# Standard Model and CKM Physics at the B Factories: Legacy for LHC 

Robert N. Cahn<br>Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA


#### Abstract

The increasing precision of measurements of $B$ decays by Belle and BaBar will provide an invaluable legacy for the LHC era whether or not deviations from the Standard Model are detected. New Physics discovered at the LHC will have to be reconciled with test that are sensitive to contributions from heavy particles of all sorts. We review with a broad brush many fundamental measurements that determine the angles and sides of the unitarity triangle.


## 1. The Kaon Legacy

The kaon taught us most of what we learned about particle physics in the twentieth century. The very discovery of "vee" articles by Rochester and Butler showed that it was possible to produce by strong interactions particles that decayed weakly. What prevented their strong decay was thus a quantity conserved in strong but not weak decays: strangeness. The Gell-Mann-Nishijima relation, $Q=I_{3}+(B+S) / 2$ showed that there were not three kaons, but four. Gell-Mann and Pais then explained that the two neutral kaons would arrange themselves into the mass eigenstates $K_{S}$ and $K_{L}$ and that produced kaons would oscillate between $K^{0}$ and $\bar{K}^{0}$. That was by no means the end of the kaon mysteries. The kaon seemed to have two different versions, evidenced in the $\tau$ (three-pion) and $\theta$ (two-pion) decays. This puzzle was nothing less than the signal of parity violation. And seven years after parity fell, CP conservation, its surviving vestige was itself brought down by the $K_{L} \rightarrow \pi \pi$ decay. Strangeness led Gell-Mann and Ne'eman to $S U(3)$ and Gell-Mann and George Zweig to quarks. And the absence of strangeness-changing neutral weak currents evidenced by the lack of decays like the $K_{L} \rightarrow \mu^{+} \mu^{-}$presaged the discovery of charm.

Will the $B$ be the $K$ of the twenty-first century? It is too soon to know, but either the discovery of New Physics in $B$ mesons or setting very stringent limits on violations of the Standard Model will likely have dramatic consequences once New Physics makes its (inevitable?) appearance at the LHC.

## 2. Flavor-changing Neutral Currents

The absence of flavor-changing neutral currents is a central feature of the Standard Model. Prior to the development of the Standard Model, the absence of flavor-changing neutral currents was a well established principle since decays like $K_{L} \rightarrow \mu^{+} \mu^{-}$weren't observed. The importance of the qualification "flavor-changing" was sometimes neglected and there was a general suspicion that there were no neutral currents at all. This led to skepticism about the Standard Model initially, because it required neutral currents, those that coupled to the $Z$. The discovery of neutral currents in neutrino interactions thus had a profound effect on acceptance of the Standard Model. The $Z$ couples only to combinations like $d \bar{d}, u \bar{u}, c \bar{c} \ldots$, which don't change flavors.

The reason that we don't see $K_{L} \rightarrow \mu^{+} \mu^{-}$is not just that the $Z$ can't couple to $d \bar{s}$ but because even with second order weak interactions, the contribution is suppressed. The contribution from the annihilation of $s \bar{d}$ through a $u$ quark is just about cancelled by the contribution from the annihilation through a $c$ quark. See Fig. 1.

The cancellation occurs because the combinations of charge $2 / 3$ quarks, $u^{\prime}, d^{\prime}, t^{\prime}$ to which the $d, s$, and $b$ are coupled by the weak interactions are simply (complex) rotations of the $u, c$, and $t$. If the masses of the charge $2 / 3$ quarks were identical, we could rename them so that each charge $-1 / 3$ quark would couple to a single charge $2 / 3$ quark. Then the box diagram would vanish. Of course the quarks aren't degenerate, but when the momenta in the loop are large, we can ignore the quark masses. This cuts off the contribution of the loop diagram, which would otherwise diverge. This is the GIM (Glashow-Iliopoulos-Maiani) mechanism. Without it, the Standard Model wouldn't be finite. You couldn't have a Standard Model with just the $u$, $d$, and $s$ quarks, the only ones known at the


Figure 1: Left: The flavor-changing neutral current process $K_{L} \rightarrow \mu^{+} \mu^{-}$, which has a very suppressed branching fraction: $6.9 \times 10^{-9}$ ). In the Standard Model, the dominant contributions are from the $u$ and $c$ quarks. These would completely cancel if the $u$ and $c$ quark masses were the same. Right:B-mixing through the box diagram. If the $u, c, t$ quarks all had the same mass, we could redefine them so that the couplings of the $W$ were just $d \rightarrow u, s \rightarrow c$, and $b \rightarrow t$. Then the box diagram would vanish. When the momenta in the box diagram are large, we can ignore the quark masses so the contribution at high momentum must vanish. This keeps the Standard Model finite. It also explains why the heavy $t$ quark dominates loop contributions to $B$ decays.
time the Standard Model was proposed. Comparing explicit calculations to the data, Gaillard and Lee anticipated that the mass of the $c$ quark was about 1.5 GeV .

This same mechanism describes one-loop $B$ decays in the Standard Model. Typically, these effects are suppressed by differences of squares of quark masses over the $W$ mass squared. Of course by itself this is not much suppression. The smallness of the off-diagonal elements o the CKM matrix are important, too. However, if new physics is introduced, in general there will be large neutral flavor-changing currents. Some mechanism will be need to suppress these below the level at which limits are set by flavor physics, be it kaon, charm, or B physics. A specific example is supersymmetry. There the suppression will need to come from the mixing matrices analogous to the CKM matrix and from ratios of differences of squares of masses over average masses. In this case, the supersymmetric particles play the roles both of the exchanged quarks in loop diagrams and the roles of the $W$ bosons.

The suppression of flavor-changing neutral weak currents is a very demanding requirement. To achieve it in supersymmetry, for example, strong measures are needed, like aligning the mixing matrices of the supersymmetric particles with the ordinary particles or demanding that the supersymmetric particles are degenerate in mass, at least at a very high energy scale with small splittings generated through renormalization effects. Similar problems are likely to beset whatever models are proposed to explain new particles found at the LHC whether supersymmetry is present or not.

## 3. Boxes

In $B$ physics there are two mechanisms that offer particularly advantageous opportunities for revealing new physics via flavor-changing neutral currents. The first of these is mixing itself, where for example we have the transition $b \bar{d} \rightarrow d \bar{b}$. The second is in so-called penguin decays: decays in which the $W$ responsible for the weak decay is reabsorbed by the quark line from which it was emitted. Examples include $b \rightarrow s \gamma$ and $b \rightarrow s$ gluon. See Fig. 2. Both mixing and penguin decays may be represented with boxes inside which the Standard Model particles are exchanged, but inside which particles not yet discovered can also be exchanged.

At LHC we expect to see new particles too massive to have been seen at the Tevatron Collider. This is not just a fond hope, but an expectation built on comparisons of data with calculations that include radiative corrections in the Standard Model, which point at least to a light Higgs boson. Much more interesting would be the appearance of a new spectroscopy, as would be afforded by supersymmetry. The new particles would then contribute inside the boxes of mixing and penguin decays. The LHC will allow us to cut open the box, just as one can cut a loop diagram when the intermediate particles are real. At the B factories we can only shake the boxes and listen for the sounds of the new particles. See Fig. 3. The sounds are flavor-changing neutral currents and the Standard Model itself produces some sounds because the charge $2 / 3$ quarks are not degenerate. (It would be enough if, alternatively, the charge $-1 / 3$ quarks were degenerate to make the Standard Model silent, i.e. free of flavor-changing neutral currents.) The


Figure 2: Both mixing and penguin decays receive contributions from virtual particles, which circulate inside a black box. Standard Model particles contribute, but if there are additional particles, they may make important contributions, too. In this way, oscillations and weak decays afford us a view of physics at very high mass scales.
challenge then is to distinguish some additional sounds, contributions from New Physics, over the sounds intrinsic to the Standard Model.

We already know that the sounds of the New Physics does not drown out the sounds of the Standard Model because measurements (described below) already show very good accord between the Standard Model and predictions for flavor-changing neutral currents in $B$ physics.

The challenge is to make measurements with the highest possible precision. Whether or not these measurements contradict the Standard Model, they will provide critical constraints on whatever models are invoked to explain the New Physics found at the LHC. Establishing a conflict with the Standard Model at present is very difficult. With so many different processed to measure, a deviation of less than $5 \sigma$ will impress few. However, LHC results may point to specific models and thus specific patterns of deviations from the Standard Model. In this circumstance, the specific patterns might be checked against results in $B$ physics with 3- $\sigma$ effects.


Figure 3: At LHC we will open the box to reveal new particles, at least those within its energy range and which have recognizable signatures. In B physics we can only shake the box and listen for the sounds of flavor-changing neutral weak currents. Nonetheless, with precision measurements we may hear sounds that betray the nature of the new particles found at the LHC or even those beyond its reach.

## 4. CKM Litany

For the sake of completeness we remind the reader that couplings of the $W$ boson to quarks is given by the Cabibbo-Kobayashi-Maskawa matrix

$$
\left(\begin{array}{ccc}
V_{u d} & V_{u s} & V_{u b}  \tag{1}\\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right)=\left(\begin{array}{ccc}
1-\frac{1}{2} \lambda^{2} & \lambda & A \lambda^{3}(\rho-i \eta) \\
-\lambda & 1-\frac{1}{2} \lambda^{2} & A \lambda^{2} \\
A \lambda^{3}(1-\rho-i \eta) & -A \lambda^{2} & 1
\end{array}\right)
$$

where the second matrix is the Wolfenstein convention, which is an expansion in $\lambda \approx 0,22$. The unitarity relation

$$
\begin{equation*}
V_{u d} V_{u b}^{*}+V_{c d} V_{c b}^{*}+V_{t d} V_{t b}^{*}=0 \tag{2}
\end{equation*}
$$

gives us the iconic unitarity triangle of Fig. 4.


Figure 4: The unitarity triangle displayed in the Wolfenstein parameterization. The angles are defined by combinations of the CKM matrix elements that are invariant under redefinition of the phases of the quark states.

While the unitarity triangle doesn't incorporate the entirety of $B$ physics, it does provide a convenient map for our strategy: determine the sides and angles in as many ways as possible, both to measure them with the best precision and to look for inconsistencies. Inconsistencies - potential failures of the Standard Model - could appear either as conflicts among the sides and angles or a conflicts in the determination of a side or angle.

## 5. Aside on phase conventions

In general, we are free to redefine any quark state or field by multiplying by a phase. This can't change the physics. Indeed, the argument of Kobayashi and Maskawa that showed that CP violation could be accommodated with three generations of quarks but not two was based on careful counting of the number of phases that could be eliminated from the CKM matrix. If we redefined the $u$ quark, say, by multiplying by $e^{i \chi}$, then we'd find $V_{u d} \rightarrow e^{-i \chi} V_{u d}$, etc. while $d \rightarrow e^{i \zeta}$ would give $V_{u d} \rightarrow e^{i \zeta} V_{u d}$. To get this right, remember that the amplitude for $d \rightarrow W^{-} u$ is proportional to $V_{u d}$, while $u \rightarrow W^{+} d$ is proportional to $V_{u d}^{*}$.

All physical observables must be independent of our arbitrary choices of phases for the quark states. The angles $\alpha, \beta, \gamma$ can be expressed in terms of the CKM matrix elements and those expressions will necessarily be phaseconvention independent.

For example

$$
\begin{equation*}
e^{-2 i \beta}=\frac{V_{c b} V_{c d}^{*} V_{t b}^{*} V_{t d}}{V_{c b}^{*} V_{c d} V_{t b} V_{t d}} \tag{3}
\end{equation*}
$$

By inspection we see that changing the phase of any of the quarks $d, c, b, t$ will not affect this ratio.
6. $\beta$

The best known CP violation measurement at B factories is in $B \rightarrow J / \psi K_{S}$. Consider specifically $\bar{B}^{0} \rightarrow J / \psi K^{0}$. One path to the final state is through the decay of $\bar{B}^{0}$ to $J / \psi \bar{K}^{0}$ directly. This is followed by oscillation of the $\bar{K}^{0}$ to
$K^{0}$. Alternatively, the $\bar{B}^{0}$ can first oscillate to $B^{0}$, which can itself decay to $J / \psi K^{0}$. Only when $K^{0}-\bar{K}^{0}$ oscillation is included is the interference possible. The CKM matrix elements from this oscillation of course make the final result independent of phase convention. See Fig. 5.

In general, the time dependence of $B^{0}$ decays is not the characteristic exponential decay, but rather as a consequence of oscillations has the form

$$
\begin{align*}
\frac{d N\left(\bar{B}^{0} \rightarrow f\right)}{d t} \propto e^{-\Gamma t}(1+S \sin \Delta m t-C \cos \Delta m t) \\
\frac{d N\left(B^{0} \rightarrow f\right)}{d t} \propto e^{-\Gamma t}(1-S \sin \Delta m t+C \cos \Delta m t) \tag{4}
\end{align*}
$$

where we have assumed the two mass eigenstates of the neutral $B$ meson have the same lifetime, $1 / \Gamma$, and where $\Delta m$ is the mass-splitting between the states. However, when there is a single weak decay mechanism, as there is for $B \rightarrow J / \psi K_{S}$, we have $C=0$. Moreover, $S$ is then simply related to an angle of the unitarity triangle. For $B \rightarrow J / \psi K_{S}, S=\sin 2 \beta$.


Figure 5: Left: The decay $\bar{B} \rightarrow J / \psi K^{0}$ through $B^{0} \bar{B}^{0}$ mixing. The amplitude is proportional to $V_{t b}^{2} V_{t d}^{* 2} V_{c b}^{*} V_{c s}$. Right: The decay $\bar{B} \rightarrow J / \psi K^{0}$ through $K^{0} \bar{K}^{0}$ mixing. The amplitude is proportional to $V_{c b} V_{c s}^{*} V_{c s}{ }^{2} V_{c d}^{* 2}$. The relative phase of the two amplitudes is $e^{-2 i \beta}$.

Representative results from BaBar are shown in Fig. 6 [1].


Figure 6: Distributions as functions of the $t_{\text {reco }}-t_{t a g}$, the difference in ps between the decay time of the reconstructed $B$ and the tagged $B$, for $B \rightarrow J / \psi K_{S}$ (upper) and $B \rightarrow J / \psi K_{L}$ (lower) for events in which the other $B$ meson is clearly tagged as a $B^{0}$ or a $\bar{B}^{0}$. Under each distribution is the asymmetry. It is clear that the asymmetry changes sign when the CP of the final state is reversed by interchanging $K_{L}$ and $K_{S}$. Data from BaBar [1].

As of the date of this lecture, the most recent results for $\sin 2 \beta$ from the B factories are $0.722 \pm 0.040 \pm 0.023$
(BaBar, [1]) and $0.652 \pm 0.039 \pm 0.020$ (Belle, [2]).
This result leaves a two-fold ambiguity since we could equally well replace $2 \beta$ with $\pi-2 \beta$ and keep $\sin 2 \beta$ the same. This ambiguity has been resolved in favor of the solution with $2 \beta<\pi / 2$ by two distinct in experiments. :in BaBar, the time-dependent angular distribution in $B \rightarrow J / \psi K^{*}$; in Belle, the time-dependent Dalitz plot for $B \rightarrow D^{(*)}\left[\rightarrow K_{S} \pi^{+} \pi^{-}\right] \pi^{0}$ and the analogous processes with $\pi^{0}$ replaced by $\eta$ or $\omega$.

## 7. Measuring $\alpha$

The description of the interference that isolates $\alpha$ is simpler than for $\beta$. See Fig. 7. Unfortunately, the simplicity here is misleading. Another mechanism, the penguin, with a different phase contributes. See Fig. 8 The consequence


Figure 7: Left: The decay $\bar{B}^{0} \rightarrow \pi^{+} \pi^{-}$without mixing. The amplitude is proportional to $V_{u b} V_{u d}^{*}$. Right: The decay $\bar{B}^{0} \rightarrow \pi^{+} \pi^{-}$through $B^{0} \bar{B}^{0}$ mixing. The amplitude is proportional to $V_{t b}^{2} V_{t d}^{* 2} V_{u b}^{*} V_{u d}$. The relative phase of the two amplitudes is $e^{-2 i \alpha}$. This simple analysis is inadequate because there is substantial contribution from the penguin mechanism.


Figure 8: The penguin contribution to $B \rightarrow \pi \pi$. The intermediate quark line contribution is dominated by the $t$ quark. The weak phase is thus given by $V_{t d}^{*} V_{t b}$.
is that the $S$ and $C$ coefficients in the time-dependent decay rate no longer have a simple relation to the angle $\alpha$. To overcome this, it is necessary to measure a variety of decay rates that are related by isospin. For example, in $B \rightarrow \pi \pi$, we need, on the one hand, to measure the asymmetry in the time dependence of $B^{0} \rightarrow \pi^{+} \pi^{-}$and $\bar{B}^{0} \rightarrow \pi^{+} \pi^{-}$, but also the decay rates for $B^{+} \rightarrow \pi^{+} \pi^{0}, B^{0} \rightarrow \pi^{+} \pi^{+}, \bar{B}^{0} \rightarrow \pi^{+} \pi^{-}, B^{0} \rightarrow \pi^{0} \pi^{0}$, and $\bar{B}^{0} \rightarrow \pi^{0} \pi^{0}$. See Fig. 9

While this seems an elegant resolution of the problem, in fact, it is fraught with difficulties. First of all, the rates for $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $\bar{B}^{0} \rightarrow \pi^{0} \pi^{0}$ aren't known separately. Indeed, the average of the two is poorly known. On top of this, we don't know how the two triangles in Fig. 9 are oriented. Are both "up,", one 'up' and one 'down'? As a result of these problems, the correction to $\alpha_{\text {eff }}$ is not well known:

$$
\begin{equation*}
\left|\alpha-\alpha_{e f f}\right|<35^{0} \quad 90 \% C L \tag{5}
\end{equation*}
$$

At the moment, the best approach is, somewhat surprisingly, through the decays $B \rightarrow \rho \rho$. A priori, this seems a hard way to go. With four particles to be observed in the final state, and some of them $\pi^{0}$ in general, the efficiencies are bound to be low and the backgrounds high. Moreover, because the $\rho$ has spin one there are three partial wave


Figure 9: The isospin triangle construction provides a means of measuring $\kappa=2 \alpha_{e f f}-2 \alpha$, the correction needed to change the observed time-dependent asymmetry $S$ in $B \rightarrow \pi^{+} \pi^{-}$into the desired quantity. Two of the four possible orientations of the triangles are shown. The angle $\kappa$ is displayed for the orientation where one triangle is up and one is down. The reader should ask herself why the rates for $B^{+} \rightarrow \pi^{+} \pi^{0}$ and $B^{-} \rightarrow \pi^{-} \pi^{0}$ must be the same.

|  | BaBar $\int \mathcal{L} d t$ |  |  | Belle |
| :--- | ---: | ---: | ---: | ---: |
| $\operatorname{BF}\left(\pi^{+} \pi^{0}\right)$ | $5.8 \pm 0.6 \pm 0.4[3]$ | 205 | $5.0 \pm 1.2 \pm 0.5[4]$ | 78 |
| $\operatorname{BF}\left(\pi^{+} \pi^{-}\right)$ | $5.5 \pm 0.4 \pm 0.3[5]$ | 205 | $4.4 \pm 0.6 \pm 0.3[4]$ | 78 |
| $\operatorname{BF}\left(\pi^{0} \pi^{0}\right)$ | $1.17 \pm 0.32 \pm 0.10[6]$ | 205 | $2.3_{-0.5}^{+0.4+0.2}[7]$ | 253 |

Table I: Recent branching fractions in units of $10^{-6}$ for various $B \rightarrow \pi \pi$ channels. Integrated luminosities for each channel are given in $\mathrm{fb}^{-1}$.
amplitudes, not just one as in $B \rightarrow \pi \pi$. It turns out, however, that that the $\rho$ s are nearly entirely longitudinally polarized as determined from measurements of the angular distributions of the final-state pions. This solves the spin problem. In addition, the branching fraction to $\rho^{0} \rho^{0}$ is very small compared to the other charge channels. This means that the isospin triangles are very flat and $\kappa$ must be small.

|  | BaBar | $\left(\mathrm{fb}^{-1}\right)$ | Belle | $\left(\mathrm{fb}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{BF}\left(\rho^{+} \rho^{0}\right)$ | $17.2 \pm 2.5 \pm 2.8[8]$ | 205 | $31.7 \pm 7.1_{-6.7}^{+3.8}[9]$ | 78 |
| $\mathrm{BF}\left(\rho^{+} \rho^{-}\right)$ | $30 \pm 4 \pm 5[10]$ | 82 | $22.8 \pm 3.8_{-2.6}^{+2.3}[11]$ | 253 |
| $\mathrm{BF}\left(\rho^{0} \rho^{0}\right)$ | $<1.1[12]$ | 205 |  |  |

Table II: Recent branching fractions in units of $10^{-6}$ for various $B \rightarrow \rho \rho$ channels. Integrated luminosities for each channel are given in $\mathrm{fb}^{-1}$.

The CP asymmetries observed by the two experiments are given in Table III.
The Heavy Flavor Averaging Group sets the current value (prior to ICHEP 2006) at $\alpha=99_{-9}^{+12} \operatorname{deg}[14]$.

| BaBar | $\left(\mathrm{fb}^{-1}\right)$ | Belle | $\left(\mathrm{fb}^{-1}\right)$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| $S_{\rho^{+} \rho^{-}}$ | $-0.33 \pm 0.24_{-0.14}^{+0.08}[13]$ | 205 | $0.08 \pm 0.41 \pm 0.09[11]$ | 22530

Table III: Measured asymmetries in $B \rightarrow \rho^{+} \rho^{-}$.


Figure 10: Left: The regions favored by the $B \rightarrow \rho \rho$ data as determined by the CKM Fitter group. Right: The regions favored by the combination of $B \rightarrow \pi \pi, B \rightarrow \rho \pi$ and $B \rightarrow \rho \rho$ as determined by the CKM Fitter group. The combination selects the solution near $100^{\circ}$.

## 8. $\gamma$

The angle $\gamma$ can be obtained through time-dependent measurements, but time-independent measurements look more promising at the moment. The underlying principle is displayed in Fig. 11. There we see decays $B^{-} \rightarrow D^{0} K^{-}$ and $B^{-} \rightarrow \bar{D}^{0} K^{-}$. These can interfere only if we observe a final state accessible to both $D^{0}$ and $\bar{D}^{0}$. An especially powerful choice is $D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$, where actually we are using $D^{0} \rightarrow \bar{K}^{0} \pi^{+} \pi^{-}$and $\bar{D}^{0} \rightarrow K^{0} \pi^{+} \pi^{-}$. Consider first the decays as if the $D$ were stable and look at the amplitude ratios:

$$
\begin{equation*}
\frac{A\left(B^{-} \rightarrow \bar{D}^{0} K^{-}\right)}{A\left(B^{-} \rightarrow D^{0} K^{-}\right)}=r_{B} e^{i \delta_{B}-i \gamma} ; \quad \frac{A\left(B^{+} \rightarrow D^{0} K^{+}\right)}{A\left(B^{+} \rightarrow \bar{D}^{0} K^{+}\right)}=r_{B} e^{i \delta_{B}+i \gamma} \tag{6}
\end{equation*}
$$

The phase $\delta_{B}$ comes from the strong final-state interaction and is thus CP invariant while the weak phase $\gamma$ of course comes in with the opposite sign for the CP conjugate process. We expect $\left|r_{B}\right|=\left|\left(V_{u b} V_{c s}^{*}\right) /\left(V_{c b} V_{u s}^{*}\right)\right| \approx$ $(0.0043 \cdot 0.97) /(0.042 \cdot 0.226) \mid \approx 0.44$. This, however, ignores the dynamics. In any event, this ratio can be measured using any decay channel for the $D^{0}$ and $\bar{D}^{0}$, provided we know the branching fractions for both channels to the particular final state. The analysis applies, mutatis mutandis, for $B \rightarrow D^{*} K$.

For a two-body decay of the neutral $D$, the full decay amplitudes are

$$
\begin{align*}
& A\left(B^{-} \rightarrow f K^{-}\right)=A\left(B^{-} \rightarrow D^{0} K^{-}\right)\left(A\left(D^{0} \rightarrow f\right)+r_{B} e^{i \delta_{B}-i \gamma} A\left(\bar{D}^{0} \rightarrow f\right)\right) \\
& A\left(B^{+} \rightarrow f K^{+}\right)=A\left(B^{+} \rightarrow \bar{D}^{0} K^{+}\right)\left(A\left(\bar{D}^{0} \rightarrow f\right)+r_{B} e^{i \delta_{B}+i \gamma} A\left(D^{0} \rightarrow f\right)\right) \tag{7}
\end{align*}
$$

If the final state is two-body, then $A\left(\bar{D}^{0} \rightarrow f\right)$ and $A\left(D^{0} \rightarrow f\right)$ are simply constants. However, for a three-body decay like $D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$they are instead functions of the location in the $\operatorname{Dalitz~plot}[15,16]$. For the decay


Figure 11: Diagrams for the decays $B^{-} \rightarrow D^{0} K^{-}$and $B^{-} \rightarrow \bar{D}^{0} K^{-}$. In the Wolfenstein parameterization both amplitudes are proportional to $\lambda^{3}$. Interference occurs only when the $D^{0}$ and $\bar{D}^{0}$ decay to a common state, like $K_{S} \pi^{+} \pi^{-}$.
$D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$the amplitudes are functions of $m_{ \pm}^{2}=\left(p_{K_{S}}+p_{\pi^{ \pm}}\right)^{2}$ and we can write

$$
\begin{align*}
A\left(D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}\right) & \equiv \mathcal{A}\left(m_{-}^{2}, m_{+}^{2}\right) \\
A\left(\bar{D}^{0} \rightarrow K_{S} \pi^{+} \pi^{-}\right) & \equiv \mathcal{A}\left(m_{+}^{2}, m_{-}^{2}\right) \tag{8}
\end{align*}
$$

if we assume that there is no CP violation in charm decays. Moreover, the decay $D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$can be studied in tremendous detail using $D^{0}$ identified through the decay of $D^{*}$. As a result it is possible to know very well the final decay amplitudes. See Fig. 12.


Figure 12: The scheme for determining $\gamma$ from decays $B^{-} \rightarrow D^{0} K^{-}, B^{+} \rightarrow \bar{D}^{0} K^{+}$, with the neutral $D$ decaying to $K_{S} \pi^{+} \pi^{-}$. The amplitudes for $D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$and $\bar{D}^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$are represented symbolically by the square root of the Dalitz plot.

The current values from BaBar $\left(70 \pm 30(\text { stat })_{-10}^{+12}(\text { syst })_{-11}^{+14}\right.$ (model) [17] and Belle $\left(53_{-18}^{+15}\right.$ (stat) $\pm 3$ (syst) $\pm$ 9 (model) $)$ [18] are similar. The BaBar results [17] in the $r_{B}-\gamma$ plane are shown in Fig. 13. Similar results are obtained with the $D^{* 0}$ in place of $D^{0}$, with the decays $D^{* 0} \rightarrow \pi^{0} D^{0}, D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$and $D^{* 0} \rightarrow \gamma D^{0}, D^{0} \rightarrow K_{S} \pi^{+} \pi^{-}$.


Figure 13: The region in the $r_{B}-\gamma$ plane allowed by BaBar data for $B \rightarrow D^{0} K[17]$. The dark and light shaded regions indicate the regions equivalent to one and two standard deviation limits.

## 9. $V_{c b}$

Measuring the magnitudes of the CKM matrix elements is made difficult by the obscuring effects of the strong interactions (QCD). These are minimized by looking at semileptonic decays, but still difficulties remain. There are two basic approaches: exclusive decays like $B \rightarrow D \ell \nu$ and inclusive processes $B \rightarrow X \ell \nu$ where the hadronic state $X$ is not observed or rather only classified by whether it does or does not have charm in it. See Fig. 14.

If an exclusive decay is used, form factors enter at the vertex of the $W$ with the initial and final hadronic states. For the decay $B \rightarrow D^{*} \ell \nu$ there are three significant form factors. These can be determined from a complete angular analysis of the decay as a function of $q^{2}=\left(p_{B}-p_{D^{*}}\right)^{2}$. Traditionally, this analysis is parameterized by the ratios $R_{1}, R_{2}$ of two of the form factors to the third, $F\left(q^{2}\right)$, and by a single parameter $\rho^{2}$ reflecting the $q^{2}$ dependence. The rate can be extrapolated to the maximal value of $q^{2}$, where it is proportional to $\left|F\left(q_{m a x}^{2}\right)^{2} V_{c b}^{2}\right|$. Only theory can provide $F\left(q_{m x}\right)$, which thus has its own uncertainty. A typical result is $F\left(q_{\max }\right)=0.91 \pm 0.04$. Recent results on the product $F\left(q_{\text {max }}^{2}\right) V_{c b}$ are $0.0355 \pm 0.0003 \pm 0.0016$ from BaBar [19] and $0.036 \pm 0.0019 \pm 0.0018$ from Belle [20].

The alternative is just what is done in deep inelastic scattering electron scattering: sum over the hadronic final states to remove the form factors and brings us into contact with the underlying quarks. As in deep inelastic scattering, the naive quark model results need to be modified for the effects of QCD. In the deep inelastic case we benefit by taking the high $q^{2}$ limit; here we benefit from taking the heavy quark limit for the $b$ and $c$ and from using the operator product expansion (OPE). In this way, one captures the QCD effects in non-perturbative parameters that reflect the properties of $B$ mesons. It turns out that theory is more accommodated to moments of distributions than to the shapes of distributions themselves. Typical results are $\left|V_{c b}\right|=\left(41.96 \pm 0.23(\exp ) \pm 0.35(\mathrm{OPE}) \pm 0.59\left(\Gamma_{s l}\right)\right) \times 10^{-3}$


Figure 14: The exclusive and inclusive approaches to measuring $V_{c b}$.

|  | $B \bar{B}$ events | Region | $\left\|V_{u b}\right\|\left(10^{-3}\right.$ |
| :--- | :---: | :---: | :---: |
| Belle | 253 M | $m_{X}<1.7 \mathrm{GeV}, q^{2}>8 \mathrm{GeV}^{2}$ | $4.70 \pm 0.37 \pm 0.31$ |
| Belle | 253 M | $m_{X}<1.7 \mathrm{GeV}$, | $4.09 \pm 0.28 \pm 0.24$ |
| Belle | 253 M | $P_{+}>0,66 \mathrm{GeV}$ | $4.19 \pm 0.36 \pm 0.28$ |
| BaBar | 210 M | $m_{X}<1.7 \mathrm{GeV}, q^{2}>8 \mathrm{GeV}^{2}$ | $4.75 \pm 0.35 \pm 0.32$ |

Table IV: Results on $\left|V_{u b}\right|$ obtained by fully reconstructing the $B$ recoiling against the semileptonic decay as compiled by Morii[23]. The data are from [24] (BaBar) and [25] (Belle).. Here $P_{+}=E_{X}-\left|\mathbf{P}_{X}\right|$ where $X$ is the hadronic system.
[21] and $\left|V_{c b}\right|=\left(41.4 \pm 0.6(\exp ) \pm 0.31\left(\Gamma_{B}\right)\right) \times 10^{-3}$ [22]. It is clear that theoretical uncertainties favor the inclusive over the exclusive approach for $V_{c b}$.
10. $V_{u b}$

Because $V_{u b}$ is much smaller than $V_{c b}$ it is much harder to measure: semileptonic decays to uncharmed states are far rarer than to charmed states. Various approaches are taken to isolate the uncharmed decays, most obviously by requiring that the charged lepton's energy be too large to permit the production of a charmed hadronic system. With enough events, it is possible to require that the $B$ meson opposite the one that decays semileptonically is fully reconstructed. Some results obtained in this way are given in Table IV [23].

## 11. Summary

This partial review does not touch upon many important topics, especially penguin decays like $B \rightarrow K^{*} \gamma, B \rightarrow \phi K$, etc. However, it does indicate the impressive results that have been obtained so far in basic measurements of sides and angles of the unitarity triangle. These are conveniently summarized in Fig. 15.

Dramatic progress was made at the Tevatron Collider by CDF and D0 shortly after this meeting. Further sharpening of the allowed contours in the plot will come with the doubling of the data set at the B factories. The great discoveries we anticipate at LHC will not reduce interest in flavor physics but rather heighten it for each new particle discovered is a potential wrecker of the delicate balance achieved in the Standard Model, especially in its exquisite suppression of flavor-changing neutral weak currents.

## 12. Listening Carefully

At B factories and LHCb we can listen for the sounds of New Physics. And if we hear no sounds that may be the important clue, as it was in Arthur Conan Doyle's the tale of Sherlock Holmes "Silver Blaze."

Gregory (Scotland Yard detective): "Is there any other point to which you would wish to draw my attention?"

Holmes: "To the curious incident of the dog in the night-time."
Gregory:"The dog did nothing in the night-time."
Holmes: "That was the curious incident."


Figure 15: A summary plot from CKM-Fitter at this time of this talk.

## References

[1] BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 94, 161803 (2005).
[2] Belle Collaboration, K. Abe, et al., hep-ex/0507037.
[3] BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 94, 181802 (2005).
[4] Belle Collaboration, Y. Chao, et al., Phys. Rev. D69, 111102 (2004).
[5] BaBar Collaboration, B. Aubert et al., hep-ex/0508046.
[6] BaBar Collaboration, B. Aubert et al., Phys. Rev. D73, 071102 (2005).
[7] Belle Collaboration, Y. Chao, et al., Phys. Rev. D71, 091106 (2004).
[8] BaBar Collaboration, G. Schott et al., Preliminary result, Rencontre de Moriond Electroweak, March 2006.
[9] Belle Collaboration, J. Zhang, et al., Phys. Rev. Lett. 91, 221801 (2003).
[10] BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 93, 231801 (2004).
[11] Belle Collaboration, A. Somov et al., Phys. Rev. Lett. 96, 171801 (2006).
[12] BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 94, 131801 (2005).
[13] BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett.95, 041804 (2005).
[14] Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/triangle/moriond2006/index.shtml
[15] A. Bondar. Proceedings of BINP Special Analysis Meeting on Dalitz Analysis, 24-26 Sep. 2002, unpublished.
[16] A. Giri, Yu. Grossman, A. Soffer, J. Zupan, Phys. Rev. D 68, 054018 (2003).
[17] BaBar Collaboration, B. Aubert et al., Phys.Rev.Lett. 95,121802 (2005).
[18] Belle Collaboration, A. Polutektov et al, Phys.Rev. D73 112009 (2006).
[19] BaBar Collaboration, B. Aubert ,Phys.Rev.D71, 051502 (2004).
[20] Belle Collaboration, K. Abe et al.,Phys.Lett.B526,247(2002).
[21] O. Buchmuller and H. U. Flächer,,Phys. Rev. D73 073008 (2006).
[22] C. Bauer et al., Phys. Rev. D70, 094017 (2004).
[23] M. Morii, invited talk at the April 2006 APS Meeting, Dallas, TX.
[24] B. Aubert, et al., hep-ex/0507017.
[25] Belle Collaboration, I Bizjak et al., Phys. Rev. Lett. 95, 241801 (2006).

