Recent developments in saturation physics

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1 Introduction

One of the most challenging problems in QCD is the understanding of its high energy limit, the so–called Regge limit in which the center–of–mass energy s is much larger than any other scales of the problem. In this limit, the most important Feynman diagrams are ladder diagrams as they are enhanced by a large logarithm $\alpha_s \ln s \sim 1$. A resummation of these logarithms $(\alpha_s \ln s)^n$ (the leading logarithmic approximation) leads to the BFKL Pomeron [1] which, however, violates the unitarity bound. There are several directions to go beyond the BKFL approach. The next–to–leading order corrections are large and important at currently accessible energies. At the same time one would probably have to consider multiple reggeon exchanges as described by the BKP equation. Another direction is the nonlinear evolution equation based on the idea of gluon saturation. Over the past decade there has been considerable theoretical progress in this direction. In this contribution I review the current understanding of the physics of gluon saturation.

2 High energy phase diagram

Fig. 1 shows the profile of a hadron as seen by a virtual photon in DIS as the rapidity $Y = \ln s = \ln 1/x$ and the virtuality Q^2 of the photon are varied. With increasing energy at fixed Q^2, the gluon distribution function increases very fast as predicted by the BFKL solution.

$$xG(x, Q^2) \sim \left(\frac{1}{x}\right)^{c\alpha_s}, \tag{1}$$

with c a constant of order unity. This means that once the energy reaches sufficiently high, gluons start to overlap in the transverse plane [3]. The occurs when

$$\frac{\alpha_s}{Q^2} xG(x, Q^2) = \pi R^2 \tag{2}$$

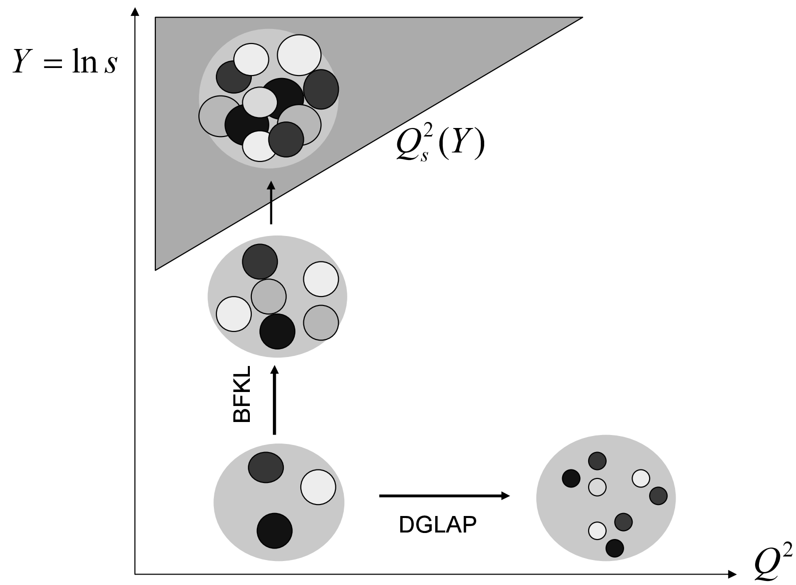

Figure 1: High energy phase diagram of DIS in the parameter space of the rapidity Y and the photon virtuality Q^2.

where the first factor on the lhs is the recombination cross section of two gluons and R is the hadron radius. Using the BFKL solution for xG, one can solve (2) for Q^2 as a function of $Y = \ln 1/x$:

$$Q^2 = Q_s^2(Y) = \Lambda^2 \exp \lambda \alpha_s Y \,, \tag{3}$$

where λ is another constant. Q_s is known as the saturation momentum. When $Q < Q_s$ (the shaded area of Fig. 1), interaction between partons inside the target becomes important, and therefore the standard factorization schemes (collinear or the k_t factorization) cannot be used.

3 The BK–JIMWLK equation

Over the past two decades, there have been many efforts to properly treat the nonlinear interaction of partons in the saturation regime, and thereby generalize the BFKL equation. The most complete equation of this kind to date is the BK–JIMWLK equation [2]:

$$\frac{\partial \langle T_{xy} \rangle}{\partial Y} = \frac{\overline{\alpha}_s}{2\pi} \int d^2 z \, \frac{(x-y)^2}{(x-z)^2(z-y)^2} \left(\langle T_{xz} \rangle + \langle T_{zy} \rangle - \langle T_{xy} \rangle - \langle T_{xz} T_{zy} \rangle \right), \tag{4}$$

where x, y, z are two dimensional transverse vectors and T_{xy} is the scattering amplitude of a color dipole (i.e., a quark–anti quark pair) of size $r = |x - y|$ off a dense target evaluated in the eikonal approximation

$$T_{xy} = 1 - \frac{1}{N_c} \mathrm{Tr}\,\{\mathrm{V_x V_y^\dagger}\}, \tag{5}$$

with

$$V_x = \mathrm{P}\exp\left(ig \int dx^- A^+(x^-, x)\right). \tag{6}$$

The DIS cross section is given by a convolution of T with the photon wavefunction. A^+ describes the target color field which is typically $\sim \mathcal{O}(1/g)$ in the saturation regime and $\langle \ldots \rangle$ in Eq. (4) denotes an averaging over the target wavefunction. With an additional assumption of factorization, $\langle TT \rangle = \langle T \rangle \langle T \rangle$, Eq. (4) becomes a closed equation. The BK–JIMWLK equation has been intensively studied both analytically and numerically. At a fixed value of the impact parameter, the amplitude saturates to the black disc limit value $T(r) \to 1$ at large r thanks to the nonlinear term $\sim T^2$. In the opposite limit $r \to 0$, T goes to zero due to color transparency. At intermediate values of r, a 'wavefront' is formed. The location of this front (the onset of strong scattering) defines the *saturation momentum* $Q_s(Y) \propto e^{\lambda Y}$: $T(r = 1/Q_s(Y)) =$ const. $\sim \mathcal{O}(1)$. When Y is sufficiently large, the strong scattering occurs in the perturbative regime $Q_s(Y) \gg \Lambda_{QCD}$. Thus it is possible to unitarize the scattering amplitude (at fixed impact parameter) in the framework of perturbative QCD. A very important property of this solution is that it shows *geometric scaling* in a wide region of the parameter space (Y, r). Namely, the dipole scattering amplitude, which is a priori a function of two variables r and Y, depends only on a single variable rQ_s; $T(r, Y) \approx (r^2 Q_s^2(Y))^\gamma$, $(\gamma \approx 0.63)$ in a wide region above the saturation momentum $r \geq 1/Q_s(Y)$.

The BK–JIMWLK equation has been thoroughly studied both numerically and analytically, and the solution has been applied to explain the experimental data at HERA and RHIC with certain success. The derivation of the next–to–leading order BK equation is currently under progress [4].

4 Fluctuations and correlations

By construction, the applicability of the BK–JIMWLK equation is limited to asymmetric scattering where the projectile is a dilute system composed of a few partons while the target is a highly evolved dense gluonic system. As such, it does not even allow one to describe the same scattering in the center–of–mass frame in which both of the hadrons contain a large number of gluons. From a diagrammatic point of view this limitation can be understood that only the gluon recombination diagrams Fig. 2(a) are summed in the BK–JIMWLK equation. For a symmetric description of the scattering, one has to sum a more general class of diagrams as shown in Figs. 2(b)

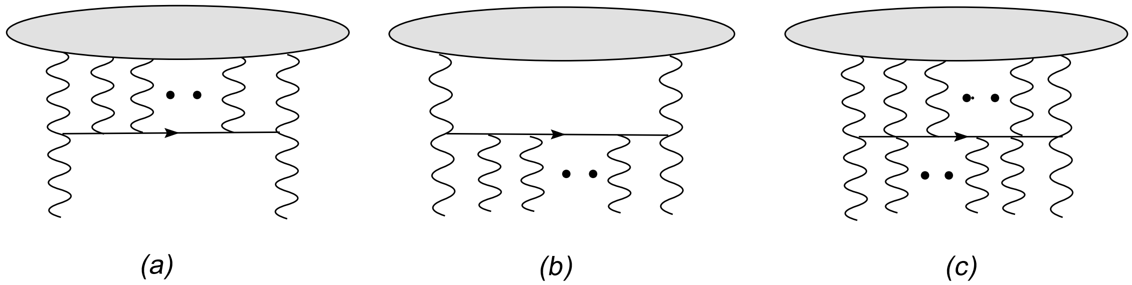

Figure 2: (a) Gluon recombination diagrams summed by the BK–JIWMLK equation. The upper blob represents the target. (b) Gluon splitting diagrams. (c) Loop diagrams.

and 2(c) in which the number of t–channel gluons changes arbitrarily (increase or decrease) as one goes down the ladder. Such diagrams are called the Pomeron loop diagrams.

Recently there has been a lot of activity trying to include Pomeron loops in the evolution. The progress in this direction can be roughly divided in two approaches: (1) To construct an effective action by integrating out the intermediate rapidity modes in the QCD functional integral with external color sources [5]. (2) Explicitly include the $2 \to 4$ gluon splitting ($1 \to 2$ dipole splitting) vertex into the evolution equation in the large N_c approximation [6]. Although these programs are still under development, several important observations regarding the nature of Pomeron loops have already been made. The diagrams Fig. 2(b), which are missing in the BK-JIWMLK equation, are responsible for generating *fluctuations* and *correlations* inside the target. Here the fluctuation refers to that of the gluon number. The gluon number as predicted from the BFKL equation is the average number. In reality, it fluctuates event–by–event because the small–x gluons are emitted with certain probability in each step of evolution. In fact this phenomenon has been known for a long time, even before the BK–JIMWLK equation, in a Monte–Carlo simulation of the BFKL evolution [7]. But the impact of the fluctuation on the asymptotic behavior of scattering amplitudes has been fully appreciated only recently. Most dramatically, it eventually violates the geometric scaling (as predicted by the BK equation) and replaces it with a new scaling law–the *diffusive scaling*. The details are reported elsewhere [8].

5 Power–law correlation in the transverse plane

Another consequence of the gluon splitting is to generate nontrivial correlation between partons in the transverse plane [9]. If the initial condition of the evolution is a dilute system (like a single dipole or a proton), small-x gluons emitted in the subse-

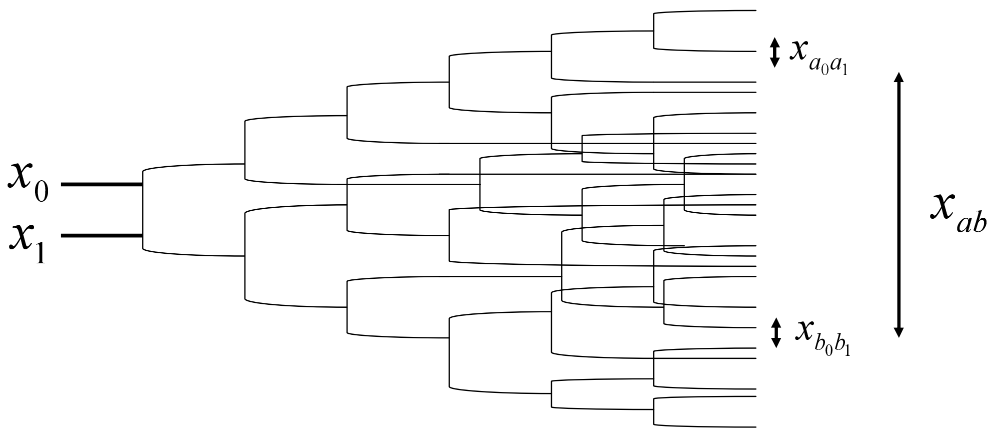

Figure 3: Cascade of dipole splitting. The parent dipole is placed at (x_0, x_1).

quent evolution are necessarily correlated because they have a common ancestor, see Fig. 3. The form of the correlation is determined by the accumulated effects of many gluon splittings whose probability nontrivially depends on the transverse coordinate. Specifically the emission probability of a gluon at the transverse coordinate z from a dipole at (x, y) due to a small increase in rapidity dY is

$$dP \propto \alpha_s \frac{(x-y)^2}{(x-z)^2(z-y)^2} dY d^2z \tag{7}$$

In [9], the leading power–law behavior of this correlation was determined in certain limits by analytically evaluating the *dipole pair density* $n^{(2)}$ which is an inclusive probability distribution to simultaneously find two dipoles in the hadron wavefunction [10]. The result is

$$n^{(2)}(x_0x_1; x_{a_0}x_{a_1}, x_{b_0}x_{b_1}) \propto \left(\frac{x_{a_0a_1}x_{b_0b_1}}{x_{ab}^2}\right)^{2\gamma_a} \left(\frac{x_{01}x_{ab}}{x_{0a}x_{1b}}\right)^{2\gamma}, \tag{8}$$

where the notation of the transverse coordinate is obvious from Fig. 3 and the BFKL anomalous dimensions γ and γ_a depend on the dipole sizes. The last factor in Eq. (8) is the anharmonic ratio which reflects the conformal invariance of the BFKL evolution. Interestingly this factor makes (8) valid both for $x_{01} \ll x_{ab}$ and $x_{01} \gg x_{ab}$.

An immediate consequence of the power–law correlation is that multiple scattering amplitudes do not factorize. Indeed, (8) implies that the two–dipole amplitude behaves as, when $x_{01} \gg x_{ab}$,

$$\langle T^{(2)}(x_{a_0a_1}, x_{b_0b_1})\rangle \sim \langle T(x_{a_0a_1})\rangle\langle T(x_{b_0b_1})\rangle \left(\frac{x_{01}}{x_{ab}}\right)^{2(2\gamma_a-\gamma)} \tag{9}$$

Note that the last factor is much bigger than 1. This should be contrasted with the fact that scattering amplitudes computed in the BK–JIMWLK framework essentially factorize

$$\langle T^{(2)} \rangle \approx \langle T \rangle \langle T \rangle + \mathcal{O}\left(\frac{1}{N_c^2}\right). \tag{10}$$

The correlation developed by the gluon splitting, Fig. 3(b), survives in the large–N_c limit, thus it represents new physics beyond the BK–JIMWLK equation. Though the assumption of no correlation (10) is probably justified for a large nucleus, it does not seem to be the case if the target is a small hadron. This means that our understanding of the asymptotic behavior of scattering amplitudes is not complete. Full inclusion of Pomeron loops in the evolution remains as a challenge.

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