Gravitational Lensing

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1. short history

2. Theory, useful concept and definitions

3. Clusters of galaxies

4. Halos of galaxies

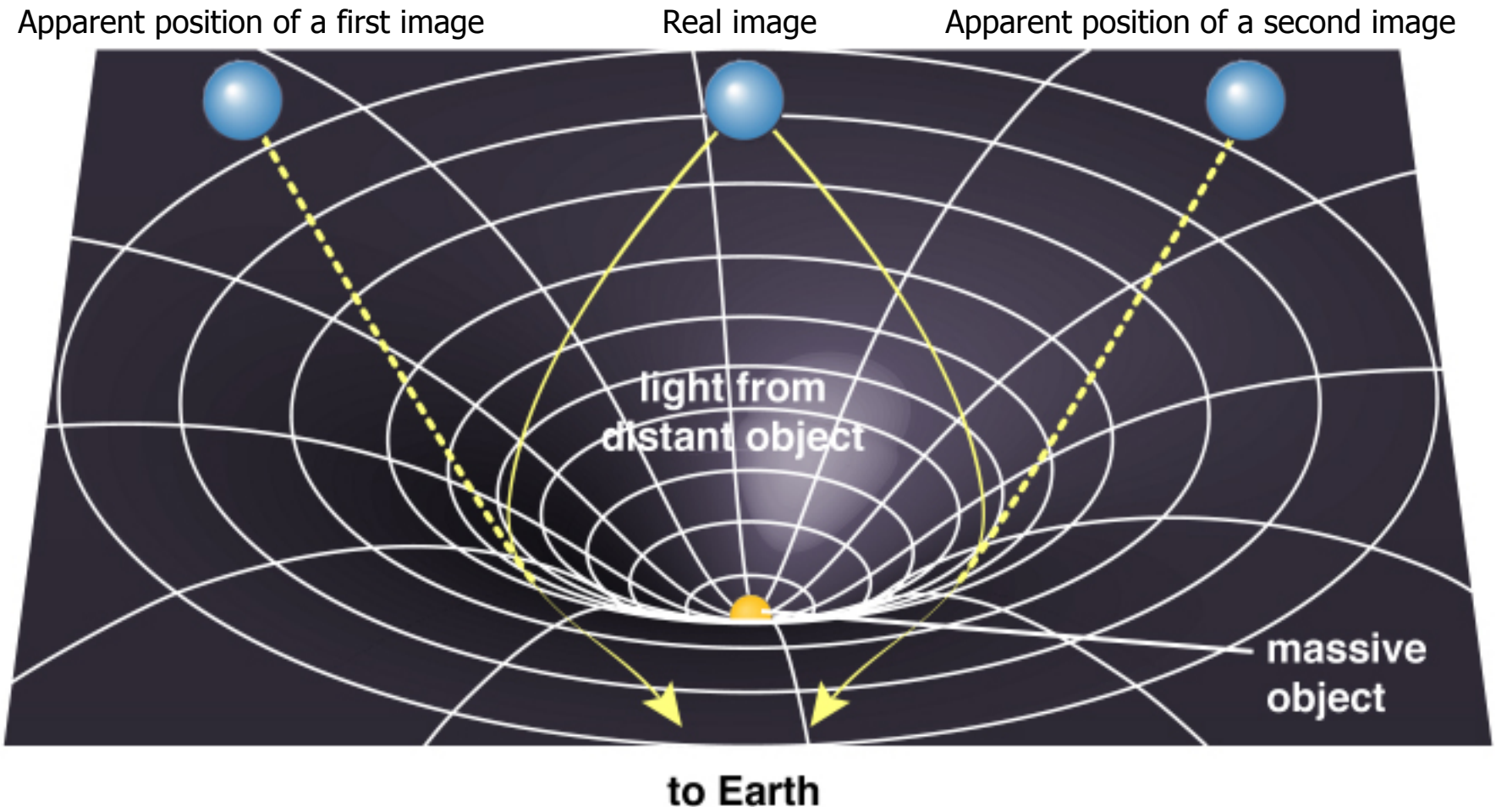
5. Cosmic shear

I. Gravitational Lensing:

theory, concepts and definitions

General relativity:

curvature of space time locally modified by mass condensation



Deflection of light, magnification, image multiplication distortion of objects : directly **depend on the amount of matter**

Gravitational lensing effect is **achromatic** (photons follow geodesics regardless their energy)

Lensing : a short history

1804: Soldner works on the deflection of light by gravity

1915: General Relativity

1919: Deflection angle of stars behind the Sun

1936: Einstein's work on lensing by stellar objects

1937: F. Zwicky visions on « extragalactic nebulae » as lenses

1964: Refsdal: time delay and H_0

1979: Multiple image of quasars (0957+561)

1987: Giant arcs in clusters of galaxies (A370, A2218, CL244)

1987: First Einstein ring

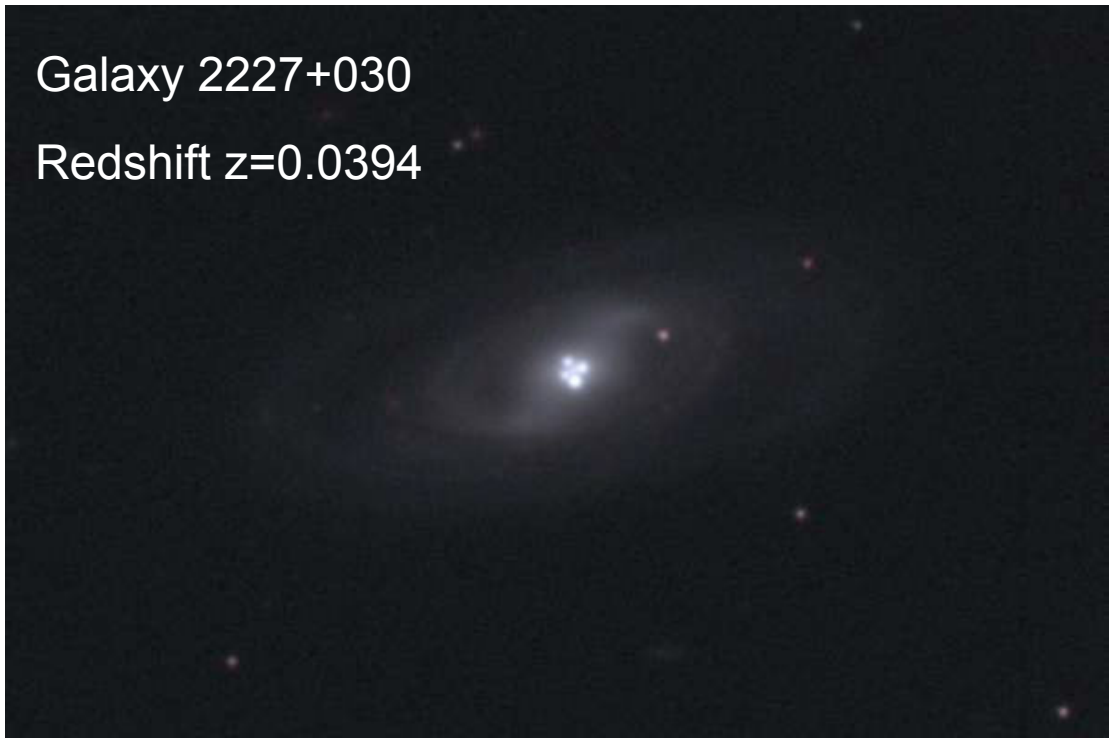
1993: EROS and MACHO microlensing experiments

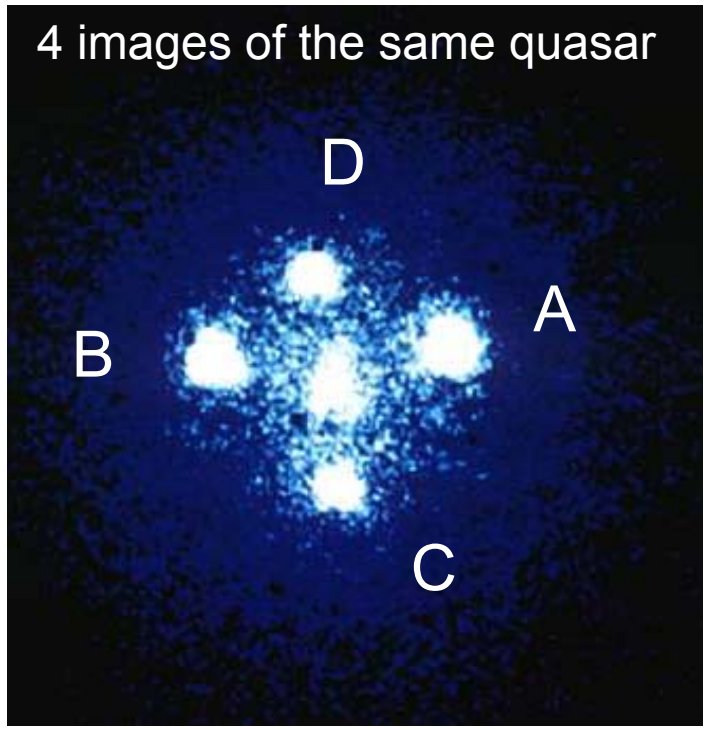
2000: Detection of cosmic shear signal

2006: The « Bullet » cluster and existence of dark matter

2007: First detection of lensing on CMB anisotropies

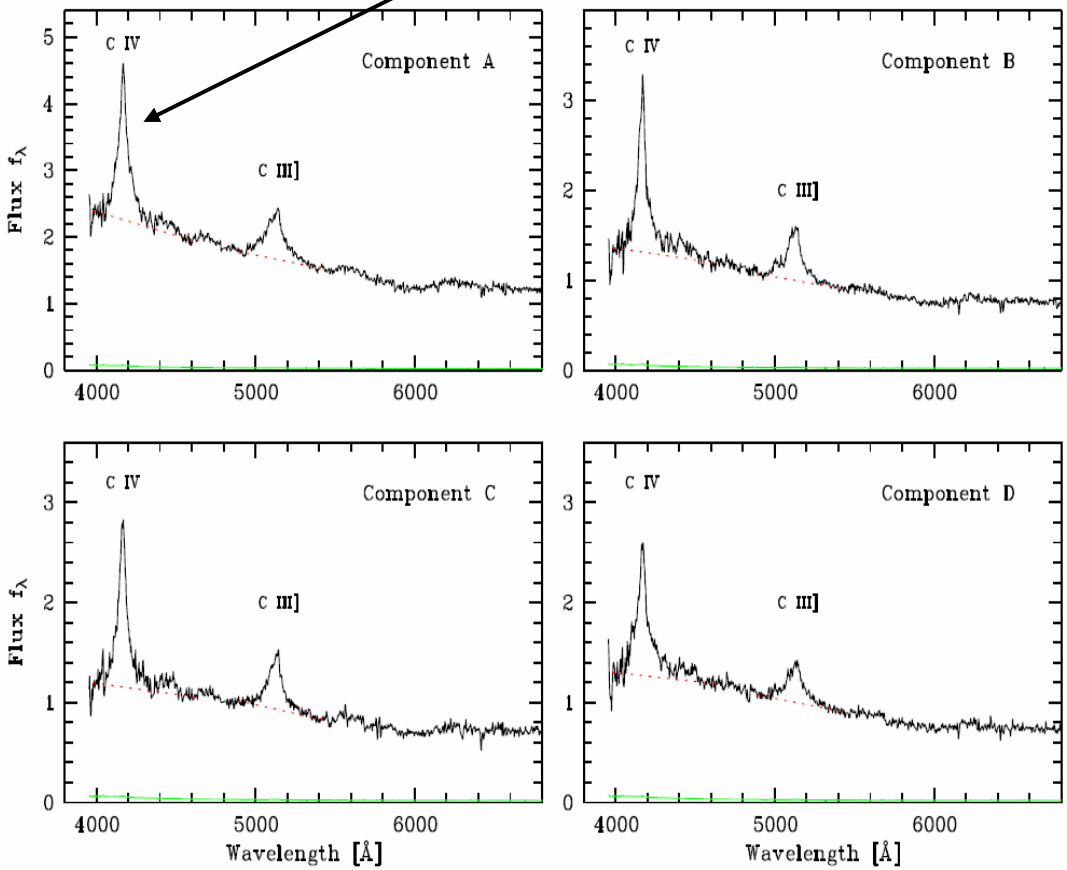
Multiple images:
The « Einstein Cross»

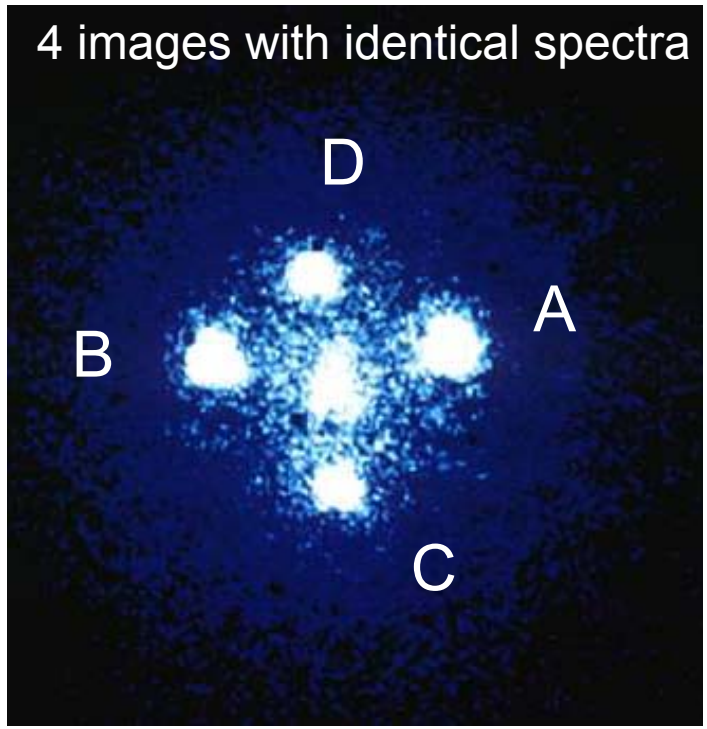


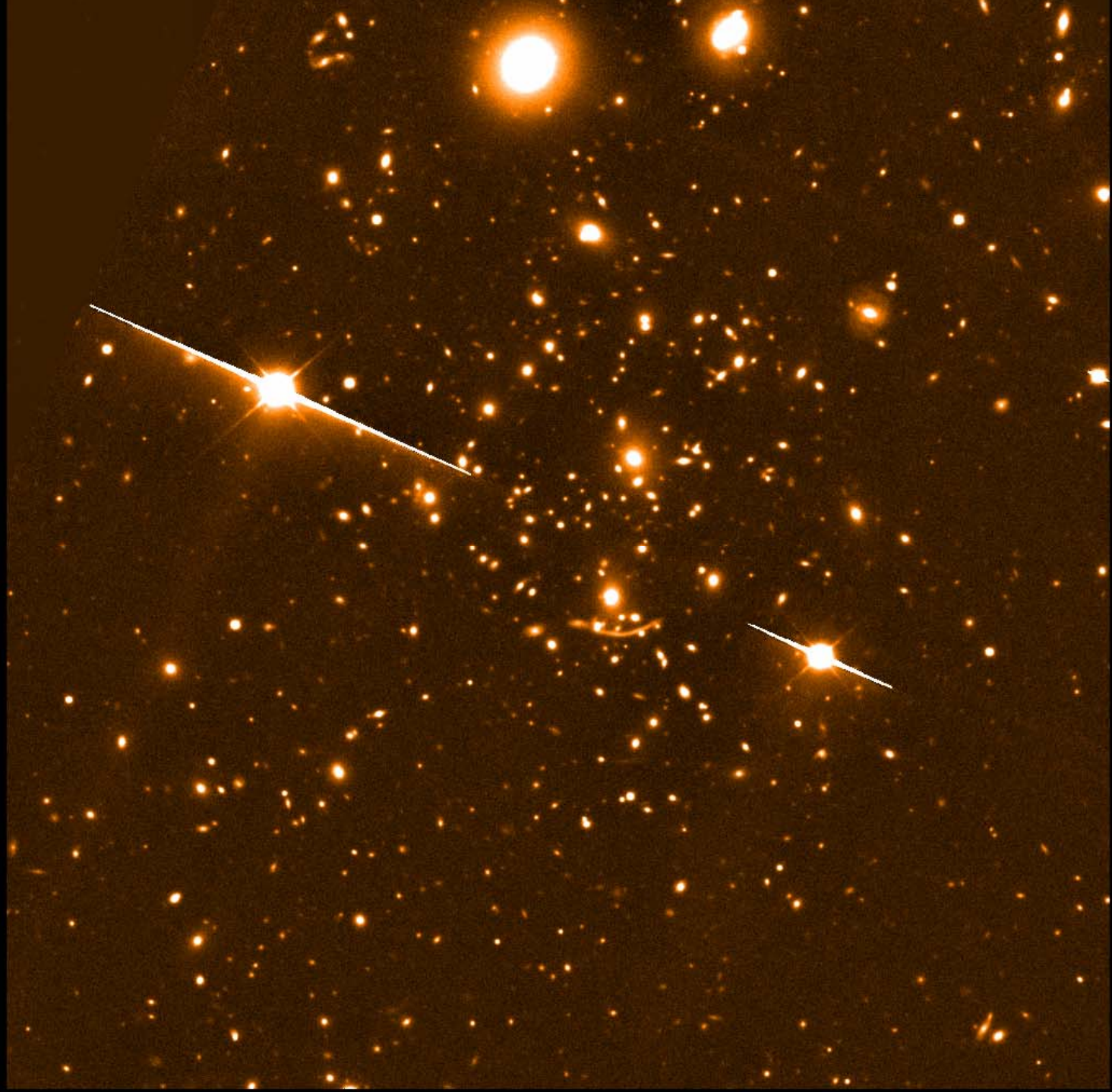


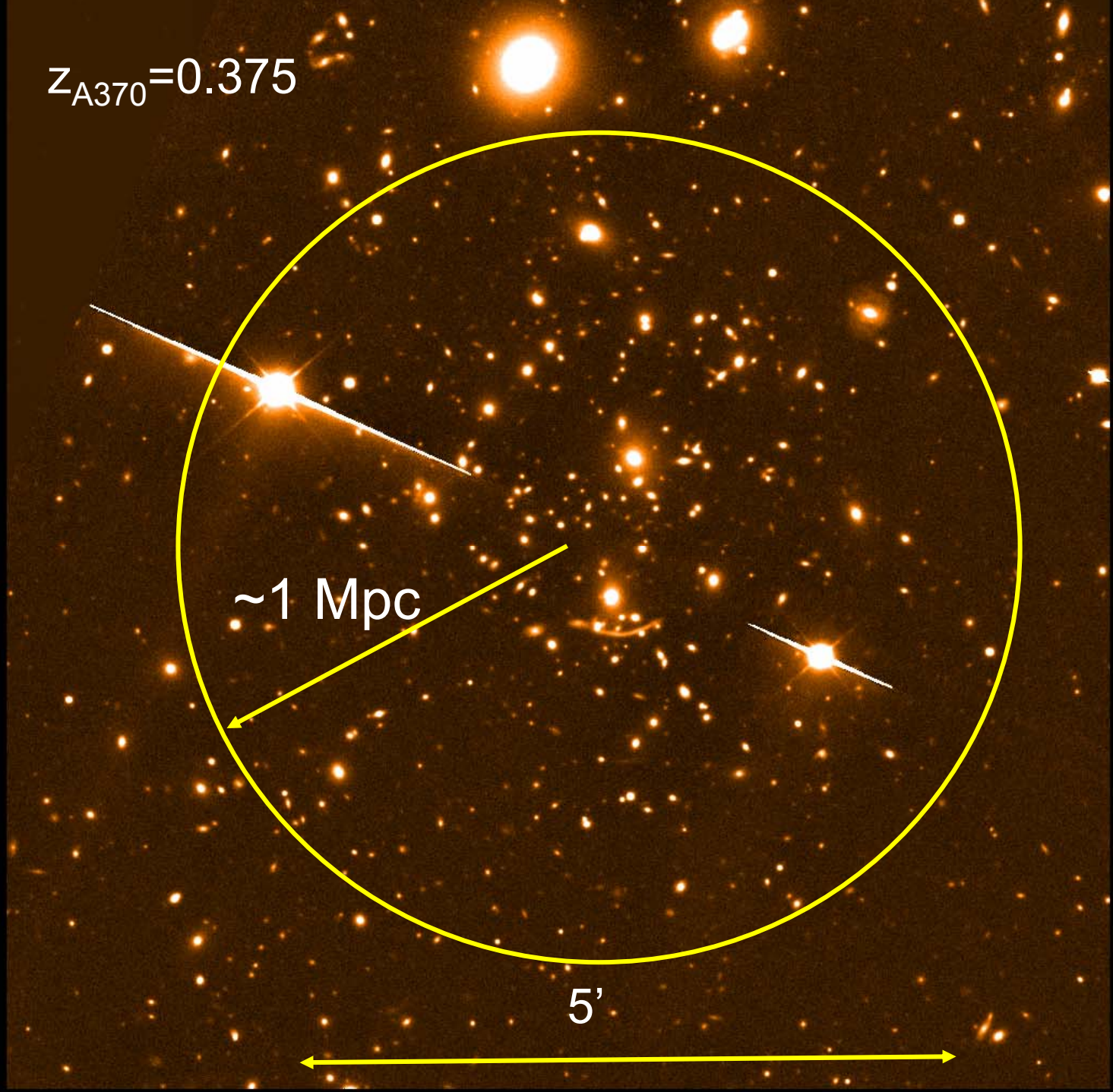
The « Einstein Cross»

CIV emission line at 154.9 nm observed 417.6 nm :z = 1.695








z_{A370}=0.375
~1 Mpc
5’

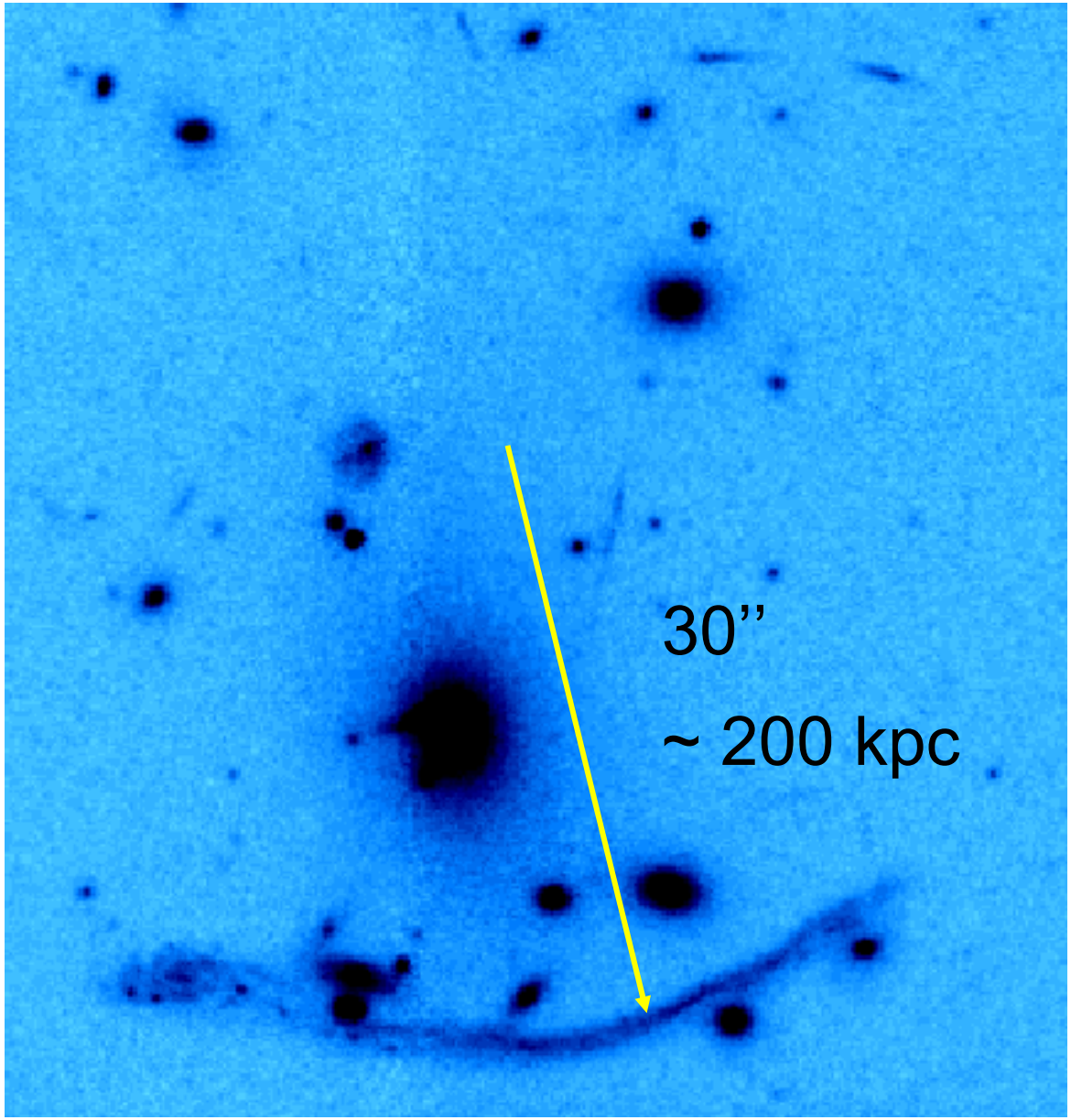
First giant arc discovered in Abell 370

Soucail et al

1987, 1988

$Z_{cluster}$ =0.375

Z_{arc}=0.720



The discovery of « arclets »

Fort et al 1988: arclets

Tyson et al 1990: first weak

Lensing mass reconstruction

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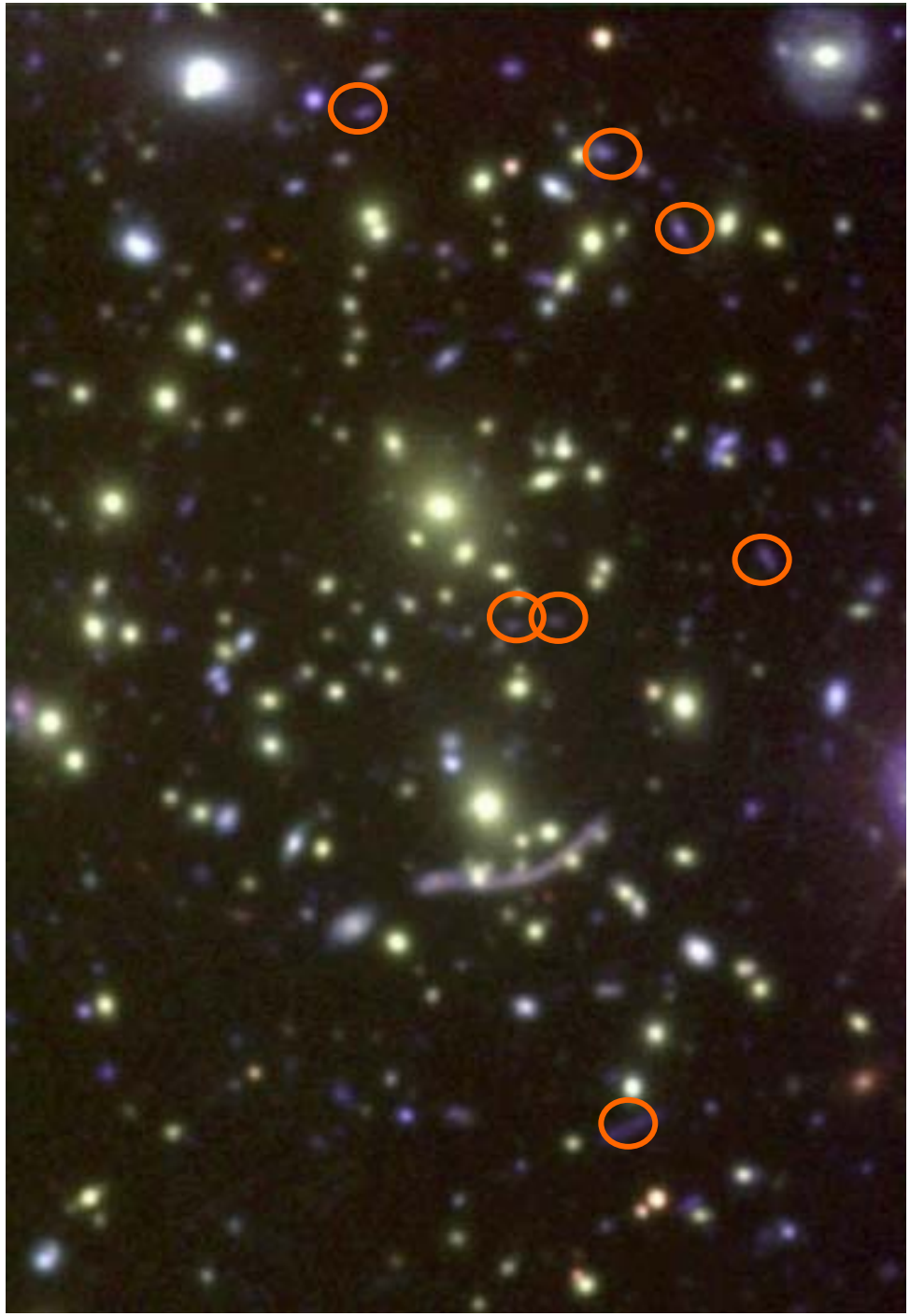
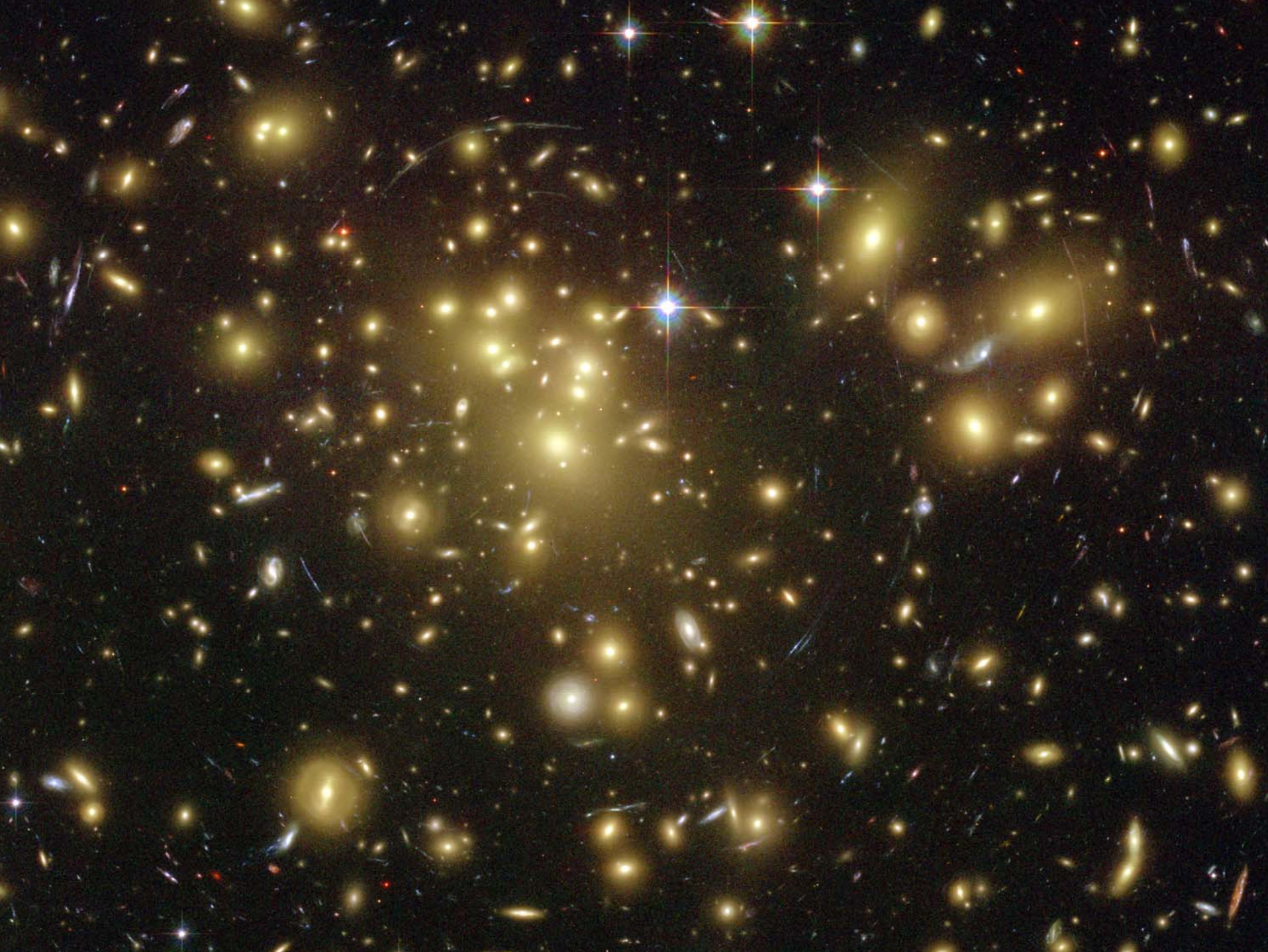




Gravitational Lensing

- Fundamental assumptions over the lecture
  - Weak field limit v^2/c^2
  - Stationnary field $t_{dyn} / t_{cross\ of\ photons}$
  - Thin lens approximation L_{lens}/L_{bench}
  - Transparent lens
  - Small deflection angle

Lens configuration and deflection angle

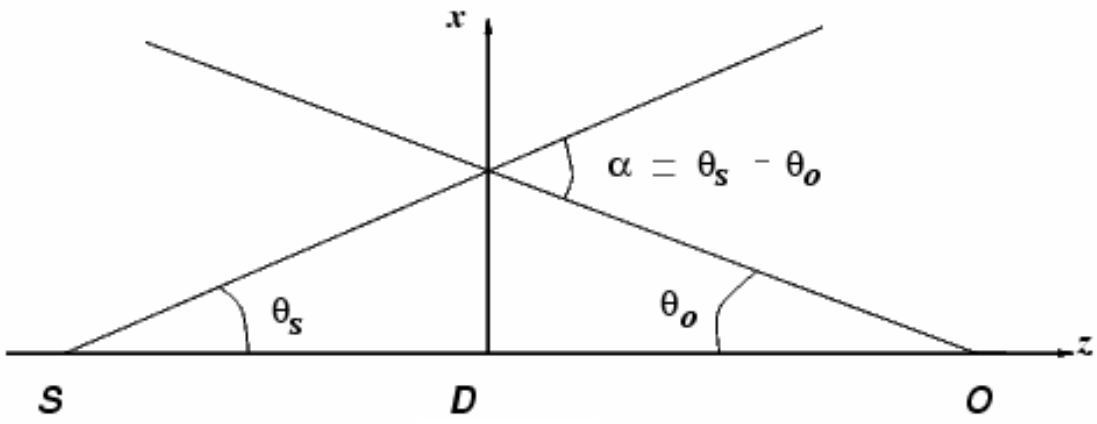


Figure 1: Configuration of the gravitational optical bench.

Deflection angle

In the weak field approximation, the metric writes

$$\mathrm{d}s^2 = c^2\left(1+\frac{2\Phi}{c^2}\right)\mathrm{d}t^2 - \left(1-\frac{2\Phi}{c^2}\right)\mathrm{d}l^2 \ .$$

For a photon $\mathrm{d}s^2 = 0$:

$$\mathrm{d}t = \frac{1}{c}\left(\frac{1-\frac{2\Phi}{c^2}}{1+\frac{2\Phi}{c^2}}\right)^{1/2}\mathrm{d}l \approx \frac{1}{c}\left(1-\frac{2\Phi}{c^2}\right)\mathrm{d}l \ .$$

Configuration: see Fig.1 .

$$\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 .$$

Deflection angle

Fermat principle: photons only follow optical paths with extrema propagation time

→The paths are those stationnary with respect to a small variation δt.

Since:
$$ct = \int \left(1 - \frac{2\Phi}{c^2}\right) \mathrm{d}l \qquad (3)$$

$$ct = \int \left(1 - \frac{2\Phi}{c^2}\right) \left[\left(\frac{\mathrm{d}x}{\mathrm{d}z}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}z}\right)^2 + 1\right]^{1/2} \mathrm{d}z\,.$$

Equation (3) corresponds to a light beam propagating in a transparent medium with refraction index :

$$n = \left(1 - \frac{2\Phi}{c^2}\right)\,.$$

Deflection angle

That is, taking into account the relation between n and Φ:

$$\alpha = -\frac{2}{c^2}\int_S^O \nabla_\perp \Phi \, \mathrm{d}l \, . \qquad (16)$$

- This is the general expression of a deflection angle for thin lenses in the weak field limit.

- Valid fo stars, galaxies, groups and clusters of galaxies and large-scale structures of the universe.

Relation with the Projected Mass Density

The gravitational force produced by the mass inside the lens is

$$\vec{F} = G \int \frac{\vec{x'} - \vec{x}}{\left|\vec{x'} - \vec{x}\right|^3} \rho\left(\vec{x'}\right) \mathrm{d}\vec{x'} \, .$$

Let us define the gravitational potential as $\Phi\left(\vec{x}\right)$

$$\Phi\left(\vec{x}\right) = -G \int \frac{\rho\left(\vec{x'}\right)}{\left|\vec{x'} - \vec{x}\right|} \mathrm{d}\vec{x'} \, ,$$

so that

$$\vec{F} = -\vec{\nabla}\Phi \, ,$$

Relation with the Projected Mass Density

By subtituting in Eq. (16) → express the deflection angle as function of the mass density field:

$$\vec{\alpha} = -\frac{2G}{c^2}\int_S^O \mathrm{d}l \int \mathrm{d}\vec{x'}\, \nabla_{\perp_{\vec{x}}} \frac{\rho\left(\vec{x'}\right)}{\left|\vec{x'}-\vec{x}\right|}$$

$$\vec{\alpha} = -\frac{2G}{c^2}\int \rho\left(\vec{x'}\right)\mathrm{d}\vec{x'} \int_S^O \frac{\left(\vec{x'}-\vec{x}\right)_{\perp}}{\left|\vec{x'}-\vec{x}\right|^3}\,\mathrm{d}l$$

Relation with the Projected Mass Density

In the *thin lens approximation*, we have

$$\int \rho\left(\vec{x}'\right) \mathrm{d}\vec{x}' = \int \rho\left(\vec{\xi}', z\right) \mathrm{d}\vec{\xi}' \, \mathrm{d}z = \Sigma\left(\vec{\xi}'\right)$$

The relation between the deflection angle and the projected mass density is therefore

$$\alpha(\xi) = \frac{4G}{c^2} \int \frac{(\xi - \xi')\,\Sigma(\xi')}{|\xi - \xi'|^2} \mathrm{d}\xi'$$

where

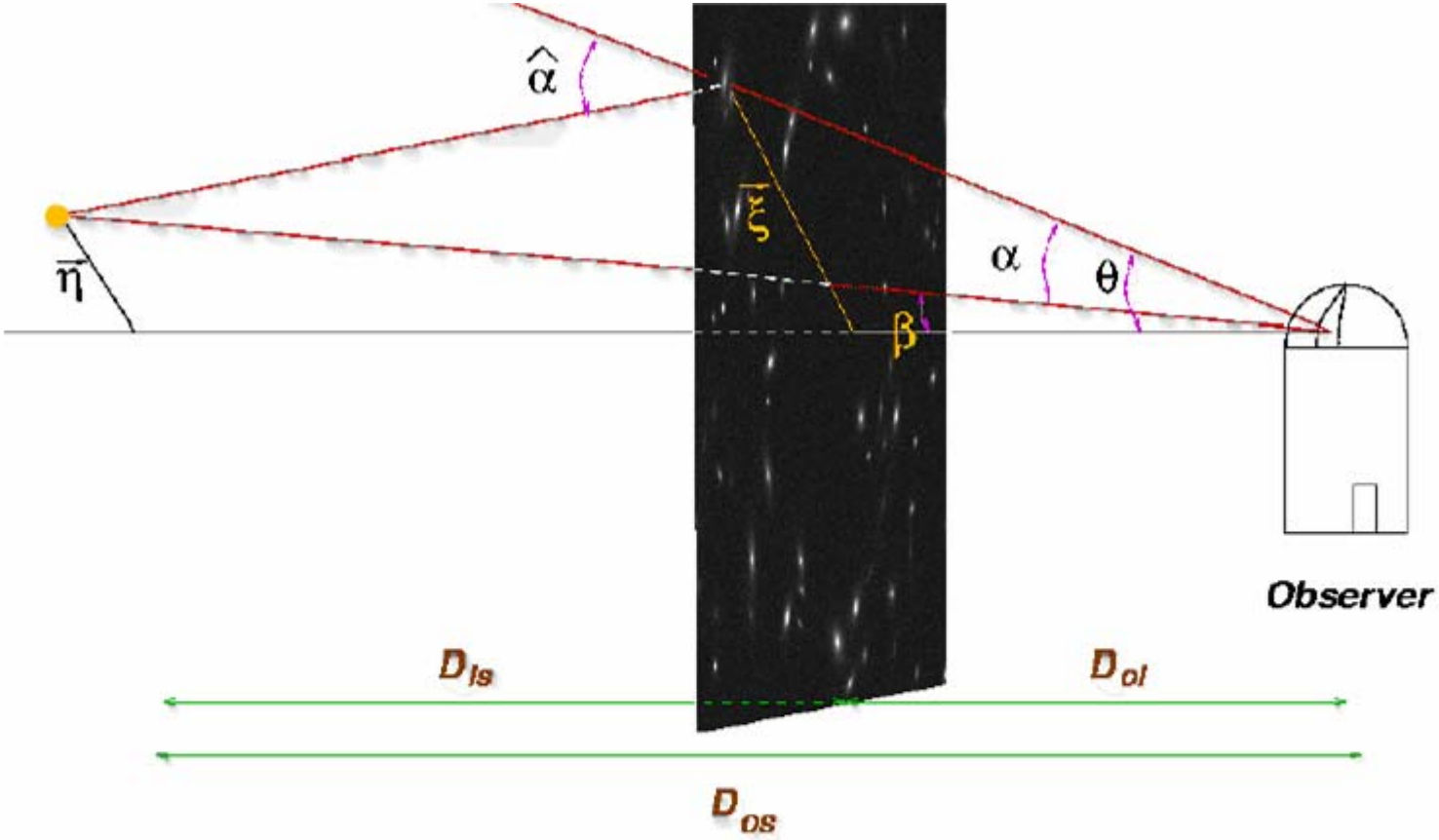
- $\Sigma(\xi)$ is the projected mass density,
- ξ is a 2-dimensional vector in the lens plane and
- the integration is done over the lens plane.

Relation with the Projected Mass Density

For a point mass of mass M: $\Sigma(\xi) = M\delta(\xi)$, then

$$\alpha(\xi) = \frac{4GM}{c^2}\frac{\xi}{|\xi'|^2}$$

Useful quantities and terminology



- Lens equation

$$\vec{\eta} = \frac{D_{os}}{D_{ol}}\vec{\xi} - D_{ls}\hat{\vec{\alpha}}\left(\vec{\xi}\right)$$

Useful quantities and terminology

- Lens equation

$$\vec{\eta} = \frac{D_{os}}{D_{ol}}\vec{\xi} - D_{ls}\hat{\vec{\alpha}}\left(\vec{\xi}\right)$$

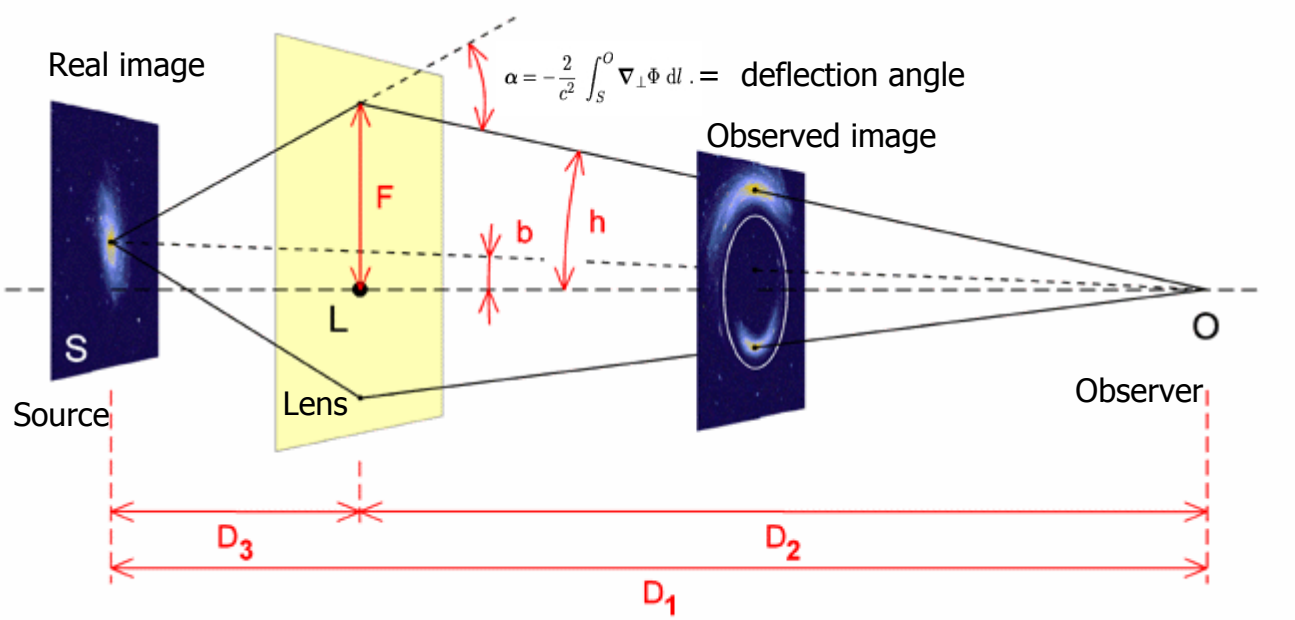
Setting $\vec{\eta} = D_{os}\vec{\beta}$ and $\vec{\xi} = D_{ol}\vec{\theta}$, one can write the lens eqation in a simple way, using the *scaled deflection angle*:

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right)$$

- Spherically symmetric lens

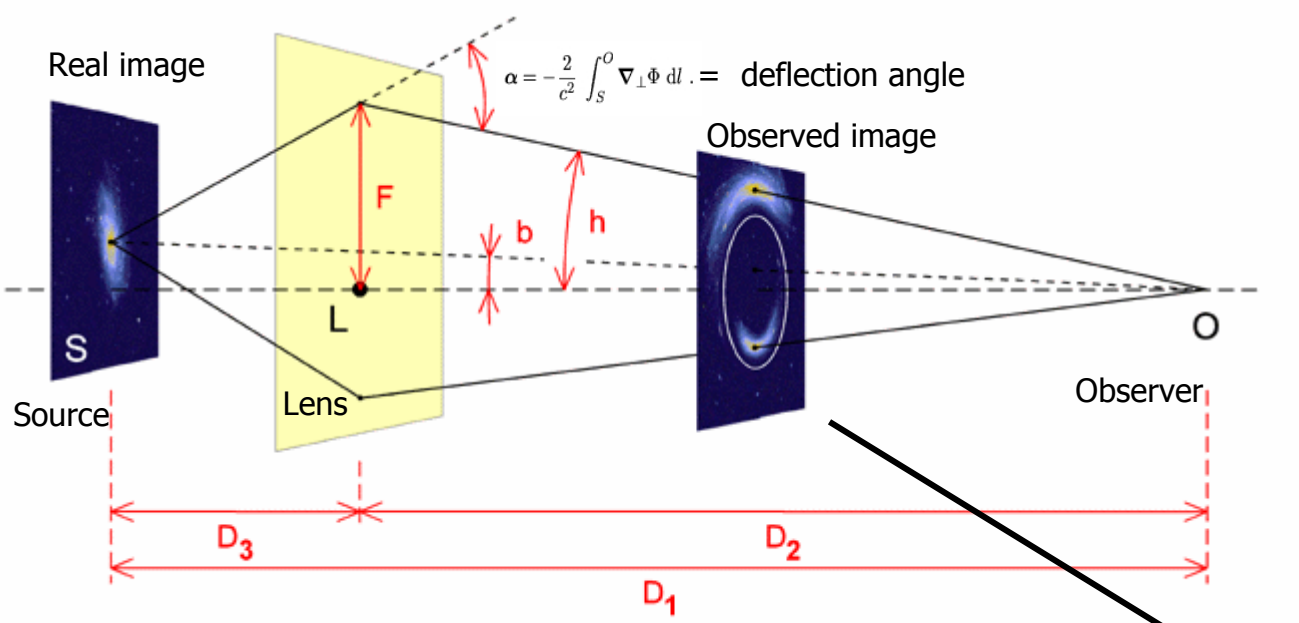
$$\beta = \theta - \frac{D_{ls}}{D_{os}D_{ol}}\frac{4GM(\theta)}{c^2\theta}$$

Perfect lens configuration



D_1, D_2, D_3 are cosmological distances: depend on the matter-energy content of the Universe

Perfect lens configuration



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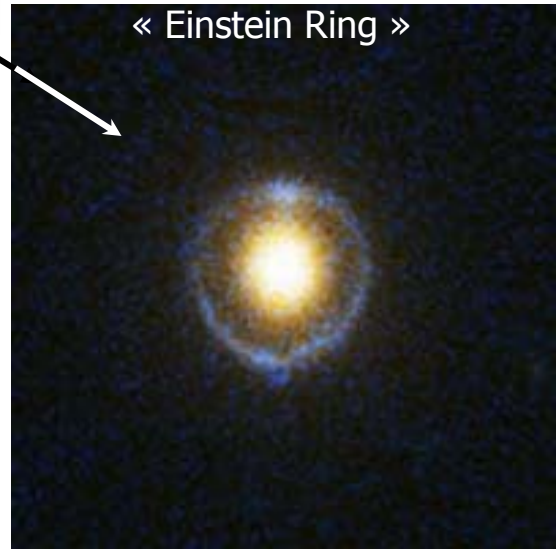


Source-Lens-Observer perfectly aligned

Einstein Rings: perfect alignement

$$\theta_E = \left(\frac{D_{ls}}{D_{os} D_{ol}} \frac{4GM(\theta_E)}{c^2} \right)^{1/2}$$

Typical values:

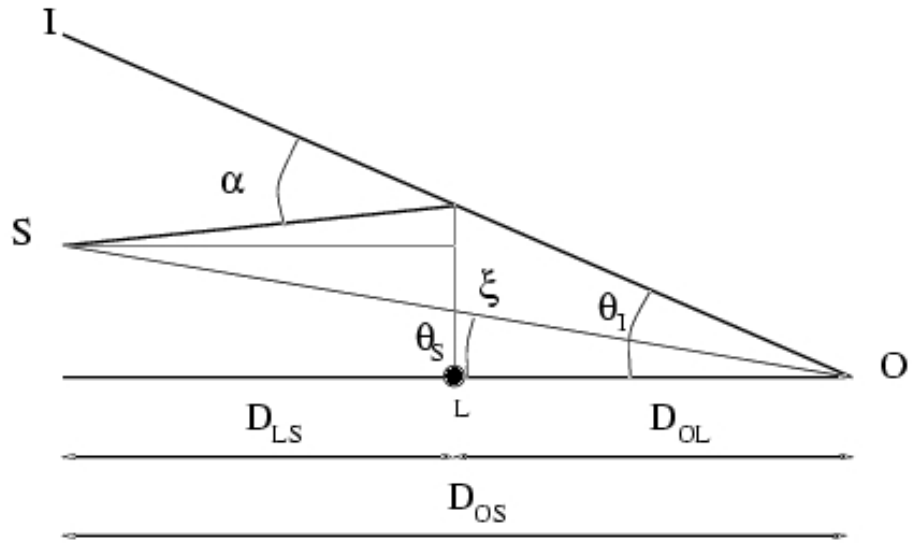
- For a lens of 1 solar mass located at 1 AU and a source a 1 kpc

  θ_E= 0.003 arc-second
- For a lens of 10^{11} solar masses located at 100kpc and a source at 300 kpc

  θ_E= 1 arc-second
- For a lens of 10^{15} solar masses located at 1Gpc and a source at 3 Gpc

  θ_E= 30 arc-second (sensitive to cosmological parameters)

Image formation, image multiplicity and image shape



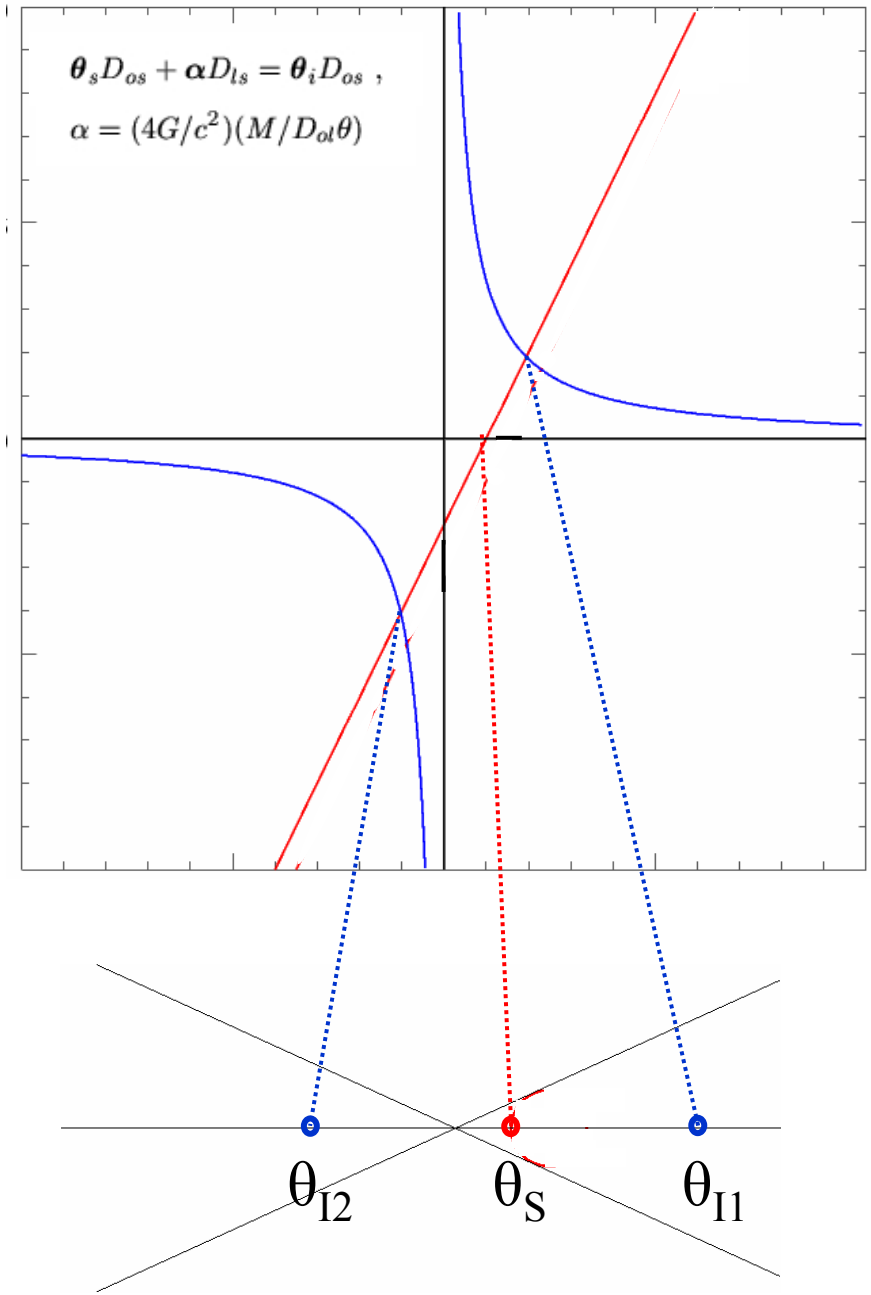
Example : a point mass:

Lens Equation $$\boldsymbol{\theta}_s D_{os} + \boldsymbol{\alpha} D_{ls} = \boldsymbol{\theta}_i D_{os} \ ,$$

Deflection angle $$\alpha = (4G/c^2)(M/D_{ol}\theta)$$

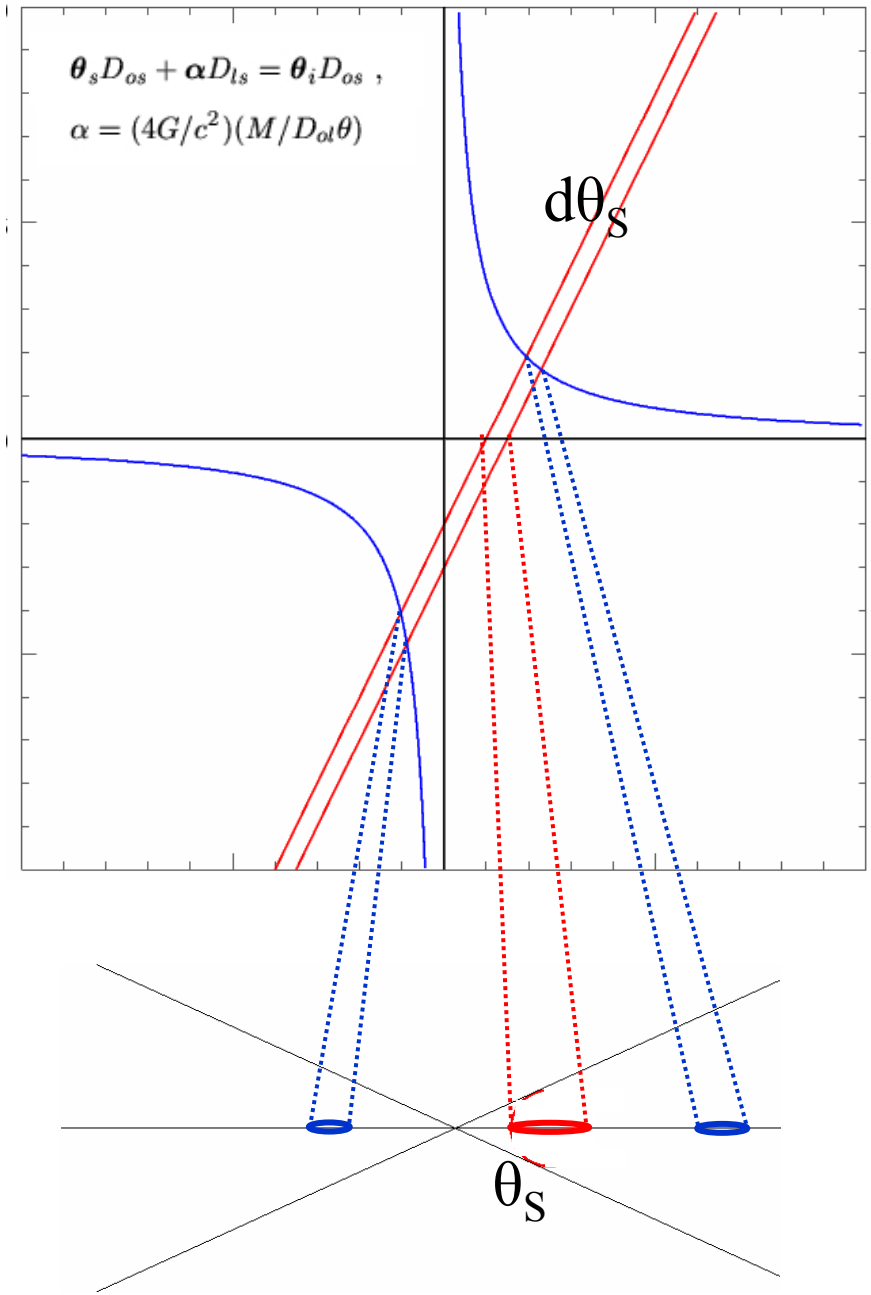
Lens: point mass

Source: point source



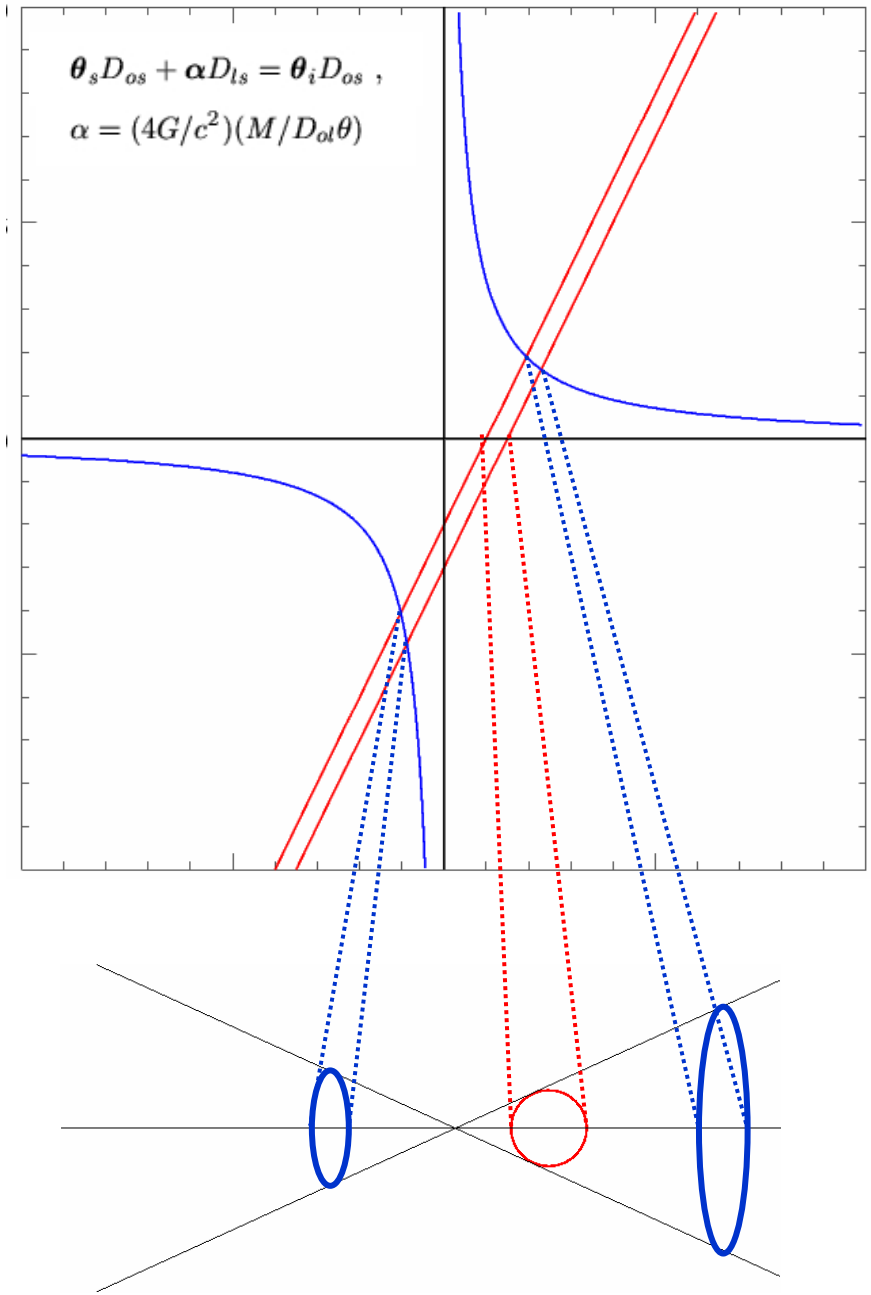
Lens : point mass

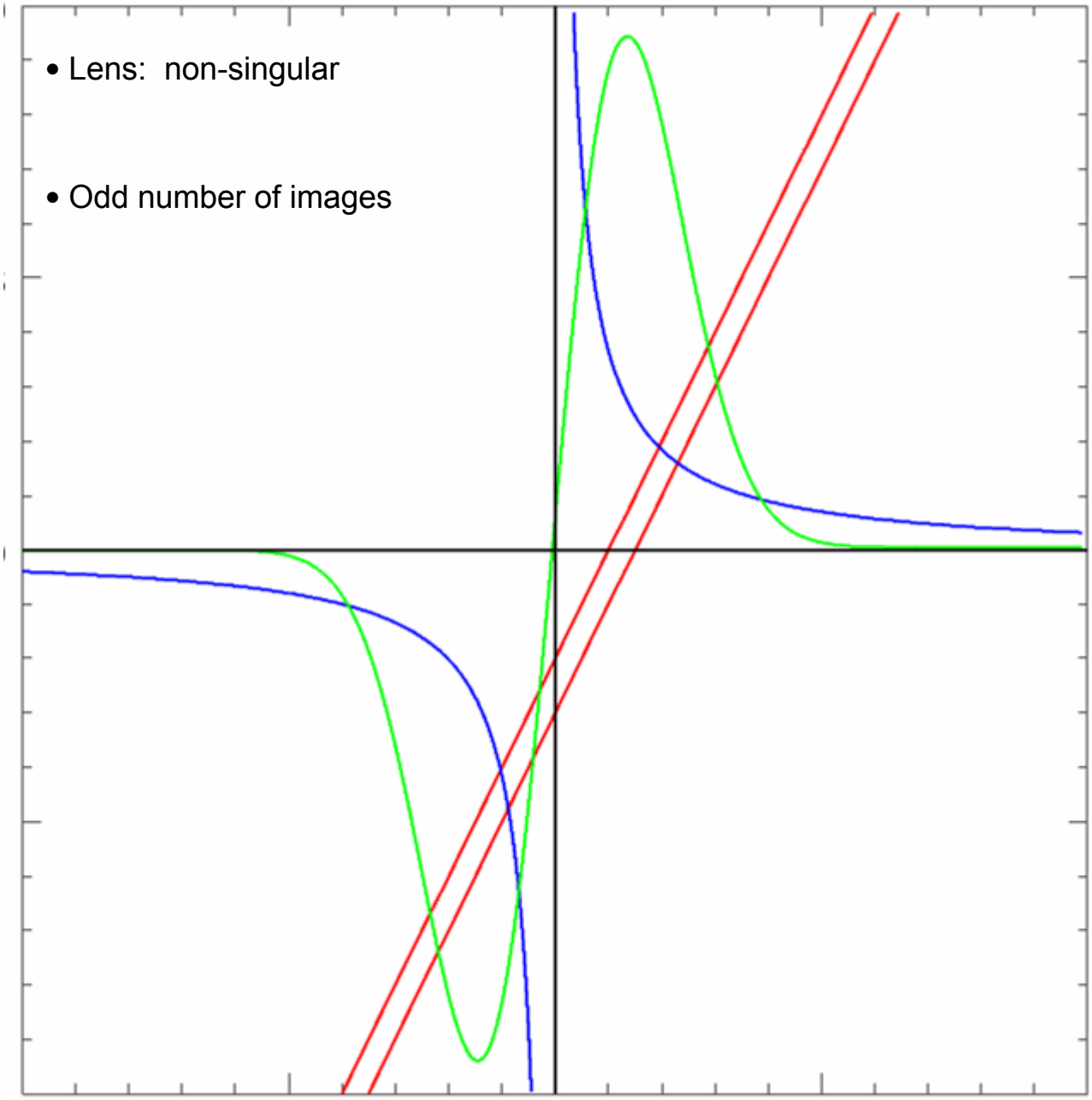
Source: line (1-D extended radially)



Lens: point mass

Source: ellipse (2-D extended)




• Lens: non-singular
• Odd number of images

Convergence and critical density:

The gravitational convergence is a dimensionless surface mass density:

$$\kappa\left(\vec{\theta}\right) = \frac{\Sigma\left(D_{ol}\vec{\theta}\right)}{\Sigma_{cr}}$$

where Σ_{crit} is the *critical surface mass density*:

$$\Sigma_{cr} = \frac{c^2}{4\pi G}\frac{D_{os}}{D_{ol}D_{ls}}$$

that defined a "strength" of the lens. Strong lensing cases have $\Sigma > \Sigma_{cr}$

Convergence and critical density

Example: lensing-cluster

- Typical value: cluster of galaxies of 10^{15} solar masses at 1Gpc and sources at 2 Gpc :

$$\Sigma_{crit} = 0.5 \text{ g/cm}^2$$

Comparison: typical mass density of a 1Mpc diameter cluster, based on its galaxy light distribution:

$$\Sigma_{\text{''light''}} = 0.02 \text{ g/cm}^2$$

Relations between deflection angle, effective potential and convergence

Since $\kappa = \Sigma / \Sigma_{crit}$, then

$$\vec{\alpha}\left(\vec{\theta}\right) = \frac{1}{\pi}\int \kappa\left(\vec{\theta}'\right)\frac{\vec{\theta}-\vec{\theta}'}{\left|\vec{\theta}-\vec{\theta}'\right|^2}\,\mathrm{d}^2\theta'$$

Using $\nabla \ln\left|\vec{\theta}\right| = \vec{\theta}/\left|\vec{\theta}\right|^2$, we derive

$$\boxed{\vec{\alpha} = \vec{\nabla}\psi}$$

Relations between deflection angle, effective potential and convergence

$$\vec{\alpha} = \vec{\nabla}\psi$$

$$\psi(\vec{\alpha}) = \frac{1}{\pi}\int \kappa(\vec{\alpha})\,\ln\left|\vec{\theta} - \vec{\theta}'\right|\,\mathrm{d}^2\theta'$$

Using $\nabla^2 \ln|\vec{\theta}| = 2\pi\delta_D\left(\vec{\theta}\right)$, where δ_D is the 2-dimension Dirac delta function:

$$\nabla^2\psi = 2\kappa$$

= 2-dimension Poisson equation

Magnification and distortion

- Jacobian of the lens mapping. Differentiting the lens equation

$$A\left(\vec{\theta}\right)=\frac{\partial\vec{\beta}}{\partial\,\theta}=\left(\delta_{ij}-\frac{\partial^2\psi\left(\vec{\theta}\right)}{\partial\,\theta_i\partial\,\theta_j}\right)=M^{-1}$$

- Convergence, Shear

$$\begin{cases}\kappa=\frac{1}{2}\left(\psi_{,11}+\psi_{,22}\right)\\ \gamma_1\left(\vec{\theta}\right)=\frac{1}{2}\left(\psi_{,11}-\psi_{,22}\right)=\gamma\left(\vec{\theta}\right)\cos\left[2\varphi\left(\vec{\theta}\right)\right]\\ \gamma_2\left(\vec{\theta}\right)=\psi_{,12}=\gamma\left(\vec{\theta}\right)\sin\left[2\varphi\left(\vec{\theta}\right)\right]\end{cases}$$

Magnification and distortion

- Magnification, Convergence, Shear

$$A = \mathscr{M}^{-1} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$\mathscr{M}^{-1} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos(2\varphi) & \sin(2\varphi) \\ \sin(2\varphi) & -\cos(2\varphi) \end{pmatrix}$$

where $\gamma = \gamma_1 + \mathrm{i}\gamma_2 = |\gamma| e^{2\mathrm{i}\varphi}$

- Amplification amplitude

$$\mu = (\det A)^{-1} = \frac{1}{\left[(1-\kappa)^2 - |\gamma|^2\right]}$$

- Eigenvalues of $\mathscr{M}^{-1}$:

$$1-\kappa+\gamma \ , \quad 1-\kappa+\gamma$$

Magnification and distortion

- Image and source From the magnification matrix,
  - κ expresses an isotropic magnification. It transforms a circle into a larger/smaller circle.
  - γ is an anisotropic magnification. It transforms a circle into an ellipse with minor and major axes :

$$b = (1 - \kappa + \gamma)^{-1} \quad , \quad a = (1 - \kappa - \gamma)^{-1}$$

- Reduced shear Let us write the magnification matrix as:

$$A = \mathscr{M}^{-1} = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

From (reduced) shear to ellipticity

where

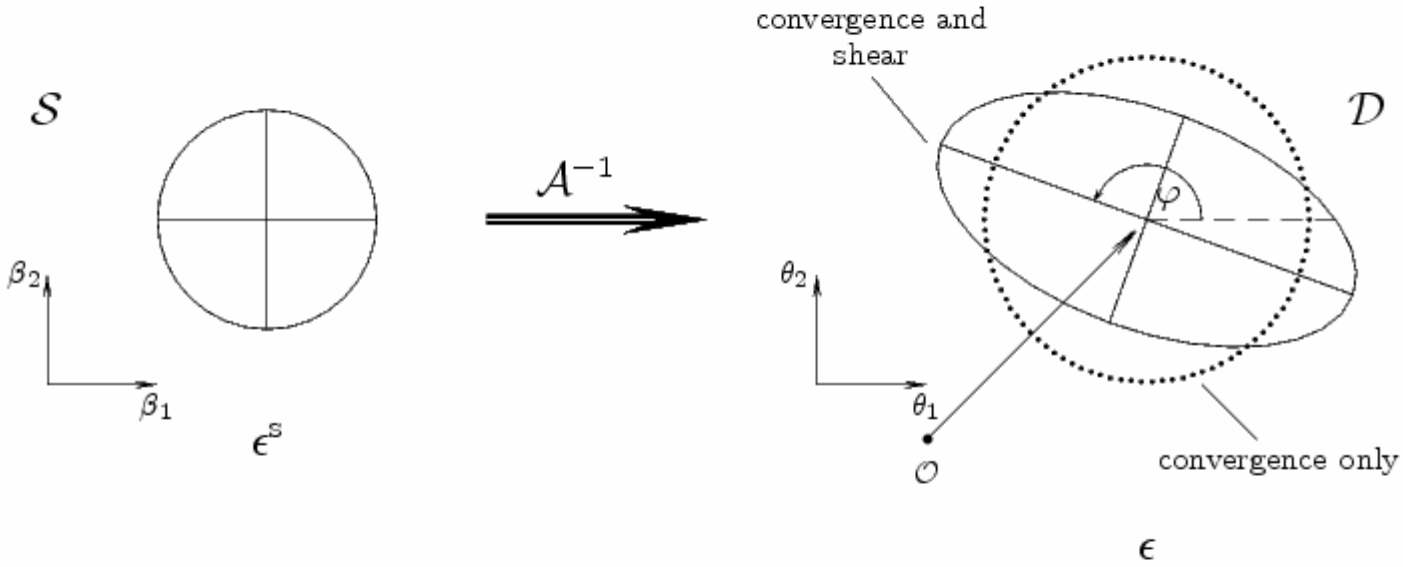
$$g\left(\vec{\theta}\right) = \frac{\gamma\left(\vec{\theta}\right)}{1-\kappa\left(\vec{\theta}\right)} = g_1 + \mathrm{i}g_2 = |g|e^{2\mathrm{i}\varphi}$$

is the *reduced shear*. It directly provides the image ellipicity induced by lensing on a circular source:

$$\frac{b}{a} = \frac{1-|g|}{1+|g|}$$

as well as the orientation of the major axis, φ.

From shear to ellipticity



Transformation of a circular source into an ellipse by the gravitational convergence and shear terms of the magnification matrix

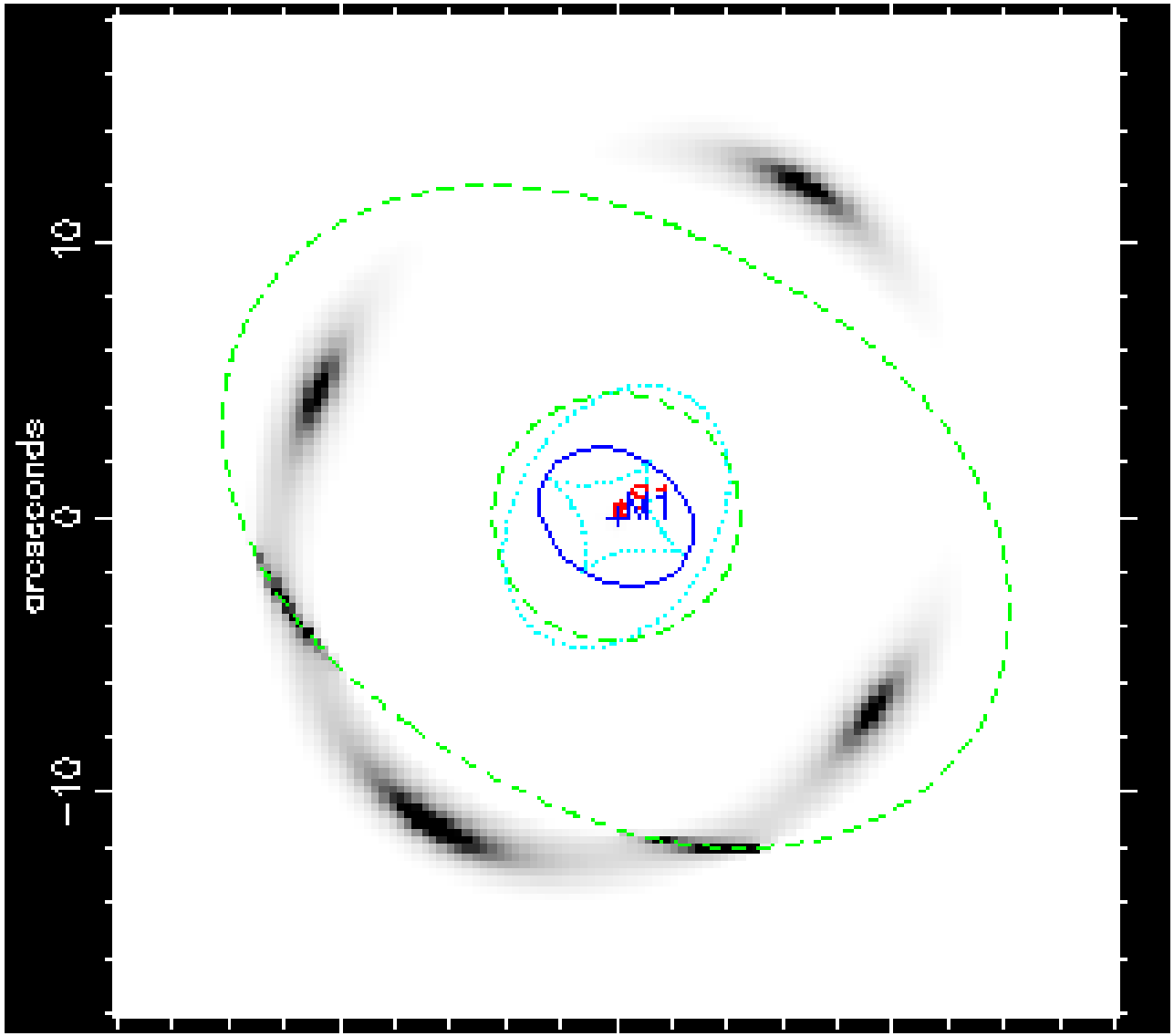
Caustic and Critical Lines

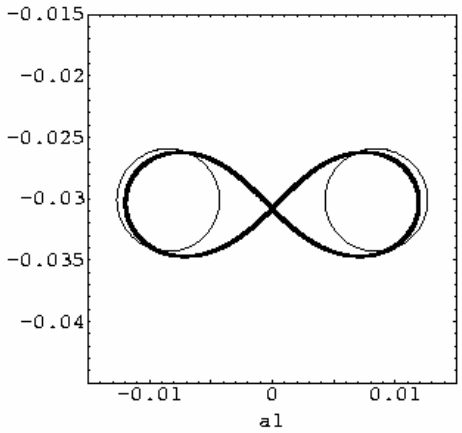
- Amplification amplitude

$$\mu = (\det A)^{-1} = \frac{1}{\left[(1-\kappa)^2 - |\gamma|^2\right]}$$

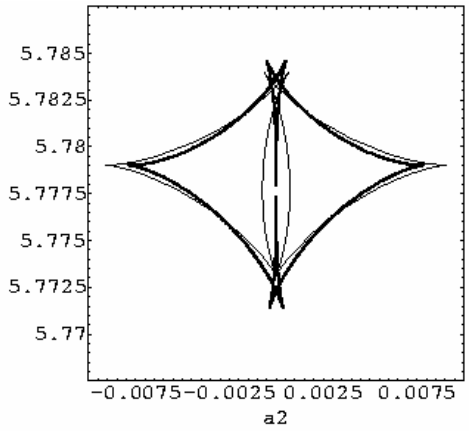
- critical lines corresponds to positions in the lens plane with $\det A = 0$
- the corresponding positions on the source plane are the caustic lines
- the positions of source points with respect to a caustic lines define de number of image multiplication and the source margnification
- when a source crosses a caustic line, its amplitication is almost infinity, and image pairs are formed

Caustic and Critical Lines

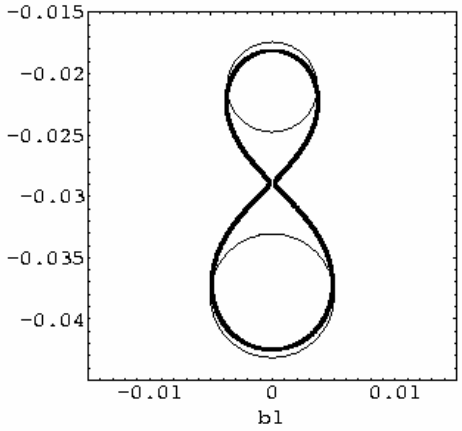




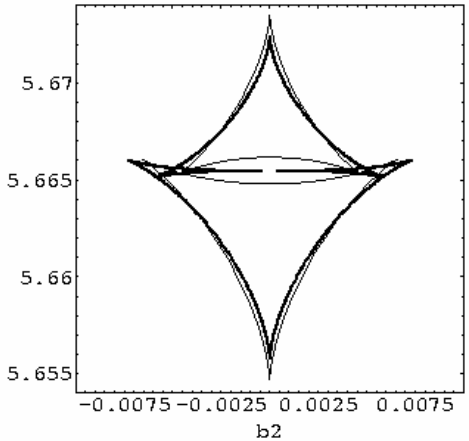
a1



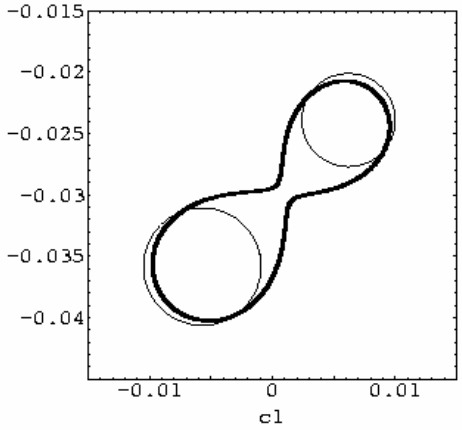
a2



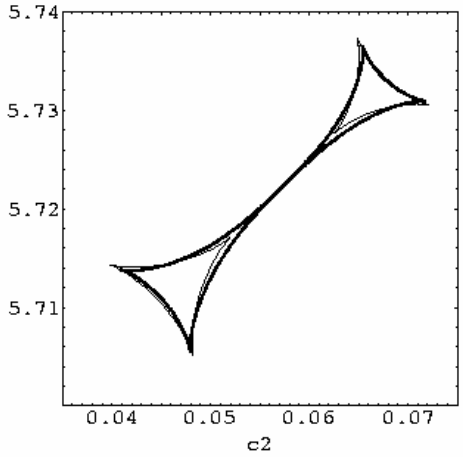
b1



b2

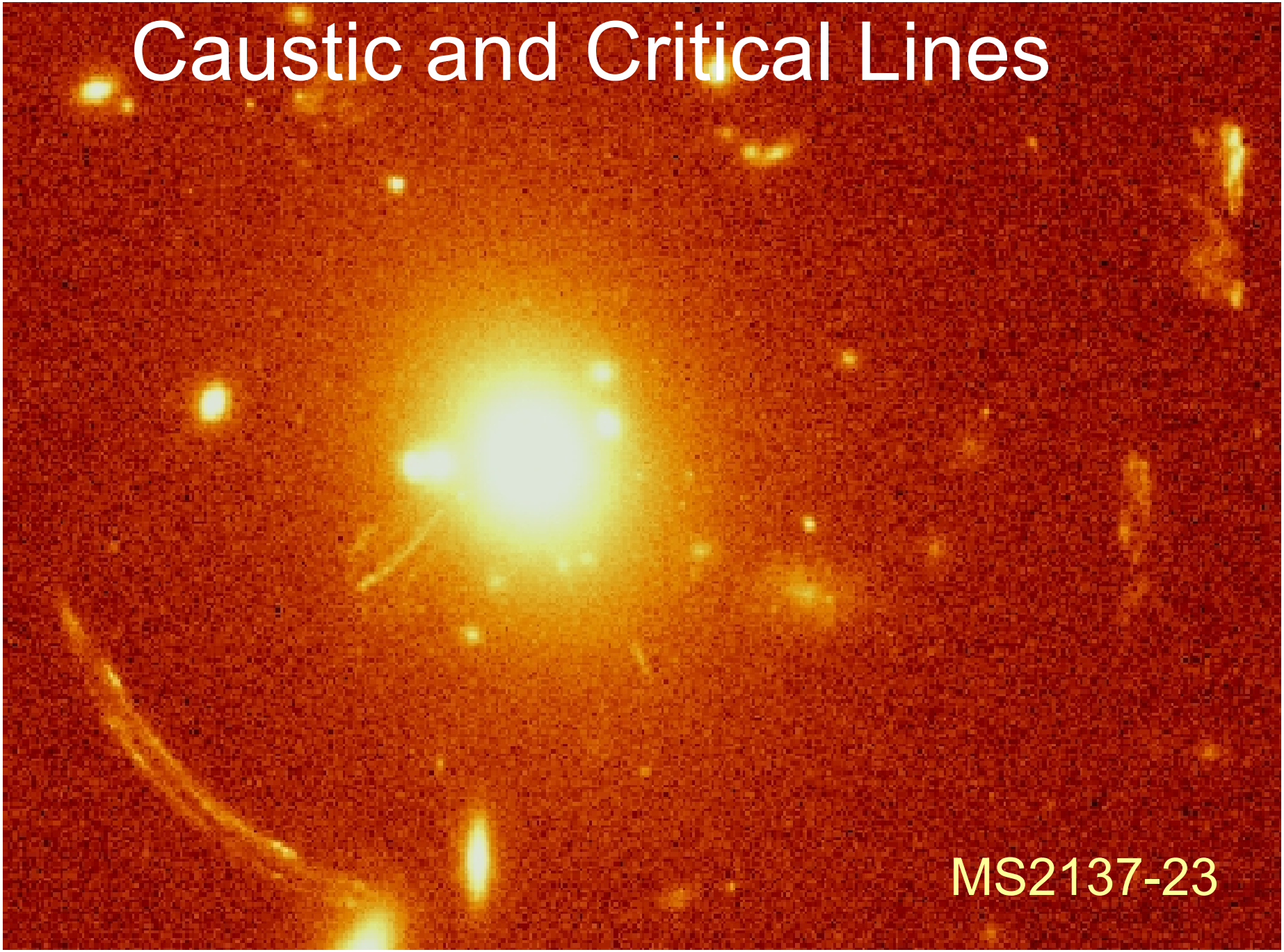


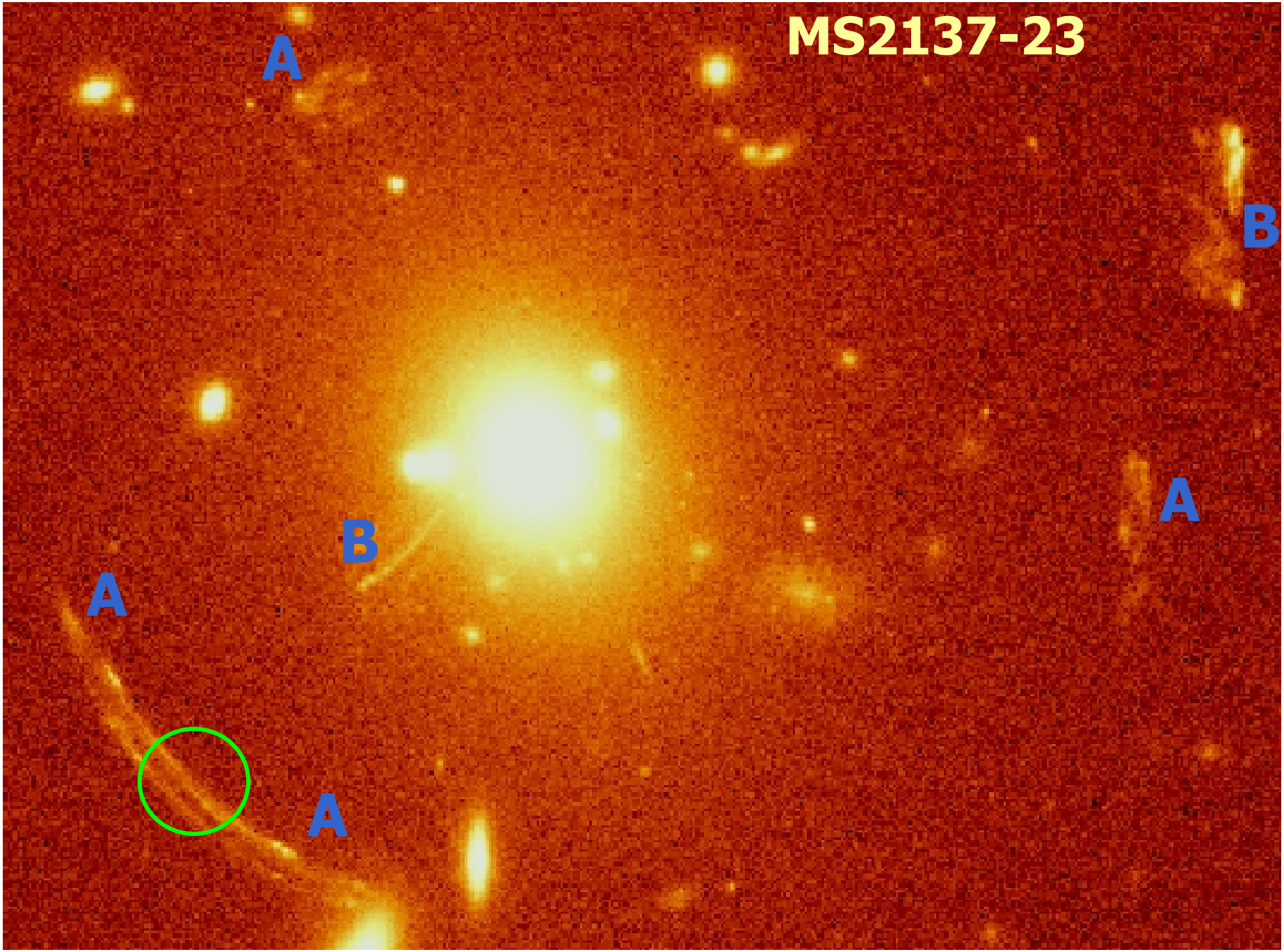
c1

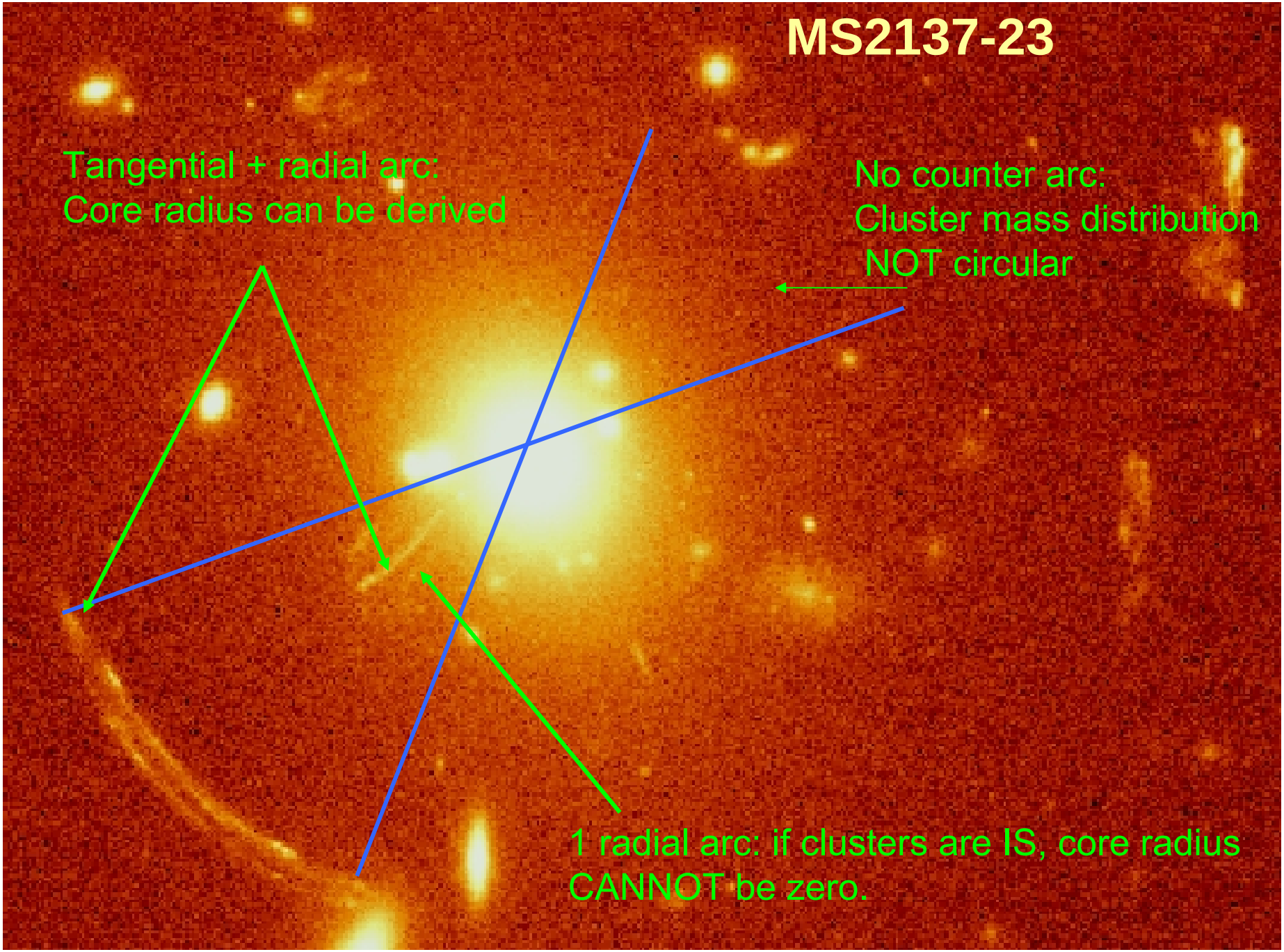


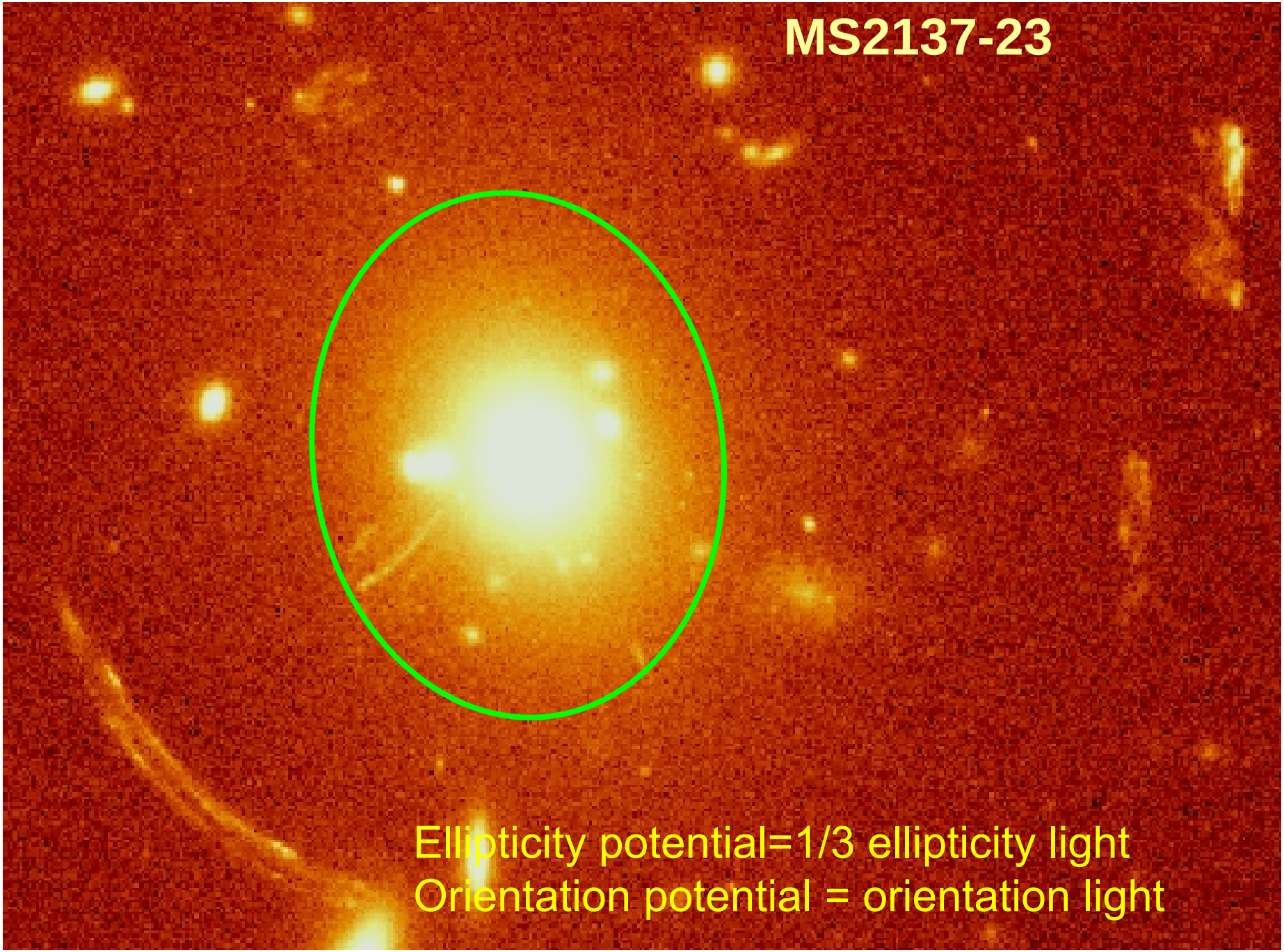
c2

Caustic and Critical Lines



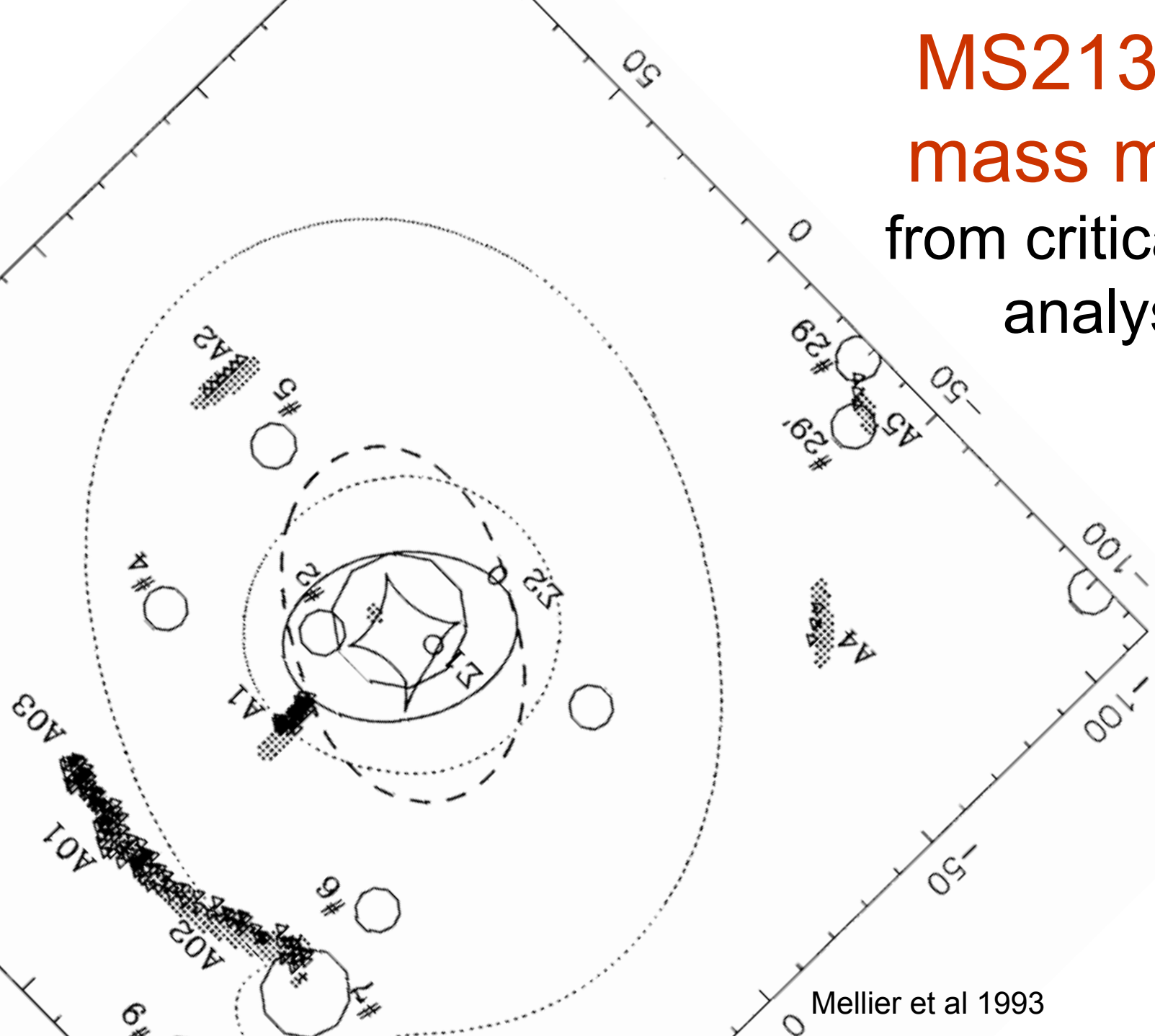

MS2137-23
A
B
A
B
A
A


MS2137-23
Tangential + radial arc:
Core radius can be derived
No counter arc:
Cluster mass distribution
NOT circular
1 radial arc: if clusters are IS, core radius
CANNOT be zero.


MS2137-23
Ellipticity potential=1/3 ellipticity light
Orientation potential = orientation light

MS2137-23 mass model

from critical lines analysis



Mellier et al 1993

Lensing properties of different mass density profiles

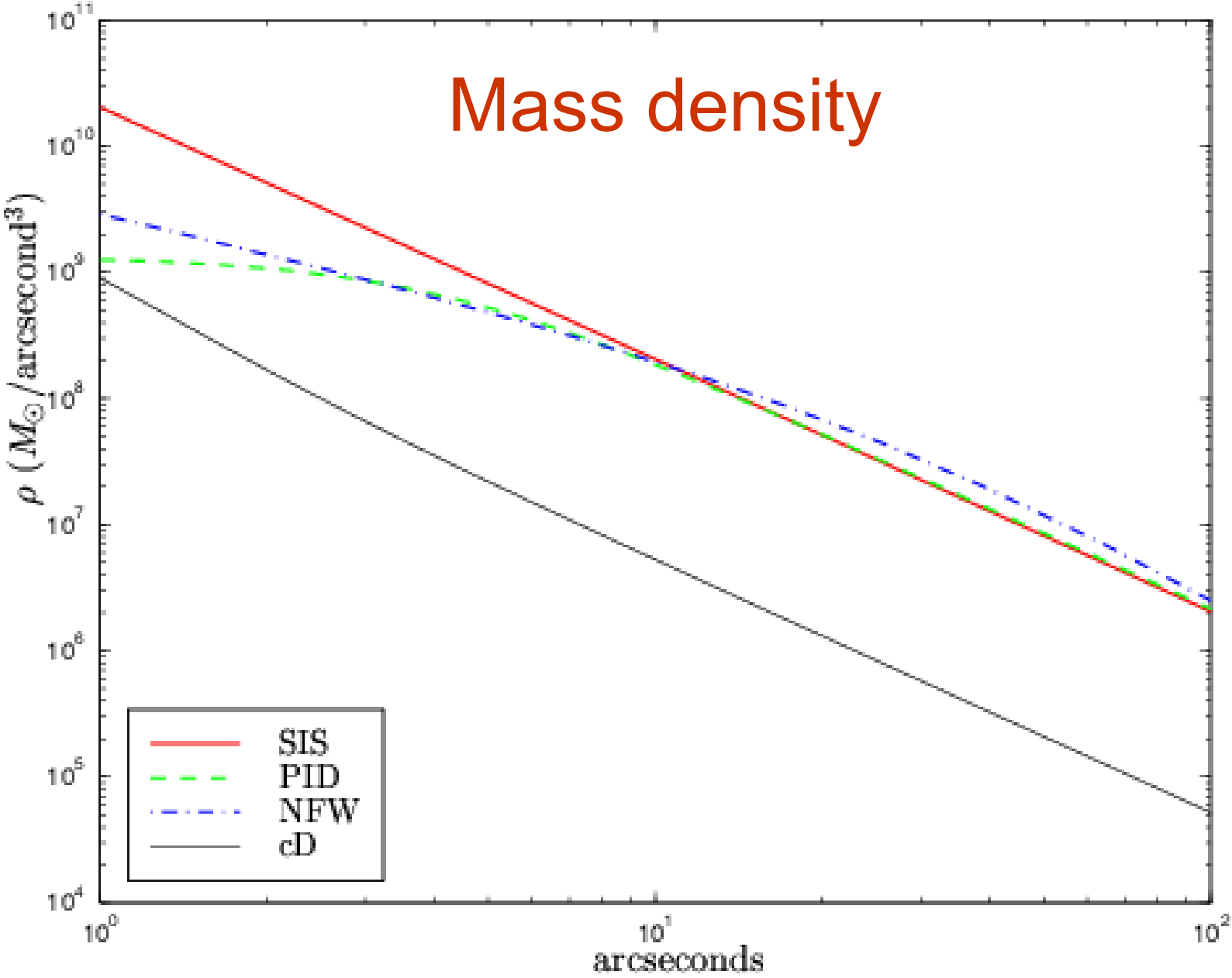
$$\overline{\rho}_{SIS}(r) = \frac{1}{r^2}$$

$$\overline{\rho}_{PID}(r; r_c) = \frac{1}{r_c^2} \frac{1}{1 + \left(\frac{r}{r_c}\right)^2}$$

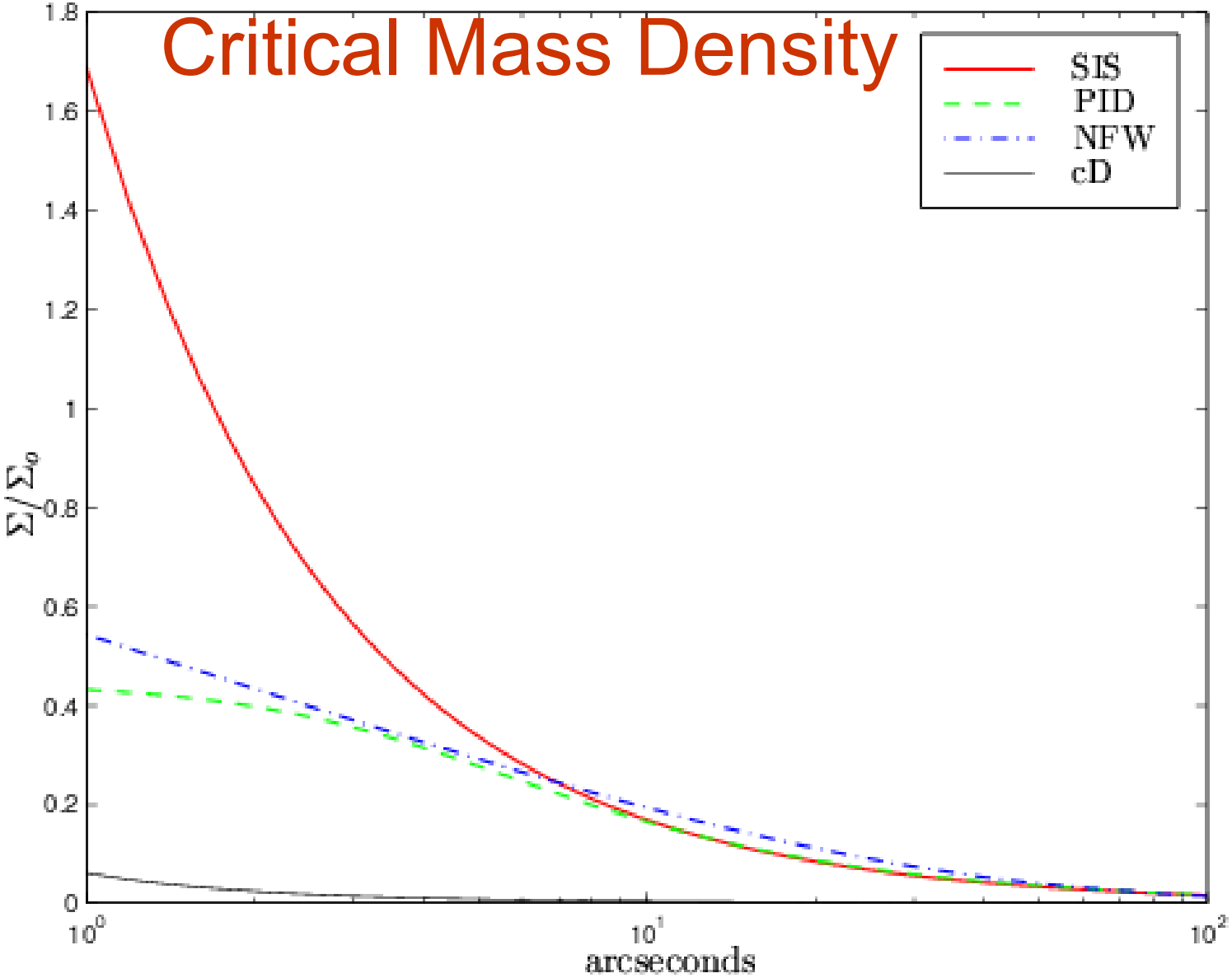
$$\overline{\rho}_{NFW}(r; r_s) = \frac{1}{r_s^2} \frac{1}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}$$

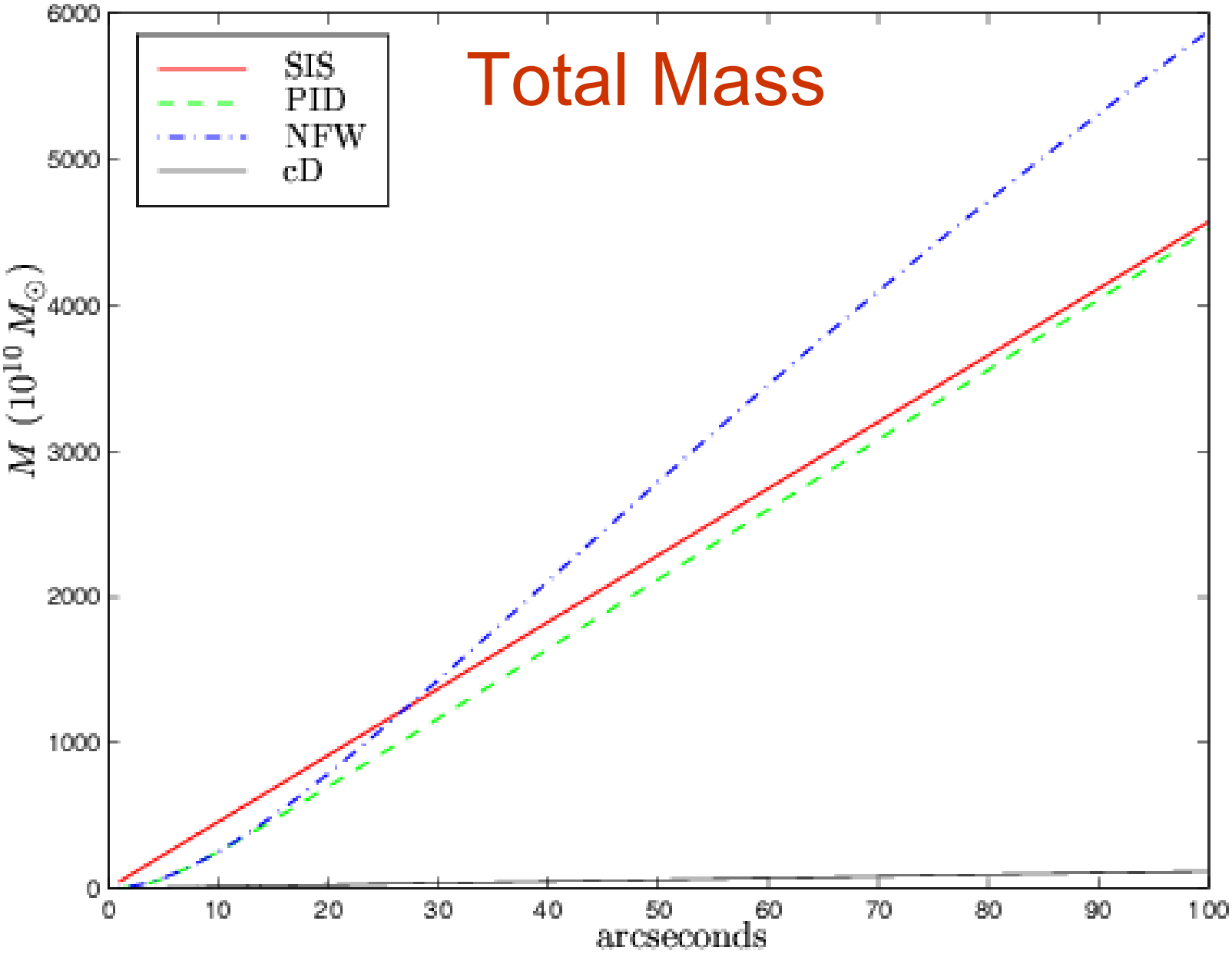
$$\overline{\rho}_{cD}(r; r_c, r_h) = \frac{1}{r_c^2} \frac{1 + \left(\frac{r}{r_c}\right)^2}{\left(1 + \left(\frac{r}{r_h}\right)^2\right)^2}$$

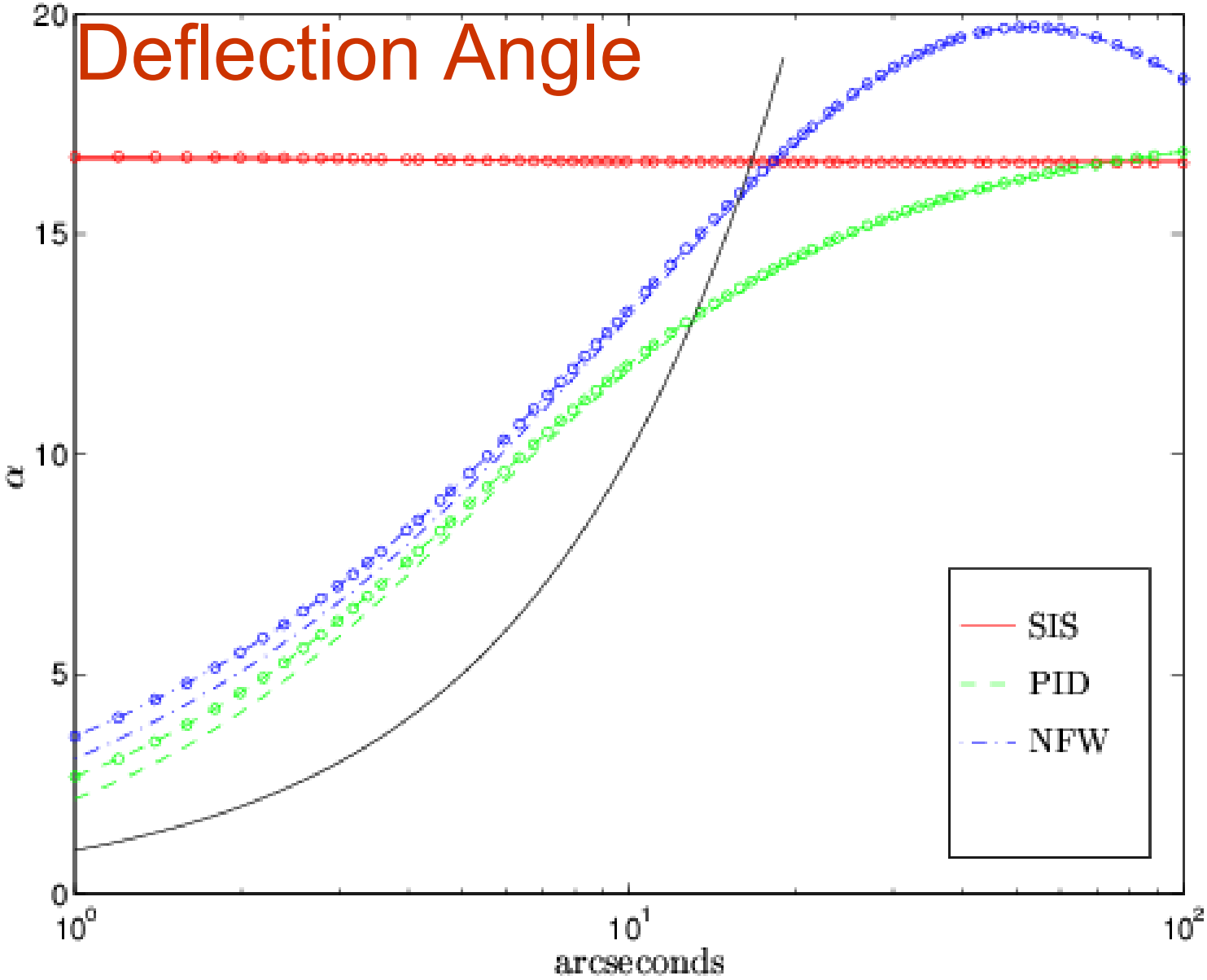
Mass density



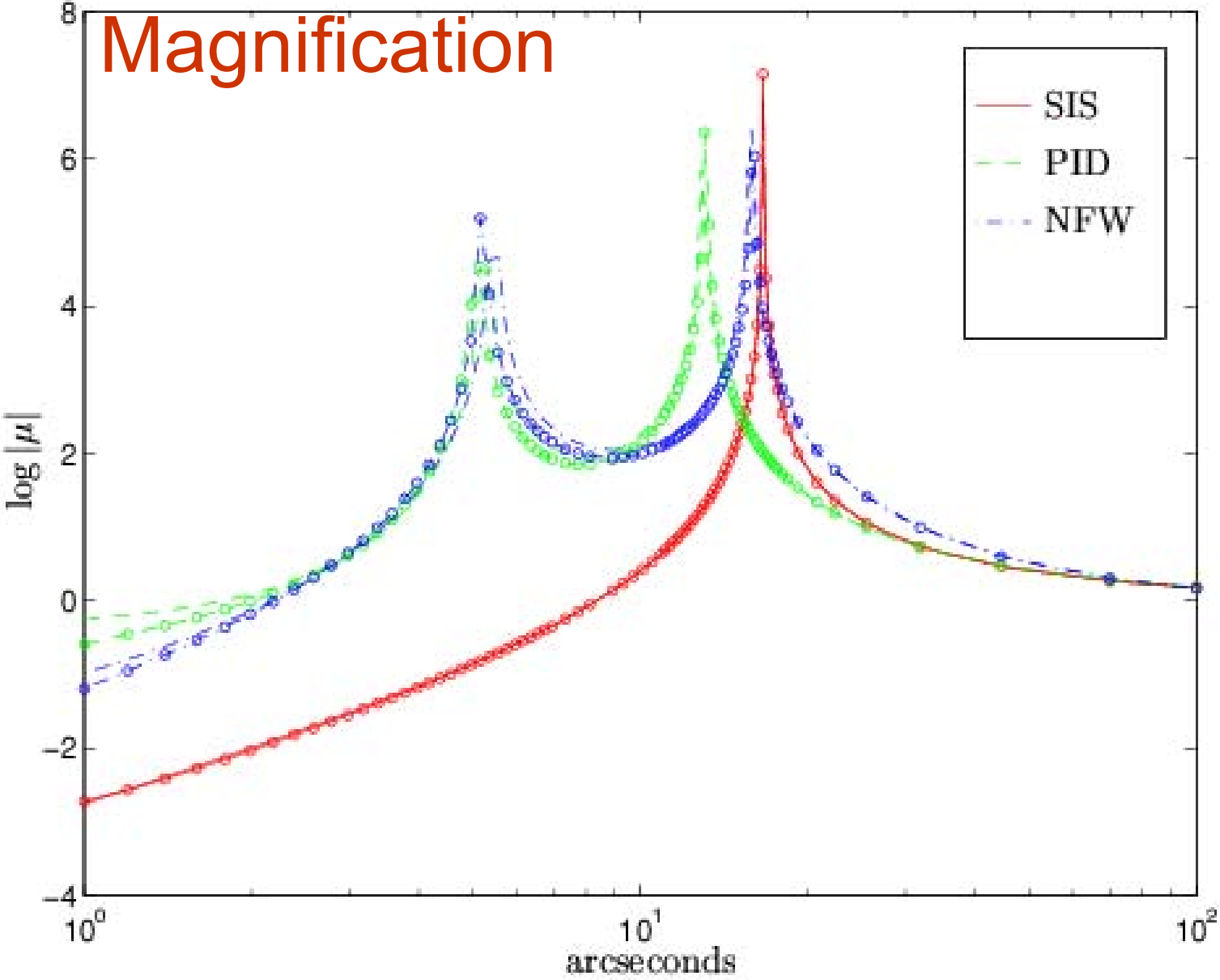
Critical Mass Density




Total Mass
SIS
PID
NFW
cD
$M\ (10^{10}\,M_\odot)$
arcseconds
0
1000
2000
3000
4000
5000
6000
0
10
20
30
40
50
60
70
80
90
100


Deflection Angle
20
15
10
5
0
α
10^0
10^1
10^2
arcseconds
SIS
PID
NFW

Magnification



The locally linearised lens equation

Consequence of the Liouville Theorem:

Etherington theorem:

Gravitational lensing effect conserves the surface brightness of lensed objects

The locally linearised lens equation

Expression of surface brightness conservation for a lensed source:

$$I\left(\vec{\theta}\right) = I^{s}\left[\vec{\beta}\left(\vec{\theta}\right)\right]$$

Assumption of the locally linearised lensing Source vs. Length scale of the Lens: the typical size of the lensed source is smaller than the scale of variation of magnification

The locally linearised lens equation

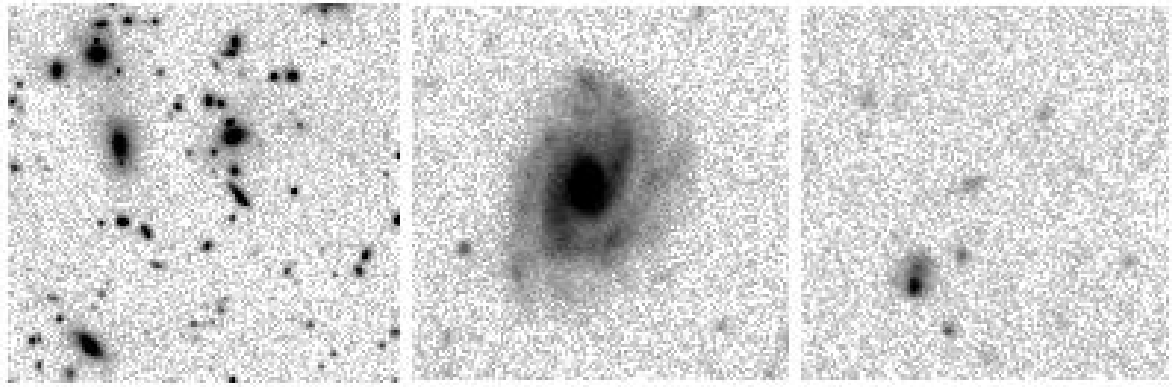
- $\Longrightarrow$ Linearisation:

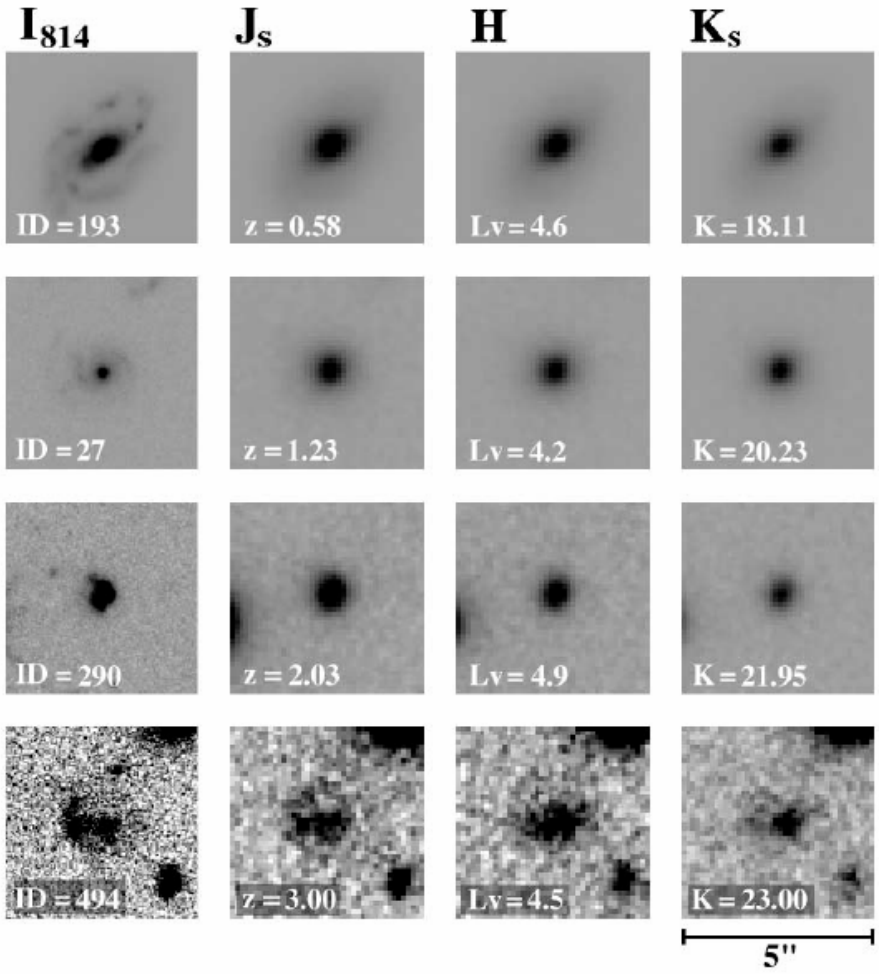
$$I\left(\vec{\theta}\right) = I^{s}\left[\vec{\beta}\left(\vec{\theta}_0\right) + A\left(\vec{\theta}_0\right)\ \left(\vec{\theta} - \vec{\theta}_0\right)\right]$$

- $\Longrightarrow$ an elliptical image of a source is transformed into an ellipse. A circular source is transformed into an ellipse with major axis $(1-\kappa-\gamma)^{-1}$ and minor axis $(1-\kappa+\gamma)^{-1}$.

We can relate lensing effect to shapes of lensed objects

The shape of galaxies





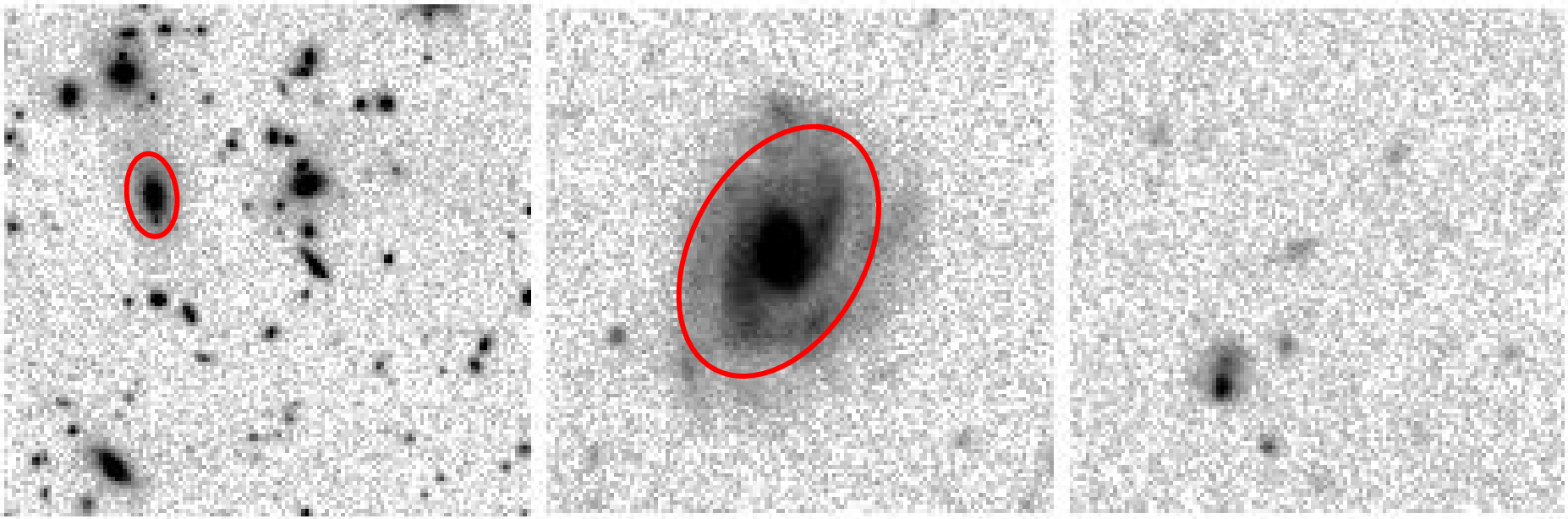
Change with

- PSF
- image sampling
- optical aberrations
- filter
- morphological type
- redshift (shift+evolution)
- signal-to-noise ratio

The shape of galaxies

To first order: all galaxies are ellipses, more or less noisy

Can be characterised by their second moments



The shape is weighted by the galaxy surface brightness

The shape of galaxies

Assume $I\left(\vec{\theta}\right)$ is the surface brigthness in an image. The centroid of the image can be defined as

- Centroid of a galaxy:

$$\vec{\theta}_0 = \frac{\int \mathrm{d}^2\theta \;\; \vec{\theta} \;\; I\left(\vec{\theta}\right) \;\; q_I\left[I\left(\vec{\theta}\right)\right]}{\int \mathrm{d}^2\theta \;\; I\left(\vec{\theta}\right) \;\; q_I\left[I\left(\vec{\theta}\right)\right]}$$

where q_I is a weight function (*e.g.* the Heaviside function).

The shape of galaxies

- Second moments On can then define the second brightness momment tensor

$$Q_{ij} = \frac{\int \mathrm{d}^2\theta \;\; (\theta_i - \theta_{0,i}) \;\; (\theta_j - \theta_{0,j}) \;\; I\left(\vec{\theta}\right) \;\; q_I\left[I\left(\vec{\theta}\right)\right]}{\int \mathrm{d}^2\theta \;\; I\left(\vec{\theta}\right) \;\; q_I\left[I\left(\vec{\theta}\right)\right]}$$

So, a circular object has

$$Q_{11} = Q_{22} \;\; \text{and} \;\; Q_{12} = 0$$

Likewise, for the source, taking into account the surface

The shape of galaxies

Surface brightness conservation:

$$Q_{ij}^{S} = \frac{\int \mathrm{d}^2\beta \;\; (\beta_i - \beta_{0,i}) \;\; (\beta_j - \beta_{0,j}) \;\; I^S\left(\vec{\theta}\right) \;\; q_I\left[I^S\left(\vec{\beta}\right)\right]}{\int \mathrm{d}^2\,\beta \;\; I^S\left(\vec{\theta}\right) \;\; q_I\left[I^S\left(\vec{\beta}\right)\right]}$$

using lensing equation and the locally linearised lens aquation, we have

$$\mathrm{d}^2\beta = \det A \; \mathrm{d}^2\theta \;\; \text{and} \;\; \vec{\beta} - \vec{\beta}_0 = A\left(\vec{\theta} - \vec{\theta}_0\right)$$

we then find the

- Relation between source and image shapes

$$Q^S = A\left(\vec{\theta}_0\right) \; Q \, A^T\left(\vec{\theta}_0\right)$$

Definition(s) of ellipticity

- Current definition(s) of ellipticities:

$$\varepsilon = \frac{1-\frac{b}{a}}{1+\frac{b}{a}} e^{2i\phi} \;;\; \chi = \frac{1-\left(\frac{b}{a}\right)^2}{1+\left(\frac{b}{a}\right)^2} e^{2i\phi}$$

with

$$a = (1-\kappa-\gamma)^{-1} \text{ and } b = (1-\kappa+\gamma)^{-1}$$

ε is directly the reduced shear

$$|g| = |\varepsilon| = \frac{|\gamma|}{1-\kappa}$$

Ellipticity

- Relations with second moments:

$$\varepsilon = \frac{Q_{11} - Q_{22} + 2\mathrm{i}Q_{12}}{Q_{11} + Q_{22} + 2\left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}}$$

$$\chi = \frac{Q_{11} - Q_{22} + 2\mathrm{i}Q_{12}}{Q_{11} + Q_{22}}$$

so,

$$\varepsilon = \frac{\chi}{1 + \left(1 - |\chi|^2\right)^{1/2}} \quad \text{and} \quad \chi = \frac{2\varepsilon}{1 + |\varepsilon|^2}$$

From ellipticity to shear

$$Q^S = A\,Q\,A$$

$$Q^S = [1-\kappa]^2 \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \times \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} \times \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

Hence

$$\frac{Q^S_{11}}{(1-\kappa)^2} = (1-g_1)^2 Q_{11} - 2g_2(1-g_1)Q_{12} + g_2^2 Q_{22}$$

$$\frac{Q^S_{22}}{(1-\kappa)^2} = g_2^2 Q_{11} - 2g_2(1+g_1)Q_{12} + (1+g_1)^2 Q_{22}$$

From ellipticity to shear

$$\frac{Q_{11}^S + Q_{22}^S}{(1-\kappa)^2} = (Q_{11} + Q_{22}) \times \left(1 + |g|^2 - 2g_1\chi_1 - 2g_2\chi_2\right)$$

We can now express the ellipticy of the source as function of the ellipticity of the image and the gravitational shear:

$$\chi^S = \frac{-2g + \chi + g^2\chi^*}{1 + |g|^2 - 2\Re\left(g\chi^*\right)}$$

From ellipticity to shear

and since

$$\varepsilon = \frac{\chi}{1 + (1 - |\chi|^2)^{1/2}}$$

we have

$$\begin{cases} \varepsilon^s = \dfrac{\varepsilon^i - g}{1 - g^* \varepsilon^i} & \text{for } |g| < 1 \\ \varepsilon^s = \dfrac{1 - g\left(\varepsilon^i\right)^*}{\left(\varepsilon^i\right)^* - g^*} & \text{for } |g| > 1 \end{cases}$$

Two shapes: tangential/radial shape

The weak lensing regime

- From ellipticity to shear

$$\varepsilon^s = \begin{cases} \frac{\varepsilon^i - g}{1 - g^* \varepsilon^i} & \text{for } |g| < 1 \\ \frac{1 - g(\varepsilon)^*}{(\varepsilon)^* - g^*} & \text{for } |g| > 1 \end{cases}$$

- Weak Lensing regime : $\kappa << 1, \ |\gamma| << 1 \ |g|^2 \simeq 0$ Therefore

$$g = \frac{\gamma}{1 - \kappa} \simeq \gamma$$

In that case

$$\chi \simeq \chi^S + 2\left(g - \chi^S \Re\left(g\chi^*\right)\right)$$

The weak lensing regime

In we decompose one component , for example

$$\chi_1 = \chi_1^S + 2g_1\left(1 - \chi_i^S\chi_1\right) - 2g_2\chi_1^S\chi_2$$

So, we can write the relation as follow:

$$\chi_i = \chi_i^S + 2\left(\delta_{ij} - \chi_i^S\chi_j\right)\gamma_j$$

- Sources orientation is isotropically distributed ←

$$\langle\chi^S\rangle \simeq 0$$

which implies!

$$\boxed{\langle\chi\rangle \simeq 2\gamma = 2g}$$

Therefore, the image ellipticity of galaxies provide an unbiased estimate of the gravitational shear.

Definitions:

tangential and cross shear components

- Cartesian coordinates:

$$\gamma = \gamma_1 + \mathrm{i}\gamma_2 = |\gamma| e^{2\mathrm{i}\varphi}$$

Tangential and Cross components

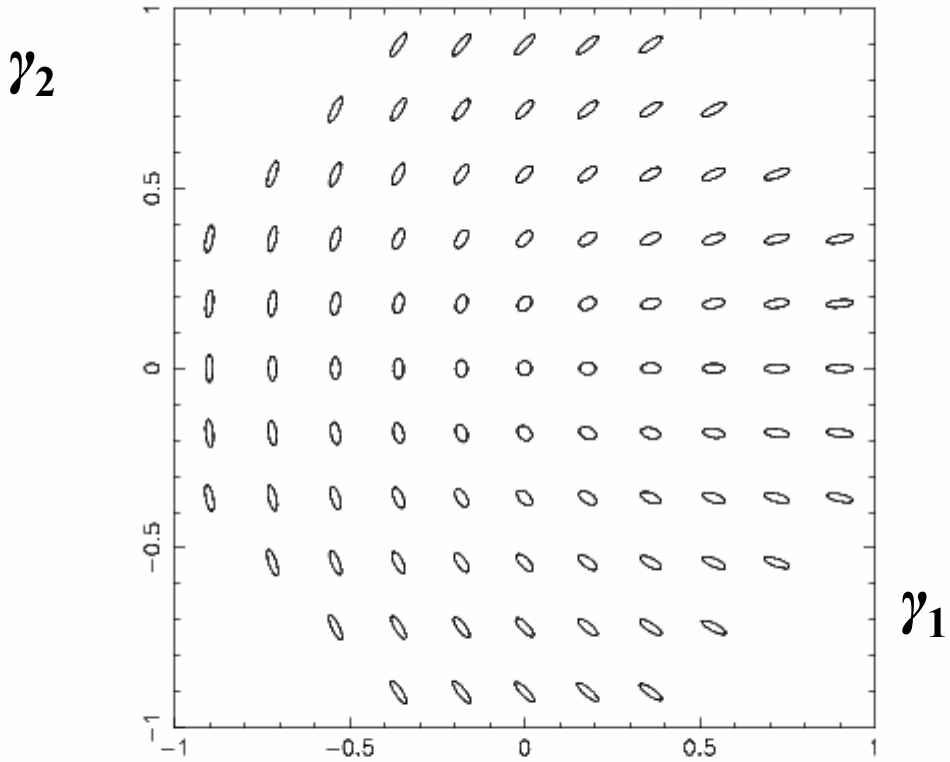


Figure 4: Orientation of the ellipses given by the Cartesian coordinates γ_1 (x-axis) and γ_2 (y-axis), with the polar angle ranging from 0 ti 2π.

Definitions:

tangential and cross shear components

- Cartesian coordinates:

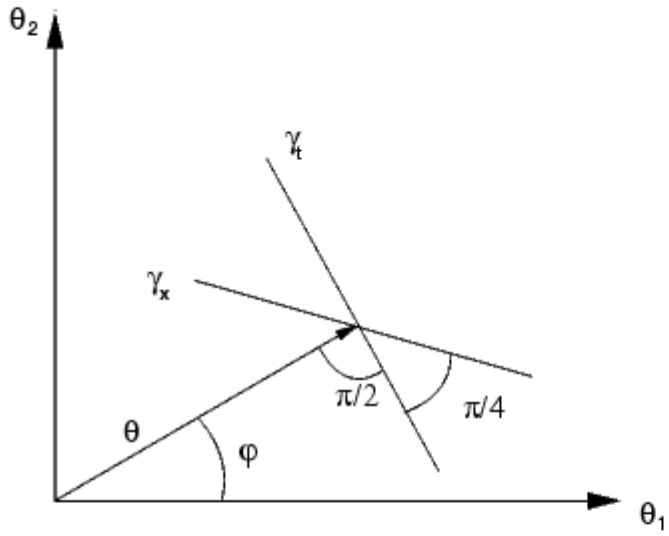
$$\gamma = \gamma_1 + \mathrm{i}\gamma_2 = |\gamma| e^{2\mathrm{i}\varphi}$$

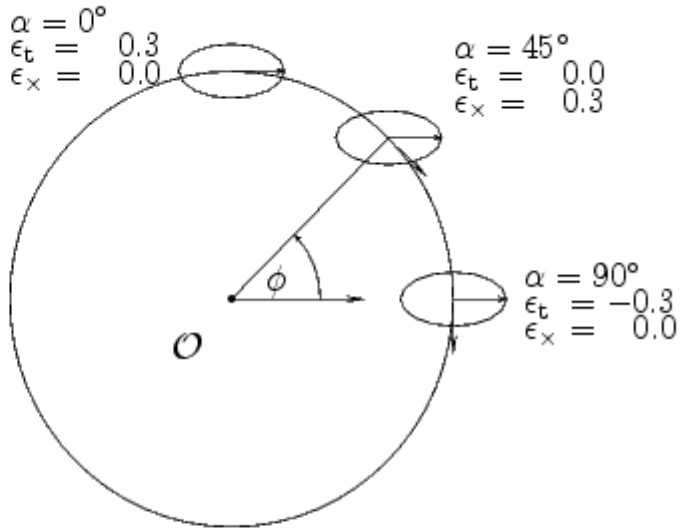
The tangential and cross components of the shear with respect to a direction ϕ is

$$\gamma_t = -\Re\left[\gamma \mathrm{e}^{-2\mathrm{i}\phi}\right] \;\; ; \;\; \gamma_x = -Im\left[\gamma \mathrm{e}^{-2\mathrm{i}\phi}\right]$$

Likewise, one can define the tangential and cross image ellipticity components, ε_t and ε_x.

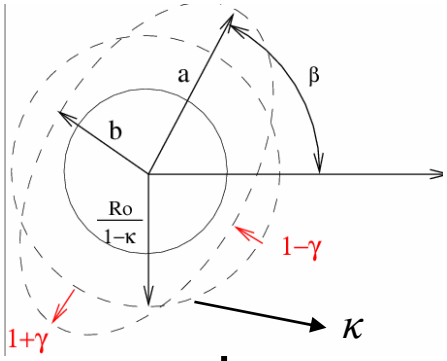
Tangential and Cross components

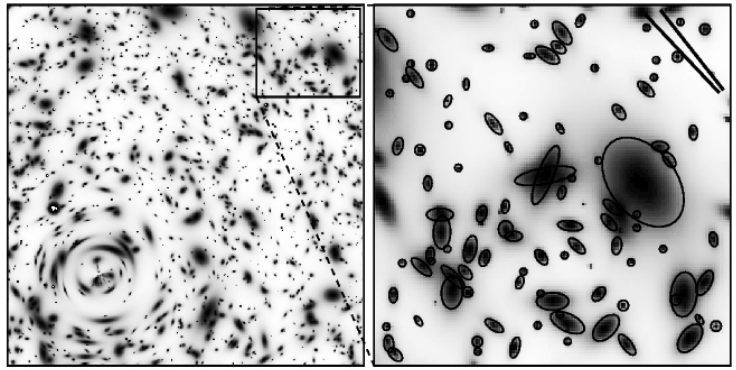




With these definition, for a spherical mass overdensity, the shear will be tangential with $\gamma_t > 0$ and $\gamma_x = 0$.

Summary: from shear to galaxy ellipticity





$$M_{ij} = \frac{\int I(\boldsymbol{\theta})\,\theta_i\,\theta_j\,\mathrm{d}^2\theta}{\int I(\boldsymbol{\theta})\,\mathrm{d}^2\theta}$$

$$\boldsymbol{\delta} = \frac{2\boldsymbol{\gamma}\,(1-\kappa)}{(1-\kappa)^2+|\gamma|^2} = \frac{a^2-b^2}{a^2+b^2}$$

$$\frac{a^2-b^2}{a^2+b^2}$$

$\boldsymbol{\delta} \sim 2\boldsymbol{\gamma}$ (weak lensing regime)

$= \boldsymbol{\varepsilon_s} + \boldsymbol{\varepsilon_i}$ + noise + systematics….

Reliability of results: depends on PSF analysis

Assume sources orientation is isotropic:

Weak lensing regime : $\delta \sim 2\gamma = \langle \varepsilon_{Shear} \rangle_\theta$ + noise

Mass reconstruction from weak lensing

We have shown that

$$\alpha(\theta) = \frac{1}{\pi}\int \kappa(\theta')\frac{\theta-\theta'}{|\theta-\theta'|^2}\,\mathrm{d}\theta^2$$

that is

$$\psi(\theta) = \frac{1}{\pi}\int \kappa(\theta')\,\ln|\theta-\theta'|\,\mathrm{d}\theta^2$$

where the amplification matrix writes

$$A(\theta) = \frac{\partial\beta}{\partial\theta} = \left(\delta_{ij} - \frac{\partial^2\psi(\theta)}{\partial\theta_i\theta_j}\right)$$

Mass reconstruction from weak lensing

or, as function of the convergence and shear components

$$\begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

Let set

$$\begin{cases} \gamma = \gamma_1 + i\gamma_2 = |\gamma| e^{2i\psi} \\ \gamma_1 = \dfrac{1}{2}(\psi_{,11} + \psi_{,22}) \\ \gamma_2 = \psi_{,12} \end{cases}$$

then

$$\gamma = \left(\frac{\partial_1^2 - \partial_2^2}{2} + i\partial_1\partial_2 \right) \psi(\theta)$$

Mass reconstruction from weak lensing

So, we can express γ as function of κ as follows :

$$\gamma(\theta) = \frac{1}{\pi} \int \kappa(\theta') F(\theta - \theta') \, \mathrm{d}\theta^2 \qquad (98)$$

with

$$F(\theta) = \frac{\theta_1^2 - \theta_2^2 + 2i\theta_1\theta_2}{|\theta|^4} = F_1(\theta) + iF_2(\theta)$$

Let write this relation in Fourier space:

$$\kappa(\theta) = \frac{1}{(2\pi)^2} \int \hat{\kappa}(\mathbf{k}) e^{i\mathbf{k}.\theta} \, \mathrm{d}^2 k$$

Eq. (98)= convolution →

$$\hat{\gamma}(\mathbf{k}) = \frac{1}{\pi} \hat{\kappa}(\mathbf{k}) \hat{F}(\hat{k})$$

Mass reconstruction from weak lensing

By inverting this relation, one can then reconstruct the κ field:

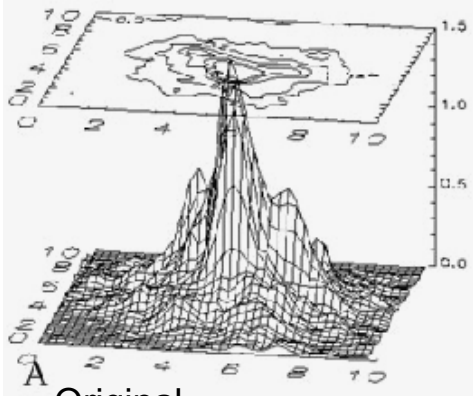
$$\kappa(\theta) = \frac{1}{\pi}\int \hat{F}^*(\theta - \theta')\,\gamma(\theta')\;\mathrm{d}\theta^2 + \kappa_0$$

where the real part is the matter field. *Application:*

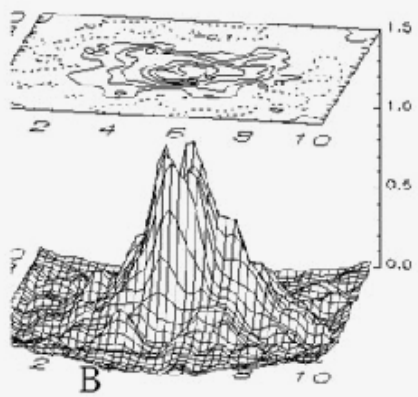
$$\Sigma(\theta) - \Sigma_0 = \Sigma_{critic}\,\frac{1}{\pi}a^2 \sum_{i,j} \Re\left(\hat{F}^*(\theta - \theta_{i,j})\,\bar{\varepsilon}(\theta_{i,j})\right)$$

where a is the distance between grid points

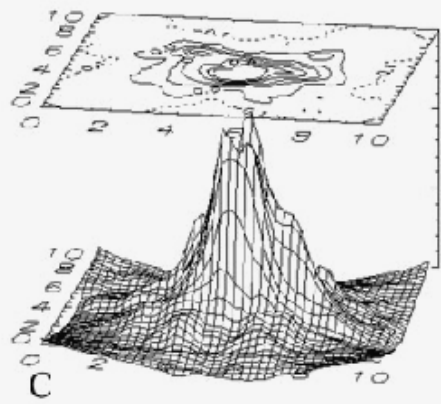
Mass reconstruction algorithms

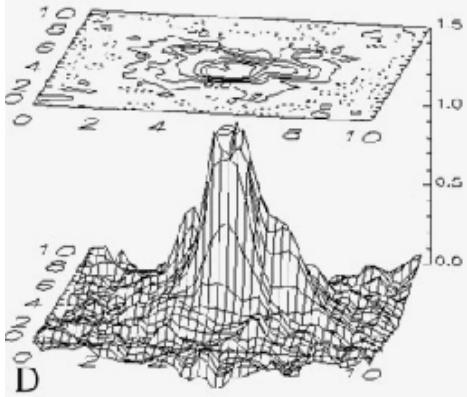


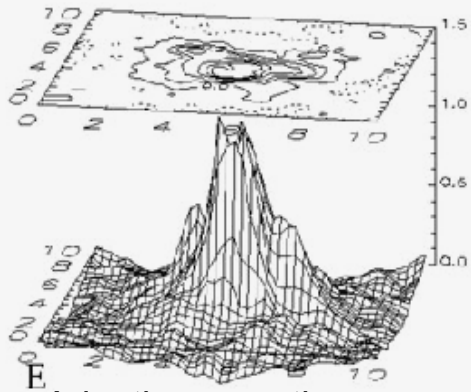
Original



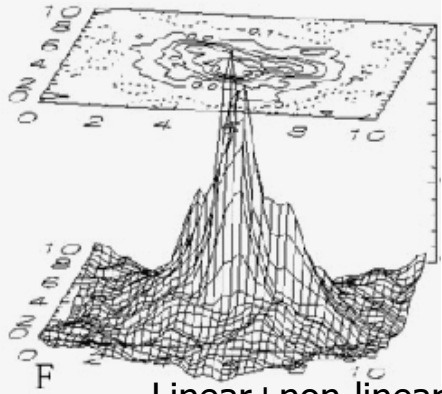
KS93 (FFT)



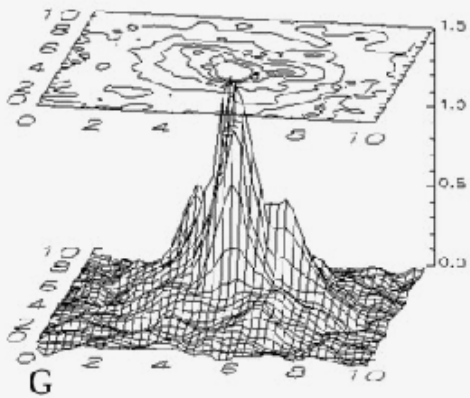


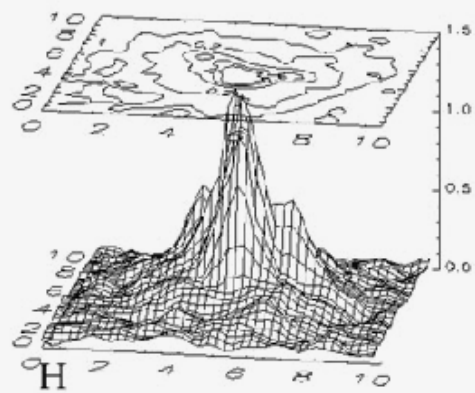


Adaptive smooting

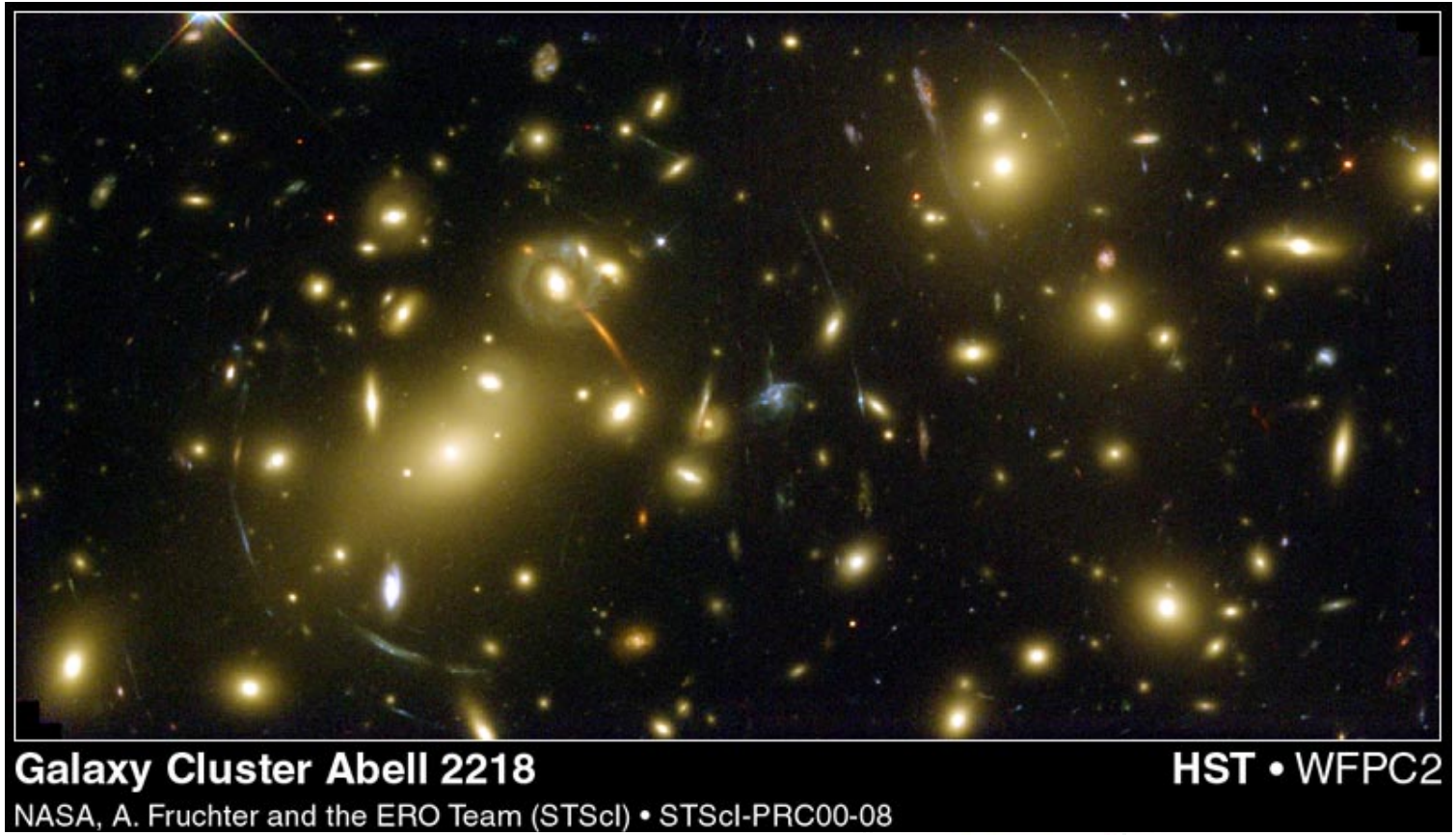


Linear+non-linear WL regimes

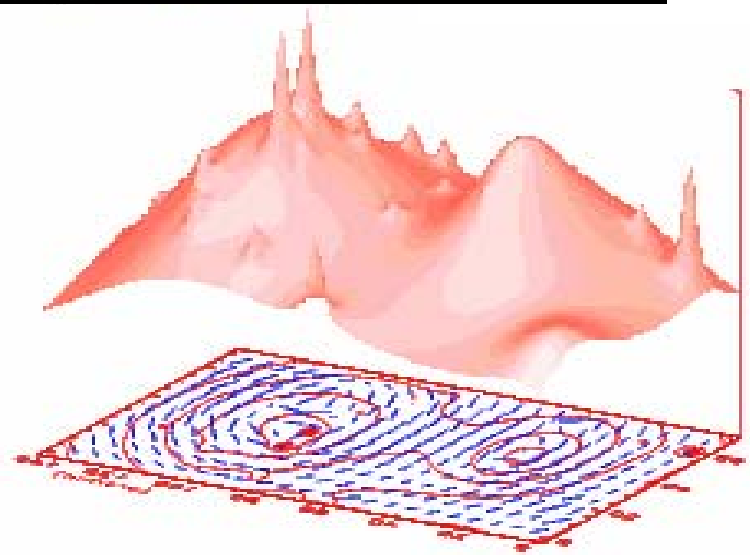




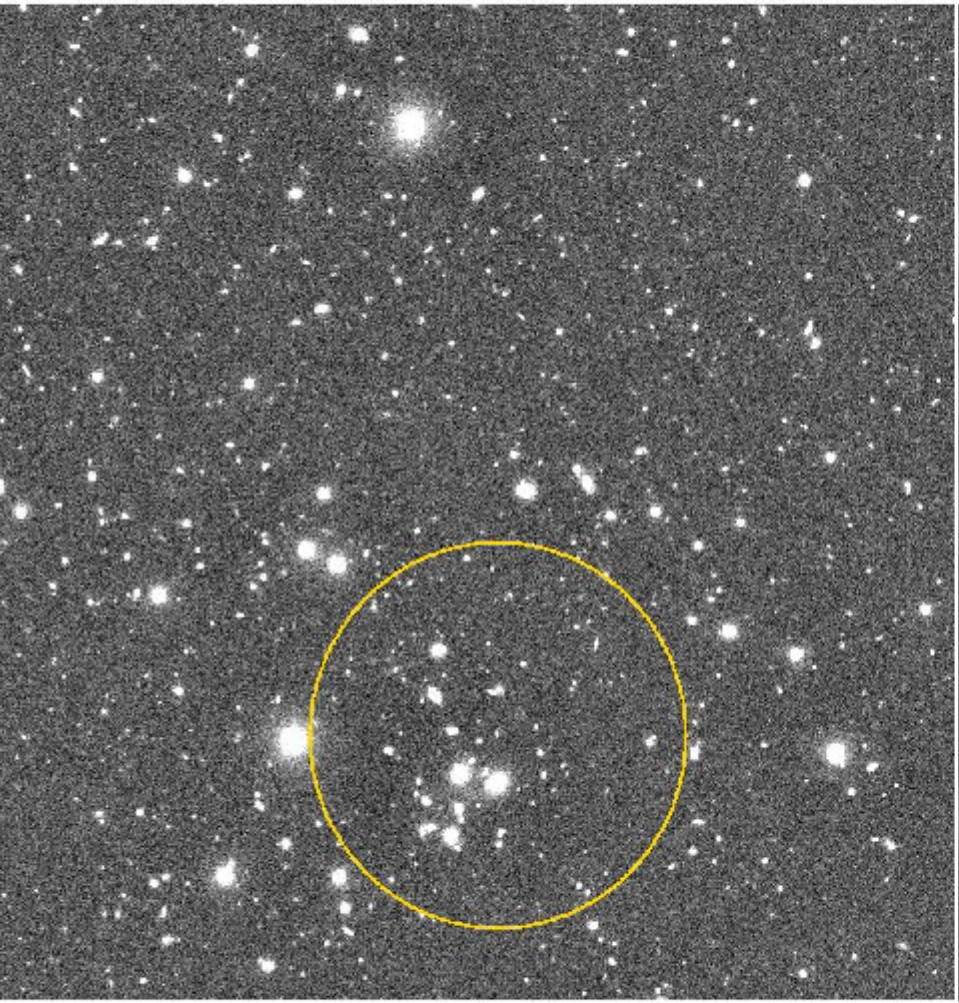
=F + mass=0 a the boundaries



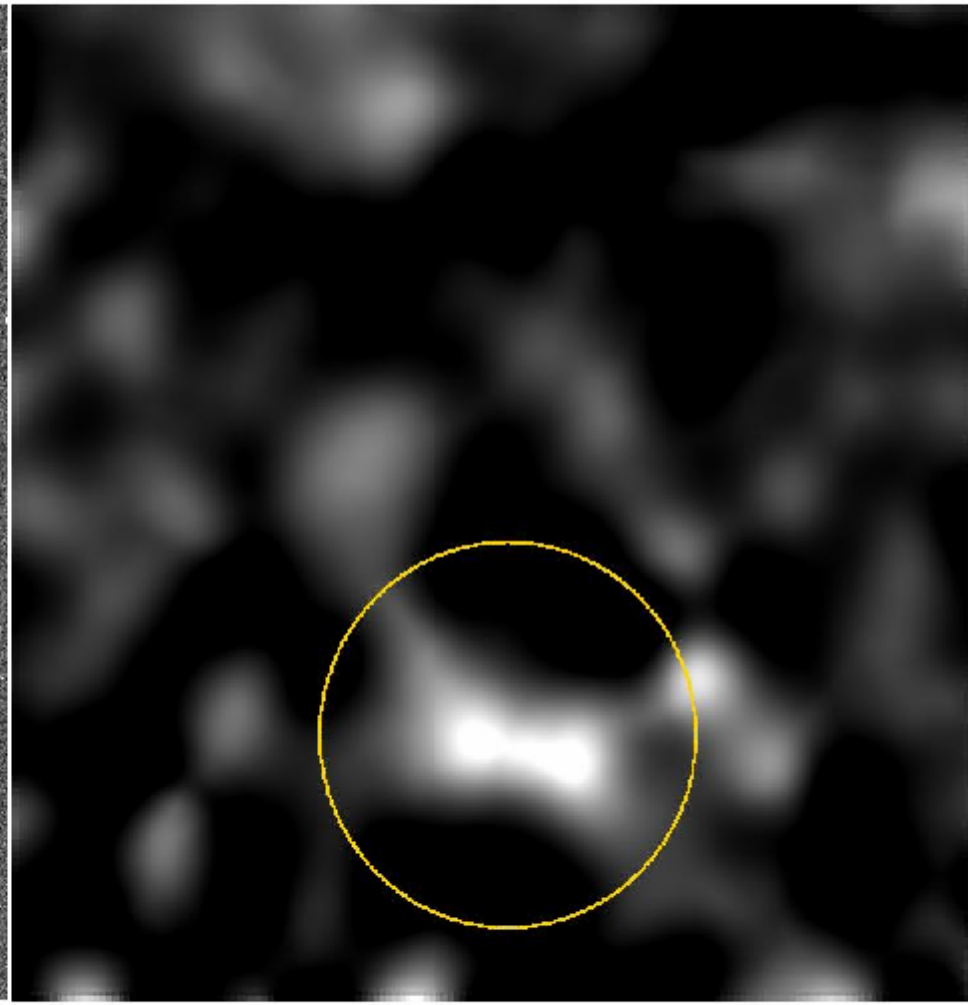
Mass reconstruction of the Lensing cluster Abell 2218 (z=0.18)



Finding clusters from dark matter detection only:



VLT I-band Image: 36 mn exposure



Dark Matter reconstruction

Getting the absolute mass

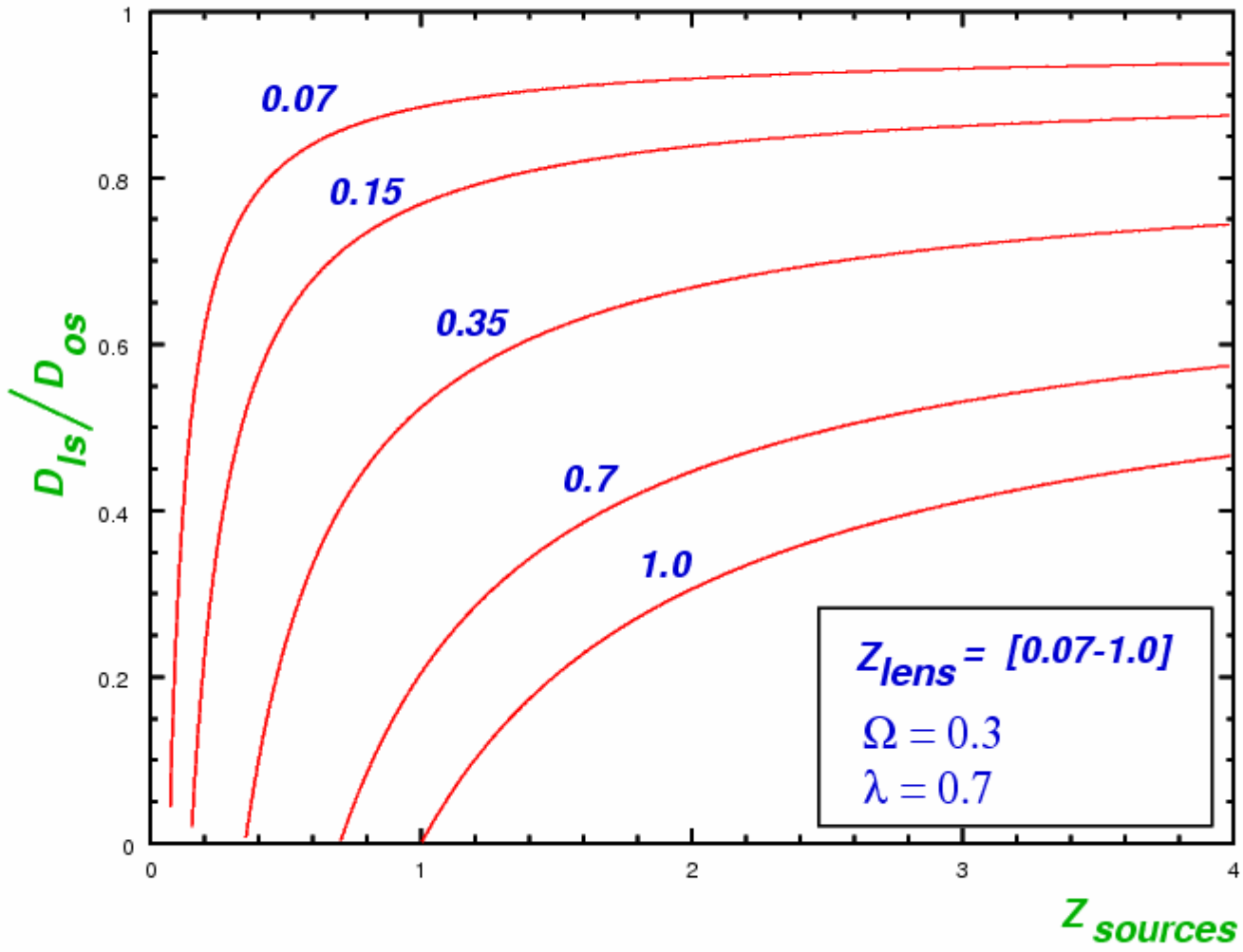
- The mass reconstruction provides the shape of the projeted mass distribution but not the absolute scale.
- Need the redshift of the sources:

$$\kappa\left(\vec{\theta},z\right)=\frac{\Sigma\left(\vec{\theta}\right)}{\Sigma_{critic}\left(z\right)}=\Sigma\left(\vec{\theta}\right)\frac{4\pi G}{c^2}D_{ol}\left[\frac{D_{ls}}{D_{os}}\right]$$

$$\gamma\left(\vec{\theta},z\right)=\frac{4\pi G}{c^2}D_{ol}\left[\frac{D_{ls}}{D_{os}}\right]\gamma\left(\vec{\theta}\right)$$

- Very sensitive to the redshift distribution for high-z lenses

Getting the absolute mass



Example: the Singular Isothermal Sphere (SIS)

- Mass density:

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2} \tag{111}$$

- Projected mass density:

$$\Sigma(\xi) = \frac{\sigma_v^2}{2G\,\xi} \tag{112}$$

- Mass inside ξ

$$M(\xi) = \int_0^{\xi} 2\pi\xi' \Sigma(\xi')\, \mathrm{d}\xi' \tag{113}$$

The Singular Isothermal Sphere

- Angular Einstein radius

$$\theta_E = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{ls}}{D_{os}}$$

- Lensing equation

$$\vec{\beta} = \vec{\theta} - \theta_E \frac{\vec{\theta}}{|\vec{\theta}|}$$

- Lensing properties

$$\kappa(\theta) = \frac{\theta_E}{2\theta}; \qquad |\gamma(\theta)| = \frac{\theta_E}{2\theta}; \; \alpha\left(\vec{\theta}\right) = \theta_E \frac{\vec{\theta}]}{|\vec{\theta}|}$$

The Singular Isothermal Sphere:

Expected Signal-to-Noise from weak lensing studies of clusters/groups of galaxies

$$S/N = 10 \left(\frac{n}{30\ \mathrm{arcmin}^2}\right)^{1/2} \left(\frac{\sigma_\varepsilon}{0.2}\right)^{-1} \left(\frac{\sigma_v}{600\ \mathrm{km/s}}\right)^{2} \left(\frac{\ln\left[\frac{\theta_2}{\theta_1}\right]}{\ln 10}\right)^{1/2} \left\langle\frac{D_{ls}}{D_{os}}\right\rangle$$

Therefore to increase the S/N

- Go deep: increase *n*
- Observe massive clusters: increase σ^2
- Reduce intrinsic ellipticity dispersion: space better

The mass sheet degeneracy

- Changing $a \longrightarrow \lambda a$ and $b \longrightarrow \lambda b$ keeps ε unchanged.
- $\Longleftrightarrow \gamma \longrightarrow \lambda \gamma$ and $\kappa \longrightarrow \lambda \kappa$.
- $\Longleftrightarrow$ changing $\kappa \longrightarrow \kappa' = \lambda \kappa + (1 - \lambda)$: rescaling κ and adding a constant mass density.
- This mass sheet degeneracy results from the fact that the gravitational distortion is sensitive to the gradient of the projected mass density.
- We are only measuring ellipticity modification, no size modification of galaxies

Solve the mass sheet degeneracy using magnification

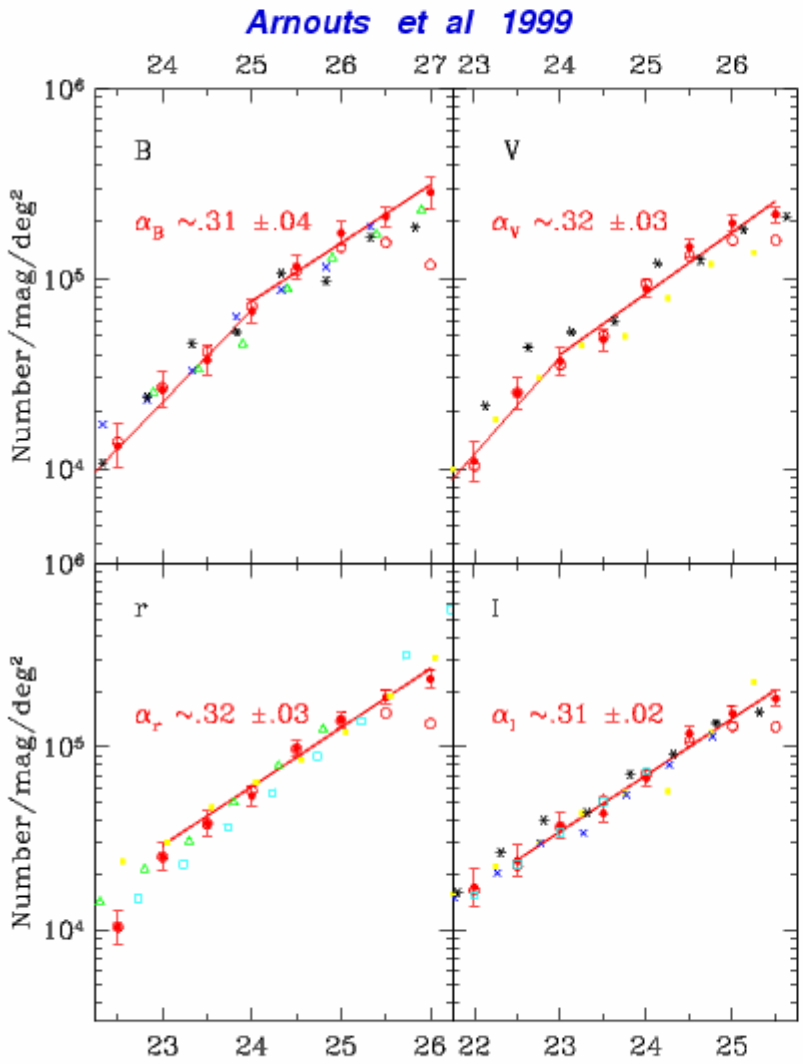
- Magnification: increases the flux received from galaxies $\longrightarrow$ more galaxies visible and ALSO magnifies by the same amount the area of the projected lensed sky $\longrightarrow$ decreases the galaxy number density (Broadhurst 1995):

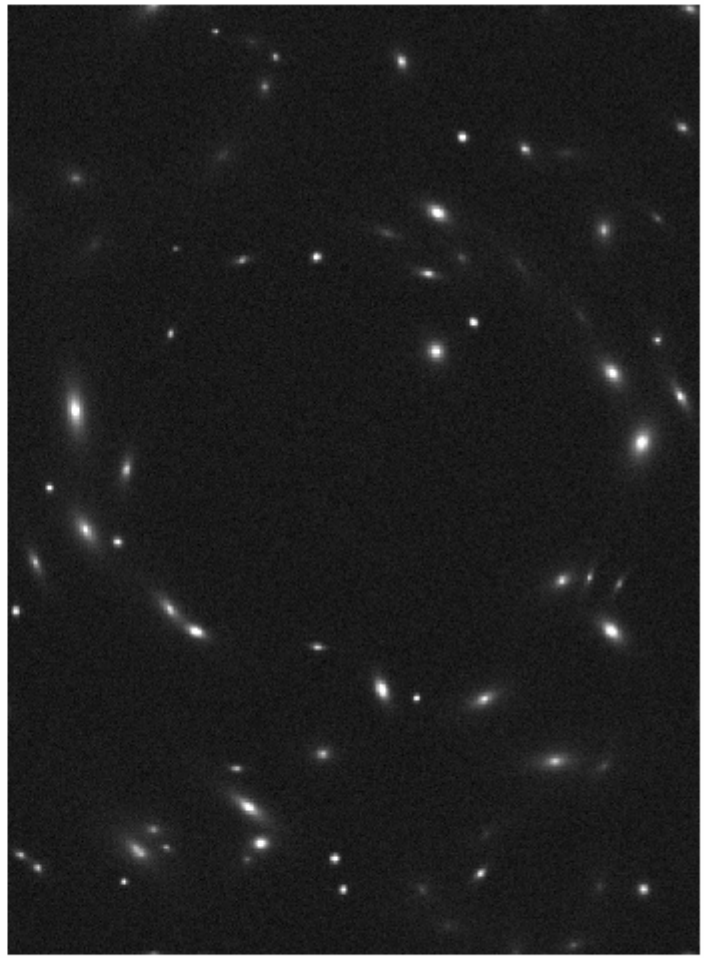
$$N(<m,r) = N_0(<m)\,\mu(r)^{2.5\alpha-1} \approx N_0\,(1+2\kappa)^{2.5\alpha-1} \tag{127}$$

$\mu(r)$ is the magnification, $N_0(<m)$ the intrinsic (unlensed) number density, and

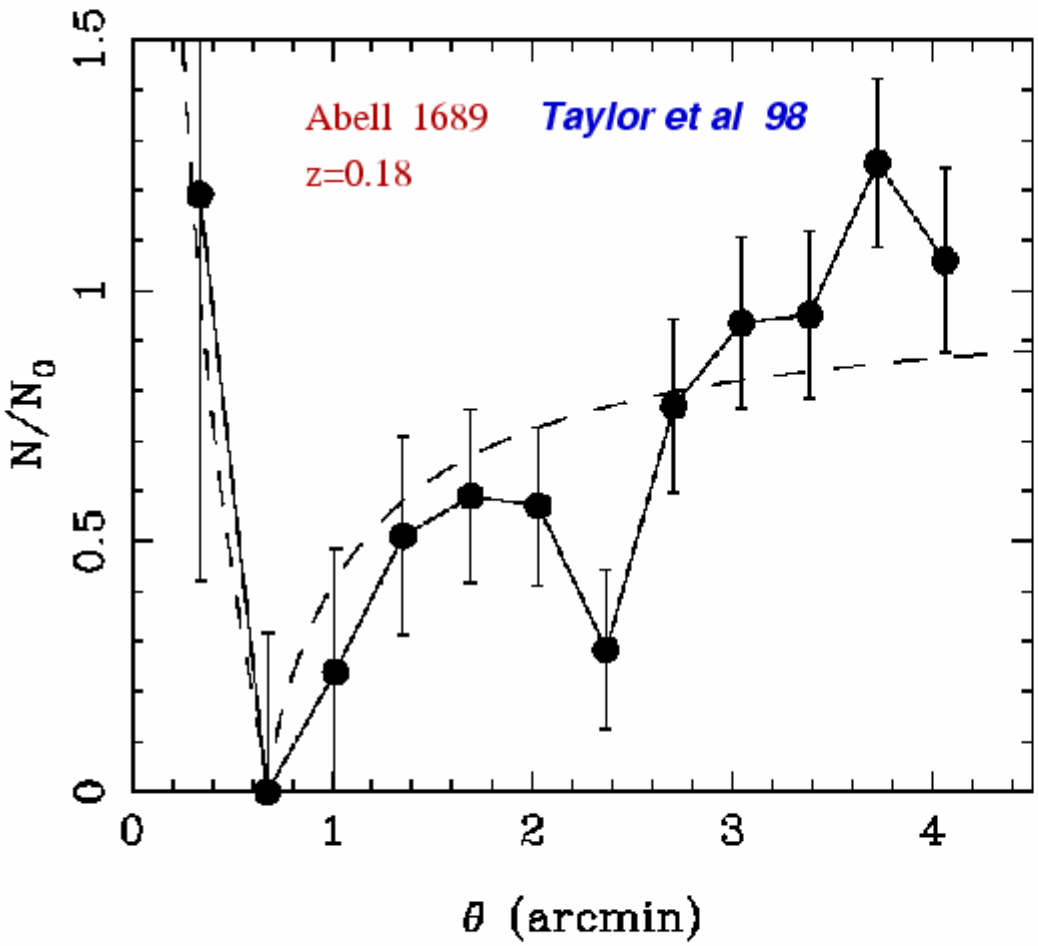
$$\alpha = \frac{\mathrm{d}logN(<m)}{\mathrm{d}m} \tag{128}$$

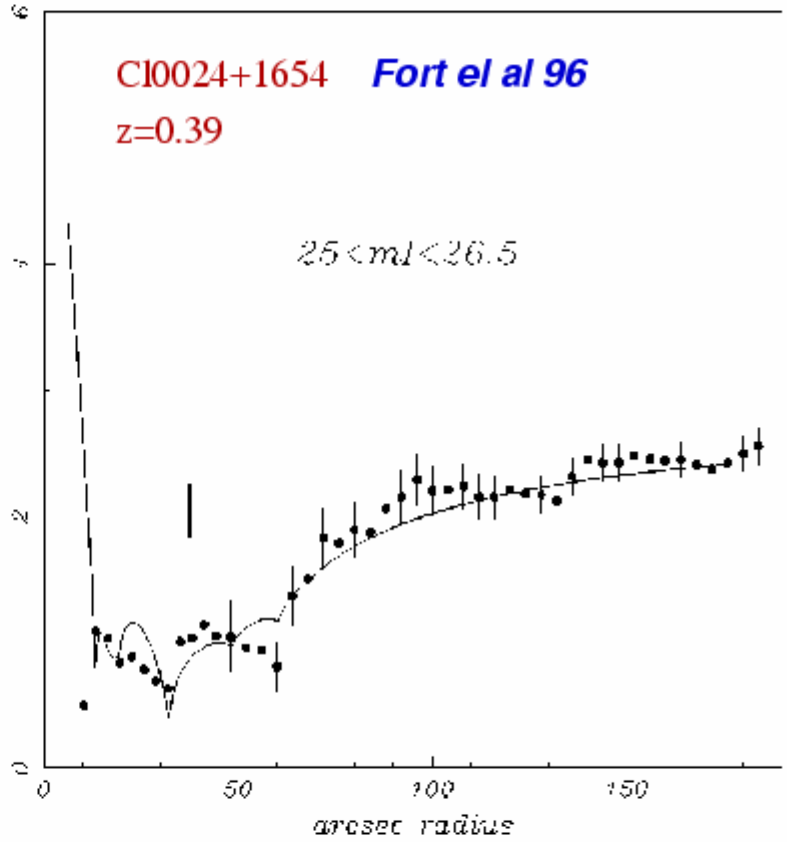
Magnification and galaxy depletion





Magnification and galaxy depletion





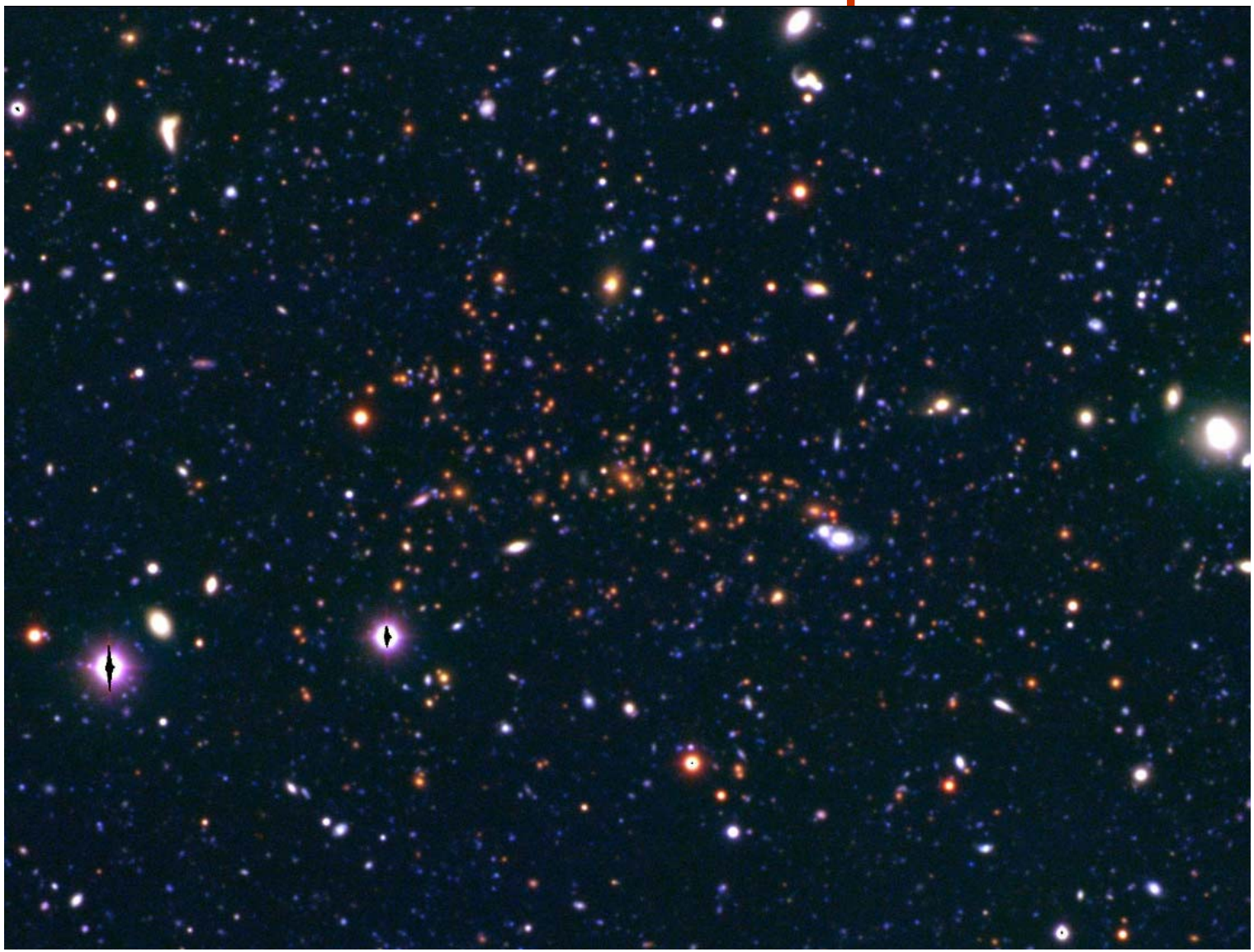
II. Gravitational Lensing:

applications to clusters of galaxies

From cluster galaxy observation to cluster mass map

MS1054
HST

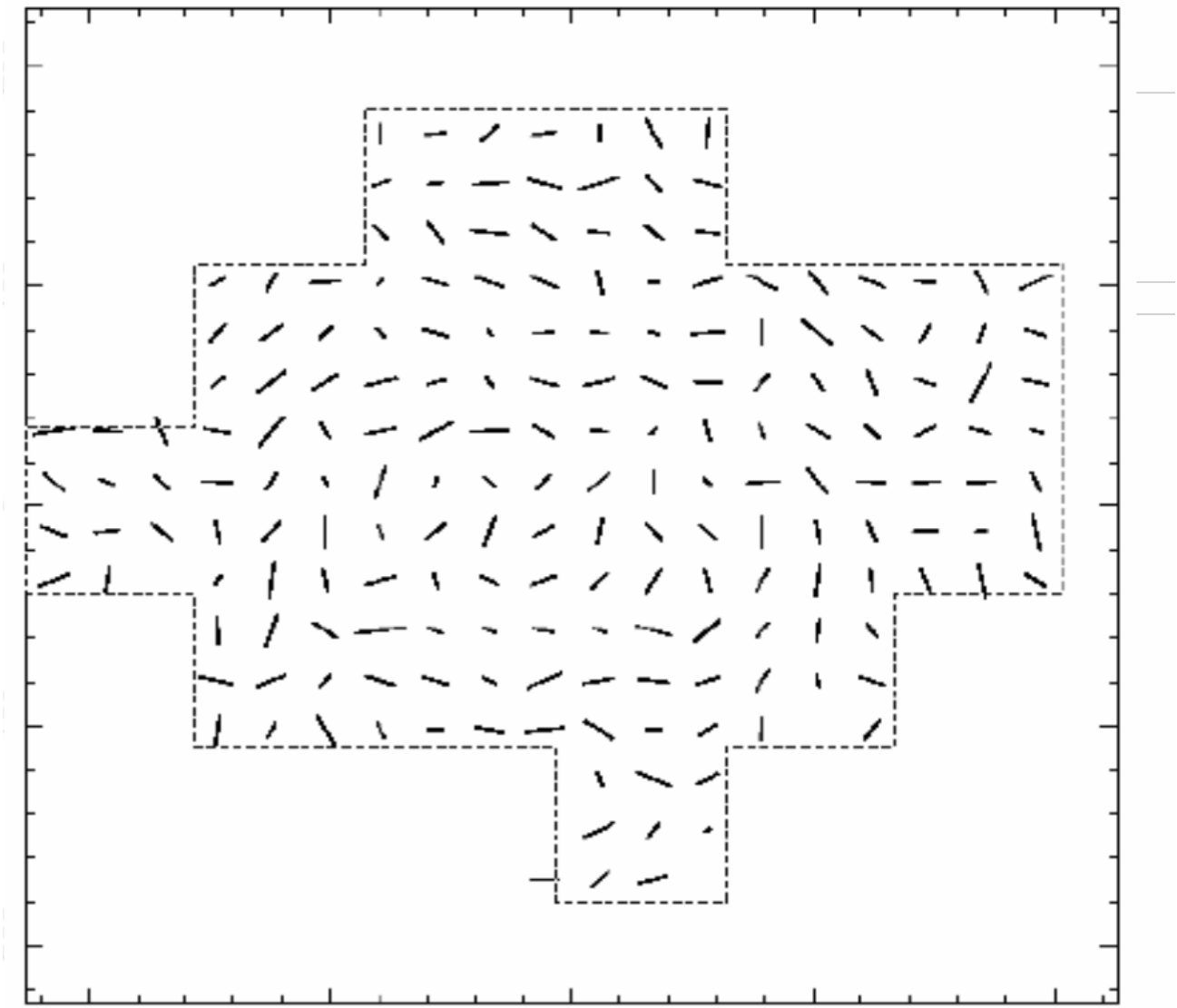
Hoekstra
et al 1998



From cluster galaxy observation to cluster mass map

MS1054
HST

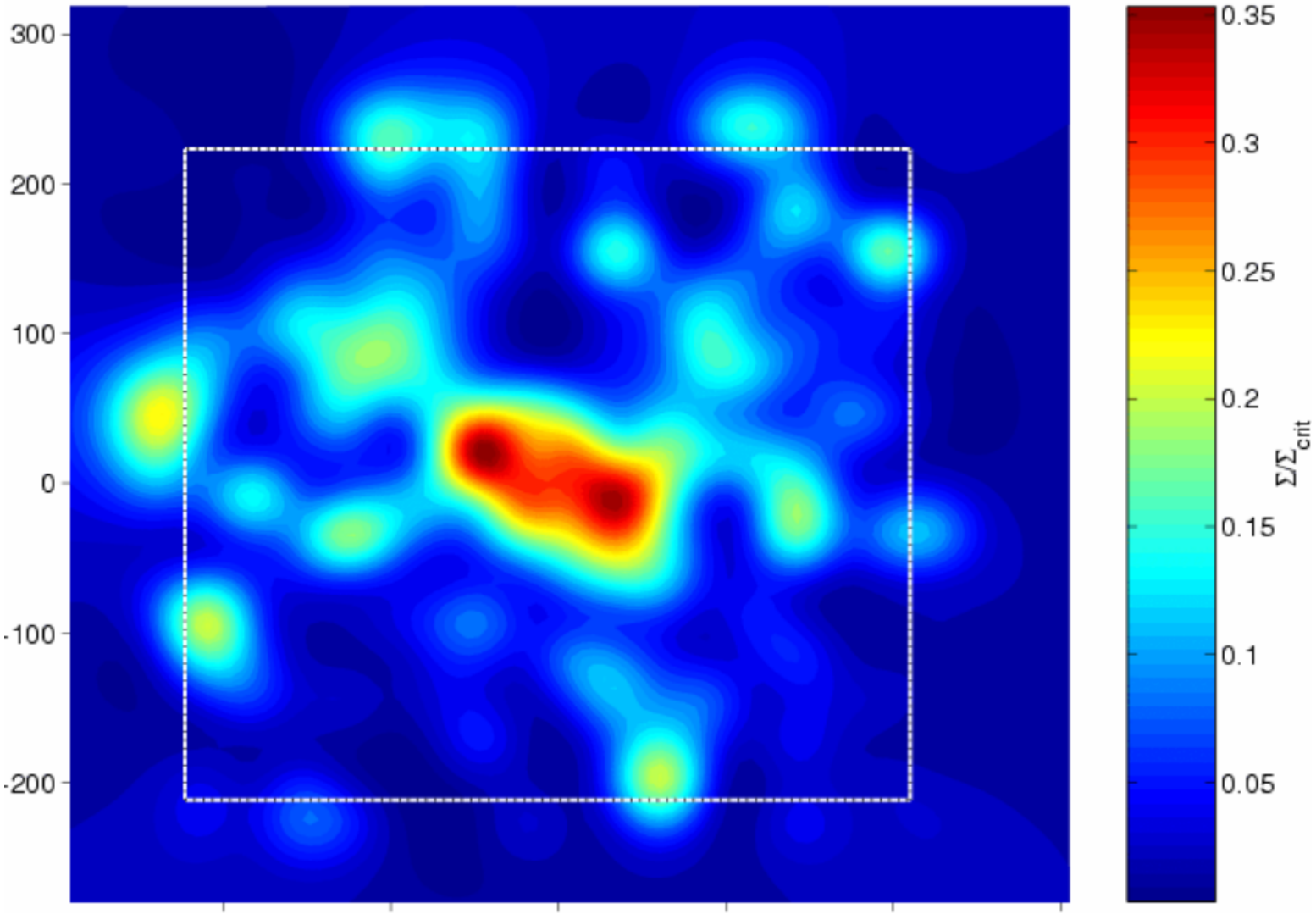
Hoekstra et al
1998

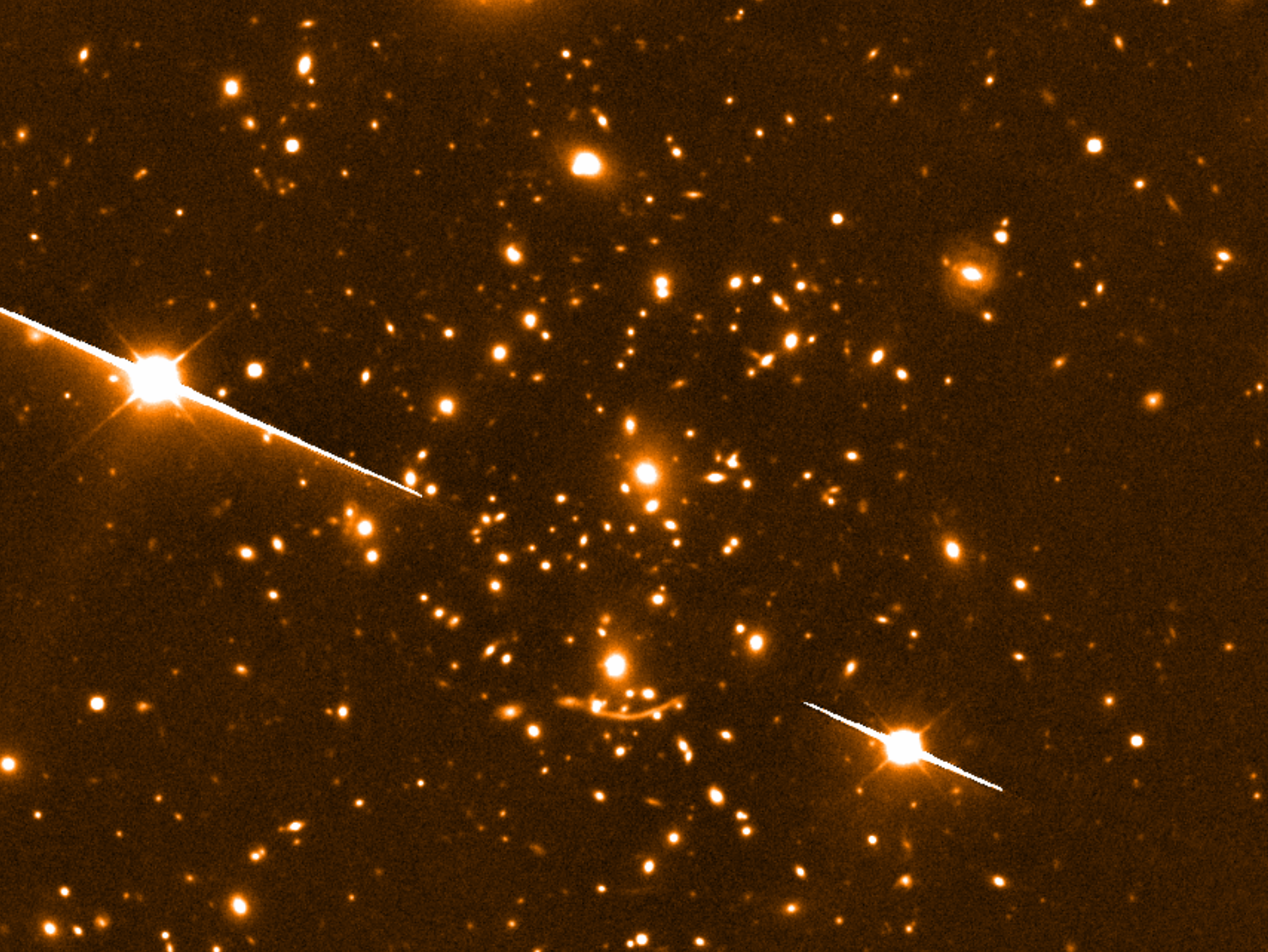


From cluster galaxy observation to cluster mass map

MS1054
HST

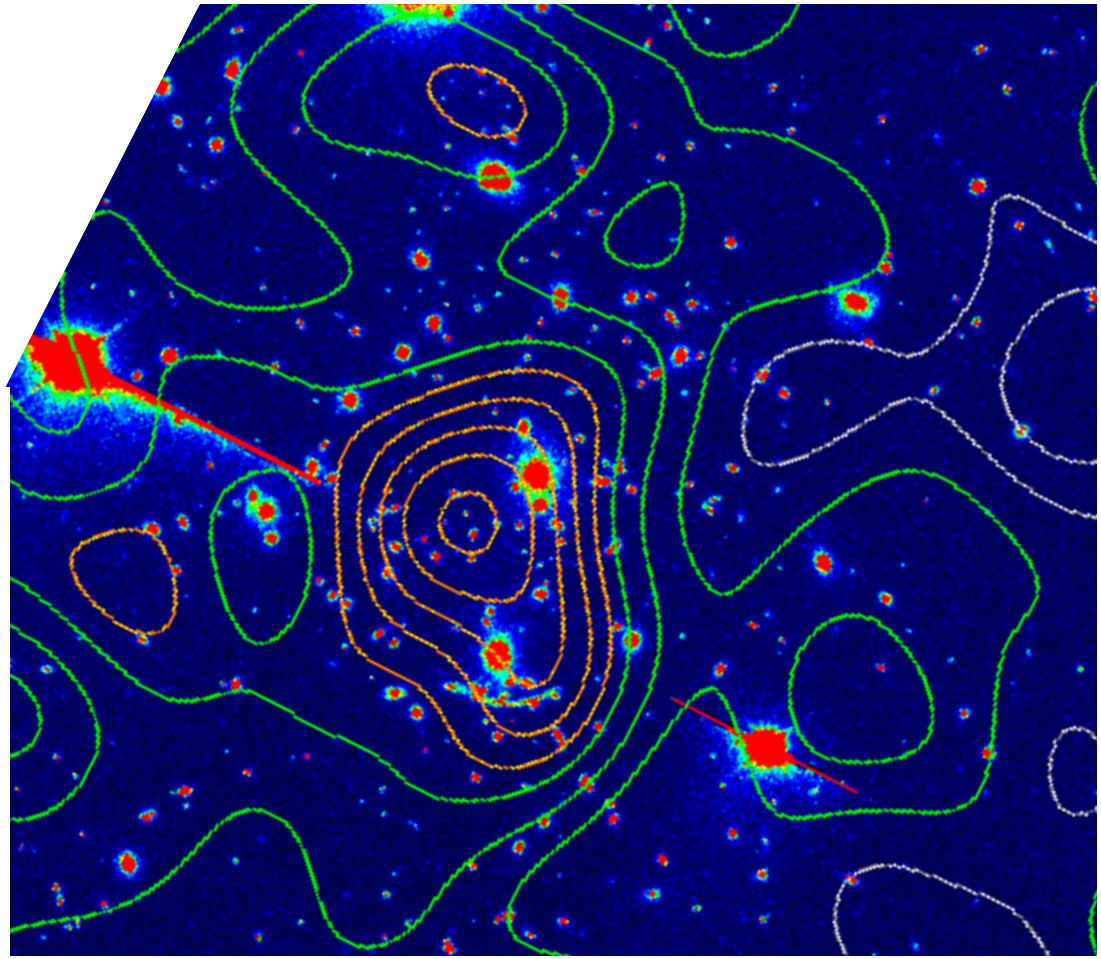
Hoekstra
et al 1998





First giant arc discovered in Abell 370

$Z_{cluster}$ =0.374 ,
Z_{arc}=0.720



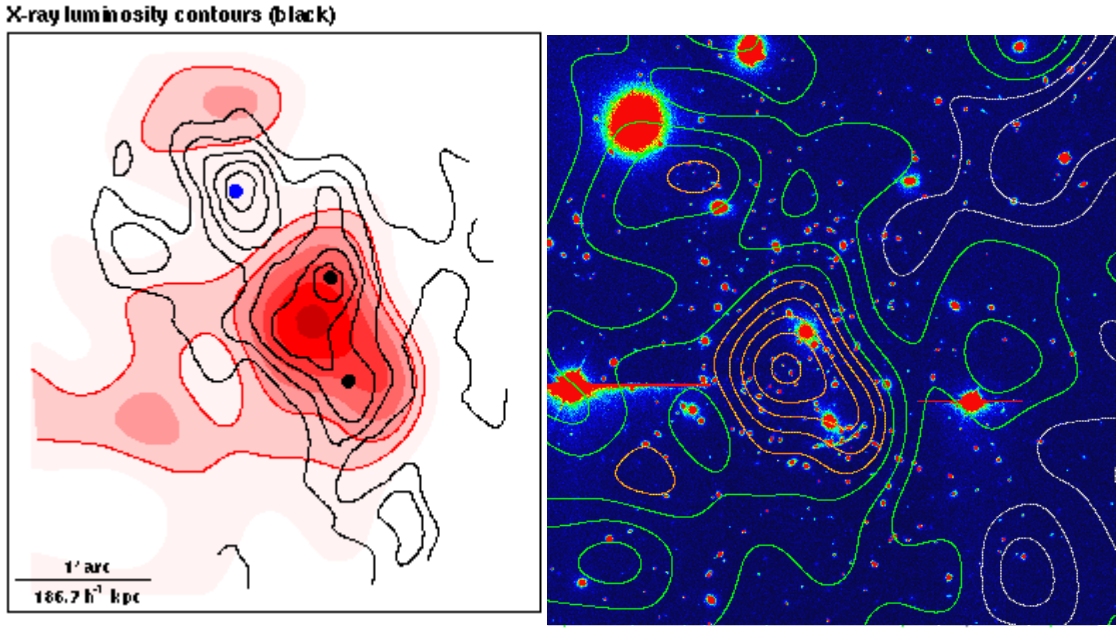
Abell 370

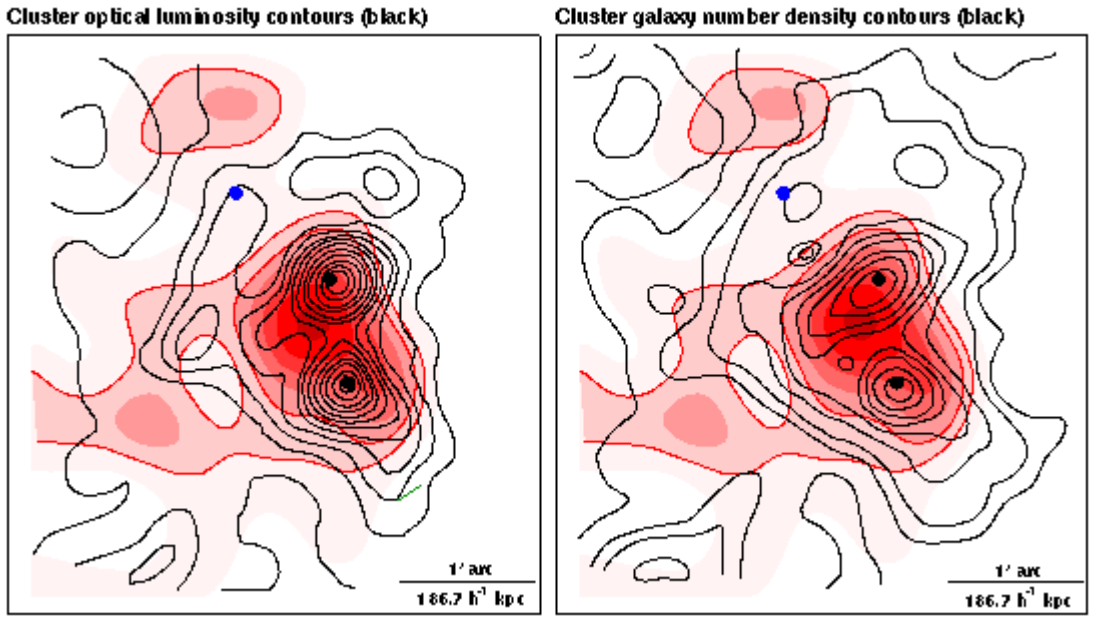
$Z_{cluster}$ =0.374 , Z_{arc}=0.720

Does light trace mass?

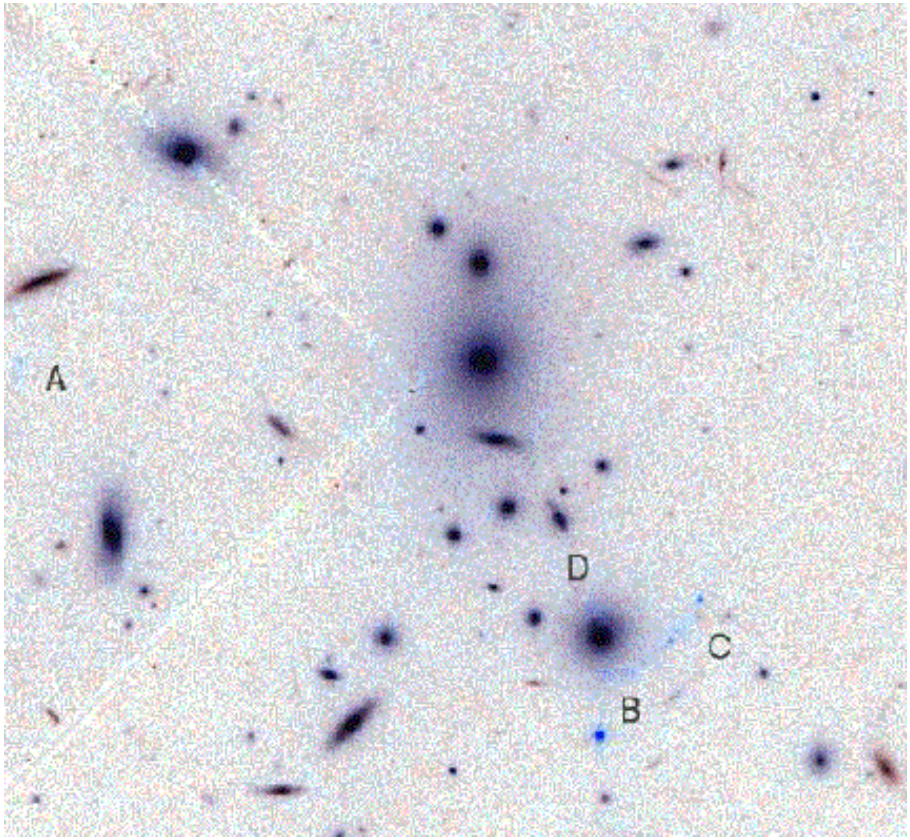
X-ray vs. Mass : yes

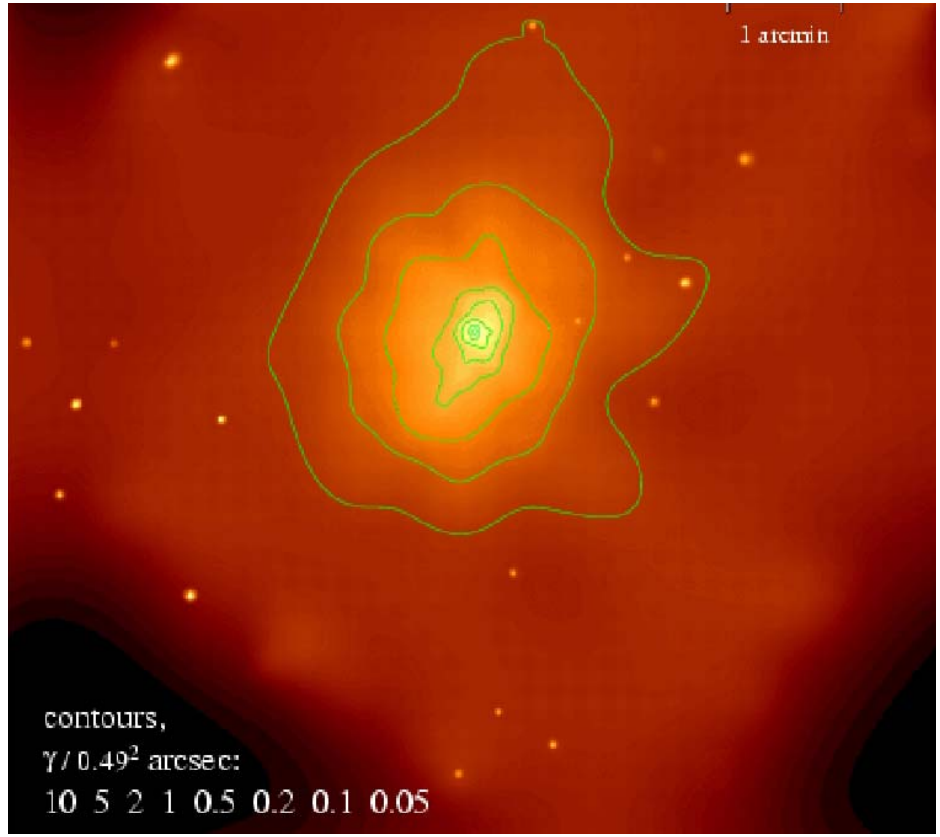
Red galaxies vs. Mass: yes





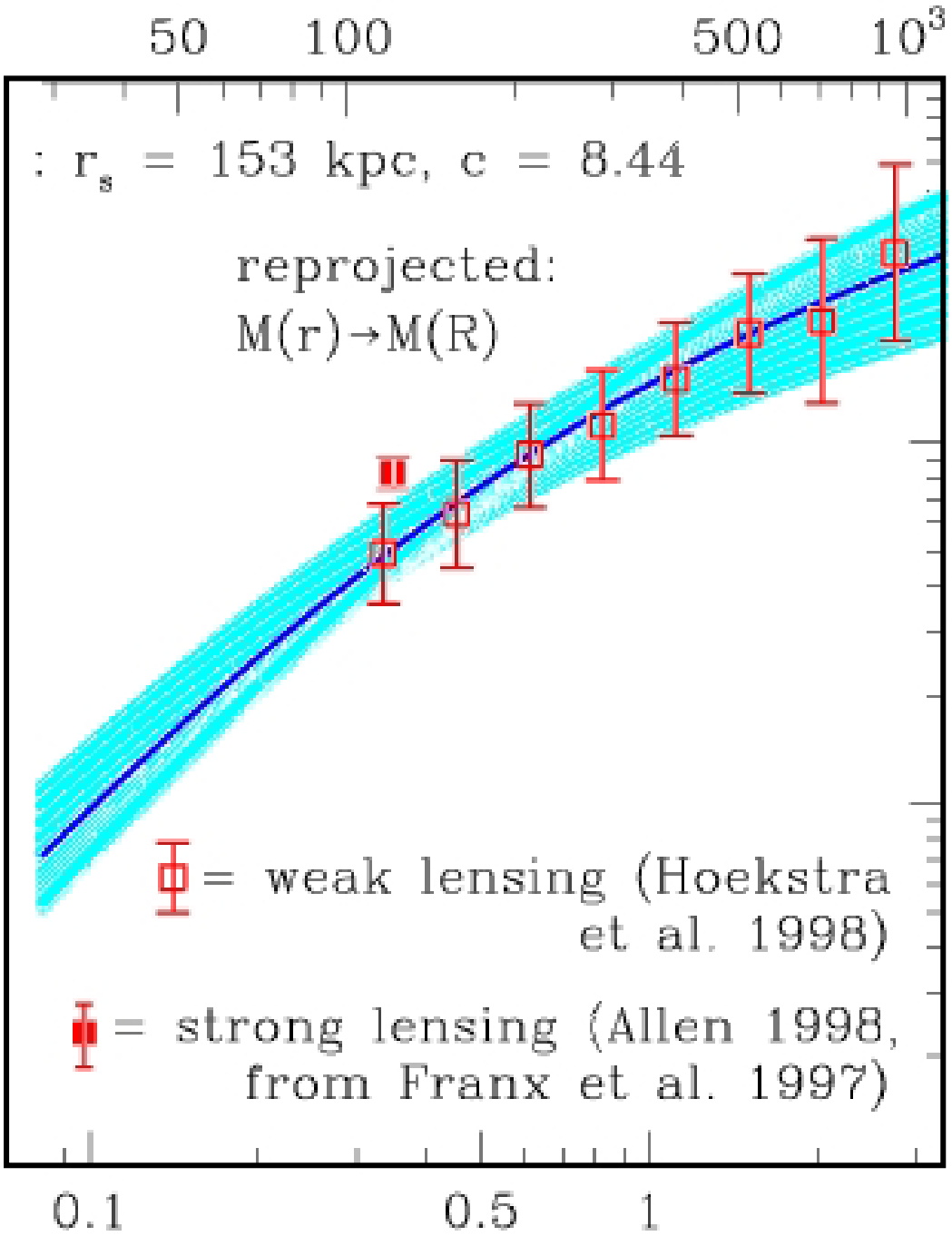
MS1358+6245





HST

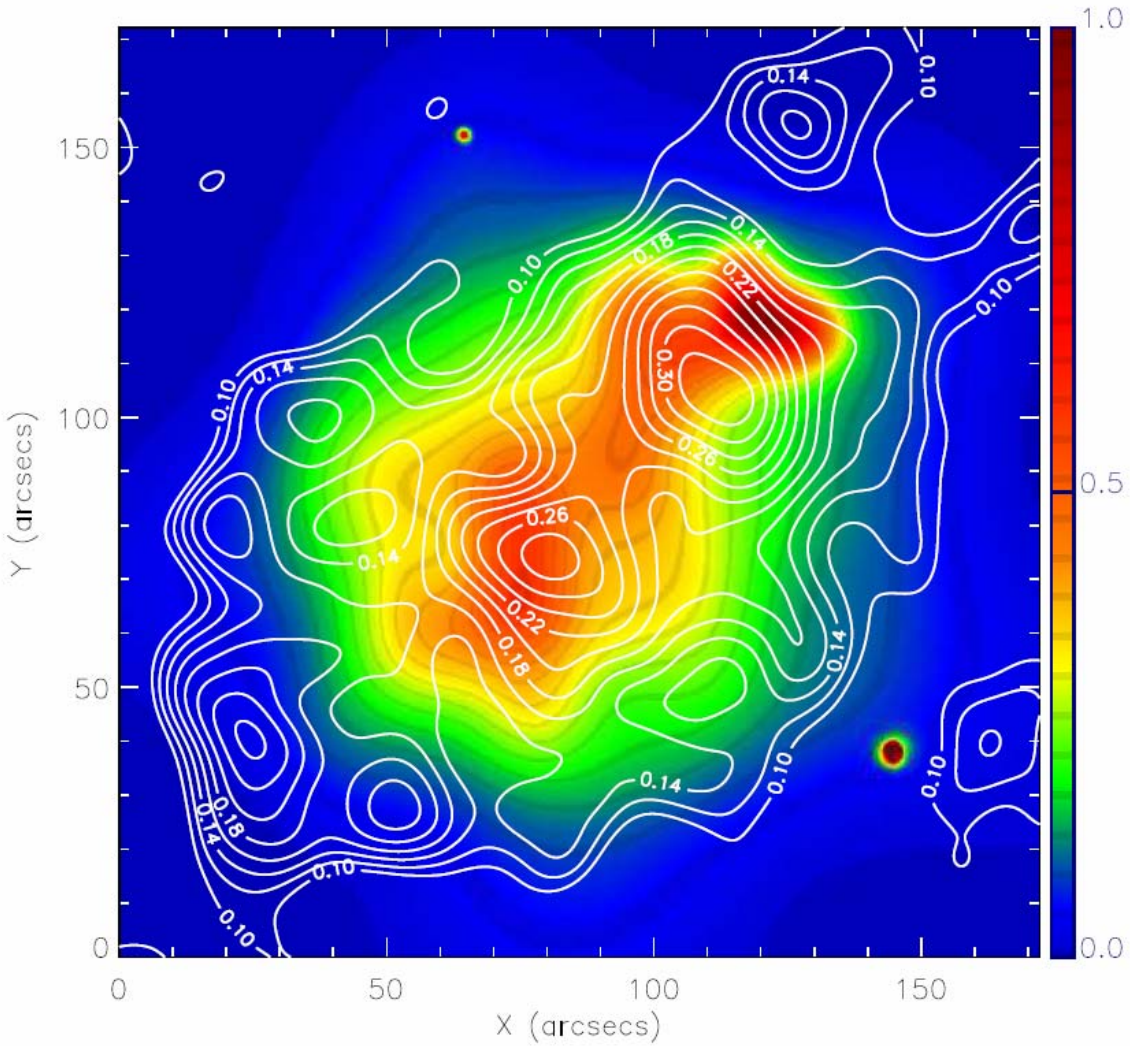
Chandra



X-ray Chandra: baryonic versus dark matter from strong / weak lensing

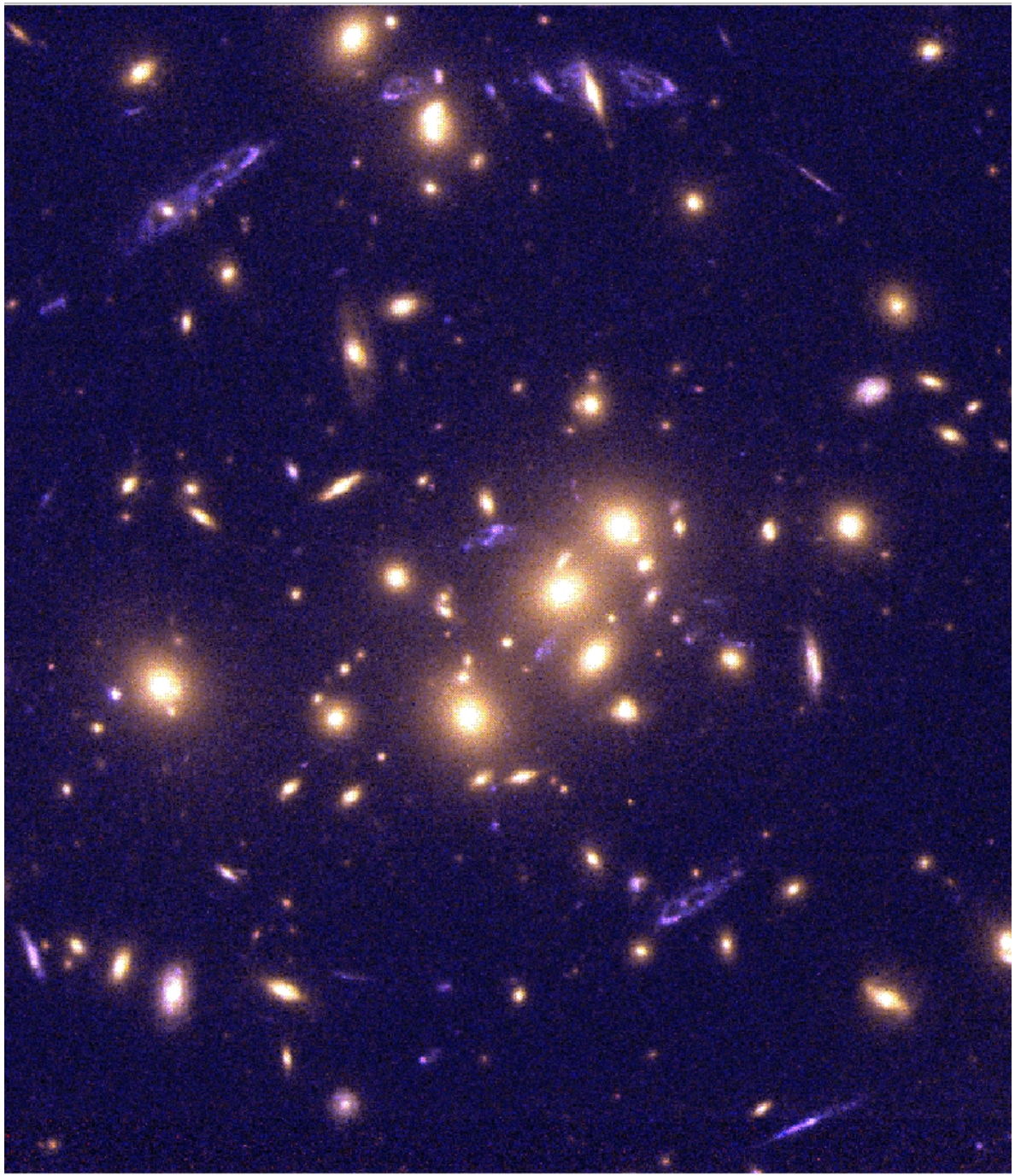
Cluster of galaxies MS1358+6254

MS1054: Chandra and weak lensing

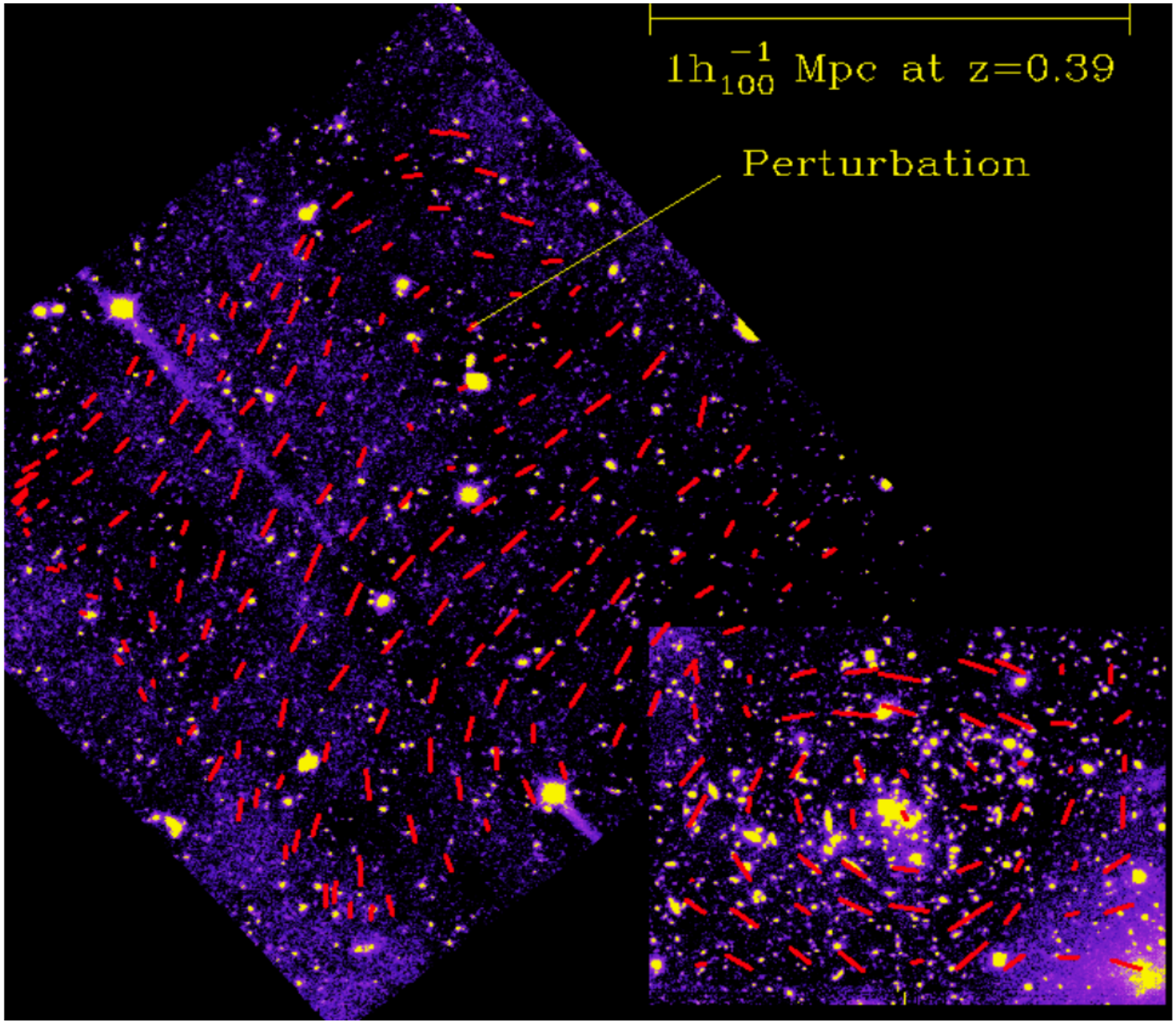


baryonic and dark matter distribution maps

Jee et al 2007

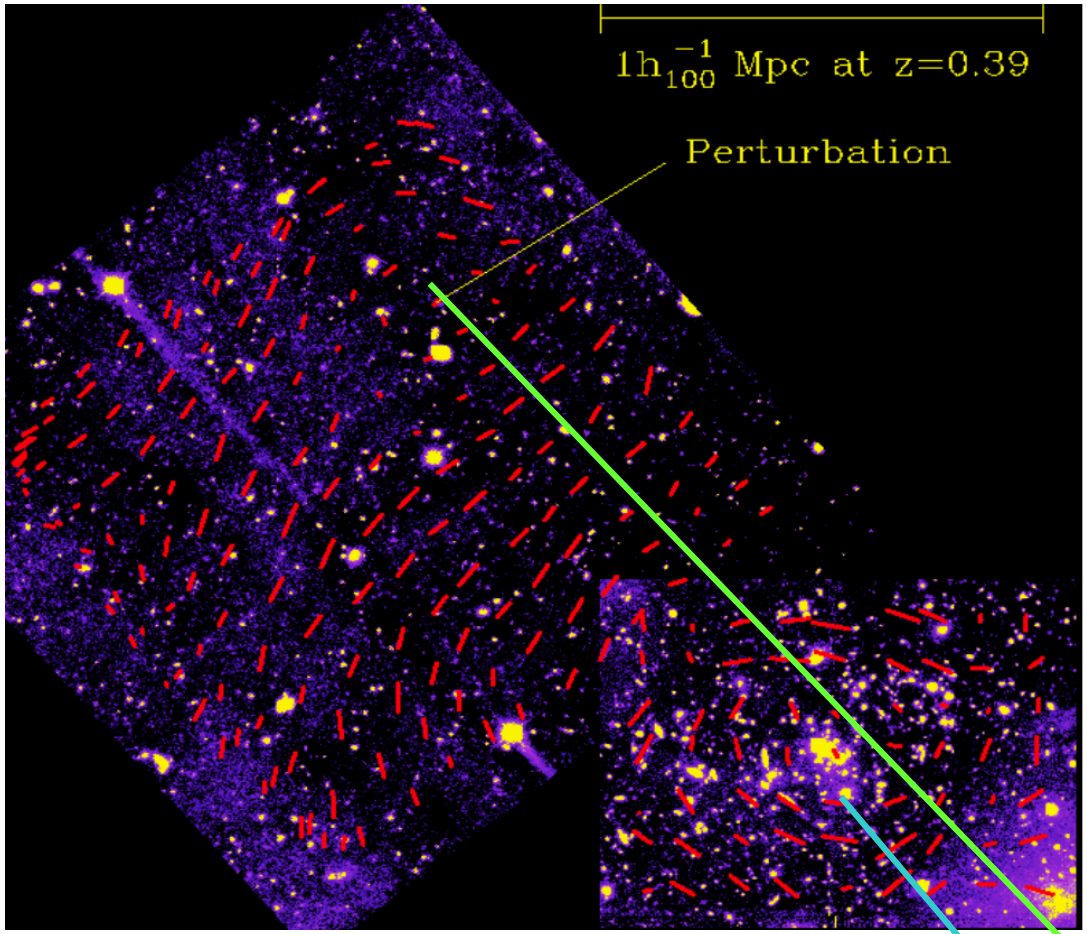


Cl0024+1654
z=0.55



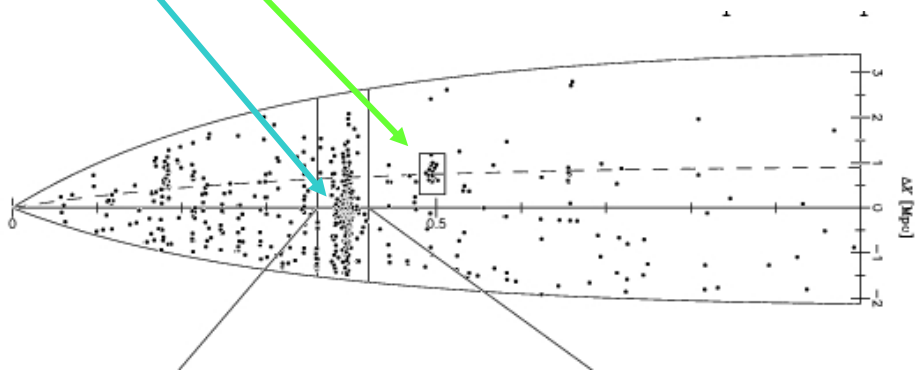
Cl0024+1654
z=0.55

Bonnet, Mellier, Fort 1994



Cl0024+1654
z=0.55

Spectroscopic detection of a group at the location of the WL perturbation



Summary mass from cluster WL reconstruction

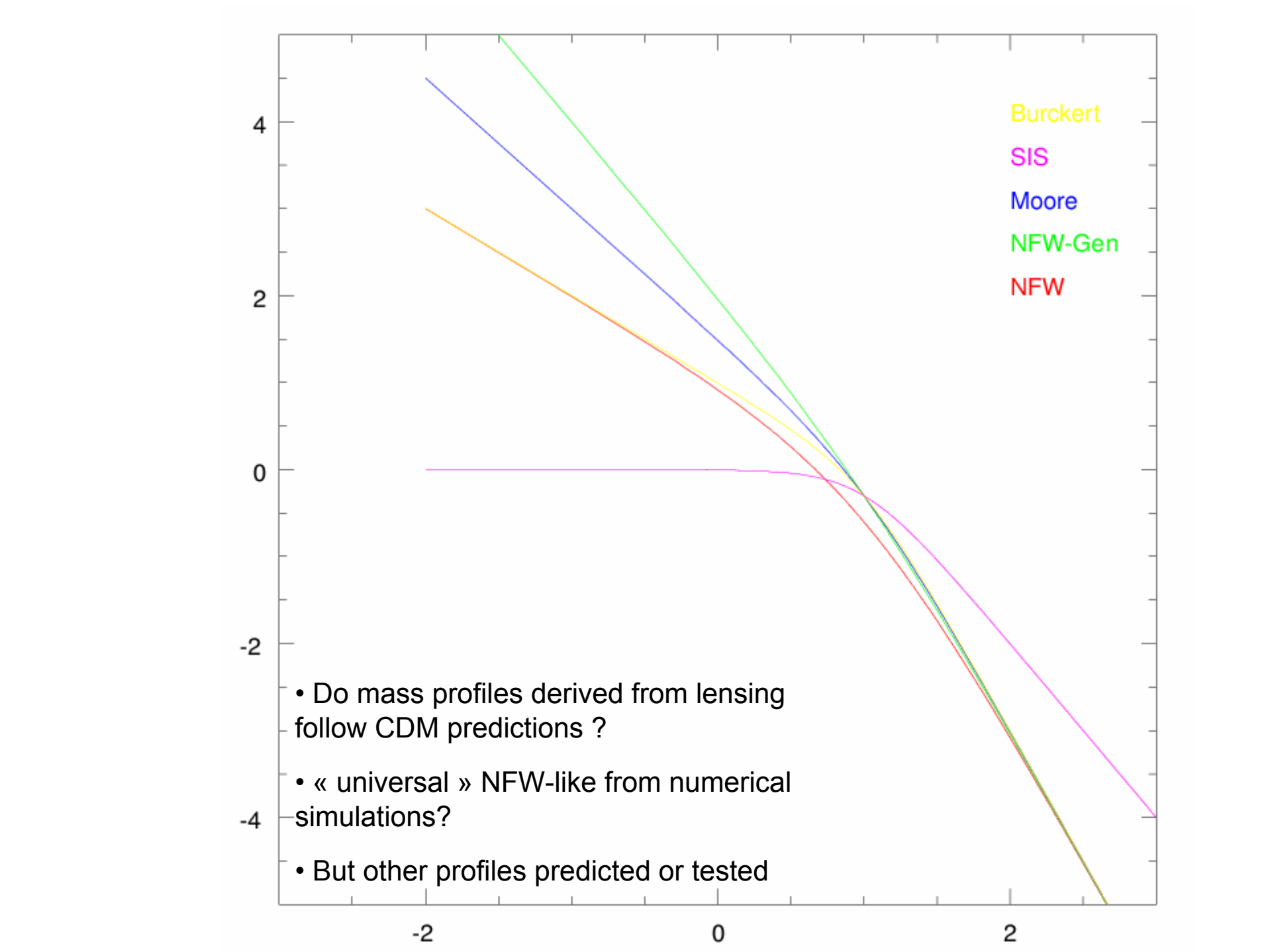
May 2007:
147 clusters
76 publications
Dahle 2007

- **Median σ_v:**
  ~ 1000 km/sec
- **Median *M/L*** :
  ~ 350 *h*

Cluster	z	σ_{obs} (kms^{-1})	σ_{wl} (kms^{-1})	M/L (h_{100})	Scale (h_{100}^{-1} Mpc)	Tel.	Ref.
A2218	0.17	1370	1000-1400	$\approx$ 300	0.5	CFHT	Squires et al (19)
			-	310	0.1	HST	Smail et al (20)
A1689	0.18	2400	1200-1500	-	0.5	CTIO	Tyson et al (21)
			-	400	1.0	CTIO	Tyson & Fischer (22)
			1030	-	1.0	ESO/2.2	Clowe & Schneider (23)
A2163	0.20	1680	740-1000	300	0.5	CFHT	Squires et al (25)
A2390	0.23	1090	$\approx$1000	320	0.5	CFHT	Squires et al (24)
< 8 *clusters* $>$	$< 0.2 >$	-		$< 295 >$	1.0	CTIO	Wittman et al (26)
MS1455+22	0.26	1133	-	1080	0.4	WHT	Smail et al (27)
AC118	0.31	1950	-	370	0.15	HST	Smail et al (20)
MS1008-1224	0.31	1054	940	340	0.5	VLT	Athreya et al (28)
			850	$\approx$ 320	0.5	VLT	Lombardi et al (29)
MS2137-23	0.31	-	950	300	0.5	VLT	Gavazzi et al (in prep.)
MS1358+62	0.33	940	780	180	0.75	HST	Hoekstra et al (30)
MS1224+20	0.33	802	-	$\approx$ 800	1.0	CFHT	Fahlman et al (31)
			1300	890	1.0	MDM2.2	Fischer (32)
Q0957+56	0.36	715	-	-	0.5	CFHT	Fischer et al (33)
Cl0024+17	0.39	1250	-	150	0.15	HST	Smail et al (20)
			1300	$\approx$900	1.5	CFHT	Bonnet et al (10)
Cl0939+47	0.41	1080	-	120	0.2	HST	Smail et al (20)
			-	$\approx$250	0.2	HST	Seitz et al (34)
Cl0302+17	0.42	1080		80	0.2	HST	Smail et al (20)
RXJ1347-11	0.45	-	1500	400	1.0	CTIO	Fischer & Tyson (35)
3C295	0.46	1670	1100-1500	-	0.5	CFHT	Tyson et al (21)
			-	330	0.2	HST	Smail et al (20)
Cl0412-65	0.51	-	-	70	0.2	HST	Smail et al (20)
Cl1601+43	0.54	1170	-	190	0.2	HST	Smail et al (20)
MS0016+16	0.55	1230	-	180	0.2	HST	Smail et al (20)
			740	740	0.6	WHT	Smail (36)
			800	-	0.6	Keck	Clowe et al (6)
MS0451	0.55	1371	980	-	0.6	Keck	Clowe et al (6)
Cl0054-27	0.56	-	-	400	0.2	HST	Smail et al (20)
MS2053	0.59	820	730	-	0.5	Keck	Clowe et al (6)
			886	360	0.5	HST	Hoekstra et al (40)
MS1137+60	0.78	884	1190	270	0.5	Keck	Clowe et al (37,6)
RXJ1716+67	0.81	1522	-	190	0.5	Keck	Clowe et al (37)
			1030	-	0.5	Keck	Clowe et al (6)
MS1054-03	0.83	1360	1100-2200	350-1600	0.5	UH2.2	Luppino & Kaiser (38)
			1310	250-500	0.5	HST	Hoekstra et al (39)
			1080	-	0.5	Keck	Clowe et al (6)
< 10 *clusters* $>$	$< 0.5 > -$	-			1.0	VLT	White et al (2002)
EDICS	$- < 0.8 >$	-					and Clowe et al (2002)

Mass profile

NFW, SIS, POW, Generalised-NFW, Moore, Buckert, de Vaucouleur??


Burckert
SIS
Moore
NFW-Gen
NFW
4
2
0
-2
-4
-2
0
2
• Do mass profiles derived from lensing follow CDM predictions ?
• « universal » NFW-like from numerical simulations?
• But other profiles predicted or tested

The Navarro-Frenk-White profile

$$\rho(r) = \frac{\delta_c \ \rho_{rc}(z)}{(r/r_s)\ (1 + r/r_s)^2}$$

$$\rho_{rc}(z) = \frac{3H(z)^2}{8\pi G}$$

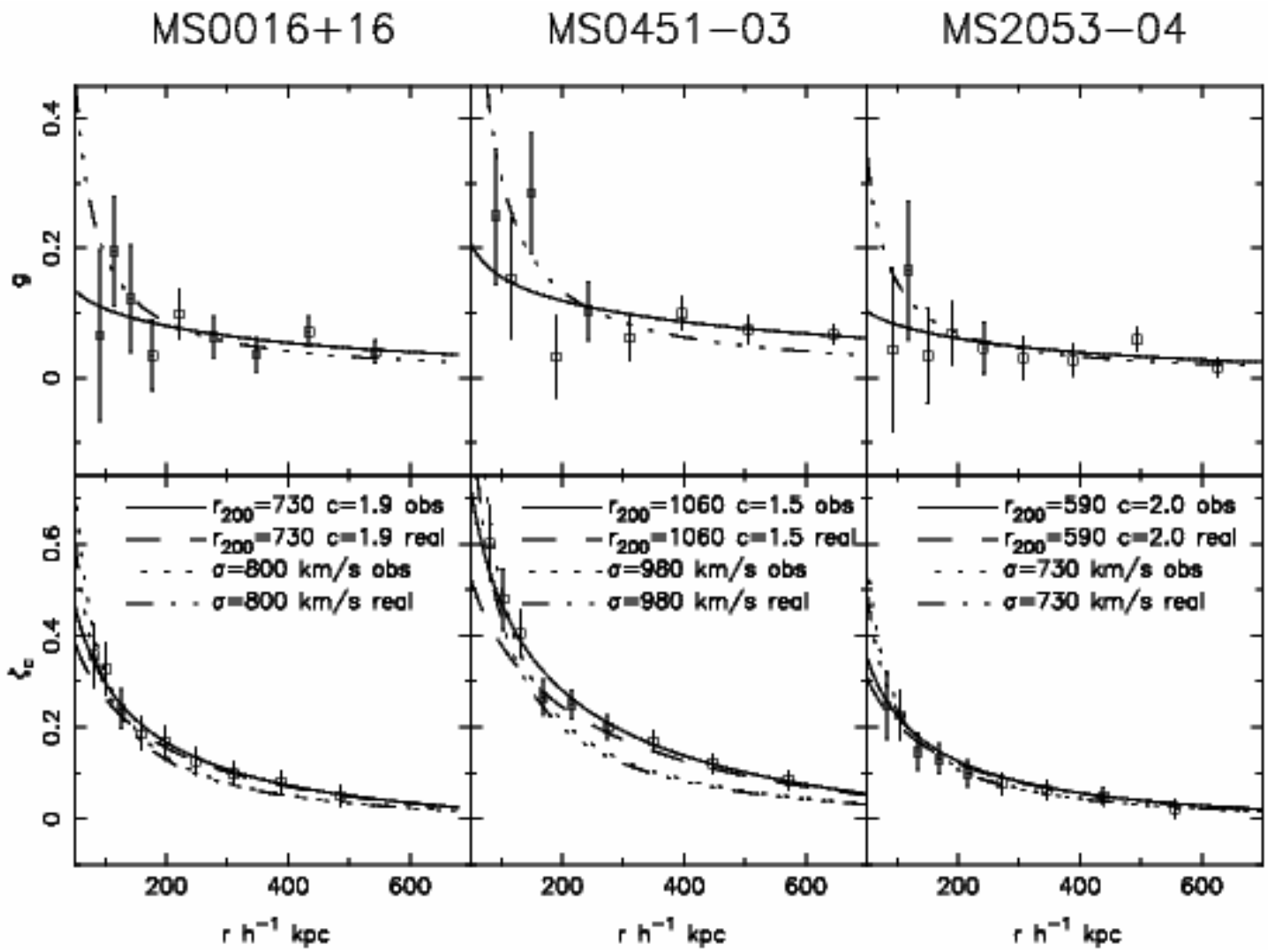
Virial radius r_{200}: radius inside which mass density equals 200 $\rho_{rc}(z)$

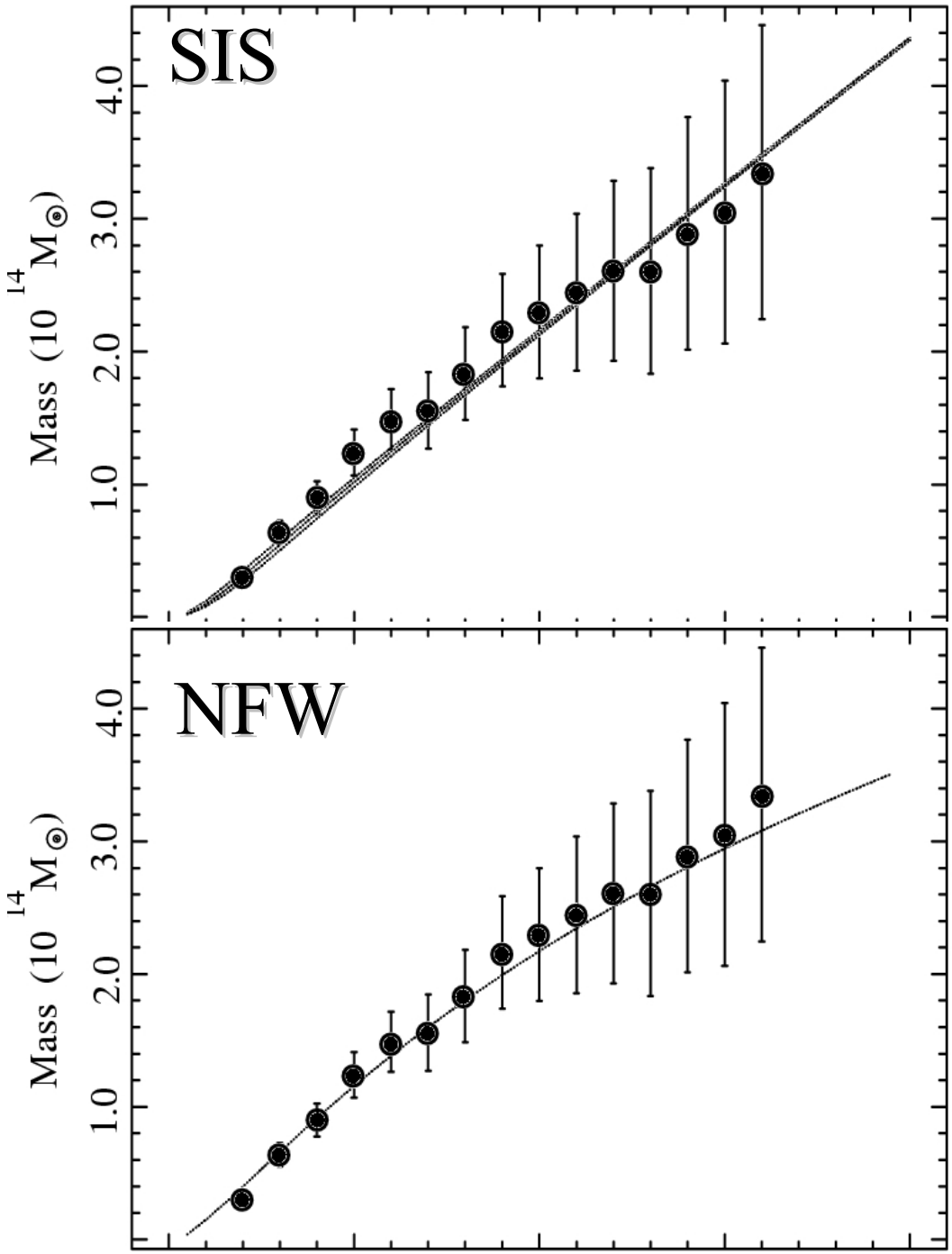
C = concentration parameter

$$\delta_c = \frac{200}{3} \frac{C^3}{\ln(1+C) - C/(1+C)}$$

NFW or SIS ?

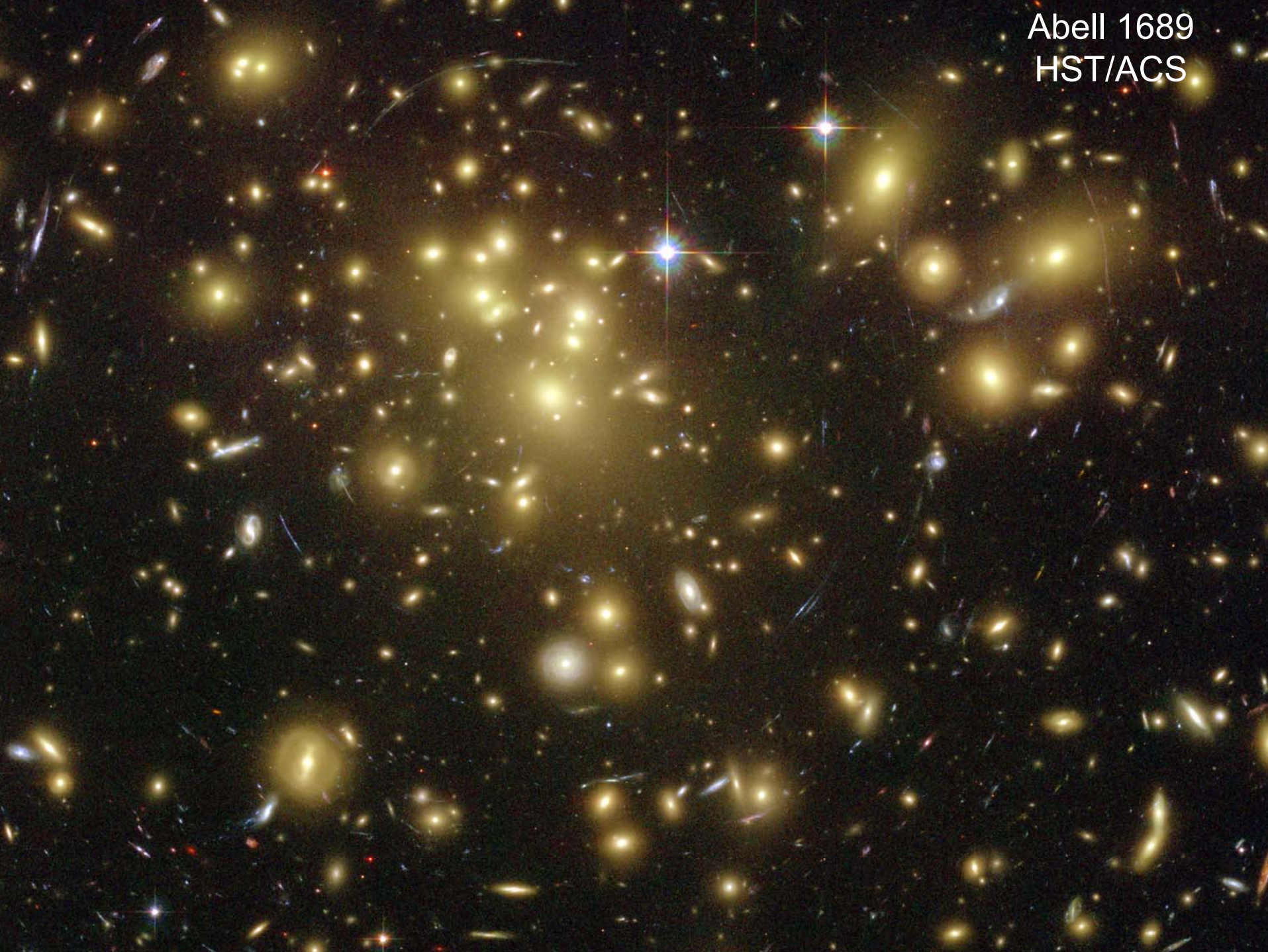
Clowe et al 2004



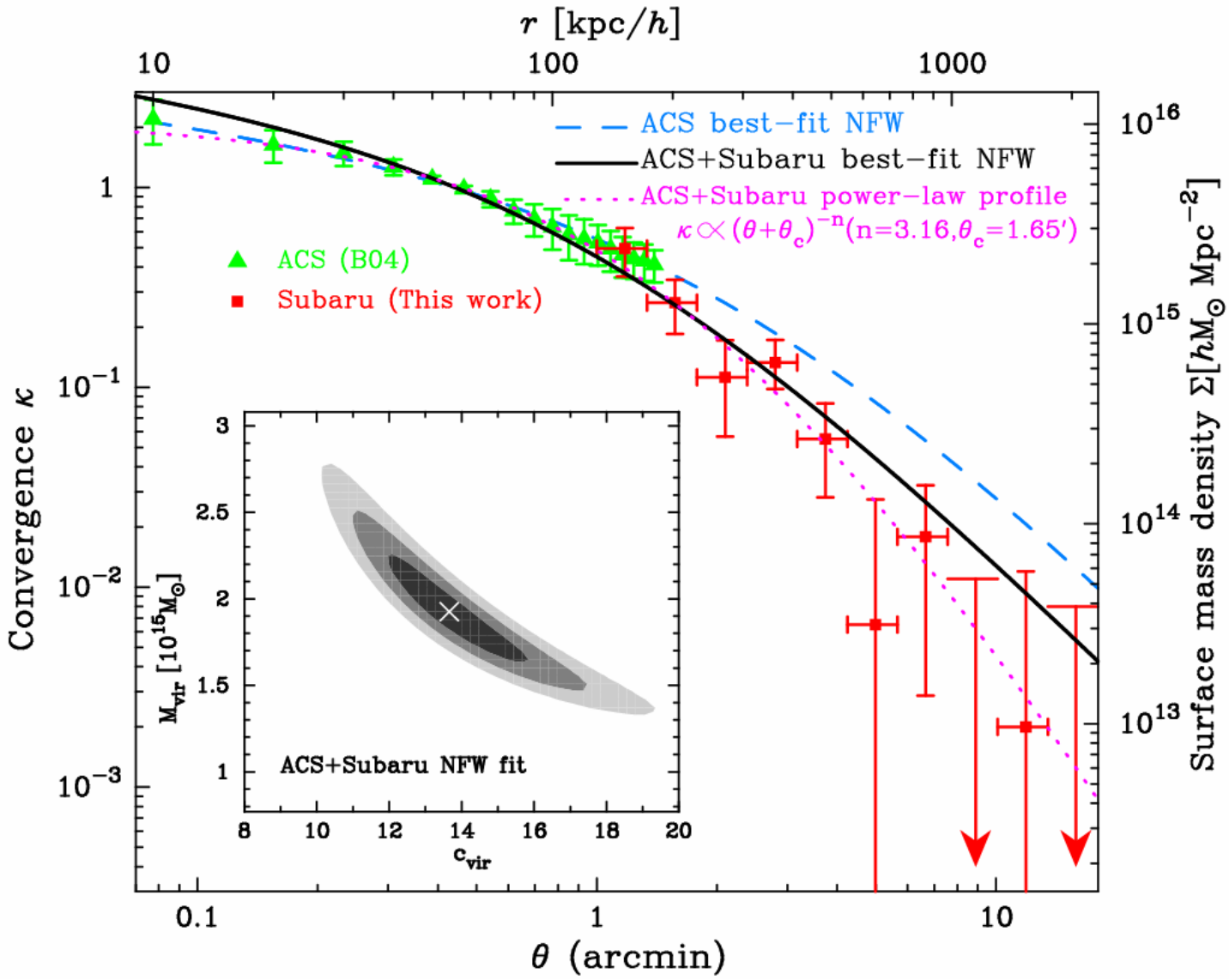


MS1008-1224:

Athreya et al 2002 ,
Lombardi et al 2001


Abell 1689
HST/ACS

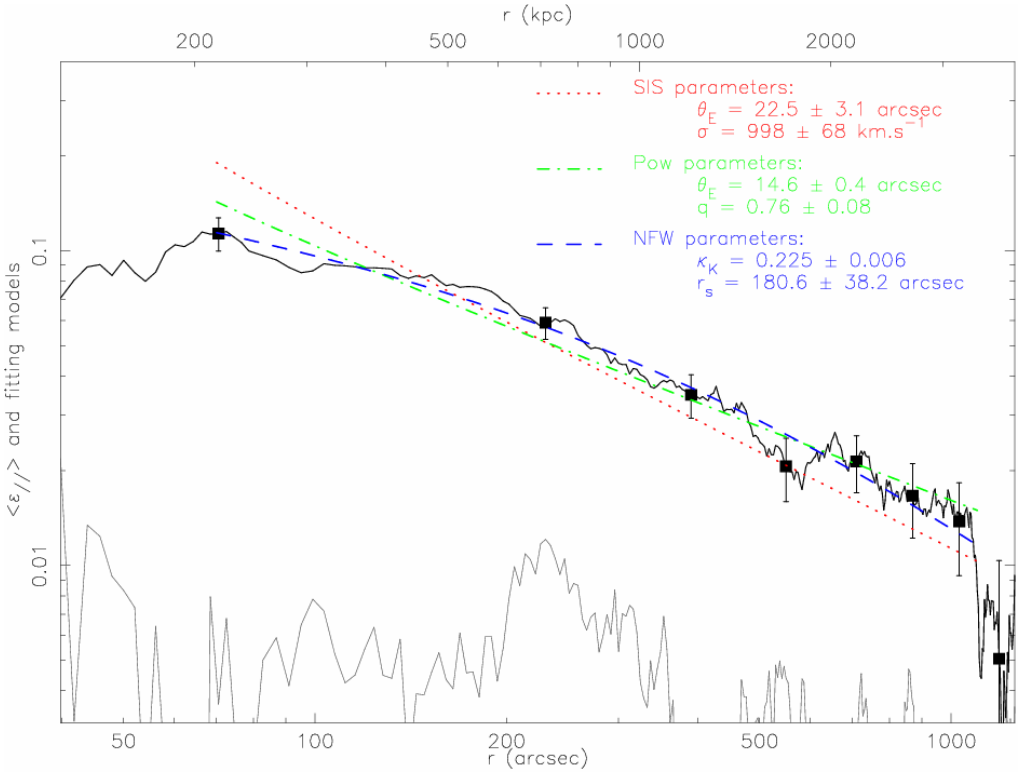
Abell 1689
HST/ACS

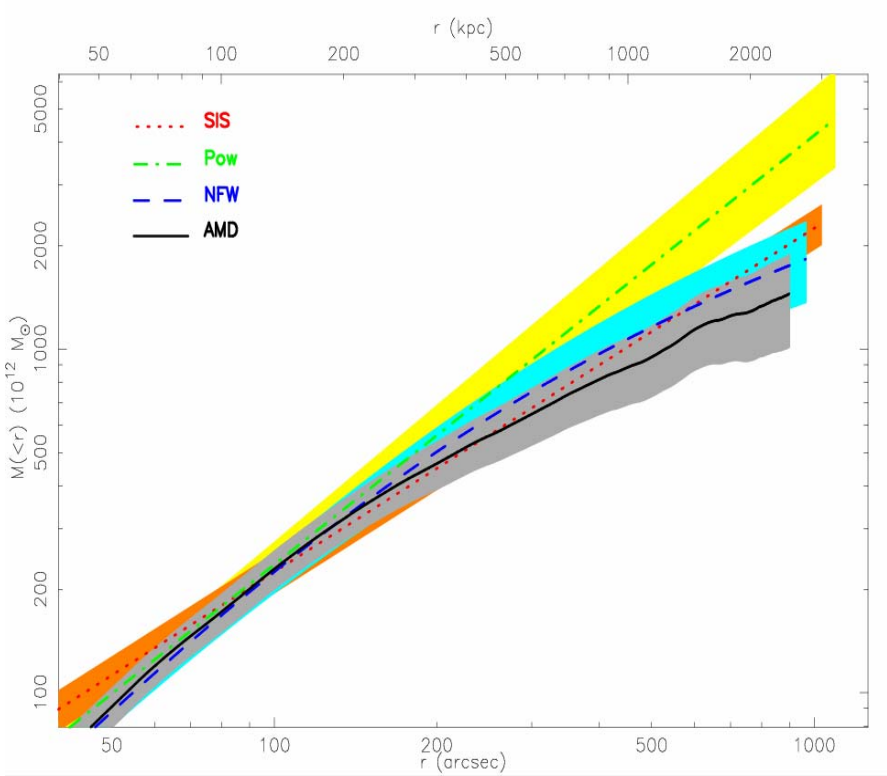


Broadhurst et al 2005

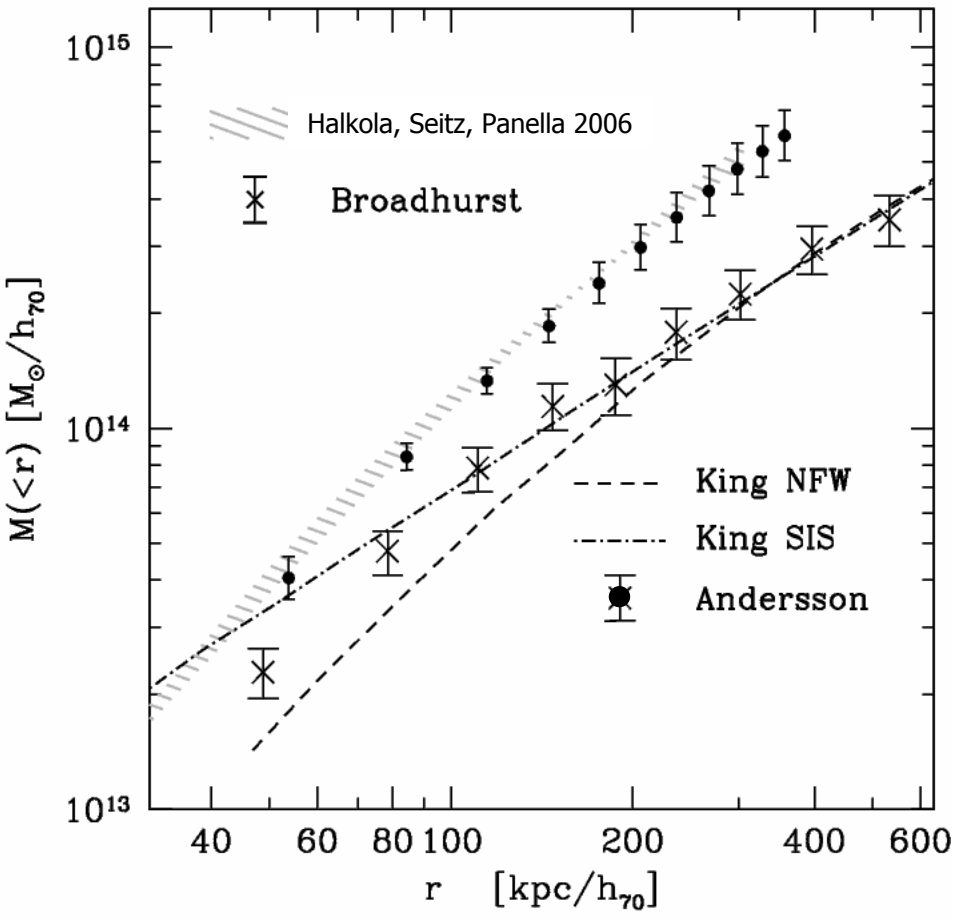
Abell 1689 ; CFH12k

Bardeau et al 2004





Abell 1689 HST/ACS revisited



Halkola, Seitz, Panella 2006

NFW Parameters

Method	C	r_{200} (Mpc)	Reference
SL	6.0±0.5	2.82±0.11	this work
SL	$6.5^{+1.9}_{-1.6}$	2.02	Broadhurst et al. 2005a
X-ray	$7.7^{+1.7}_{-2.6}$	1.87±0.36	Andersson & Madejski 2004
WL	4.8	1.84	King et al. 2002

NSIE Parameters

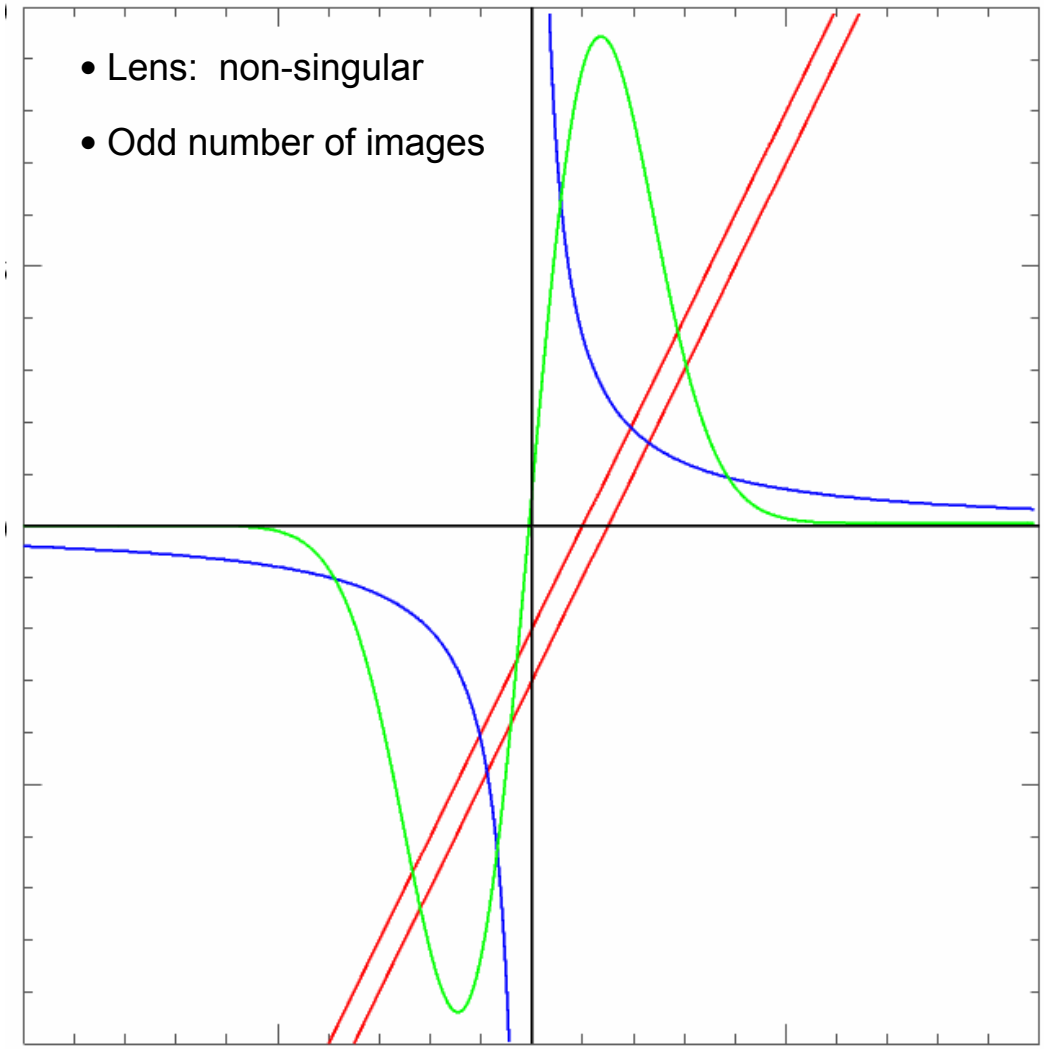
Method	σ (km/s)	r_c (kpc)	Reference
SL	1514±18	71±5	this work
SL	1390	60	Broadhurst et al. 2005a
X-ray	918±27	SIS	Andersson & Madejski 2004
X-ray	1190	27	Andersson & Madejski 2004*
WL	998^{+33}_{-42}	SIS	King et al. 2002
LOSVD	1429^{+145}_{-96}	-	Girardi et al. 1997

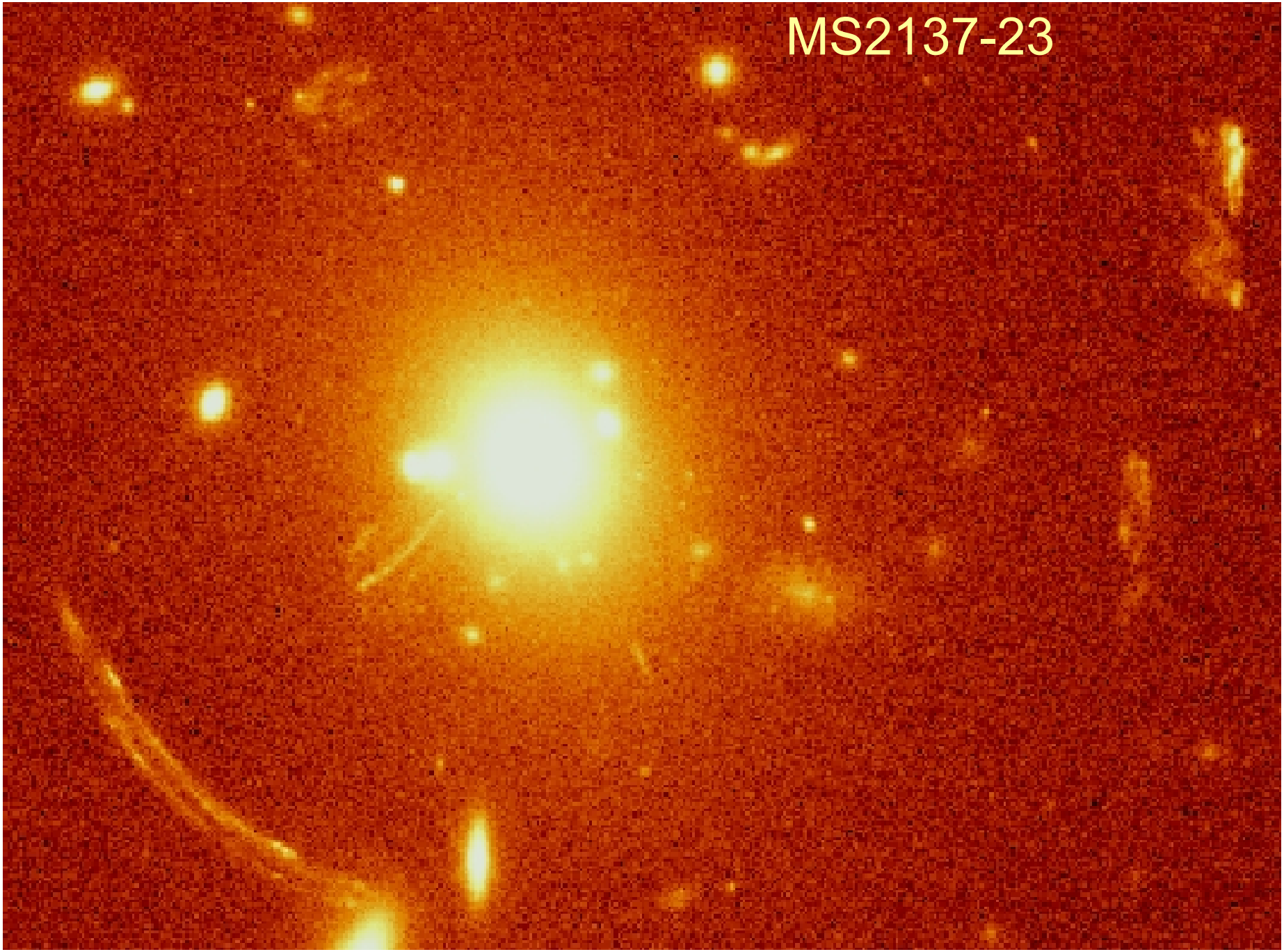
* data from Andersson & Madejski 2004, fitting done in this work.

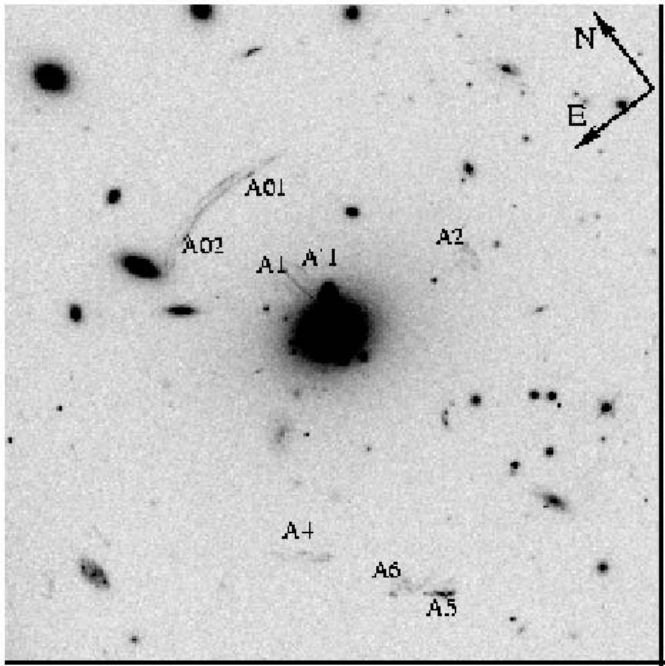
Table 6. Comparison between mass estimates for Abell 1689 from different methods. The mass measured by Andersson & Madejski (2004) are underestimates of the total mass if the cluster is undergoing a merger. For our work the mass at r=0.25 Mpc/h_{70} is an extrapolation since the multiple images do not extend to such large clustercentric radii.

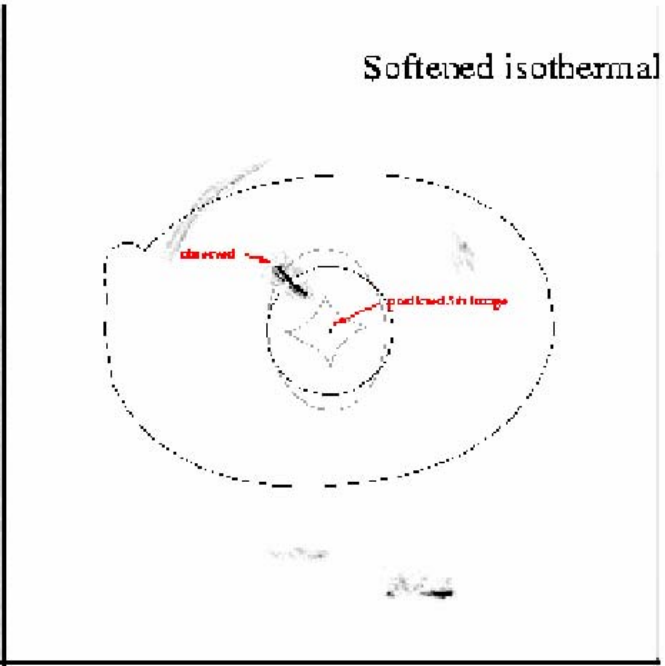
M(<r) (10^{15} $M_\odot$ h_{100}^{-1})	r (Mpc h_{100}^{-1})	Reference
0.14 ± 0.01	0.10	this work, Model III
0.082 ± 0.013	0.10	Andersson & Madejski (2004)
0.43 ± 0.02	0.24	Tyson & Fischer (1995)
0.20 ± 0.03	0.25	Andersson & Madejski (2004)
0.37 ± 0.06	0.25	this work, Model III
0.48 ± 0.16	0.25	Dye et al. (2001)

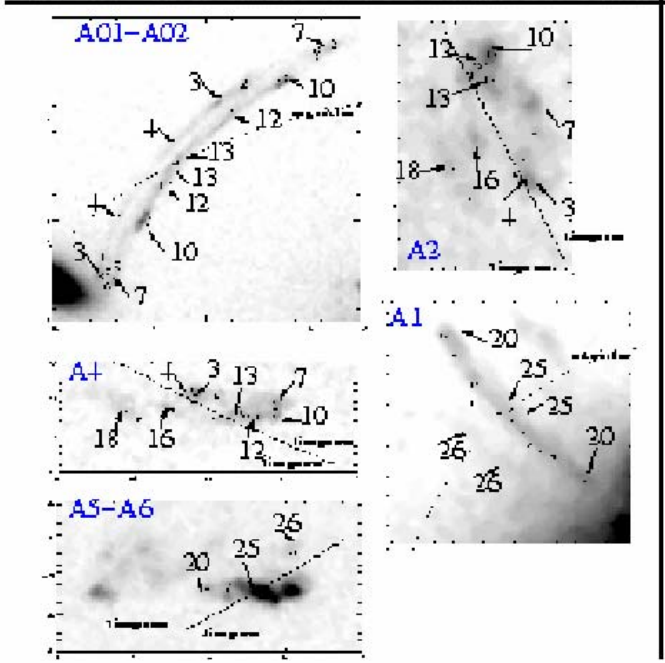
Using radial arcs to constrain mass profile

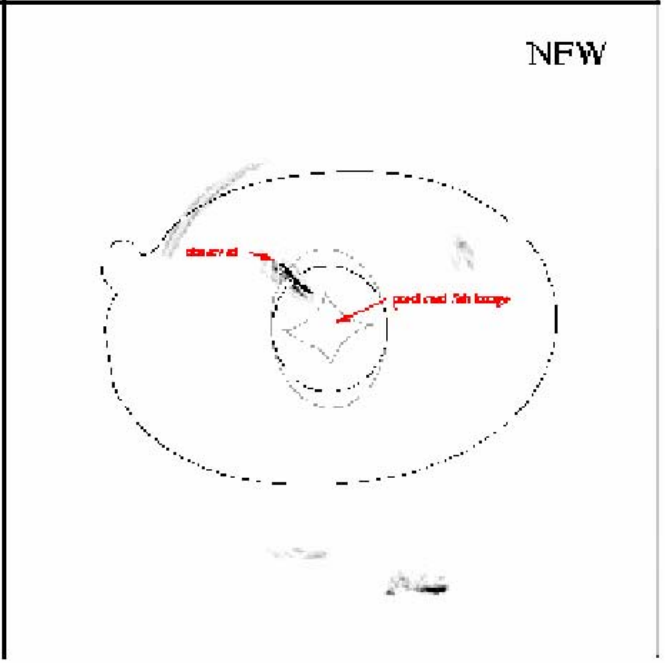



MS2137-23





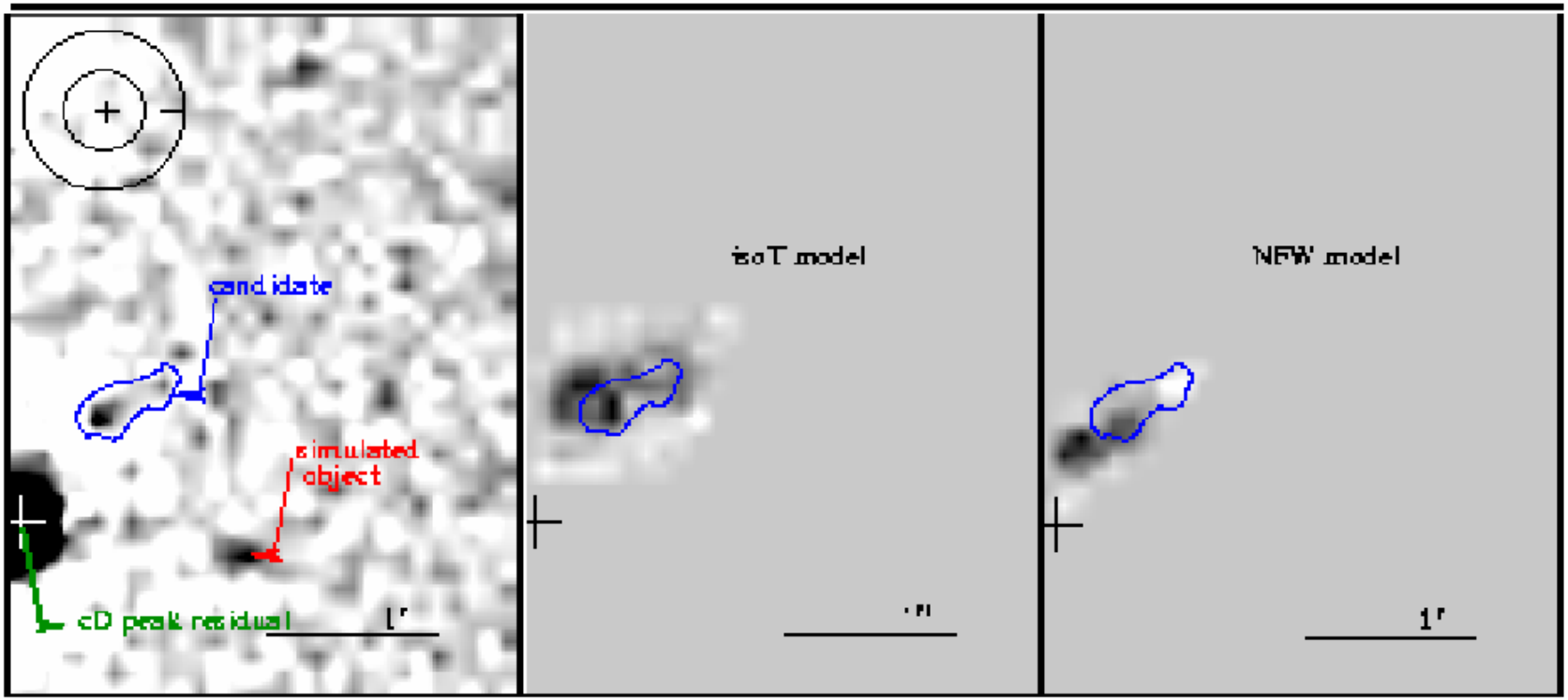




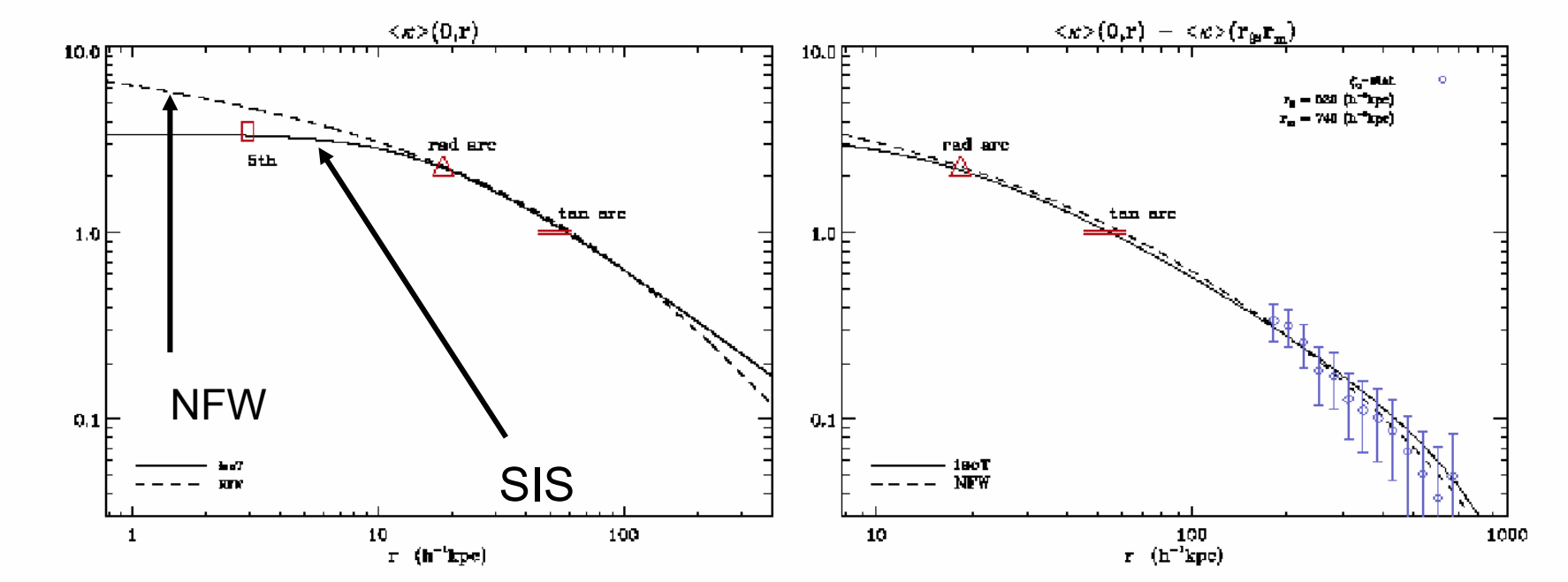
MS2137-23

Gavazzi et al 2004

MS2137-23 Gavazzi et al 2004



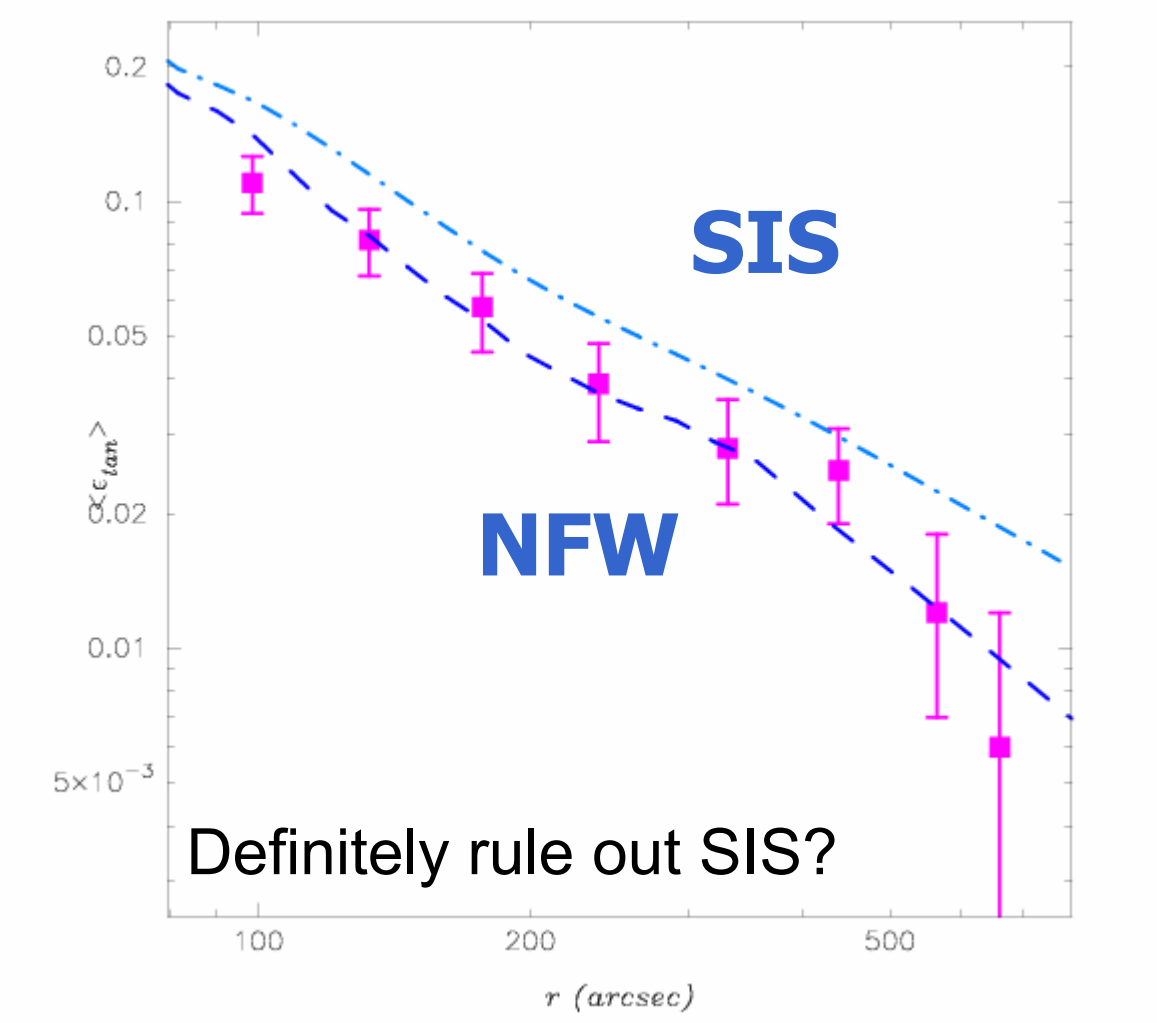
MS2137-23: strong + weak with the 5th image



Gavazzi et al 2004

Cl0024 HST wide field data

Kneib et al 2003



Summary

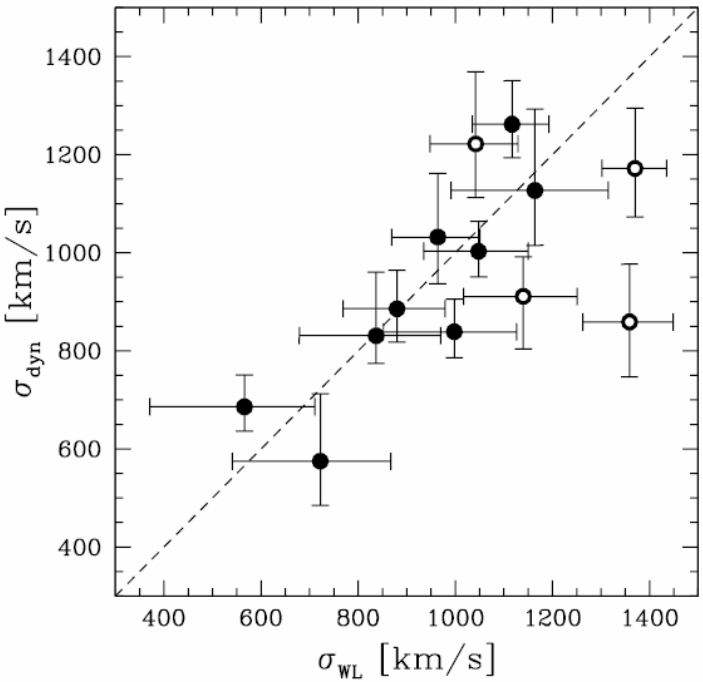
- Most Weak and/or Strong lensing data agree with NFW and SIS

- NFW not rejected (all data compatible with CDM predictions)

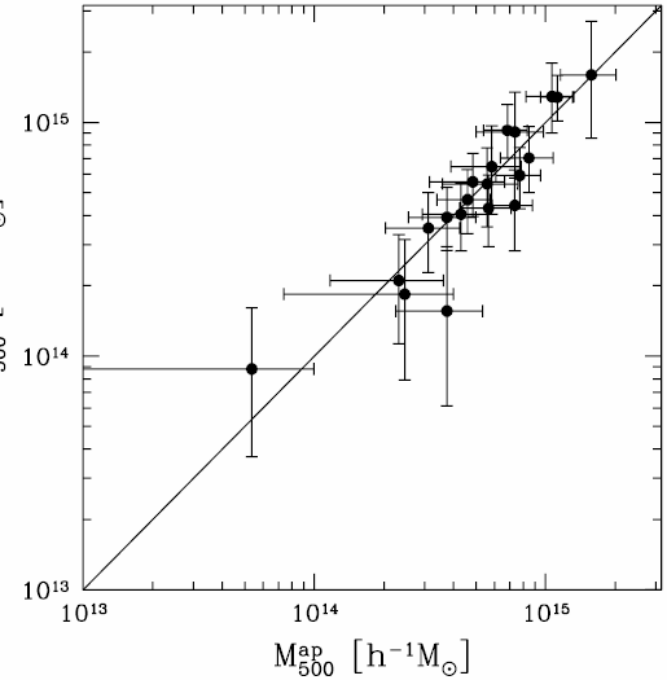
- Still hard to get a final answer, if any…

Most recent cluster survey

Spectroscopic redshift



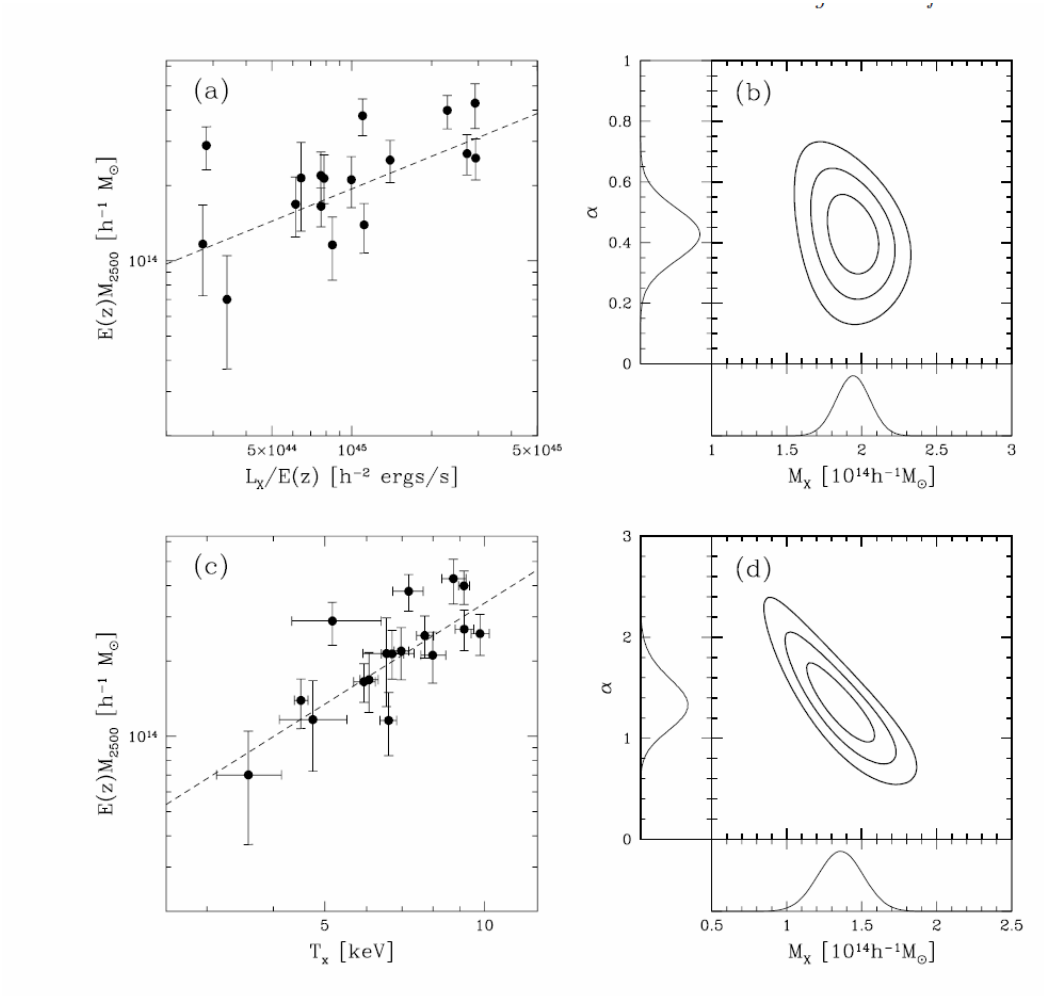
NFW predictions



Map

Hoekstra 2007

Most recent cluster survey: mass vs. Lx and Tx relations

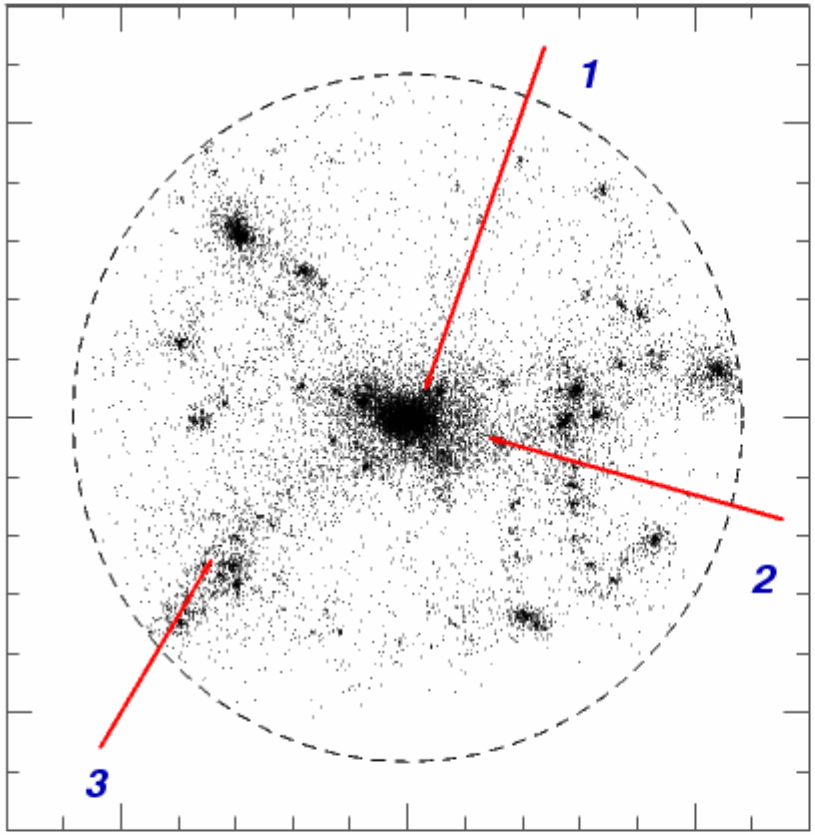


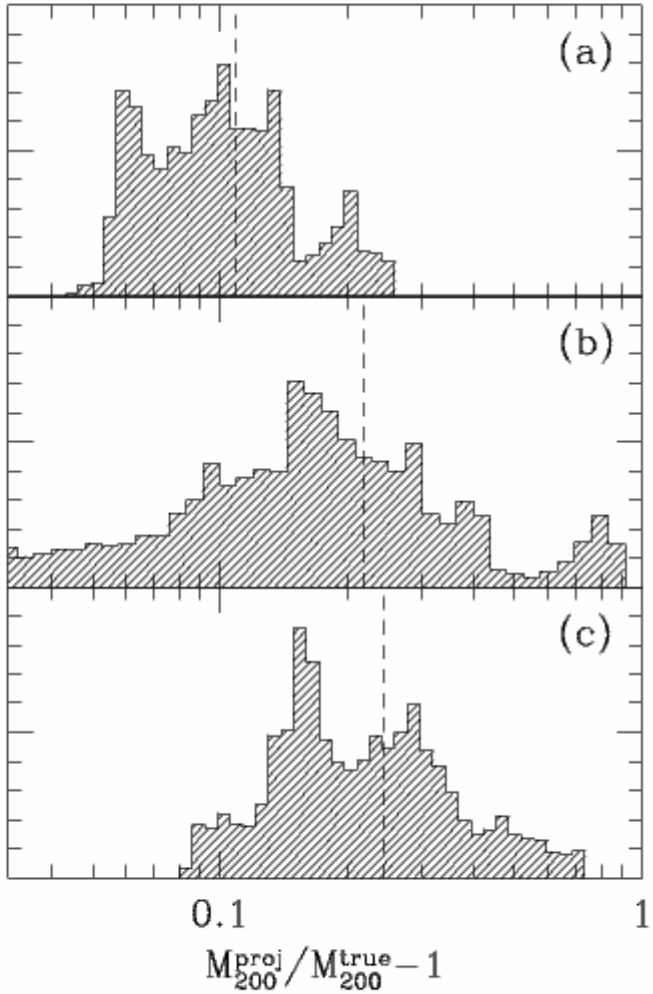
Hoekstra 2007

Projection effects:

non negligible contamination

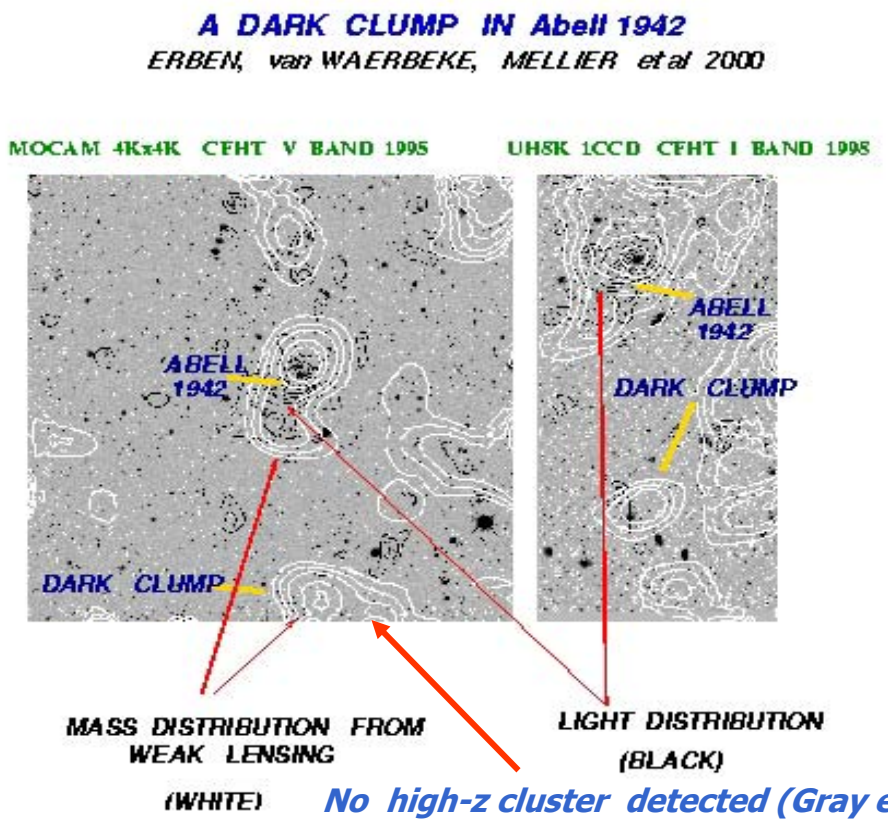
Metzler et al 2000

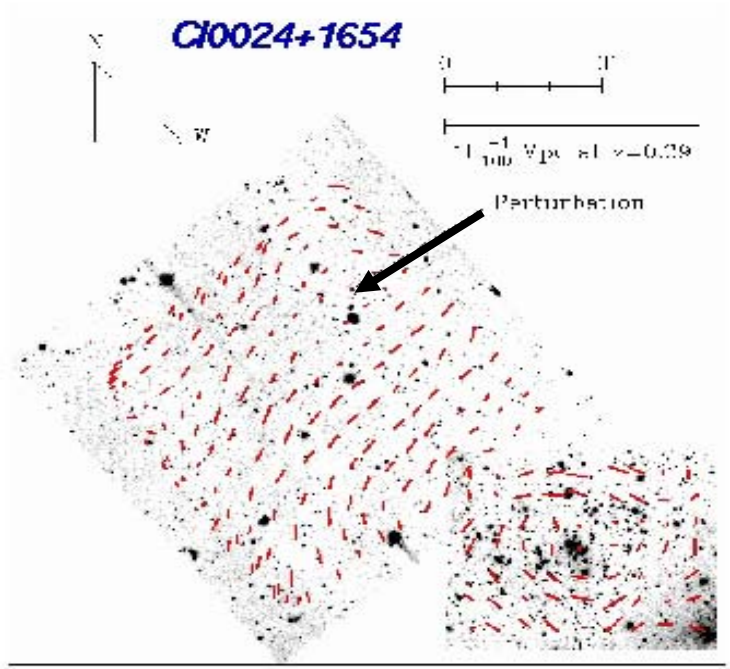




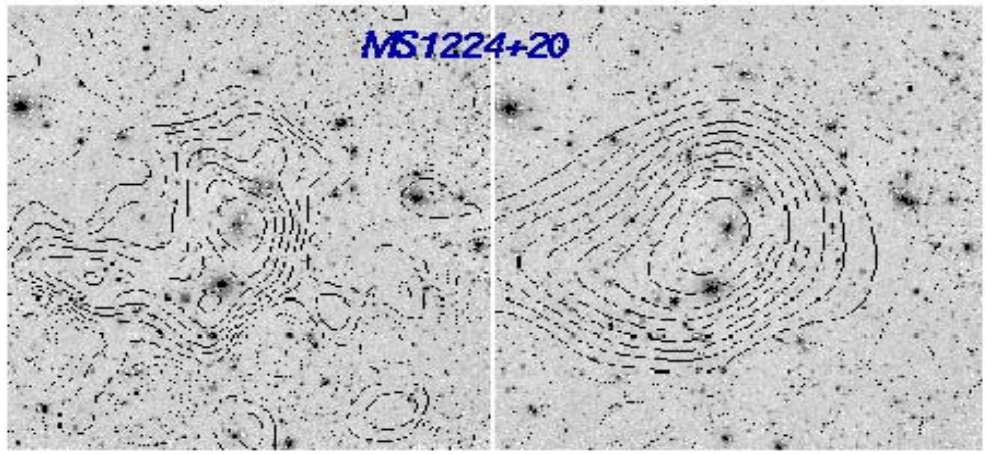
Dark Cluster ?

Dark Clusters ?



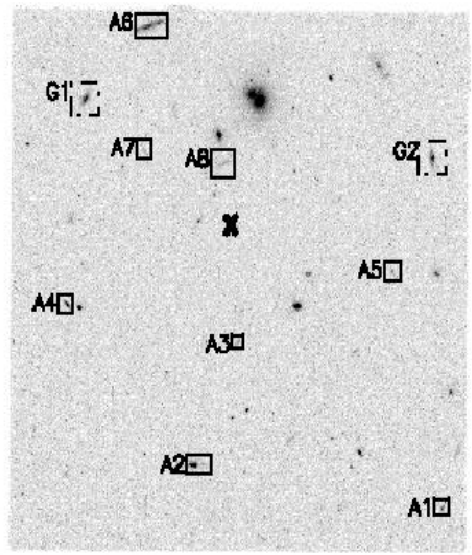


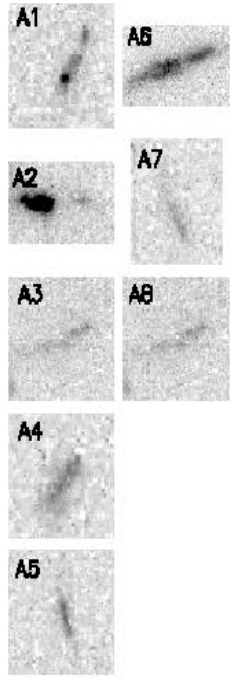
No high-z cluster detected (Gray et al 2000) Bonnet, Mellier, Fort 1994

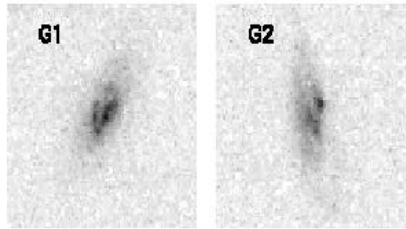


Fischer 2000 (see also Fahlman et al 1994)

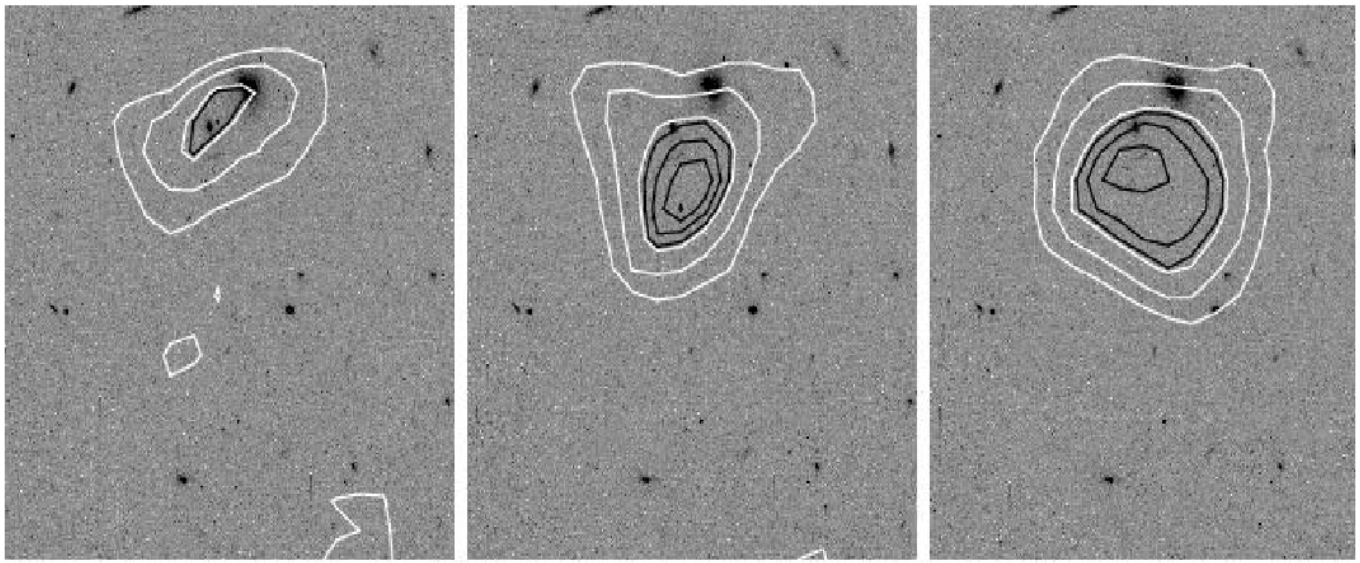
An impressive candidate with a remarkable alignment of galaxies and coherent mass map…..







Miralles, Erben, Haemmerle et al 2002



HST/STIS data

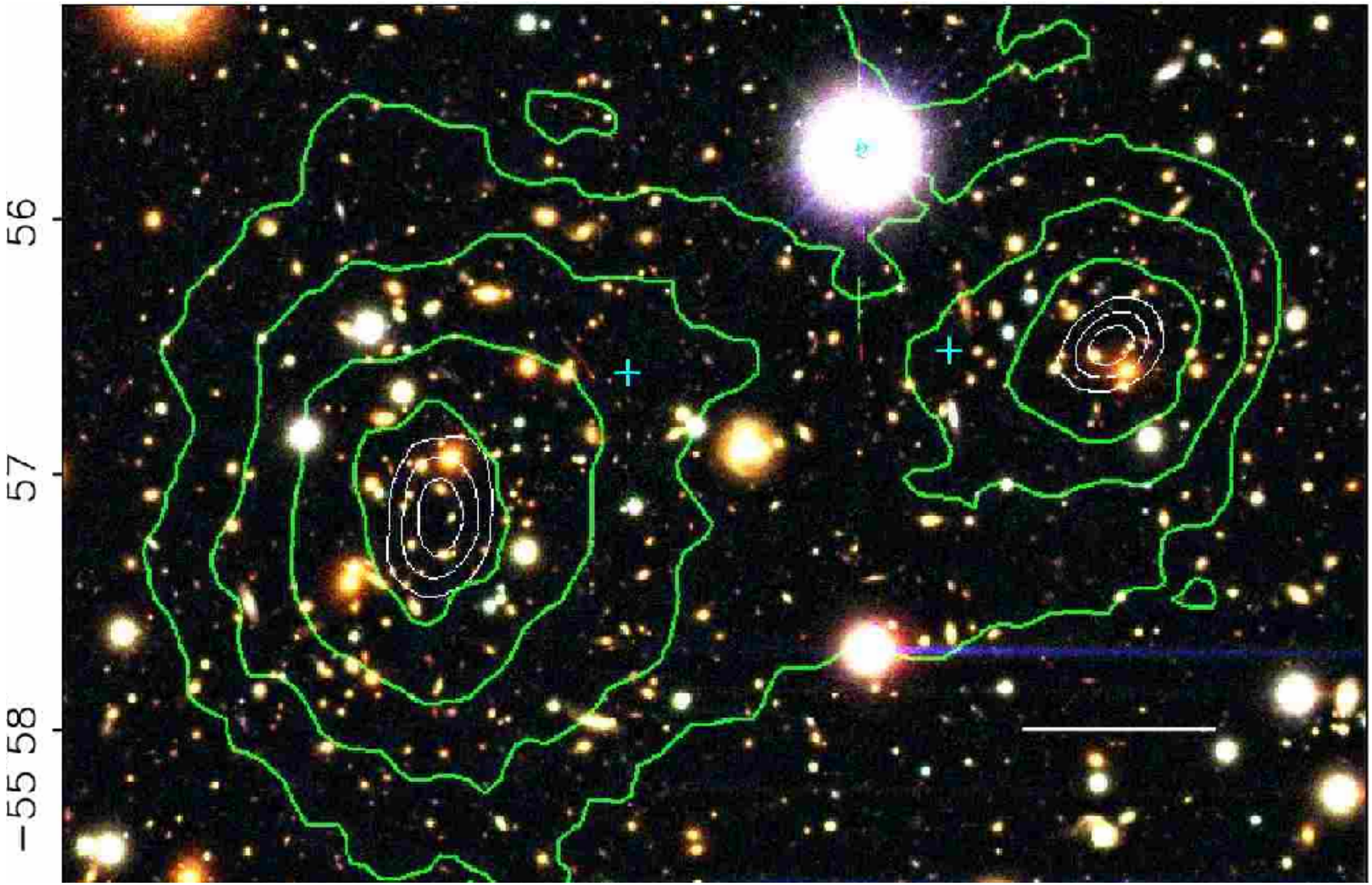
Dark clusters…

not yet convincing evidence they exist…

The « Bullet » cluster

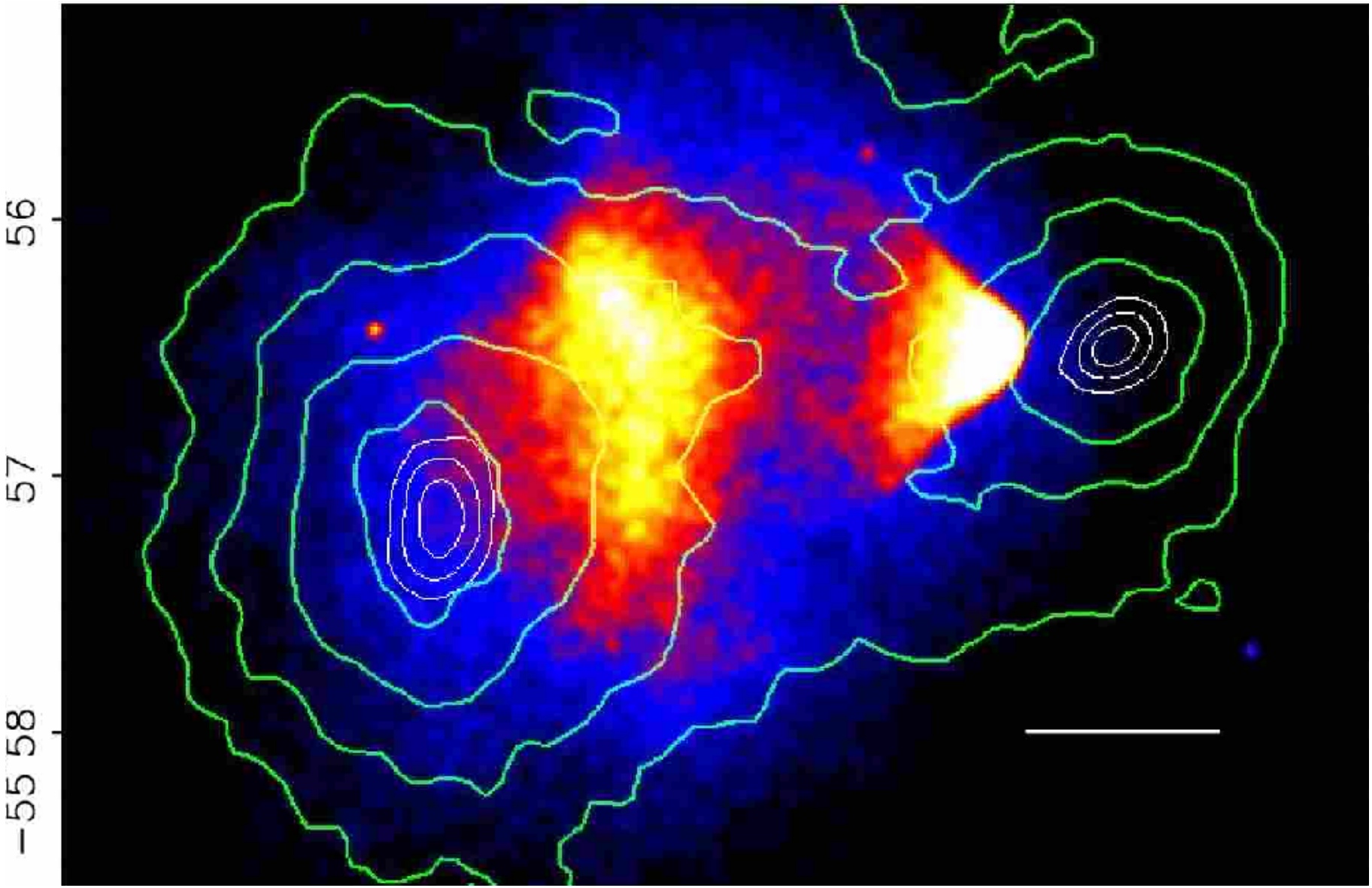
Bullet cluster

Clowe et al 2006



Bullet cluster

Clowe et al 2006



Bullet cluster : example of the interest of lensing

Clowe et al 2006

Mass map from Ellipticity->convergence: total matter

X-ray: baryonic matter

Bullet: baryonic matter decoupled from total mass

-> Clowe et al (2006): Dark matter exists!

Q1: X-ray emissivity is thermal bremsstralhung? … or shock?

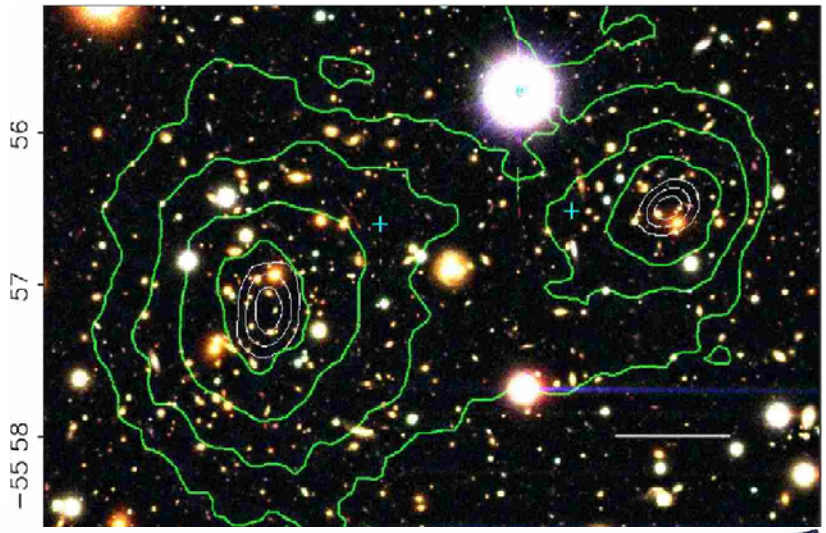
Q2: If shock, X-ray from baryons should be visible anyway and center mass positions, but not visible at the kappa center positions

Q3: Is ellipticity->gamma-> kappa calibration correct?

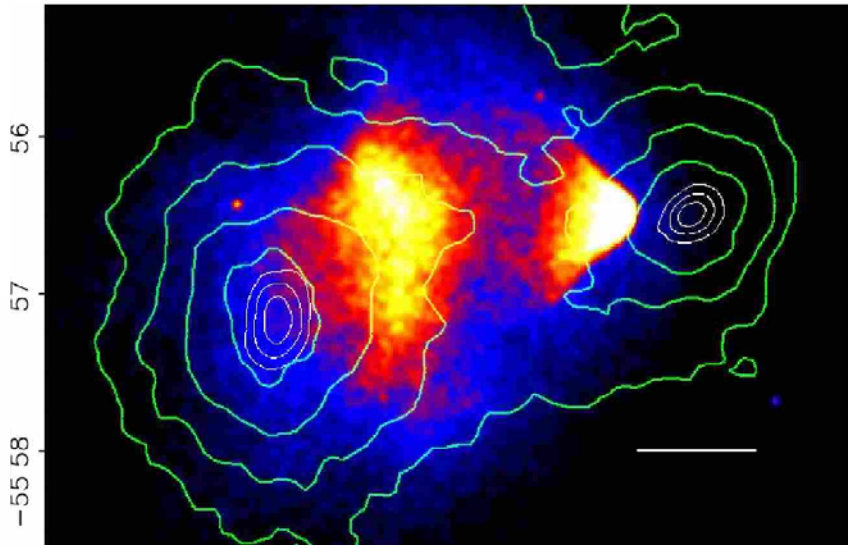
Q4: Is kappa=mass density (MOND: Berkenstein; Angus et al 2006)

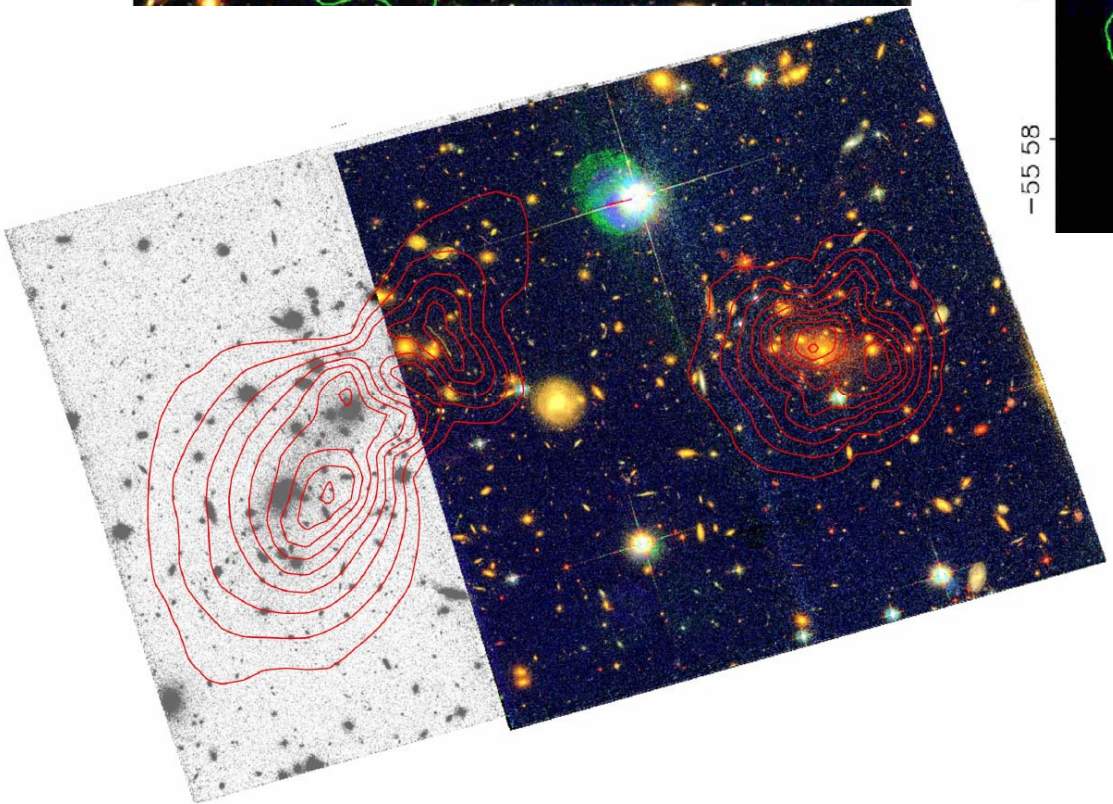
Robust statement: the cluster is dominated by collisionless matter, but could be CDM or neutrino with mass~2eV?

Ground: weak



Chandra+weak





HST: strong+weak
(importance of depth: sampling the mass density)

DM or collisionless matter (neutrino)?

Clowe et al: ground +weak +Chandra: DM exists! MOND rejected!

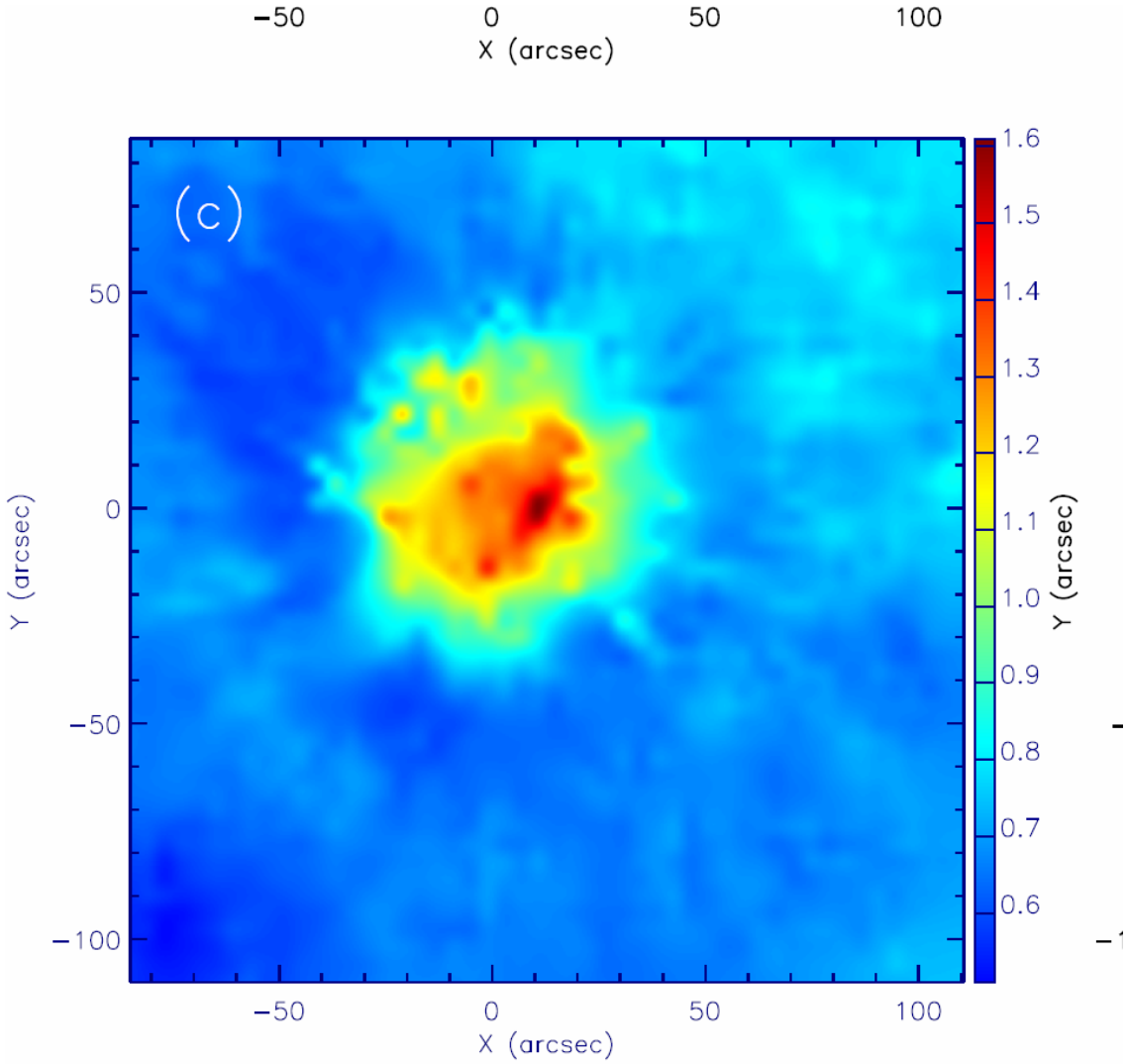
Angus et al: ground data + weak + chandra compatible with modifies gravity AND neutrino with mass ~ 2 eV

Clowe et al : HST strong+Weak + chandra : new mass distribution would imply neutrino with mass of 3.6 eV. Already rulled out !

Bartelmann et al 2007: confirm Clowe et al

The « Ring » in Cl0024+1654

The « Ring » in Cl0024+1654



Jee et al 2007

III. Gravitational Lensing:

galaxy-size halos

Galaxy-Galaxy lensing

Distortion of foreground halos on the background galaxies

In the weak lensing regime:

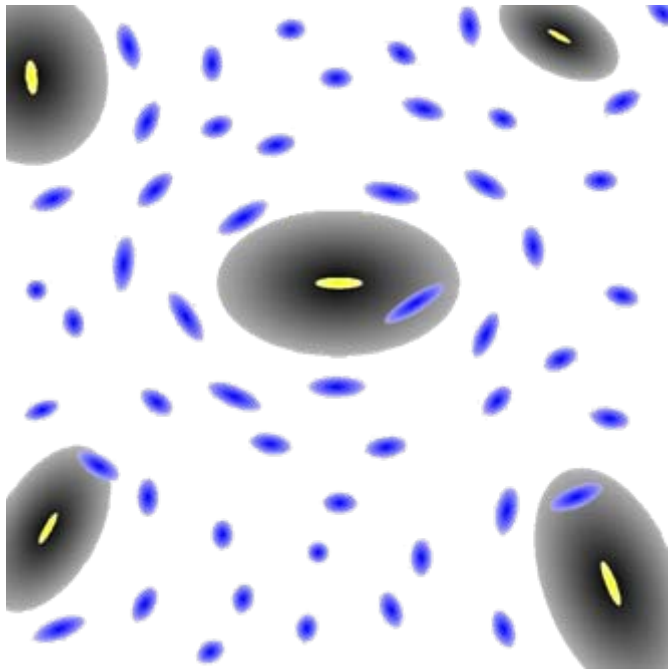
$$\epsilon^s = \epsilon - \gamma$$

Therefore:

$$p(\epsilon) = p^{(s)}(\epsilon - \gamma) = p^{(s)}(\epsilon) - \gamma_\alpha \frac{\partial}{\partial \epsilon_\alpha} p^{(s)}(\epsilon)$$

$$\int \mathrm{d}^2\epsilon \; p(\epsilon) = 2\pi \int \mathrm{d}\epsilon \; \epsilon \; p(\epsilon) = 1$$

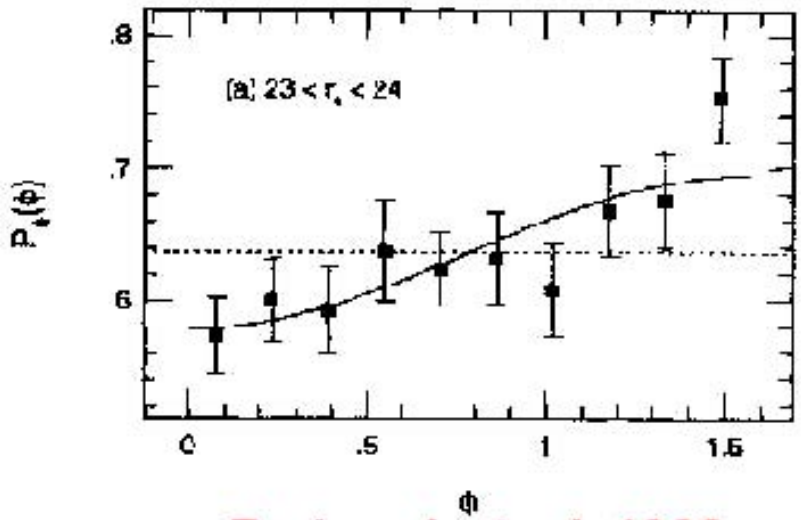
$$\left\langle \frac{1}{\epsilon} \right\rangle = \int \mathrm{d}^2\epsilon \; \frac{p(\epsilon)}{\epsilon} = 2\pi \int \mathrm{d}\epsilon \; \epsilon \; \frac{p(\epsilon)}{\epsilon}$$



Galaxy-Galaxy lensing

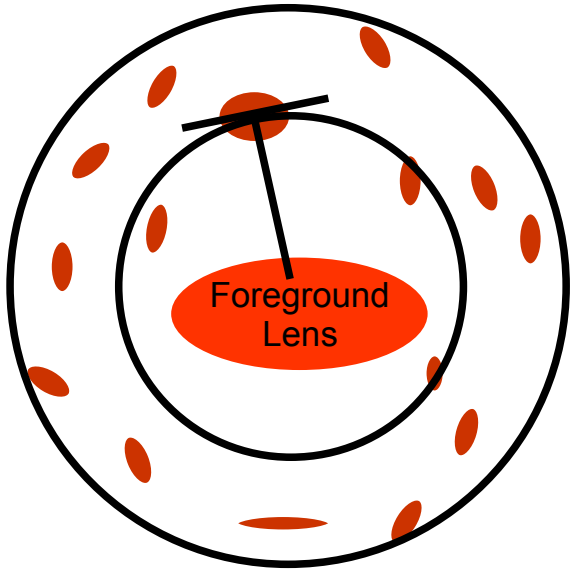
One can then express the modification of the orientation distribution produced by the foreground

$$p(\phi) = \frac{2}{\pi}\left[1 - \langle p \rangle \cos(2\phi) \ \left\langle \ \frac{1}{\epsilon} \ \right\rangle\right]$$

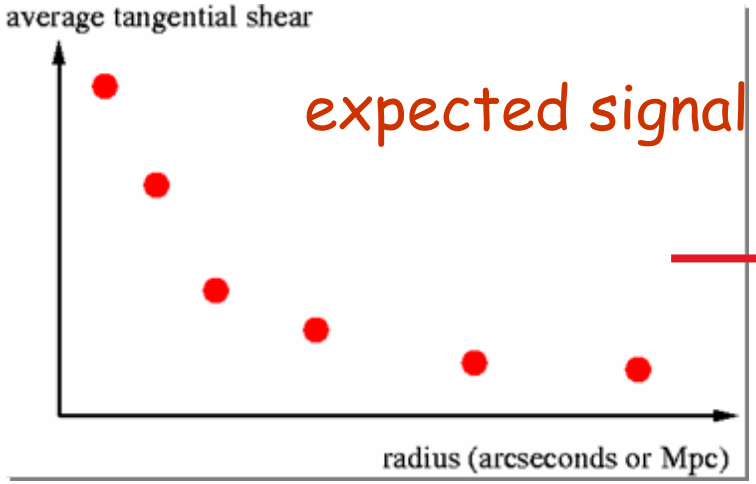


Brainerd et al 1996

Galaxy-Galaxy lensing



- Measure tangential component of shape in bins
- Need to stack signal around MANY foreground lenses

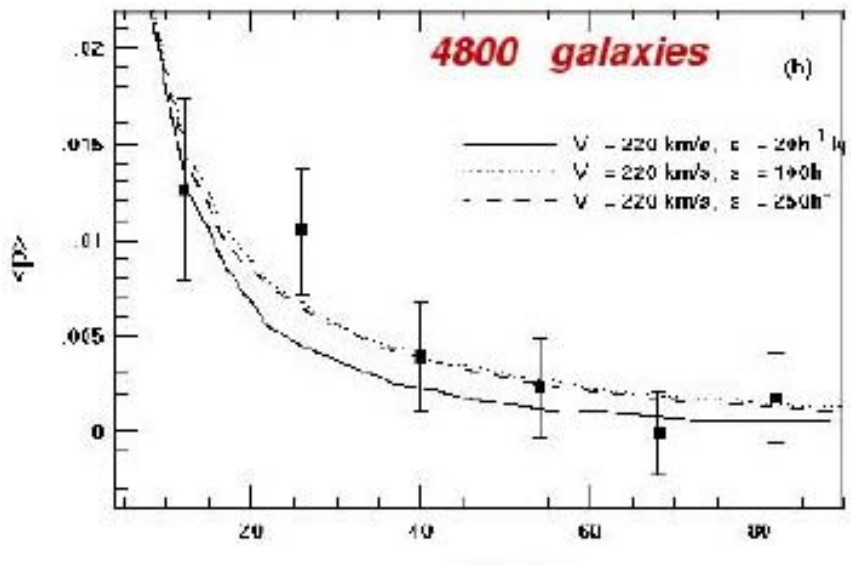


fit signal with your favourite mass profile

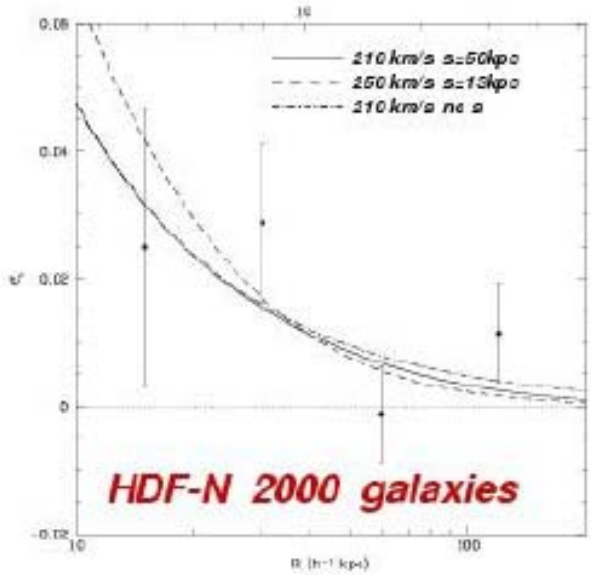
(From Parker 2007)

Galaxy-Galaxy lensing

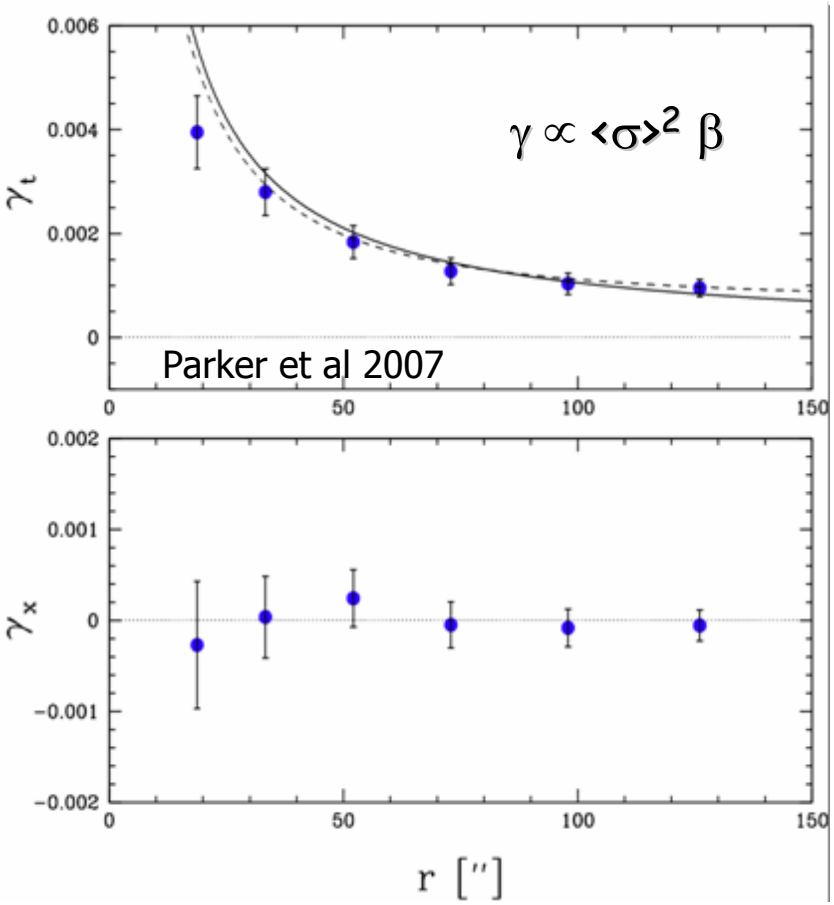
Brainerd et al 1996



Hudson et al 1999.



Galaxy-Galaxy lensing in CFHTLS



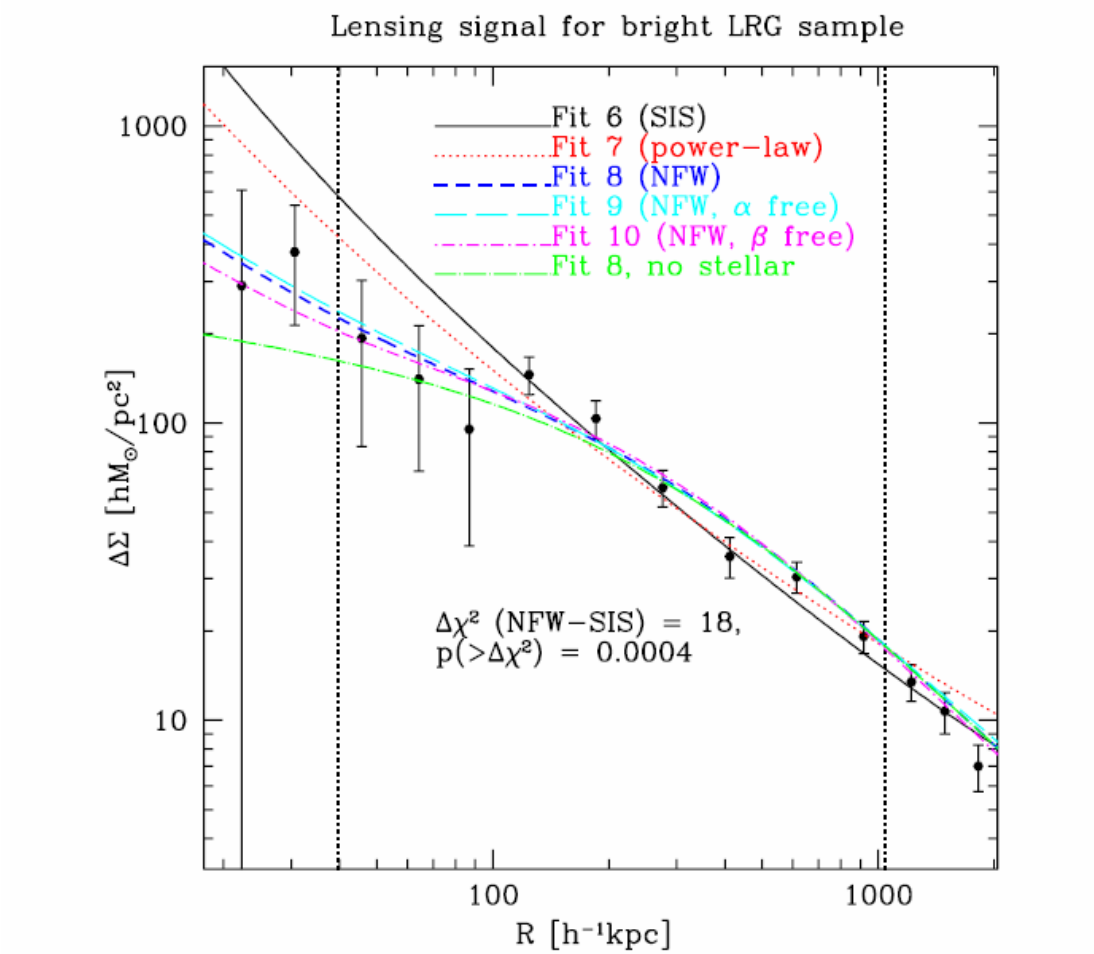
- Velocity dispersion depends on the lens sample.
- Must scale to some typical L* galaxy, based on an assumed relation between L and velocity (σ prop.to $L^{0.25}$, for example)
- Use σ^* to estimate the total mass of the halo assuming a cut-off radius

<σ> km/s	<σ>* km/s	Mass total	Mass at r_{200}	<M/L> R-
132+/-10	137+/-11	2.2e12	1.1e12	170+/-30

- well-fit with a SIS with a velocity dispersion of 132 +/- 10 km/s
- best fit NFW (dashed) has r_{200} = 150 h^{-1}kpc

(From Parker 2007)

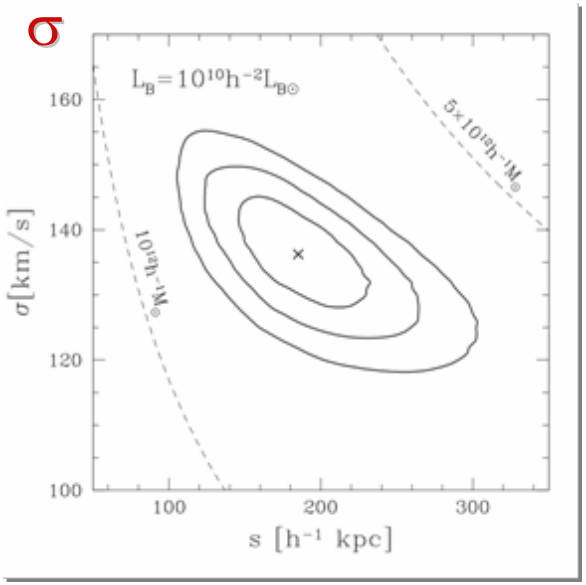
Mass density profile of Luminous Red Galaxies in SDSS using galaxy-galaxy lensing



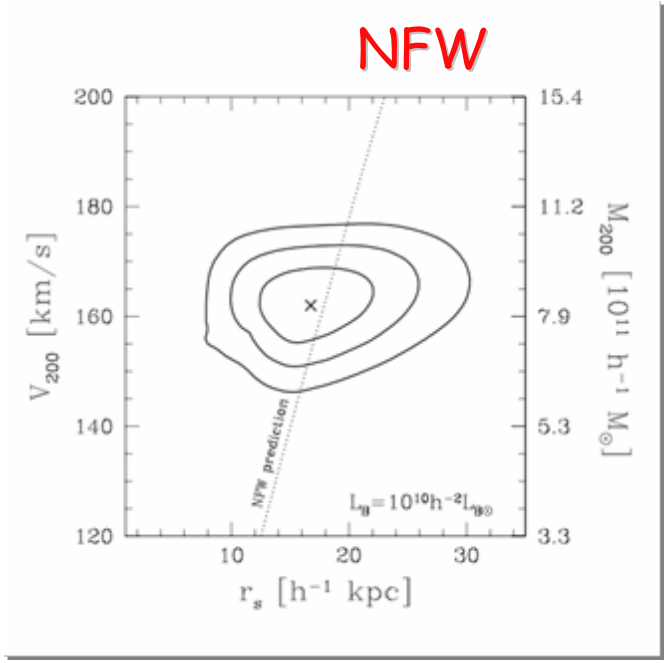
Maudelbaum et al 2006: reject SIS and favor NFW

Extents of halos: CFHTLS

- Maximum likelihood technique to fit for halo model
- For each source you determine the influence from all nearby foreground lenses with a parameterised lens model

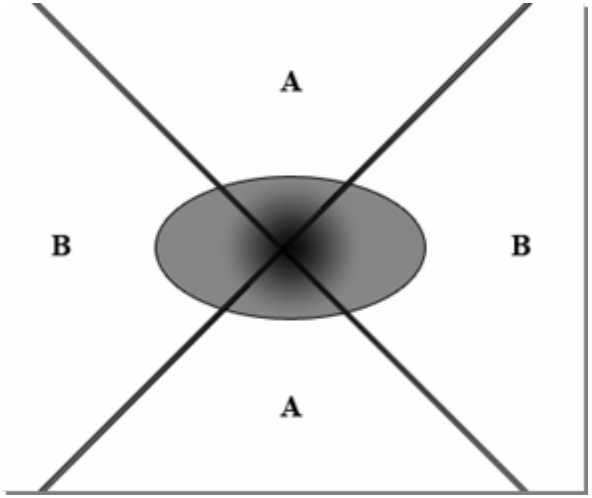


$$\rho(r) = \frac{\sigma^2 s^2}{2\pi G r^2 (r^2 + s^2)}$$

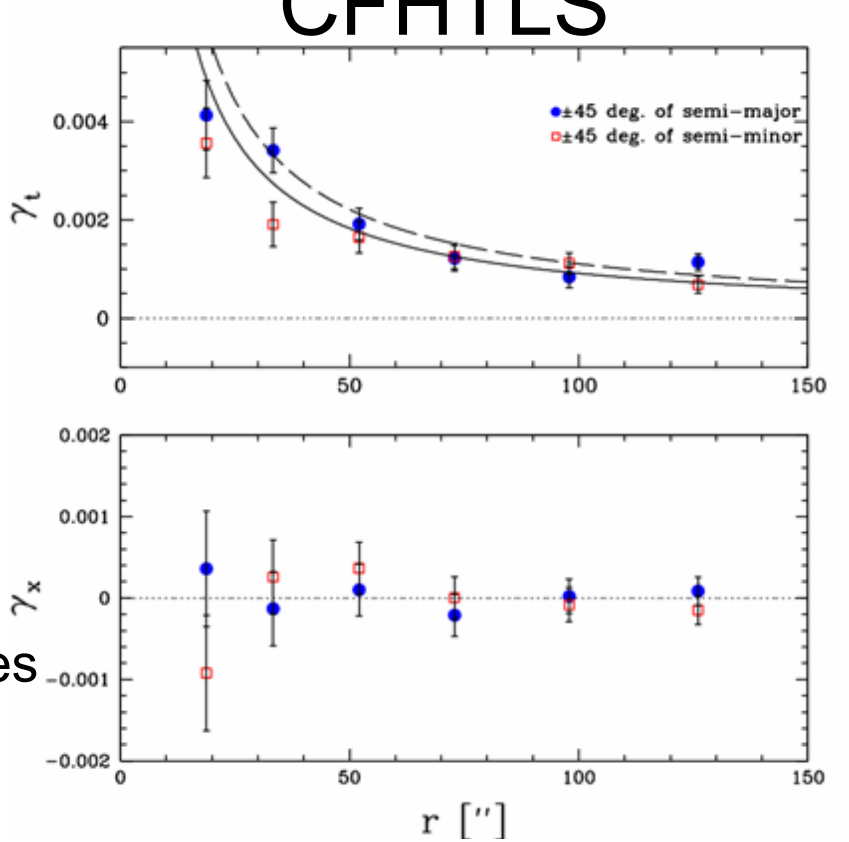


$$p(r) = \frac{\delta_c \rho_c}{(r/r_s)(1 + r/r_s)^2}$$

Halo shapes from galaxy-galaxy lensing



CFHTLS

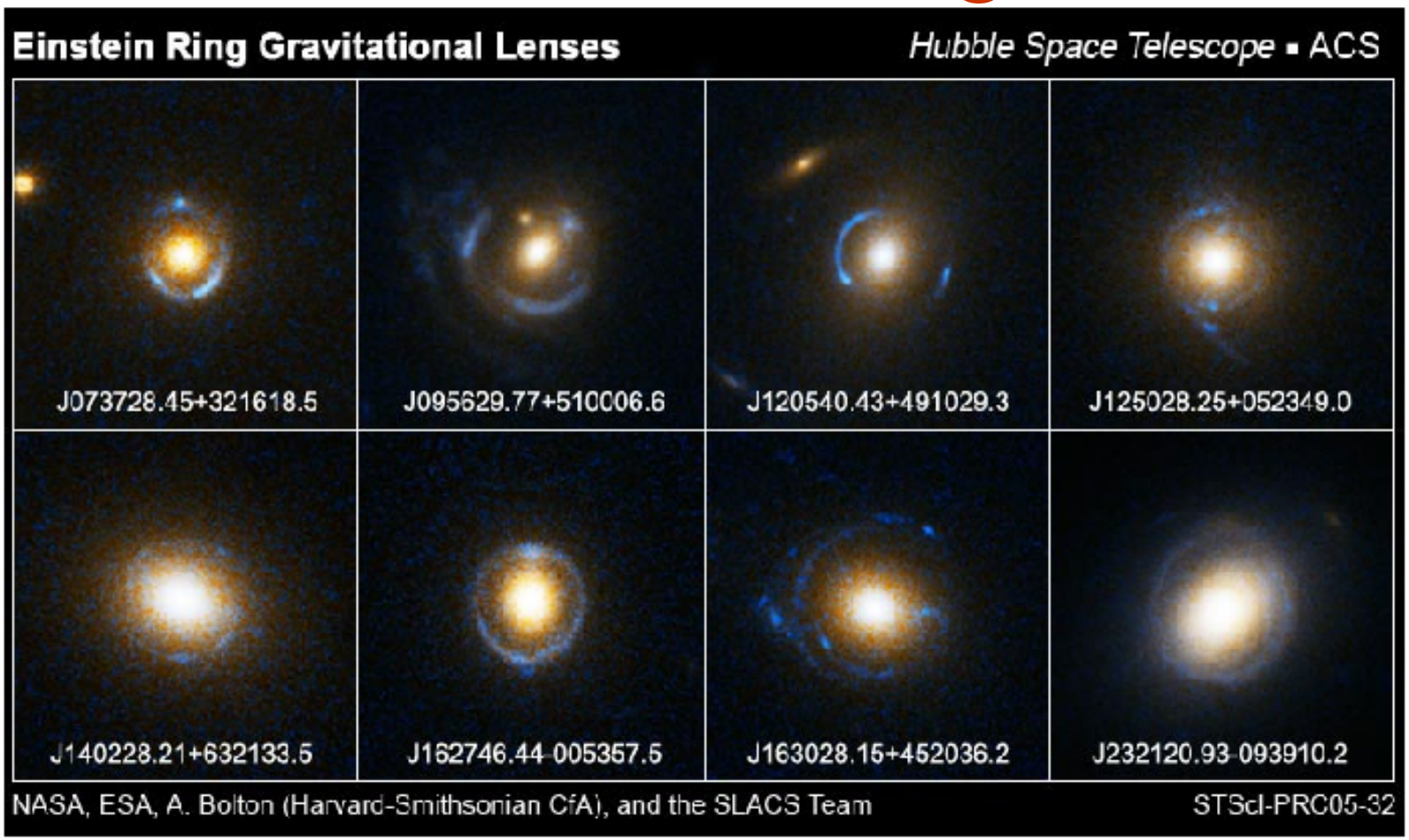


- Look for non-spherical halo shapes by comparing the tangential shear from sources near the major axis to those near the minor

- Halo flattening was observed in a weak lensing analysis of RCS data (Hoekstra et al., 2004), but not in SDSS by Mandelbaum et al. (2005)

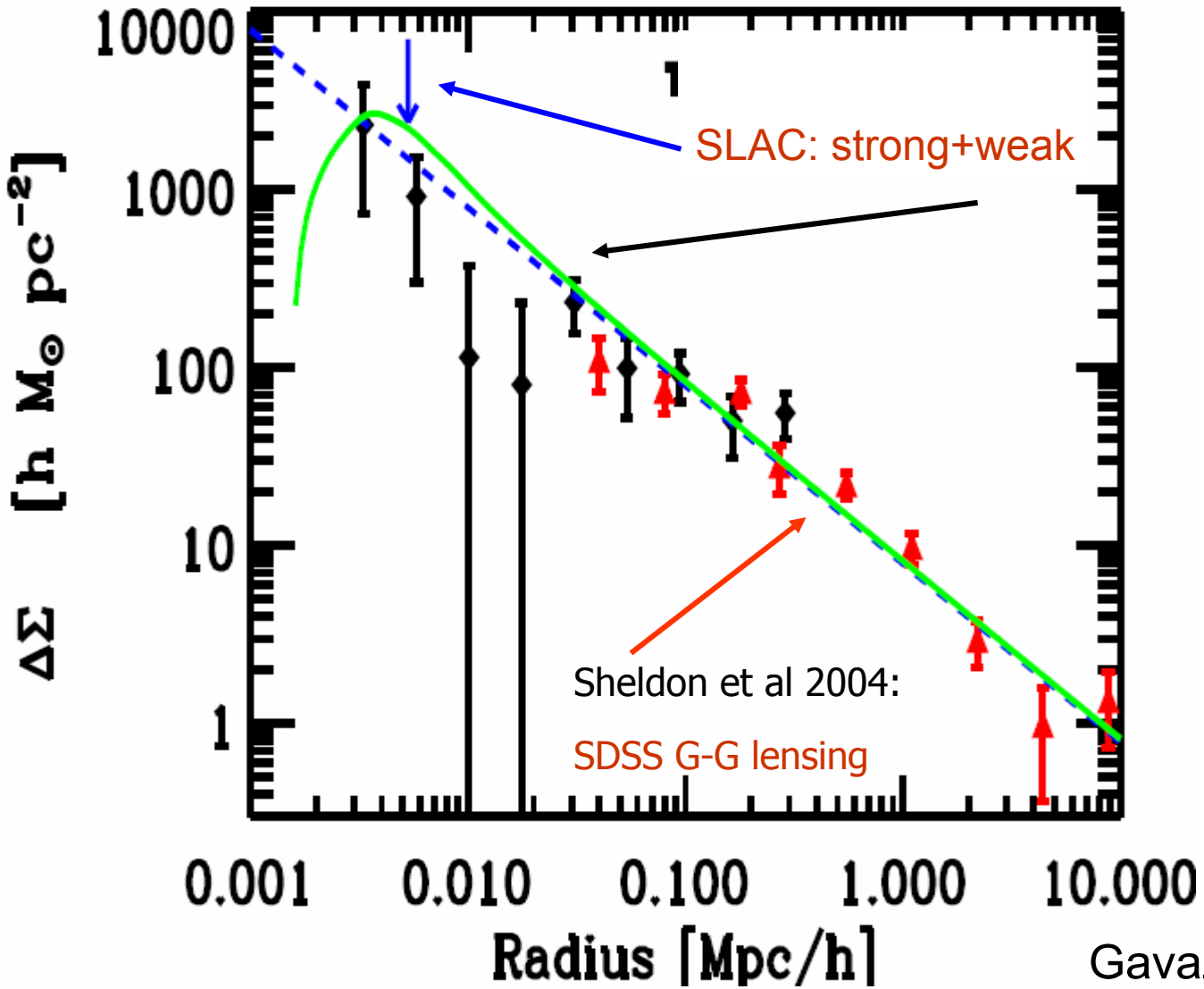
(Parker 2007)

Properties of galaxy halos using Einstein Rings



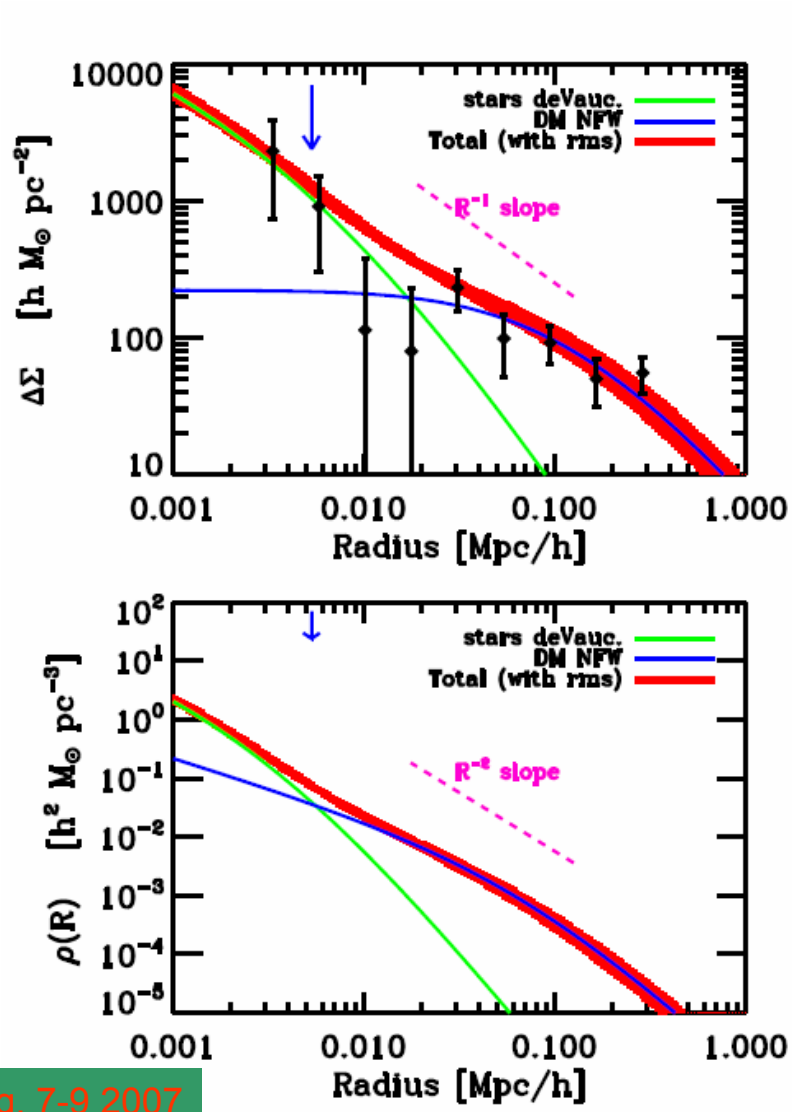
SLOAN Lens ACS Survey: Bolton et al 2006, Treu et al 2006, Koopmans et al 2006

Strong+Weak lensing of galaxies with rings in SLAC



Gavazzi et al 2007

Strong+Weak lensing of galaxies with rings in SLAC



Gavazzi et al 2007:

- Fit SIS
- Does not fit NFW

- Contradict Mandelbaum et al 2006?

- Why? :