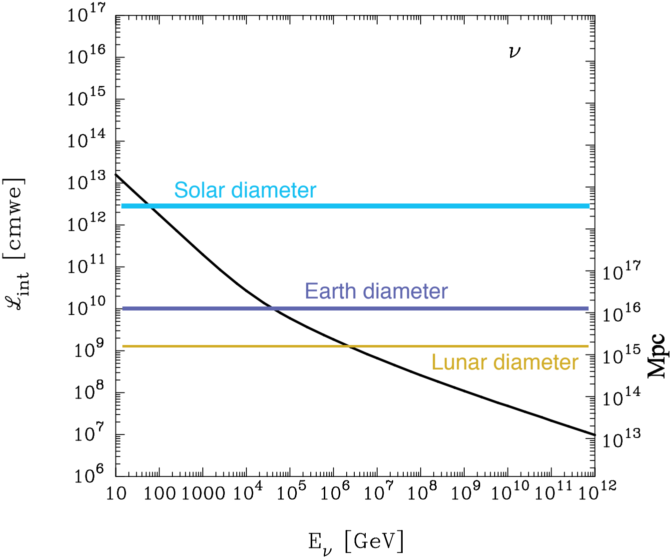
$$x = Q^2/2M\nu \quad y \equiv \nu/E_\nu \quad \nu \equiv E_\nu - E_\mu$$

$\nu N \rightarrow \mu + \ldots$ Cross Sections



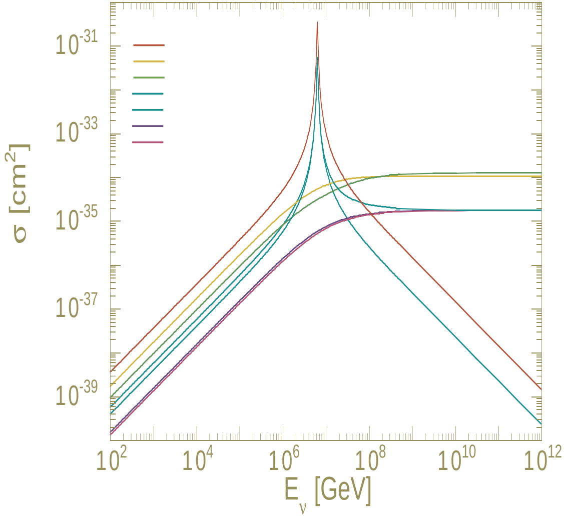
M. H. Reno

$\nu N \to \mu + \ldots$ Interaction Lengths



$n_{B0} \approx 2.5 \times 10^{-7}\ \mathrm{cm}^{-3}$

νe cross sections . . .



At low energies: $\sigma(\bar{\nu}_e e \to$ hadrons$) > \sigma(\nu_\mu e \to \mu\nu_e) > \sigma(\nu_e e \to \nu_e e) >$ $\sigma(\bar{(}\nu_e e \to \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) > \sigma(\nu_\mu e \to \nu_\mu e) > \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$

Influence of neutrino oscillations

Flux at Earth $\Phi = \{\varphi_e, \varphi_\mu, \varphi_\tau\} \neq \Phi^0 = \{\varphi_e^0, \varphi_\mu^0, \varphi_\tau^0\}$ source fluxes

Vacuum oscillation length is short; for $|\Delta m^2| = 10^{-5}\ \mathrm{eV}^2$,

$$L_{\mathrm{osc}} \;=\; 4\pi E_\nu/|\Delta m^2| \approx 2.5 \times 10^{-24}\ \mathrm{Mpc} \cdot (E_\nu/1\ \mathrm{eV})$$

. . . a fraction of Mpc even for $E_\nu = 10^{20}$ eV

ν *oscillate many times between cosmic source and terrestrial detector*

Also, over long paths, cosmic neutrinos are vulnerable to decay processes that would not affect terrestrial or solar experiments.

. . . Neutrino Oscillations

$$(\text{flavor})\ \nu_\alpha = \sum_i \mathcal{V}_{\alpha i}\nu_i\ (\text{mass})$$

Idealize $\sin\theta_{13} = 0$, $\sin 2\theta_{23} = 1$, write $x = \sin^2\theta_{12}\cos^2\theta_{12}$.

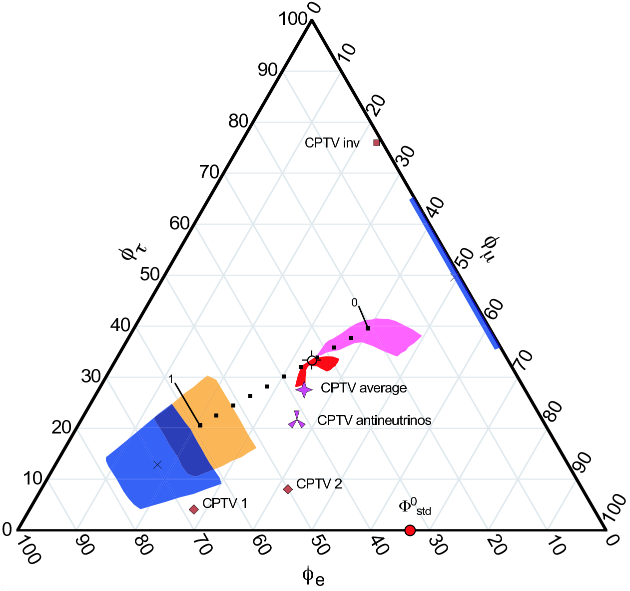
$$\mathcal{V}_{\text{ideal}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Transfer matrix $\mathcal{X}$: Φ^0 (source) → Φ (detector); Over many oscillations,

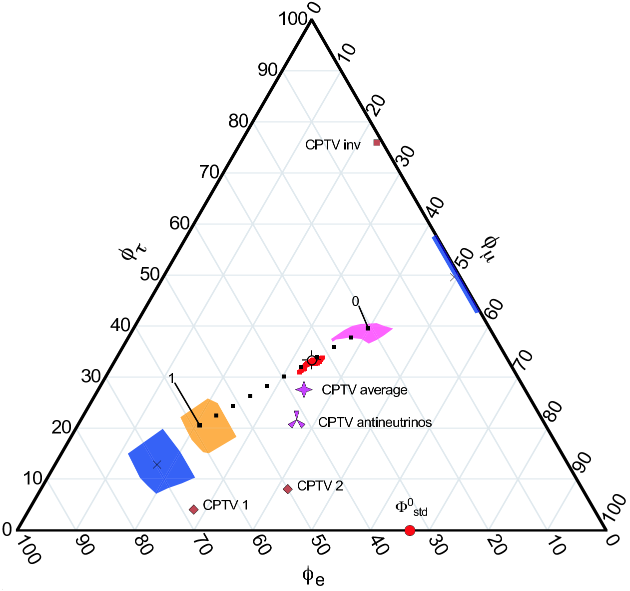
$$\mathcal{X}_{\mathrm{ideal}} = \begin{pmatrix} 1-2x & x & x \\ x & \frac{1}{2}(1-x) & \frac{1}{2}(1-x) \\ x & \frac{1}{2}(1-x) & \frac{1}{2}(1-x) \end{pmatrix}$$

$$\boxed{\mathcal{X}_{\mathrm{ideal}} : \Phi^0_{\mathrm{std}} \rightarrow \{\varphi_e = \tfrac{1}{3}, \varphi_\mu = \tfrac{1}{3}, \varphi_\tau = \tfrac{1}{3}\}}$$

With current constraints . . .



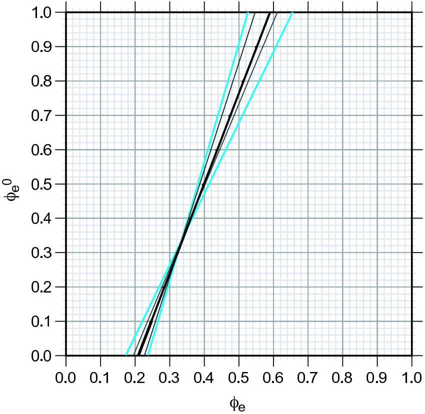
After current generation . . .



Reconstructing the ν Mixture at the Source

ν_μ, ν_τ fully mixed $\Rightarrow$ can't fully characterize Φ^0

Reconstruct ν_e fraction at source using $\mathcal{X}_{\mathrm{ideal}}$: $\varphi_e^0 = (\varphi_e - x)/(1 - 3x)$



Extreme φ_e implicates unconventional physics

Influence of neutrino decays

Nonradiative decays $\nu_i \rightarrow (\nu_j, \bar{\nu}_j) + X$ not very constrained:

$\tau/m \gtrsim 10^{-4}$ s/eV

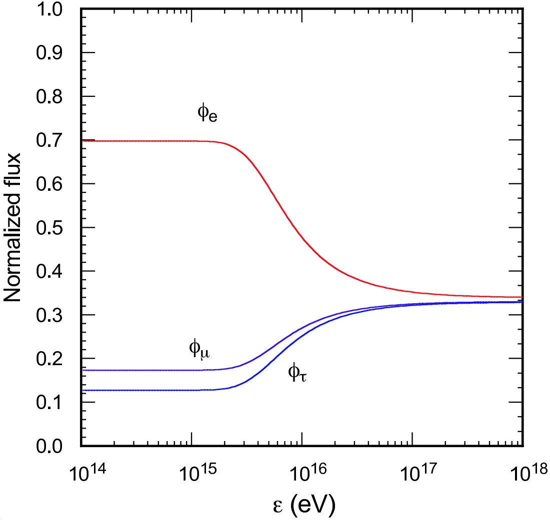
If only lightest neutrino survives, flavor mix at Earth is independent of composition at source

Normal hierarchy
$m_1 < m_2 < m_3$:
$\varphi_\alpha = |U_{\alpha 1}|^2$
$\Phi_{\text{normal}} \approx \{0.70, 0.17, 0.13\}$

Inverted hierarchy
$m_1 > m_2 > m_3$:
$\varphi_\alpha = |U_{\alpha 3}|^2$
$\Phi_{\text{inverted}} \approx \{0, 0.5, 0.5\}$

$$\boxed{\text{far from } \Phi_{\text{std}} = \{\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\}}$$

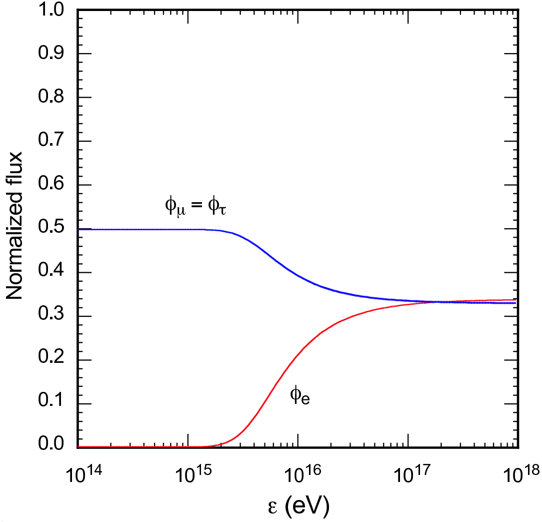
Energy-dependent composition as marker of ν decays



$E_\nu = \varepsilon(1\ \mathrm{s/eV})/(\tau_\nu/m_\nu)\cdot L/(100\ \mathrm{Mpc})$

Normal hierarchy

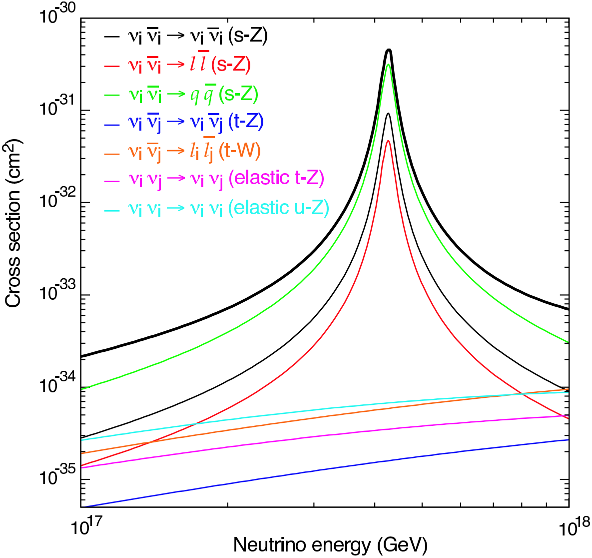
Energy-dependent composition as marker of ν decays



$E_\nu = \varepsilon(1\ \mathrm{s/eV})/(\tau_\nu/m_\nu) \cdot L/(100\ \mathrm{Mpc})$

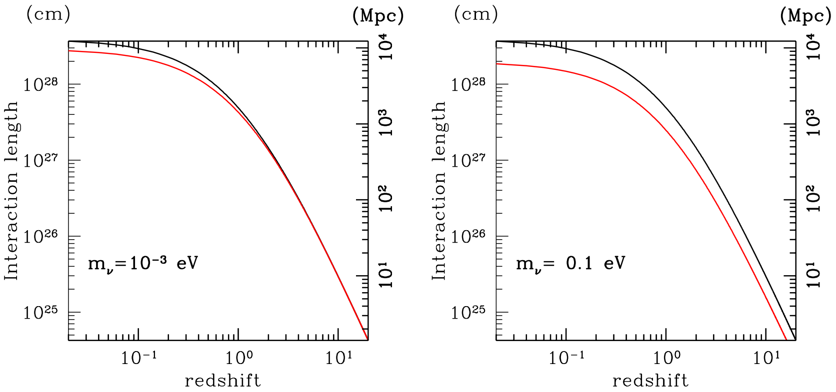
Inverted hierarchy

UHE ν annihilation on ν relics



$m_\nu = 10^{-5}$ eV

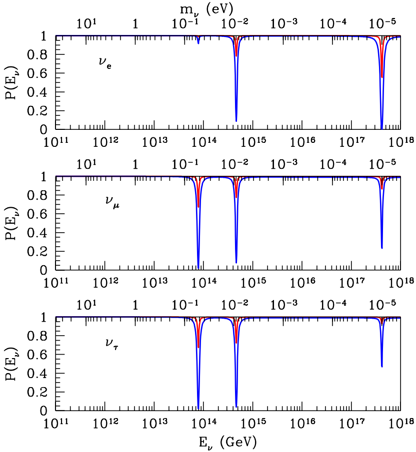
Interaction lengths on Z^0 resonance: Dirac Majorana



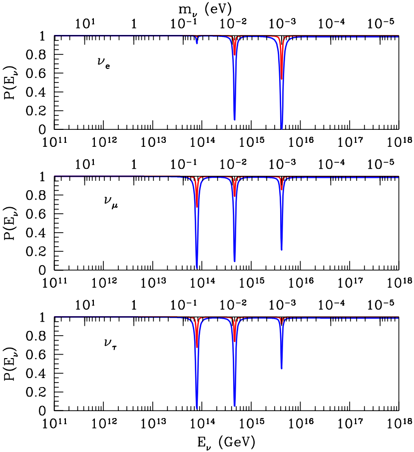
ν number density now: 56 cm^{-3}, $\propto (1+z)^3$

1.2×10^4 Mpc $=$ 39 Gly

Fable: l-o-o-o-o-o-n-g path (10^4 or 10^5 Mpc) in current U

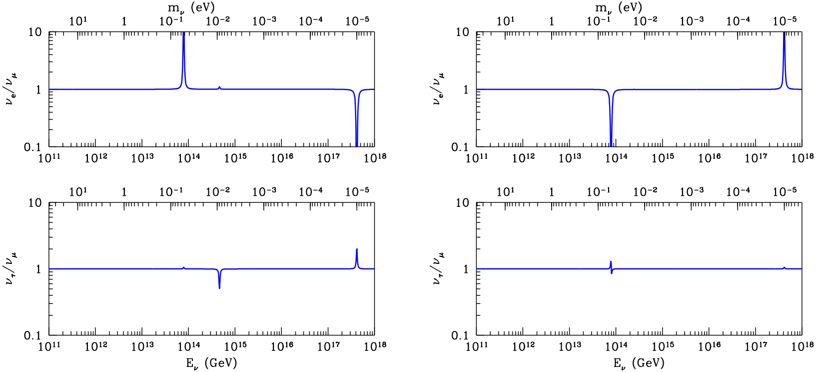


$m_\ell = 10^{-5}$ eV



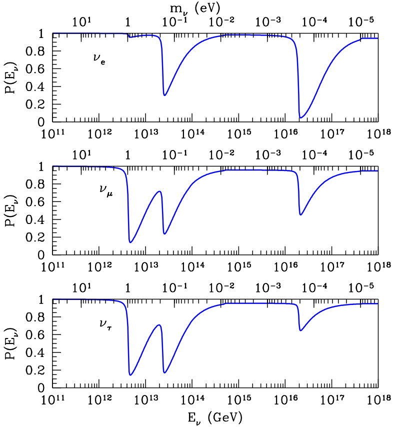
$m_\ell = 10^{-3}$ eV

Flavor ratios probe the mass hierarchy . . .

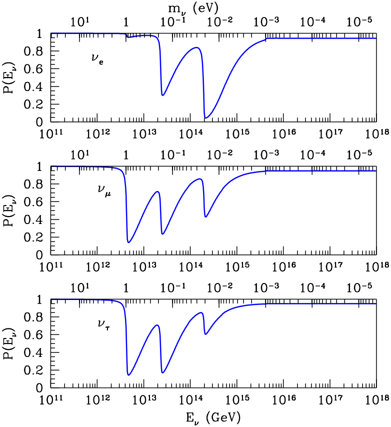


Normal $m_\ell = 10^{-5}$ eV Inverted

Incorporate evolution of U back to $z = 20$

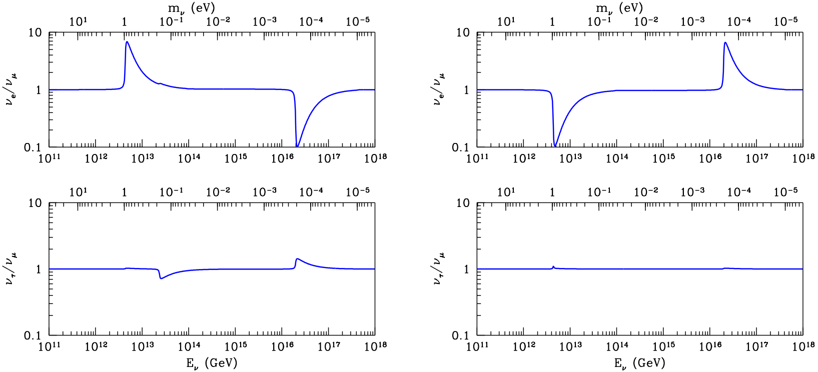


$m_\ell = 10^{-5}$ eV



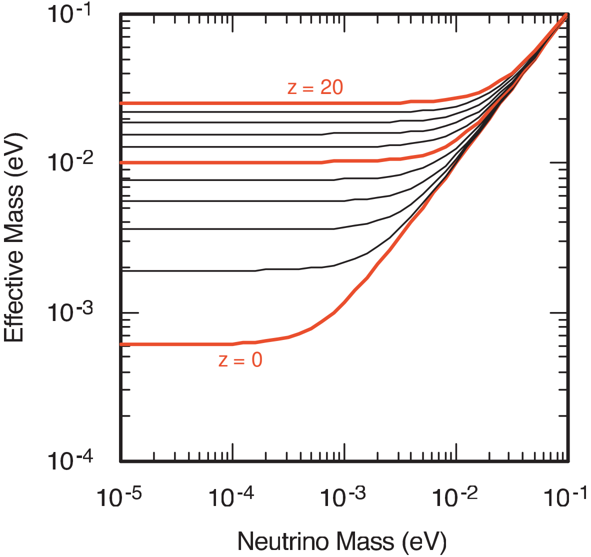
$m_\ell = 10^{-3}$ eV

Flavor ratios probe the mass hierarchy($z \lesssim 20$)



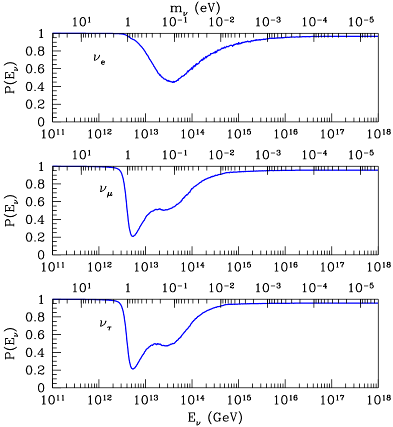
Normal $m_\ell = 10^{-5}$ eV Inverted

Relic neutrinos are moving targets . . .

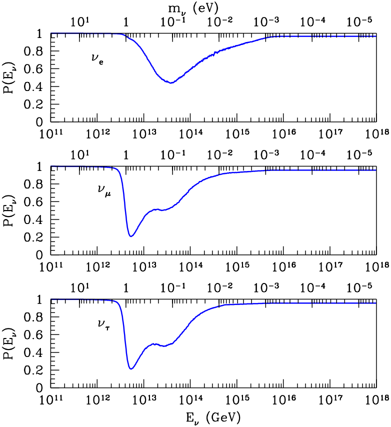


$$T_{\nu 0} = 1.945\ \mathrm{K} = 1.697 \times 10^{-4}\ \mathrm{eV} \qquad T_\nu \propto (1+z)$$

Incorporate evolution of U back to $z = 20$, Fermi motion

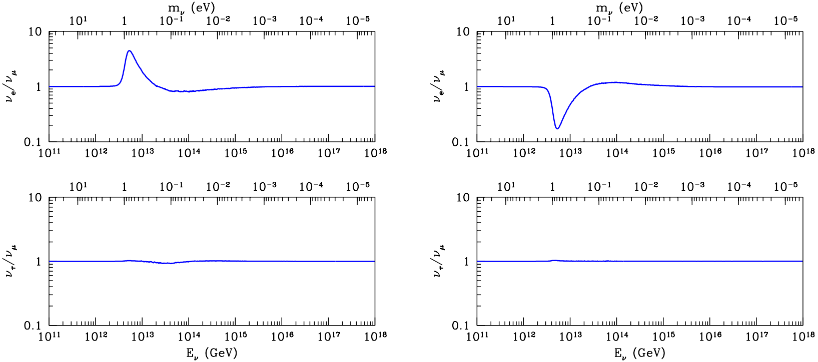


$m_\ell = 10^{-5}$ eV



$m_\ell = 10^{-3}$ eV

Flavor ratios probe mass hierarchy($z \lesssim 20$, Fermi motion)



Normal $m_\ell = 10^{-5}$ eV Inverted

Dark Matter: The Contest

What should be the theme of SSI 2010?

Propose a title and give a two-sentence description to appear on the SSI 2010 web page.

Justify your choice in one tightly reasoned paragraph.

Winners receive valuable prizes and untold glory

Deadline for entries: 17h00 Thursday

Award ceremony: Friday morning