Dirac structure of quark-hadron vertex in Bethe-Salpeter wave function under the framework covariant instantaneous ansatz

S Bhatnagar[1] *and S-Y Li*[2]
[1] *Department of Physics, Addis Ababa University, Ethiopia*
[2] *Department of Physics, Shandong University, PRC and CERN PH-TH, Suisse*

Abstract

In the framework of covariant instantaneous ansatz, the general form of the quark-hadron vertex of certain kind of mesons is provided; how to arrange the Dirac covariants of the complete set by the power counting rule is discussed. Employing the leading order Dirac covariants for vector and pseudoscalar mesons, their decay constants are calculated. This proceeding paper is complementary to our other two previous papers, emphasizing on some conceptual discussions.

The strong interaction is special for its basic degree of freedoms are confined in hadrons, preventing the measurement of the "really-free" quanta, i.e., quarks and gluons in quantum chromodynamics (QCD). Hence, a description for the asymptotic states, the hadrons, in the strong interactions/collisions, with the language of the basic quanta/fields, quarks, and gluons, is a necessary but challenging task. In the framework of relativistic quantum field theory, at least one "standard" way to describe bound states, i.e., the Bethe-Salpeter wave function (BSW), which is the solution of the Bethe-Salpeter integral equation (BSE), seems available. Through an extensive employment of various techniques in the quantum field theory, such as the Dyson-Schwinger Equations, one may expect to know the movement of the quarks and gluons in hadrons, and to calculate various kinds of interactions/collisions in various energy scales with such knowledge. This is one of the most important purposes of studying the hadron structure. However, a full understanding of the physical picture of the BSW and a satisfactory way to solve the BES in QCD are still beyond approach. Even, one is not able to give the "rigorous" form of the kernel of the BSE without practical non-perturbative techniques in QCD. More specifically, one has a very powerful picture/tool in high energy hadronic interactions, i.e., the perturbative QCD inspired parton model, but how to approach this parton picture starting from low energy BSW of hadron is an open question. At the same time, the standard factorization scheme is often facing danger in various special processes [1, 2]. So it is very much justified to put more labour on the study of BSW/BSE, especially their

application to high energy hadronic interactions, in various cases where factorization scheme is in danger, before more powerful/standard ways in quantum field theory burst out. One is also reminded that, *en route* on exploring deeper structures of matter, it may be exposed that the elementary particles are bound state—e.g., the possible case of the Higgs—one had better well prepare for such a possible condition.

As stated above, it is not possible to give the rigorous QCD kernel of the BSE, so generally we rely on specific models, from which the most effective way to gain deep understanding of QCD is to apply a particular framework to a diverse range of phenomena. That is, on various kinds of hadrons and into various kinds of collisions (decays) in various energies. By comparing with data, improving the models, one may get more understanding on QCD, and may get some more powerful methods in dealing with bound states.

In this paper, we discuss some development on covariant instantaneous ansatz (CIA) framework of BSE/BSW. That is, using the power counting rule to arrange the Dirac covariants of the vertex function. First an outline of some general properties in BSW, with the framework of CIA as well, then the application to vector/scalar meson is sketched. To be complementary to our paper [3, 4], some details of discussions and calculations presented there are omitted and referred to the reader to [3, 4]. While some concepts and thinking related to this framework are discussed in details.

The definition of BSW we adopt is (taking the $q\overline{q}$ bound states as an example, in this paper, if no special statements; and the notations are standard),

$$\chi(x_1, x_2) =< 0|T(\Psi(x_1)\overline{\Psi}(x_2))|B > \tag{1}$$

(The summation on colours and flavours is implicitly indicated), whose Fourier transformation to momentum space $\chi(P, q)$ is the solution of the BSE

$$S_F^{-1}(p_1)\chi(P, q)S_F^{-1}(-p_2) = \int \frac{d^4q'}{(2\pi)^4} K(q, q')\chi(P, q'). \tag{2}$$

First thing to discuss is the problem of $SU_C(3)$ gauge-invariance (GI) for the definition of the BSW in (1), when it is of the quark/gluon bound state. To the lowest order of electro-weak interaction, for some simplest case, e.g., the hadron directly coupling to one virtual photon or $W^{\pm}$, Z^0, the invariant transition matrix T in collisions/decays is proportional to the BSW at the origin of space-time. In this case, $\chi(0, 0)$ is GI, and the results can be considered as a limiting procedure. That is, one can start the modeling and calculation of $\chi(x_1, x_2)$ at some feasible gauge, but at last take the limit to the origin of space-time. If there is no singularities in the limiting procedure, one can get a definite GI value (One thing to be paid attention to is for the contour integral of the propagator, we need to do the integral before take x_1, x_2 to be 0 to eliminate uncertainty). For more complex but still electro-weak cases, though T depends on the BSW of all the space-time points, but the starting

point in deriving T matrix is the electro-weak interaction Hamiltonian, which is GI ($SU_C(3)$), so we can expect the final result is still GI, and the integral process on space-time involving the BSW to get the T matrix could be considered as a more complex limiting procedure. However, to consider hard strong interaction process, where perturbative QCD is expected to be applied, we in fact implicitly employ an alternative definition of the BSW (defined on the asymptotic states in interaction picture or defined by the in or out fields), hence the GI is not automatic guaranteed, and we can not be sure if the latter definitions are really equivalent to the original one unless an all order provement show the perturbative calculation is reliable (i.e. the singularities safely canceled and large correction properly resummed).

In general, a BS "vertex" (BSV) $\Gamma(P,q)$ can be defined, to be (in momentum space, as an example)

$$\chi(P,q) =: S_F(p_1)\Gamma(P,q)S_F(-p_2) \tag{3}$$

In the most general way, the wave function should represent the quantum number of the hadron, so that it has the definite properties under the Parity and charge conjugation transformation [5]. By investigating the transformation properties of the FULL fermionic propagator, we can make sure that the BSV has the SAME transformation properties as the full BSW. From the definition (3), the BSV is 4×4 matrix in the spinor space, so it can be expanded by linear combination of the Dirac γ matrices, just the same as the BSW. One statement is that, though all the mesons are eigenstates of the Parity with certain eigenvalues, but this is not so for charge parity: only the "pure neutral" particles are. But one is not sure whether one should apply the restriction from it to all the others hence to reduce number of parameters or allow the others to have more freedom hence can expect a better fitting of the data.

The simple derivation about the above statements are:

$$\begin{aligned}
\chi(P,q) &= \pi_B\gamma_0\chi(\tilde{P}\tilde{q})\gamma_0 \\
S_F(p_1)\Gamma(P,q)S_F(-p_2) &= \pi_B\gamma_0 S_F(\tilde{p}_1)\Gamma(\tilde{P}\tilde{q})S_F(-\tilde{p}_2)\gamma_0 \\
&= \pi_B(\gamma_0 S_F(\tilde{p}_1)\gamma_0)\gamma_0\Gamma(\tilde{P}\tilde{q})\gamma_0(\gamma_0 S_F(-\tilde{p}_2)\gamma_0) \\
&= \pi_B S_F(p_1)\gamma_0\Gamma(\tilde{P}\tilde{q})\gamma_0 S_F(-p_2)) \\
\Gamma(P,q) &= \pi_B\gamma_0\Gamma(\tilde{P}\tilde{q})\gamma_0;
\end{aligned} \tag{4}$$

with

$$\tilde{p} = (p^0 - \vec{p}), or\ \tilde{p}^\mu = p_\mu. \tag{5}$$

In the same manner, paying attention to

$$\begin{aligned}
C\gamma_\mu^T C^{-1} &= -\gamma_\mu \\
\chi(P,q) &= \mathcal{C}_B C\chi^T(P,-q)C^{-1}
\end{aligned}$$

$$\begin{aligned} \chi(P,q) &= S_F(p_1)\Gamma(P,q)S_F(-p_2) = S_F(p_1)\Gamma(P,q)CS_F^T(p_2)C^{-1} \\ q \to -q &\Rightarrow p_1 \leftrightarrow p_2. \end{aligned} \tag{6}$$

Here superscript T refer to tranvesity.

By the above discussions, the complete Dirac structure for BSW or BSV can be obtained, and they can take slightly different forms, see eqs. (2.19),(2.24) of [5]; or see eq. (10) of [3], eq.(2.13) of [4], The coefficients of the Dirac covariants are $F(q\cdot P)$, but to pay attention that $q\cdot P$ is odd under C transformation, so for certain term of the Dirac covariants, the powers of $q\cdot P$ is restricted to be even or odd, if a Taylor expansion for F is applied.

Now few words on CIA. One of the key definition in the kinematics is

$$\hat{q}_\mu = q_\mu - \frac{q\cdot P}{P^2}P_\mu. \tag{7}$$

Hence the covariant instantaneous ansatz is

$$K(q,q') = K(\hat{q},\hat{q}'). \tag{8}$$

At first sight this is a strong assumption. However, from the BSE of Eq. (2), one is easy to see that K should be a scalar in Lorentz transformation. So it should be the same in different frame. But if it depend on P, it will change when boosted to another frame. So, subtracting the component parallel to P from q is very natural. At the same time, it is very easy to verify the q and q' in $K(q,q')$ is symmetric, by interchanging q and q' in Eq. (2).

In this framework, the derivation of the 3 D BSE, BSW, as well as the discussion on the vertex and power counting, can be found in our paper [3, 4]. It is widely accepted that, in the rest frame of the hadron, taking into account the non-perturbative effect of chiral symmetry breaking, the quark mass is at the same order of the hadron mass. Hence we can assume $q << M_{hadron}$. The power counting rule is based on that q/M is small, and terms receive suppression as powers of this factor (only not good for pion). The employment of this relates with the assumption of F. As in the CIA framework, it is considered as constant, so the number of the terms is finite. But to apply this rule to more general cases beyond CIA, we may make Taylor expansion for F, hence the number of terms is infinite.

The power counting analysis makes clear which terms is leading order. From our results [3, 4], one finds that, the introducing of other leading order Dirac covariants besides the original ones before our investigation for various mesons, improves the results dramatically. The changes can approach to around 100% for many particles. However, for most of the particles (even pion), the difference between the data and our leading order results is not very significant. Such a conclusion justifies the power counting rule, and suggests further studies.

The speaker (SL) thanks the organizer and IAP, Paris. This work is supported in part by NSFC with grant number 10775090.

References

[1] J. Collins, arXiv:0708.4410 [hep-ph].

[2] J. Collins and J. W. Qiu, Phys. Rev. D **75** (2007) 114014 [arXiv:0705.2141 [hep-ph]].

[3] S. Bhatnagar and S. Y. Li, arXiv:hep-ph/0612084.

[4] S. Bhatnagar and S. Y. Li, J. Phys. G **32** (2006) 949 [arXiv:hep-ph/0512352].

[5] C. H. Llewellyn-Smith, Annals Phys. **53** (1969) 521.

Discussion

The speaker of the authors (SL) Thanks Prof. B Müller's raising the question if the BSW can provide the complete information of the structure of hadron, on the degree of freedom of quarks and gluons. The authors think the final answer seems open. Especially taking into account the problem of the $SU_C(3)$ GI of BSW, as well as the great difficulties on rigorous kernel and the solution of BSE, which directly relating with the difficulties in non-perturbative QCD. However, the extensive application of the BSW/BSE framework to various processes in various energy scales is the only and necessary way in exposing more deeper insights in answering Prof. Müller' s question.