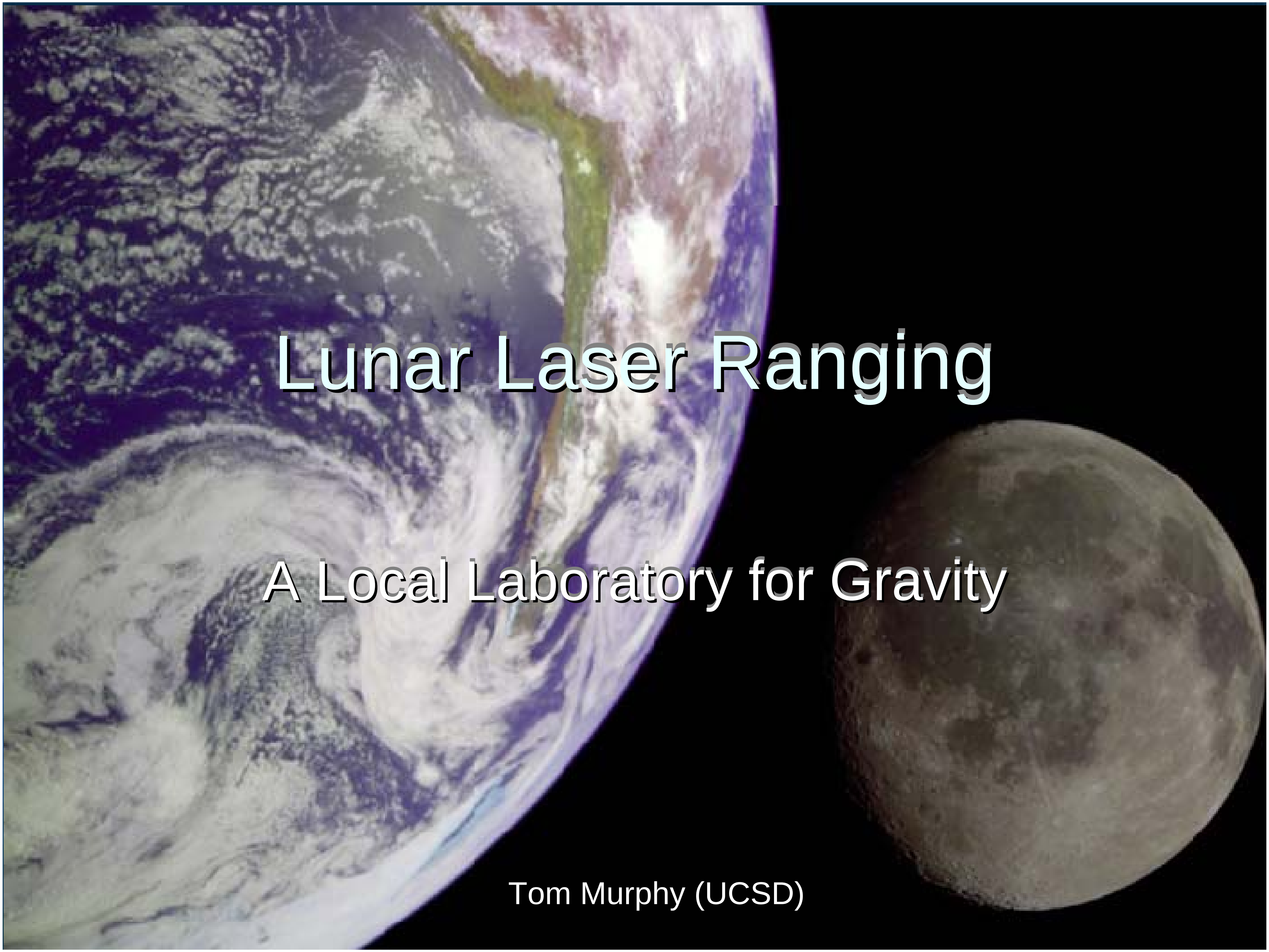


Lunar Laser Ranging

A Local Laboratory for Gravity

Tom Murphy (UCSD)

Why Test Gravity?

- Dark Matter *could* be misunderstood gravity (nobody say MOND)
- Dark Energy *could* be misunderstood gravity
- Gravity and Quantum Mechanics do not play well together
- Fundamental constants (like *G*) may change with time
- Scalar fields may lead to Equivalence Principle violation
- Extra Dimensions may impact the gravity sector
- A theory is only as believable as the backing experiment

The Full PPN Metric

- Generalized metric where little is sacred
  - GR is a special case
  - Allows violations of conservations, Lorentz invariance, etc.

$$\begin{aligned} g_{00} &= -1 + 2U - 2\beta U^2 - 2\xi\phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\phi_1 \\ &+ 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\phi_2 + 2(1 + \zeta_3)\phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\phi_4 \\ &- (\zeta_1 - 2\xi)A - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i \\ &+ \mathcal{O}(\epsilon^3) \end{aligned}$$

$$\begin{aligned} g_{0i} &= -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i \\ &- \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}) \end{aligned}$$

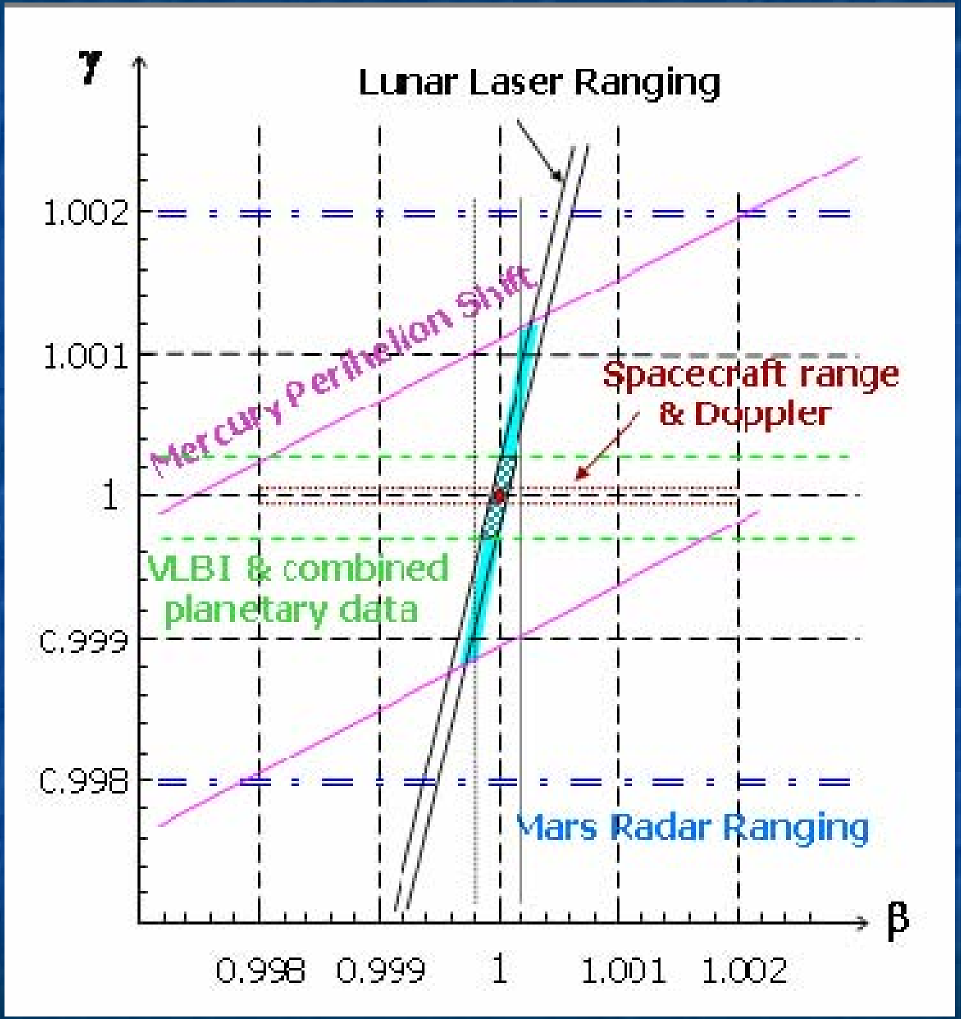
$$g_{ij} = (1 + 2\gamma U + \mathcal{O}(\epsilon^2))\delta_{ij}$$

Simplified (Conservative) PPN Equations of Motion

$$
\begin{aligned}
\ddot{\mathbf{r}}_{i_{\text{point mass}}} \quad = \quad & \sum_{j\neq i} \frac{\mu_j(\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3} \left\{ 1 - \frac{2(\beta+\gamma)}{c^2} \sum_{k\neq i} \frac{\mu_k}{r_{ik}} \right. \\
& - \frac{2\beta - 1}{c^2} \sum_{k\neq j} \frac{\mu_k}{r_{jk}} + \gamma \left(\frac{v_i}{c}\right)^2 + (1+\gamma)\left(\frac{v_j}{c}\right)^2 \\
& - \frac{2(1+\gamma)}{c^2} \dot{\mathbf{r}}_i \cdot \dot{\mathbf{r}}_j - \frac{3}{2c^2} \left[\frac{(\mathbf{r}_i - \mathbf{r}_j) \cdot \dot{\mathbf{r}}_j}{r_{ij}} \right]^2 \\
& \left. + \frac{1}{2c^2} (\mathbf{r}_j - \mathbf{r}_i) \cdot \ddot{\mathbf{r}}_j \right\} + \frac{1}{c^2} \sum_{j\neq i} \frac{\mu_j}{r_{ij}^3} \\
& \times \left\{ [\mathbf{r}_i - \mathbf{r}_j] \cdot [(2+2\gamma)\dot{\mathbf{r}}_i - (1+2\gamma)\dot{\mathbf{r}}_j] \right\} (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j) \\
& + \frac{3+4\gamma}{2c^2} \sum_{j\neq i} \frac{\mu_j \ddot{\mathbf{r}}_j}{r_{ij}}
\end{aligned}
$$

A Modern, Post-Newtonian View

- The Post-Newtonian Parameterization (PPN) describes deviations from GR
- The main parameters are γ and β
  - γ tells us how much curvature is produced per unit mass
  - β tells us how nonlinear gravity is (self-interaction)
  - γ and β are identically 1.00 in GR
- Current limits have:
  - $(\gamma-1) < 2.3\times10^{-5}$ (Cassini)
  - $(\beta-1) < 1.1\times10^{-4}$ (LLR)



Relativistic Observables in the Lunar Range

- Lunar Laser Ranging provides a comprehensive probe of gravity, boasting the best tests of:
  - Equivalence Principle (two flavors)
    - WEP: $\Delta a/a < 1.4\times10^{-13}$
    - SEP: $\eta < 4.5\times10^{-4}$
  - time-rate-of-change of G: $< 9\times10^{-13}$ fractional change per year
  - geodetic precession: confirmed to 0.6%
  - $1/r^2$ force law: $< 10^{-10}$ times strength of gravity at 10^8 m scales
  - gravitomagnetism (frame-dragging): 0.1%
- Future sensitivity to extra-dimensional modifications to gravity as proposed by Dvali et al.
  - result would be precession of lunar orbit

Two Flavors of the Equivalence Principle

- Weak EP
  - Composition difference: e.g., iron in earth vs. silicates in moon
  - Probes all interactions but gravity itself
    - Currently tested by LLR and torsion balance to $\Delta a/a < 1.4\times10^{-13}$
  - the LLR $\Delta a/a$ value is tied with torsion balance results (e.g., Eöt-Wash group), though laboratory measurements constrain WEP more meaningfully
- Strong EP
  - Applies to gravitational "energy" itself
    - Earth self-energy has equivalent mass ($E = mc^2$)
      - Amounts to 4.6×10^{-10} of earth's total mass-energy
    - Does this mass have $M_G/M_I \equiv 1.00000$?
  - Another way to look at it: gravity pulls on gravity
    - This gets at *nonlinear* aspect of gravity (PPN β)

The Strong Equivalence Principle

- Earth's energy of assembly amounts to 4.6×10^{-10} of its total mass-energy

$$M_{S.E.} = \frac{G}{c^2}\int\int \frac{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} d^3\mathbf{r}_1 d^3\mathbf{r}_2 \approx \frac{GM^2}{Rc^2}$$

The ratio of gravitational to inertial mass for this self energy is

$$\frac{M_G}{M_I} = 1 - (4\beta - 3 - \gamma)\frac{M_{S.E.}}{M} \equiv 1 - \eta\frac{M_{S.E.}}{M}$$

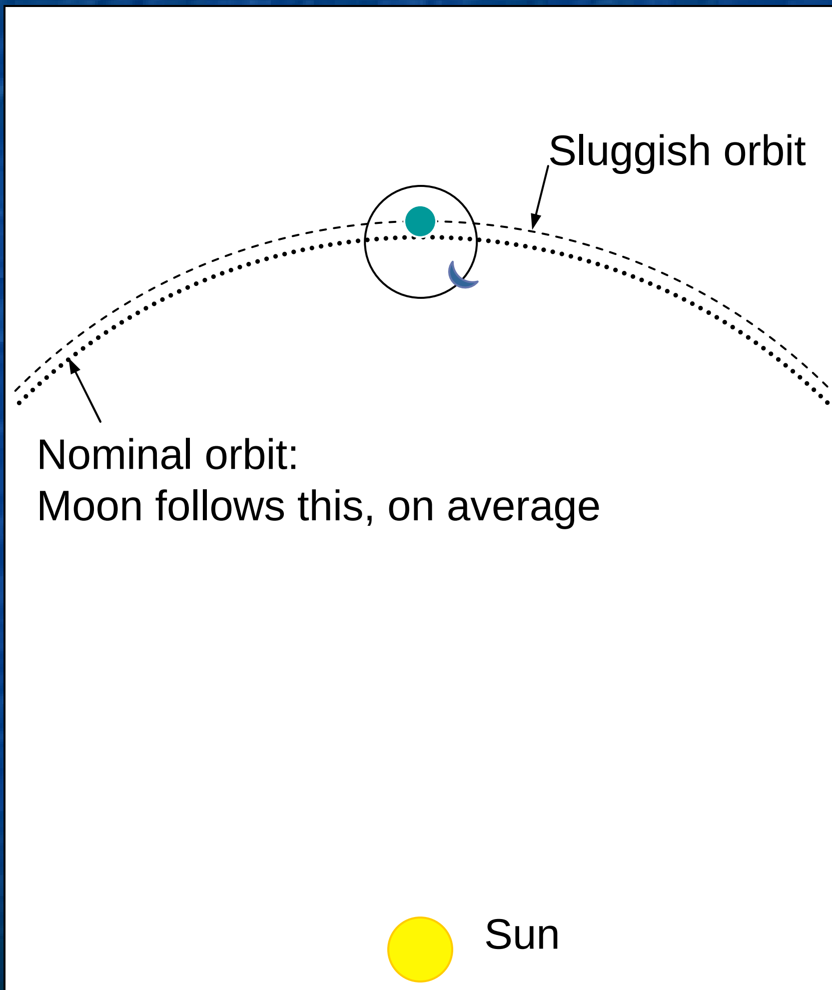
The resulting range signal is then

$$\Delta r = 13.1\eta\cos D \quad \text{meters}$$

Currently η is limited by ~cm LLR to be $\leq 4.5\times10^{-4}$

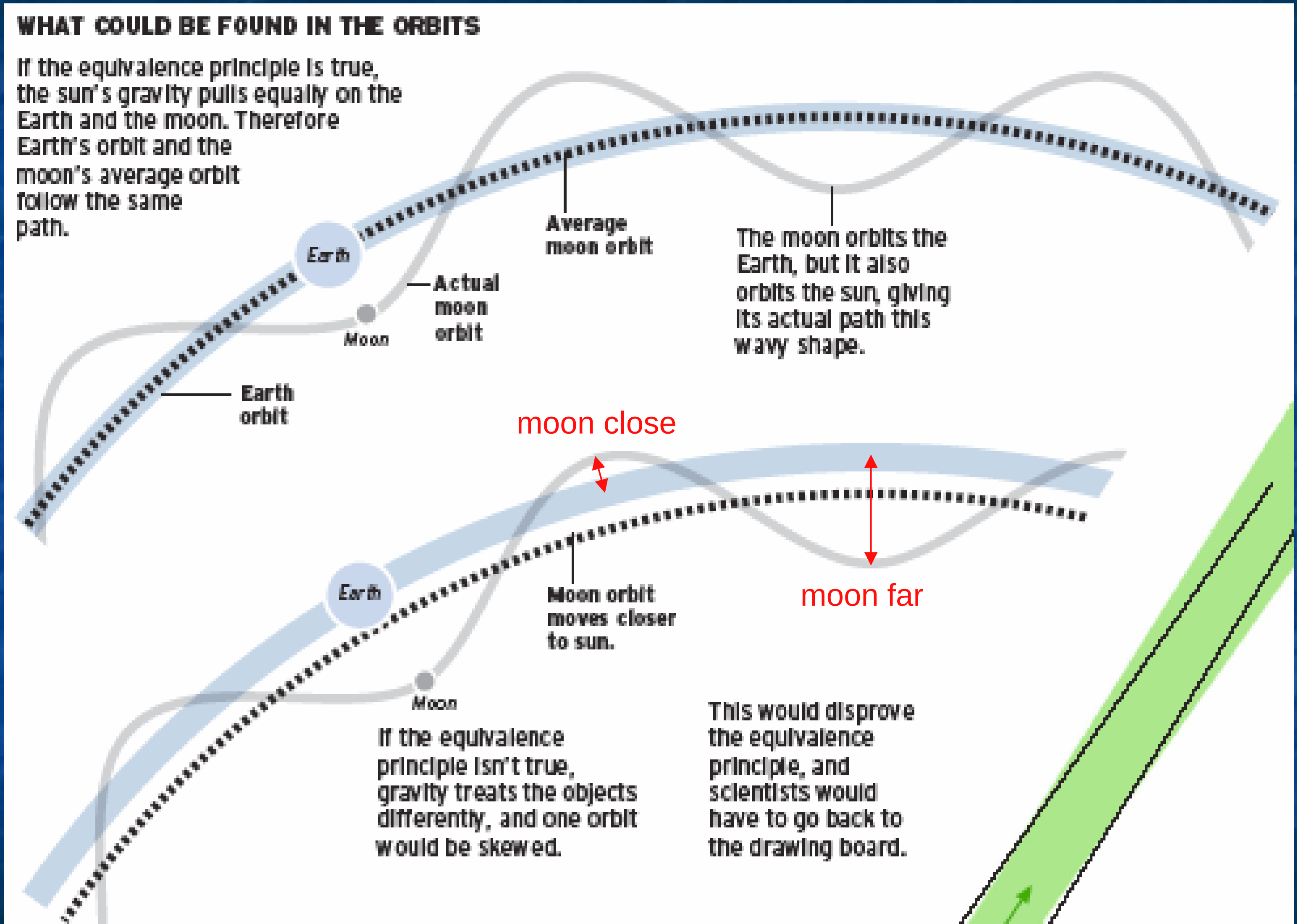
LLR is the best way to test the strong EP

Equivalence Principle Lunar Signal



- If, for example, Earth has greater inertial mass than gravitational mass (while the moon does not):
  - Earth is sluggish to move
  - Alternatively, pulled weakly by gravity
  - Takes orbit of larger radius (than does Moon)
  - Appears that Moon's orbit is *shifted* toward sun: cos*D* signal

EP Signal, Illustrated



Graphic excerpt from San Diego Union Tribune

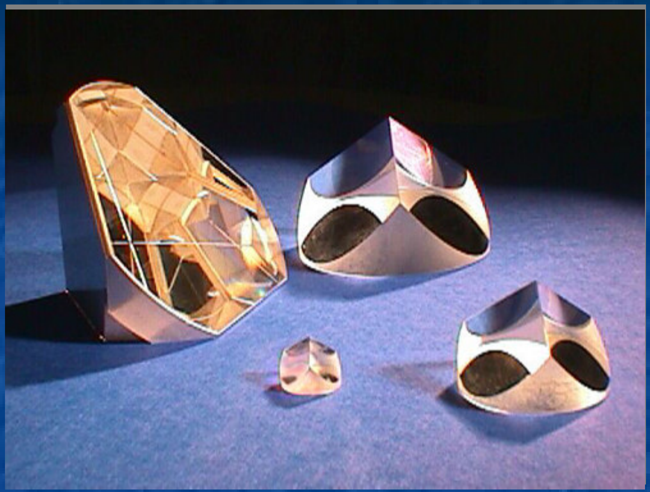
Aside on Gravitomagnetism

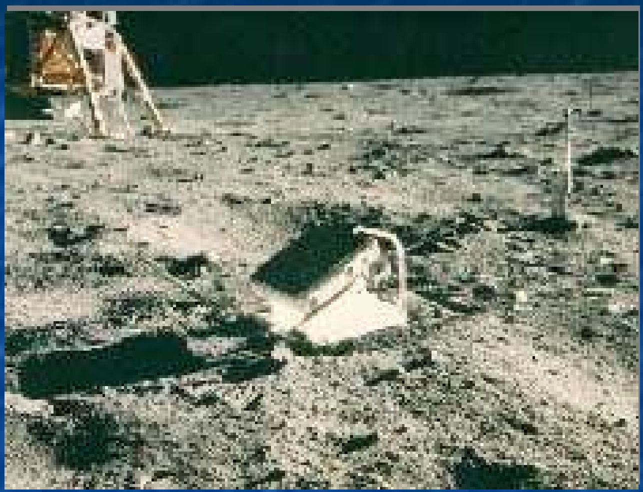
- Stems from "motional" term in equation of motion:

$$\mathbf{a}_i = -\frac{\mu_j(2+2\gamma)}{c^2 r_{ij}^3}\mathbf{v}_i \times (\mathbf{v}_j \times \mathbf{r}_{ij})$$

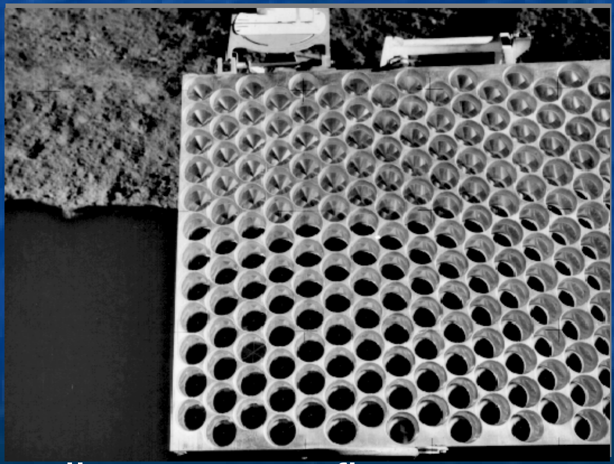
- If earth has velocity **V**, and moon is **V+u**, two terms of consequence emerge:
  - One proportional to V^2 with 6.5 meter cos2*D* signal
  - One proportional to *Vu* with 6.1 meter cos*D* signal
- LLR determines cos*D* to 4 mm precision and cos2*D* to < 8 mm
  - Constitutes a ≈ 0.1% measurement of effect
- The same exact *v*×*v*×*g* term can be used to derive the precession of a gyroscope in the presence of a spinning mass
  - recovers the full effect (0.042 mas/yr) sought by Gravity Probe-B

Lunar Retroreflector Arrays


Corner cubes

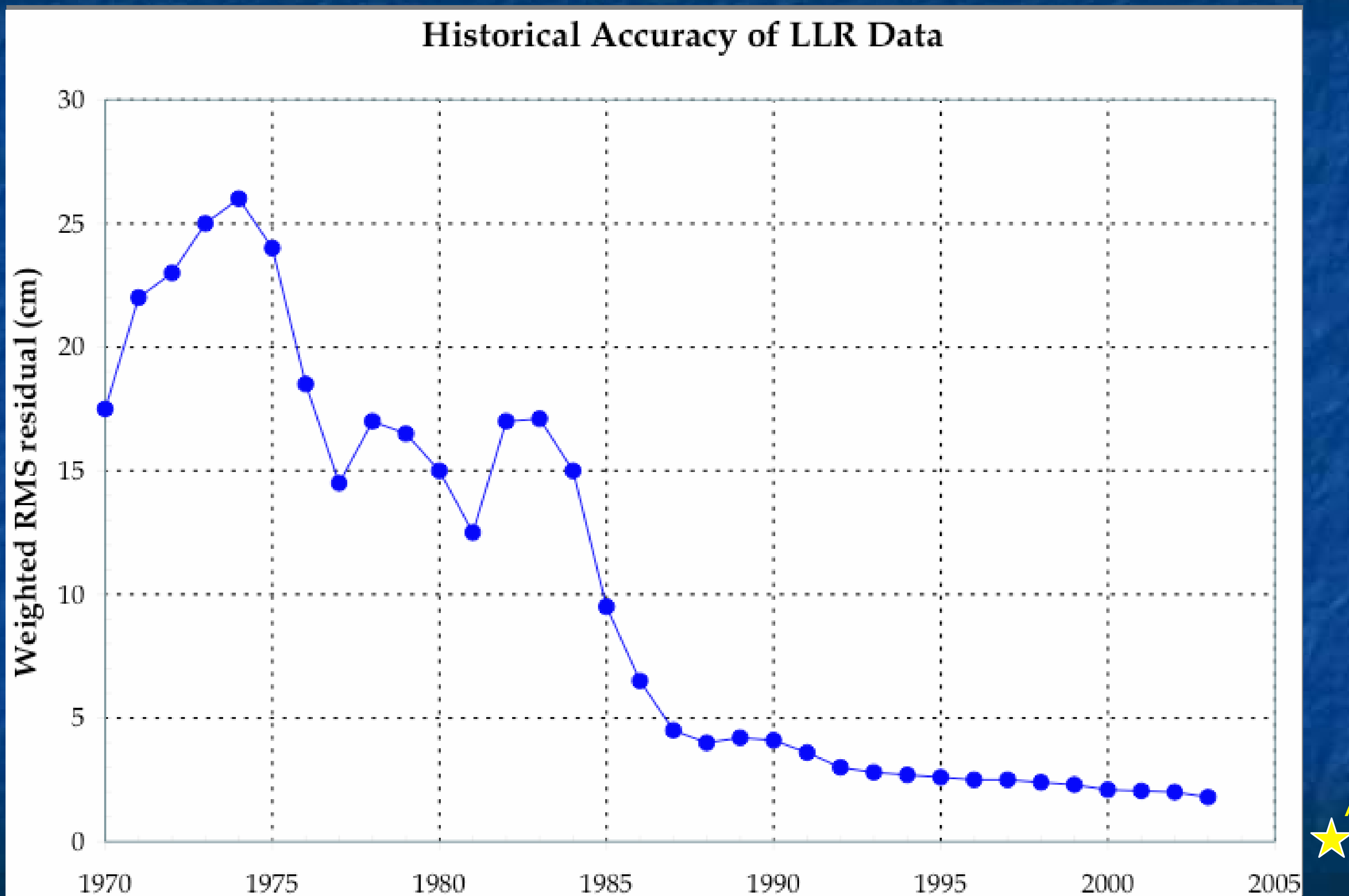

Apollo 11 retroreflector array


Apollo 14 retroreflector array

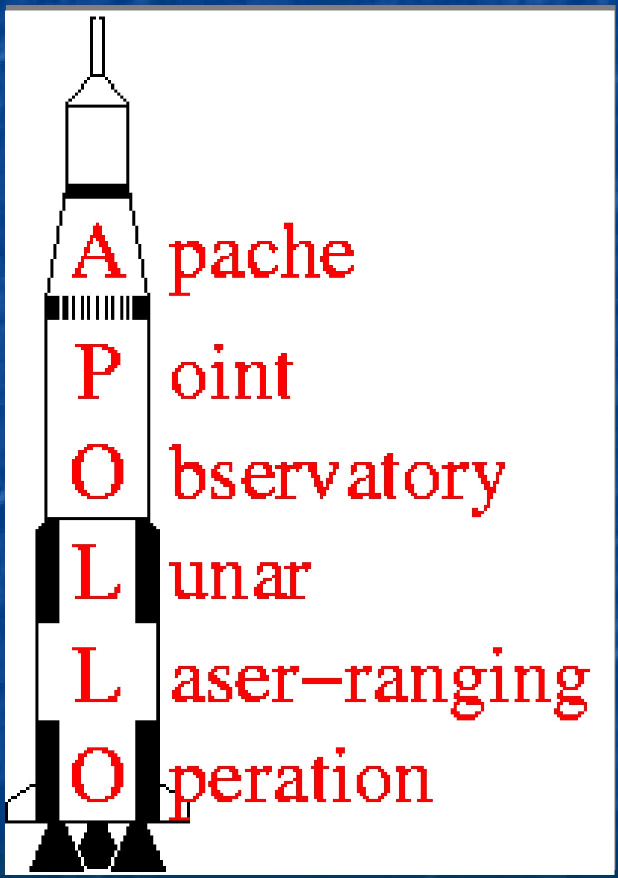

Apollo 15 retroreflector array

LLR through the decades

Previously
200 *meters*



APOLLO: the next big thing in LLR



- APOLLO offers order-of-magnitude improvements to LLR by:
  - Using a 3.5 meter telescope
  - Gathering multiple photons/shot
  - Operating at 20 pulses/sec
  - Using advanced detector technology
  - Achieving millimeter range precision
  - Tightly integrating experiment and analysis
  - Having the best acronym

The APOLLO Collaboration

UCSD:
Tom Murphy (PI)
Eric Michelsen

U Washington:
Eric Adelberger
Erik Swanson

Apache Point Obs.
Bruce Gillespie
Russet McMillan

Harvard:
Chris Stubbs
James Battat

Humboldt State:
C. D. Hoyle

Northwest Analysis:
Ken Nordtvedt

JPL:
Jim Williams
Slava Turyshev
Dale Boggs

Lincoln Lab:
Brian Aull
Bob Reich

Gigantic Laser Pointer

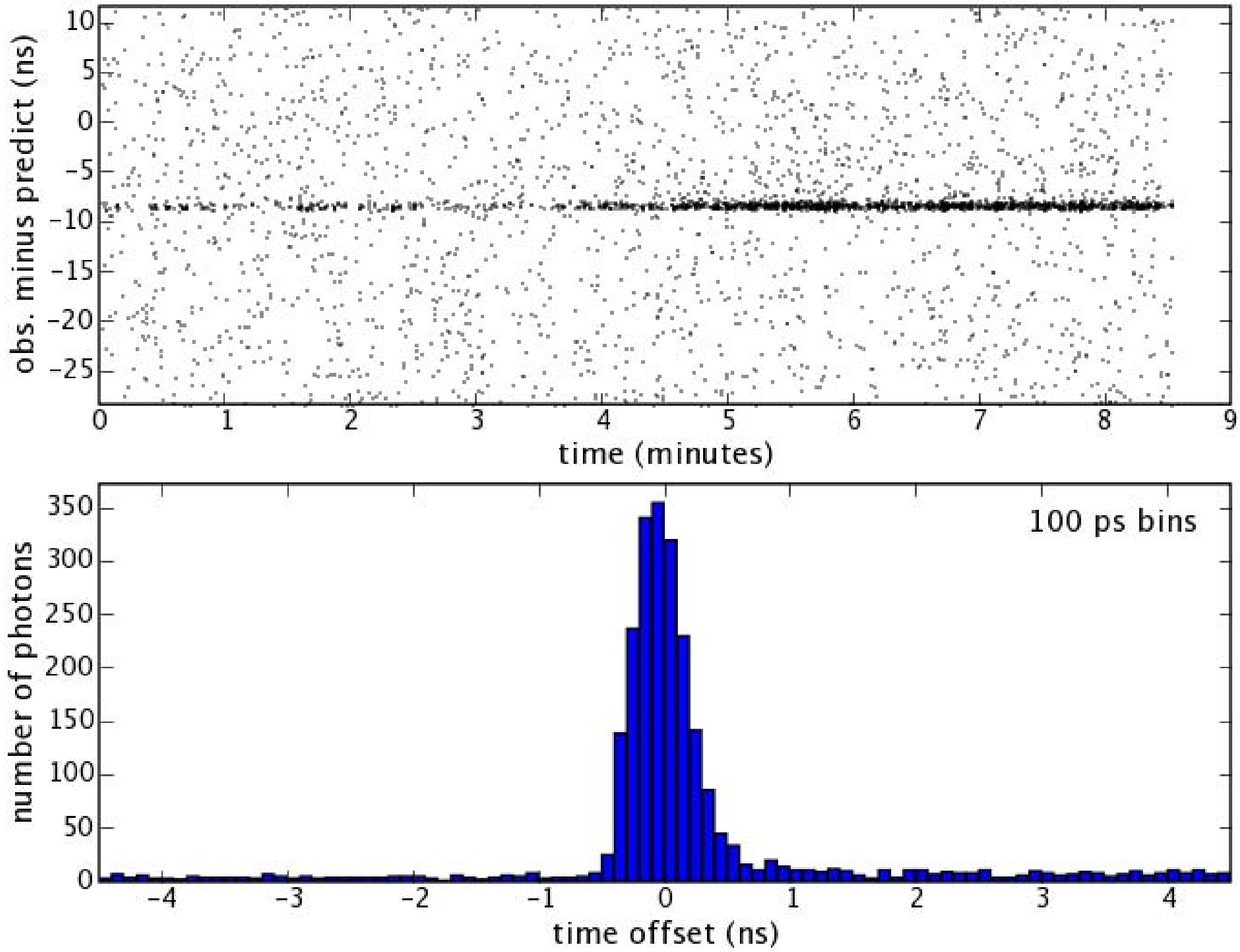
Blasting the Moon

APOLLO Random Error Budget

Error Source	Time Uncert. (ps) (round trip)	Range error (mm) (one way)
Retro Array Orient.	100–300	15–45
APD Illumination	60	9
APD Intrinsic	<50	< 7
Laser Pulse Width	45	6.5
Timing Electronics	20	3
GPS-slaved Clock	7	1
Total Random Uncert	136–314	20–47

Ignoring retro array, APOLLO system has 93 ps (14 mm) error *per photon*

Example Range Return



03 Dec., 2006

Apollo 15

2000 photons

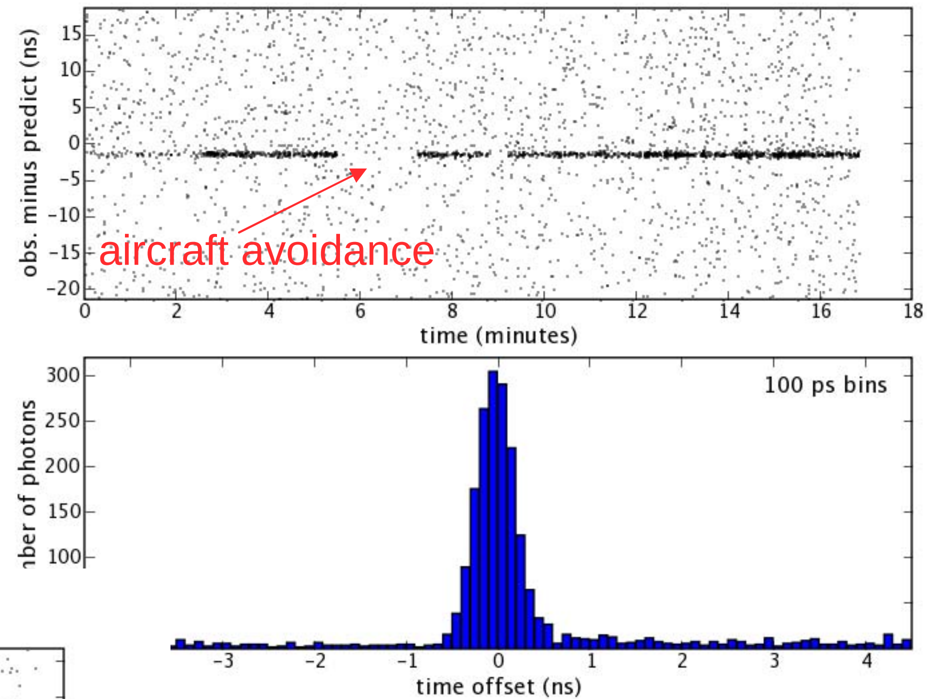
1670 in last half

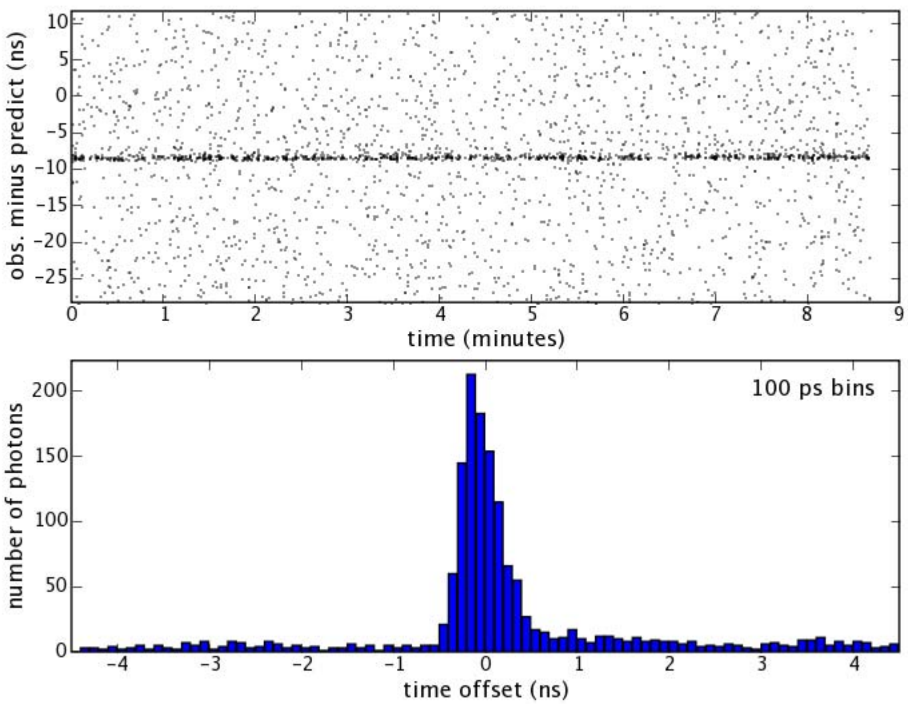
last half average: 0.33 photons per pulse

c.f. OCA: 0.01
MLRS: 0.002

More Examples

9 Jan., 2007: Apollo 15
1700 photons

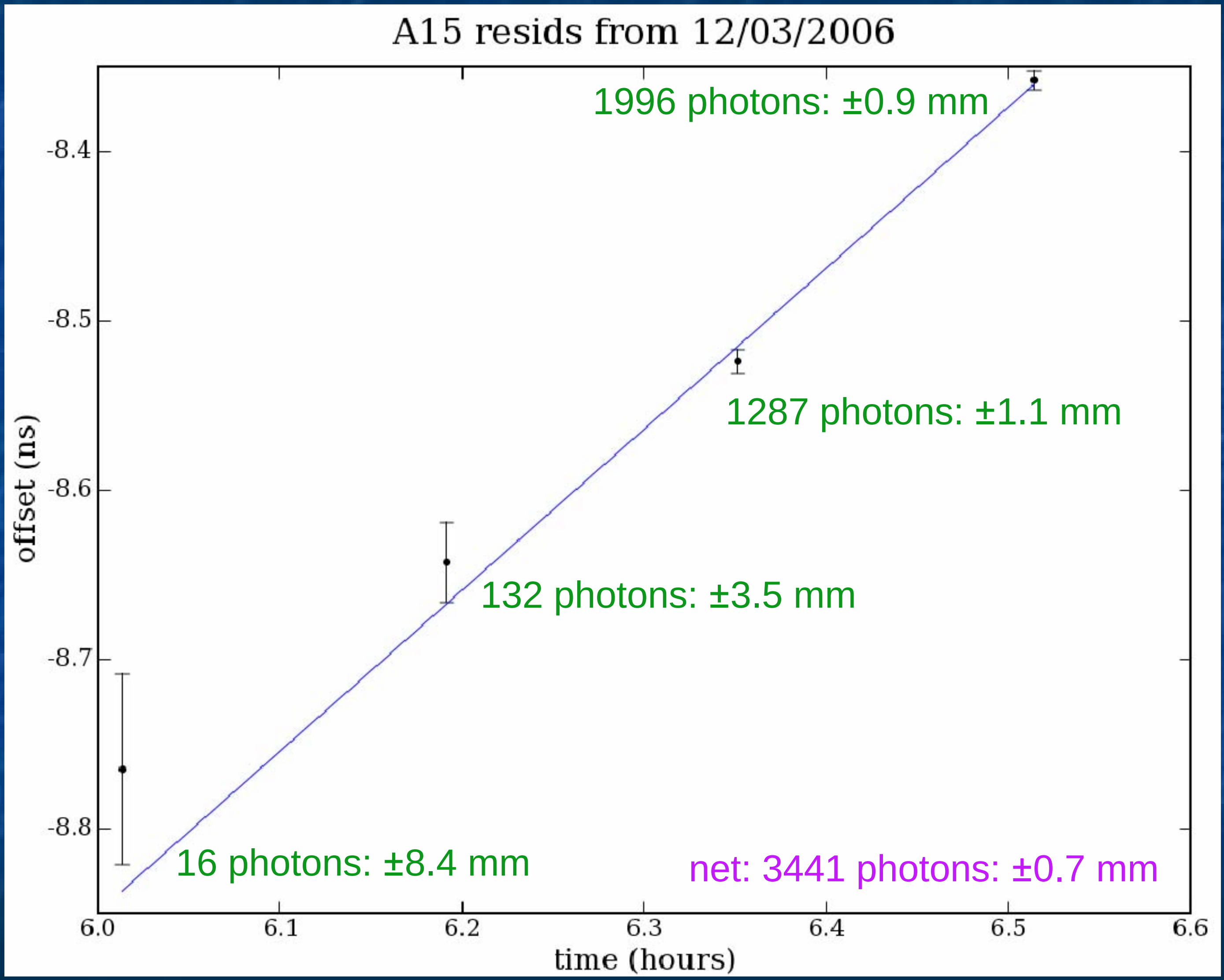




03 Dec., 2006: Apollo 11
1100 photons

Apollo 11 is 3× smaller than Apollo 15

Residuals Run-to-Run



We learn:

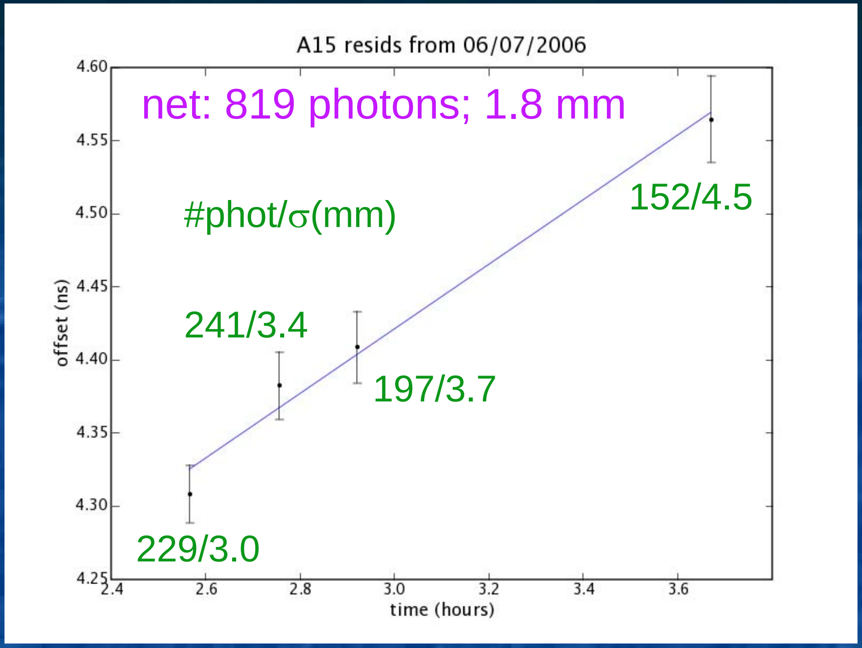
We can optimize the signal given time

We can get 1 mm range precision in single 10-minute “runs”

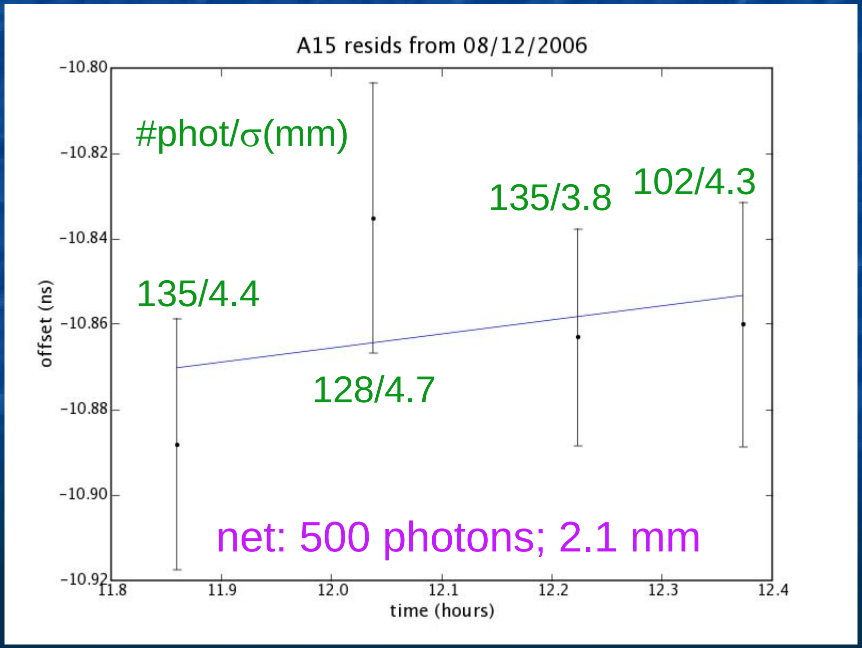
The scatter about a linear function is small: *almost* within estimated error

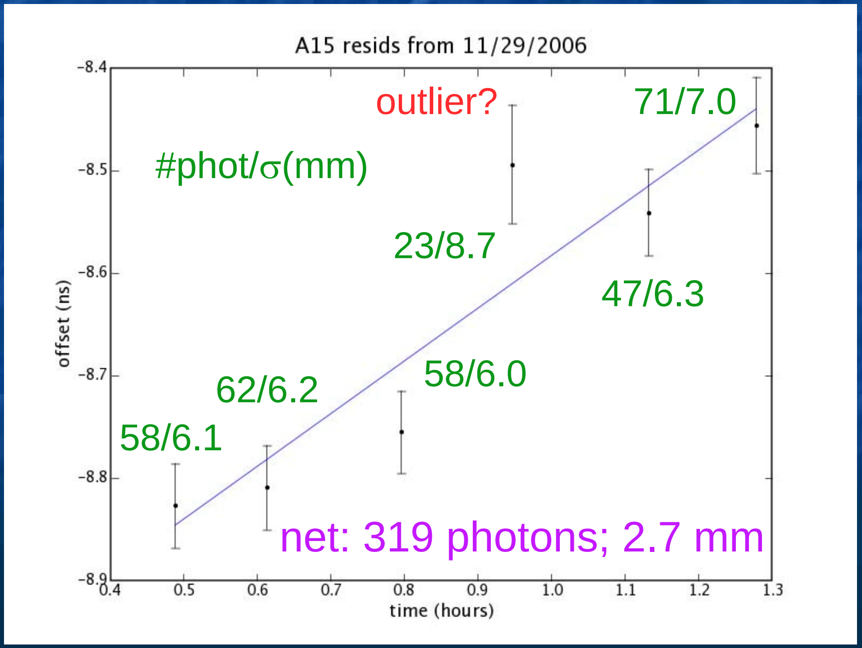
We’re producing mm-quality data

Run Residuals

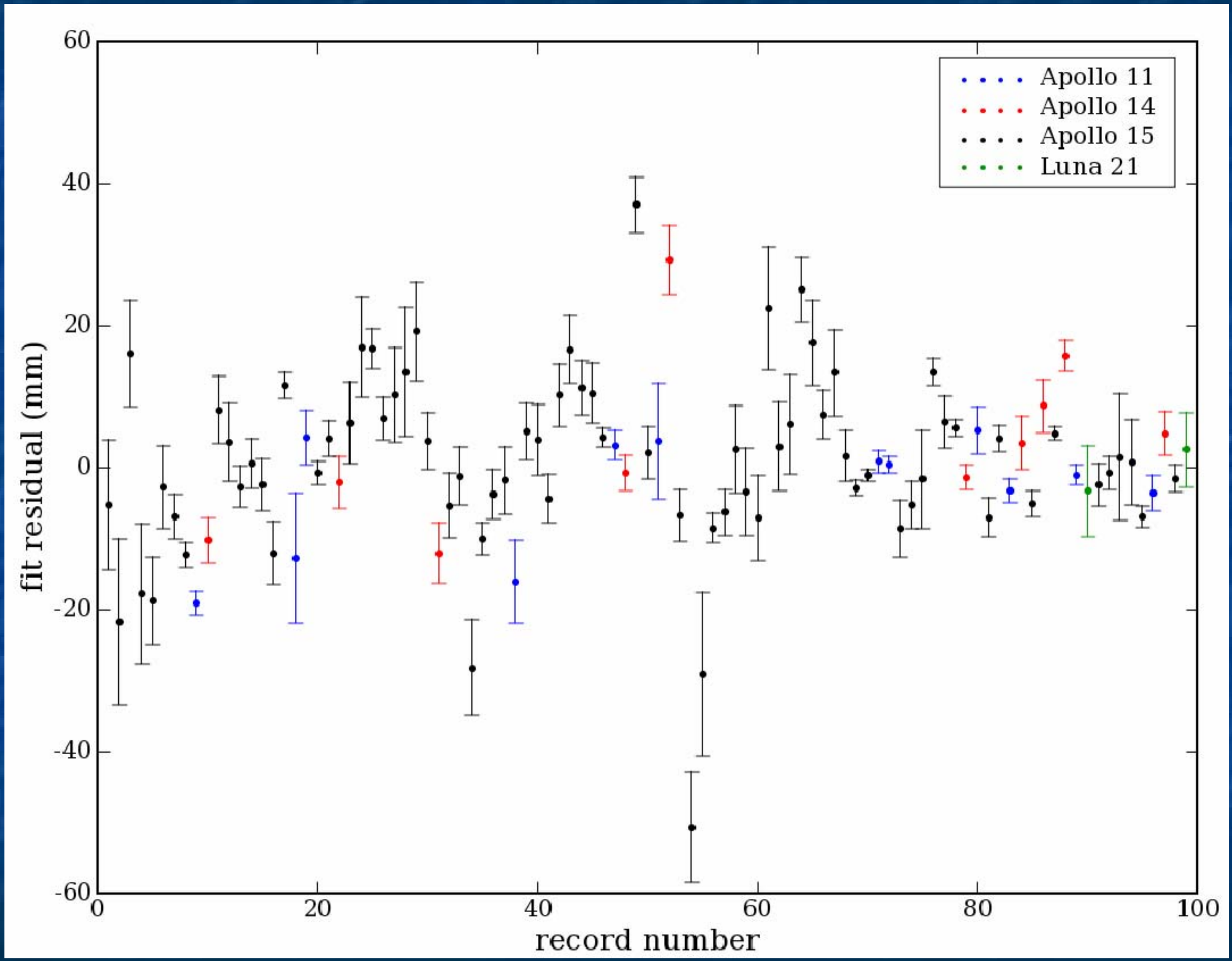


- In poor conditions, we tend to dwell on Apollo 15 (3× bigger)
- This gives us a useful internal check on scatter: is scatter consistent with estimated errors?
- Generally, it is, with occasional outliers thought to result from crude initial form of data reduc.





Residuals Against JPL Model



APOLLO data points processed together with 16,000 ranges over 35 years shows consistency with model orbit

Some outliers likely APOLLO's fault (early form of data reduc.)

Some trends likely model deficiencies

Weighted RMS is about 6–8 mm

$\chi \approx 3$ for this fit

APOLLO Superlatives

- More lunar return photons in 10 minutes than the McDonald station gets in three years
  - best single run: >2500 photons in 10,000 shots (8 minutes)
- Peak rates of >0.6 photons per shot (12 per second)
- Sustained rates typically 0.1–0.3 photons per shot
  - compare to typical 1/500 for McDonald, 1/100 for France
- Range with ease at full moon
  - APOLLO's *very first* returns were at full moon
  - other stations can't fight the high background
- As many as 9 photons detected in a single pulse!
  - In best runs, *half* of detected photons in multi-photon clumps
- Expect substantial gains in EP, gravitomagnetism within the year
  - longer for other physics

Future Directions for LLR

- Current Problems:
  - Only one one-millimeter capable station in the world
  - Array tilts dominate error budget (designed in the day of few-ns lasers)
  - Array response appears to suffer a factor of > 10 attenuation
- Future Option 1:
  - deploy new arrays that allow one to resolve individual corner cubes
  - thus take full advantage of ground capability (spurring improvements)
  - fresh arrays allow greater participation
- Future Option 2:
  - deploy transponders on lunar surface
  - $1/r^4$ passive loss becomes $1/r^2$: everyone can play
  - essential practice for more ambitious solar system ranging experiments

Future Directions Beyond LLR

- LLR tests gravity on our doorstep
- There's also a back yard: the solar system
- Interplanetary laser ranging offers another order-of-magnitude over APOLLO's capabilities:
  - Measure γ via Shapiro delay
  - Measure strong equivalence principle as Sun falls toward Jupiter
- Techniques:
  - Multi-task laser altimeters as asynchronous transponders
    - incredible demonstration to MESSENGER: 24 million km 2-way link
  - Piggyback on optical communications/navigation
- Other methods for probing local spacetime:
  - Weak equivalence principle tests (e.g., STEP)
  - Solar-induced curvature via interferometric angular measurements
  - Clocks in space to test Lorentz invariance/SME

Interplanetary Laser Link Milestone

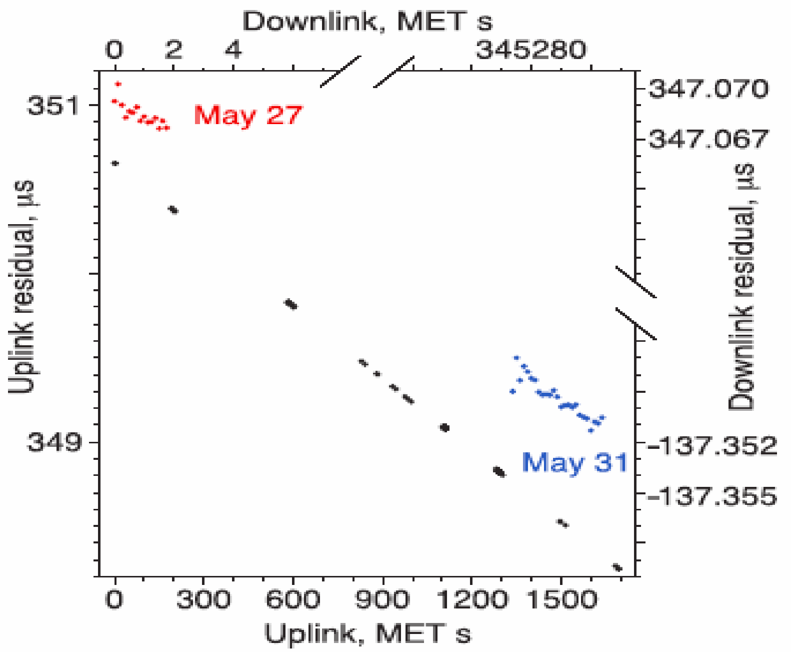
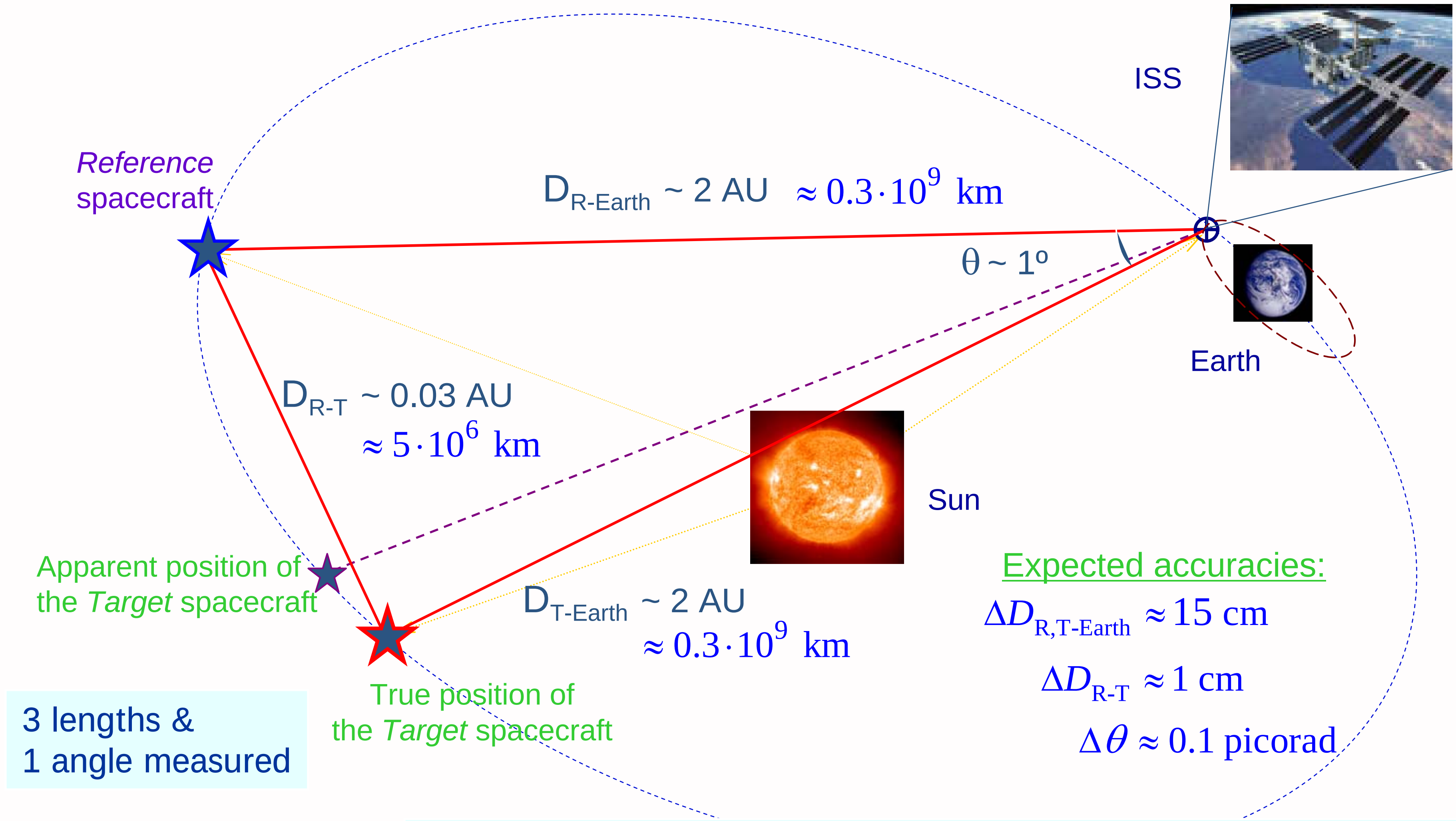


Fig. 1. Pulse-received times at MLA and GGAO. The graph shows that ground laser pulses (black symbols) were received by MLA ~0.35 ms earlier than predicted. Similarly, the ground-receive time of MLA pulses was ~0.34 ms earlier on 27 May (red symbols) but ~0.14 ms later on 31 May (blue symbols).

Smith et al., 2006: *Science*, **311**, 53

- Farthest laser link established May 2005: 24 million km
- MESSENGER spacecraft en-route to Mercury
- Used laser altimeter as asynchronous transponder
- 20 cm range precision achieved
  - 10^{-11} precision
- path-breaking result important for interplanetary ranging, laser communications, navigation

LATOR: the Laser Test of Relativity



Geometric redundancy allows for accurate measurement of relativistic gravitational light deflection to 1 part in 10^8.