Part 2: Measuring SUSY

.

Giacomo Polesello

INFN, Sezione di Pavia

Establishing SUSY experimentally

Assume an excess seen in inclusive analyses: how does one verify wheteher it is actually SUSY? Need to demonstrate that:

- Every particle has a superpartner
- \bullet Their spin differ by 1/2
- Their gauge quantum numbers are the same
- Their couplings are identical
- Mass relations predicted by SUSY hold

Available observables: • Sparticle masses, • BR's of cascade decays, • production cross-sections, • angular decay distributions

Precise measurements of such observable not completely straightforward at the LHC: develop a strategy based on detailed MC study of reasonable candidate models

Measurement of model parameters: LHC strategy

The problem is the presence of a very complex spectroscopy due to long decay chains, with crowded final states. Many concurrent signatures obscuring each other General strategy:

- Choose signatures identifying well defined decay chains
- Extract constraints on masses, couplings, spin from decay kinematics/rates
- Try to match emerging pattern to tentative template models, SUSY or anything else
- Having adjusted template models to measurements, try to find additional signatures to discriminate different options

In last ten years developed techniques for mass and spin measurements in complex SUSY decay kinematics

Progress helped by close collaboration with theory colleagues

Focus today on explaining most promising techniques for mass and spin measurements

Practical approach on Monte Carlo:

- Start from predictive models: masses and decay patterns defined in terms of few parameters. Example: mSUGRA
- For each model choose points in parameter space covering the main phenomenological scenarios (benchmark point)



Show in detail application of this program to an "easy" model point



Typical starting point: $ilde{\chi}_2^0$ decays

QCD Background: need decay chains involving leptons (e, μ), *b*'s, τ 's Consider signatures from $\tilde{\chi}_2^0$ decays:

- $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^*$ (6% BR to $(e,\mu) \tilde{\chi}_1^0$ non–resonant)
- $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z$ (6% BR to $(e,\mu)\tilde{\chi}_1^0$ resonant)

• $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 h \to \tilde{\chi}_1^0 \bar{b} b$

• $\tilde{\chi}_2^0 \to \tilde{\ell}^{\pm(*)} \ell^{\mp} \to \tilde{\chi}_1^0 \ell^+ \ell^-$ (ℓ mostly $\tilde{\tau}_1$ at high $\tan \beta$)

One or more of these decays present in all mSUGRA Points considered Abundantly produced: BR($\tilde{q}_L \rightarrow q \tilde{\chi}_2^0$) typically 30% in mSUGRA

R-parity conservation \Rightarrow two undetected LSP's per event

 \Rightarrow no mass peaks, constraints from edges and endpoints in kinematic distributions Key result: If a chain of at least three two-body decays can be isolated, can measure masses and momenta of involved particles in model-independent way.

Two-body kinematics



4-momentum conservation

$$m_a^2 = (E_b + E_c)^2 - (\vec{p_b} + \vec{p_c})^2 \quad E_{b(c)}^2 = m_{b(c)}^2 + |\vec{p_b}|^2$$

In rest frame of a: $\vec{p_b} + \vec{p_c} = 0 \Rightarrow |\vec{p_b}| = |\vec{p_c}| = |\vec{p}|$

$$m_a^2 = (E_b + E_c)^2 \qquad m_a^2 = m_b^2 + m_c^2 + 2 |\vec{p}|^2 + 2 \sqrt{m_b^2 + |\vec{p}|^2} \sqrt{m_c^2 + |\vec{p}|^2}$$

Solve for $|\vec{p}|$: $|\vec{p}|^2 = [m_b^2, m_a^2, m_c^2]$ where $[x, y, z] \equiv \frac{x^2 + y^2 + z^2 - 2(xy + xz + yz)}{4y}$ (1)

For a two-body decay the momenta of the outgoing particles are uniquely determined as a fuction of the particle masses

Cascade of successive two-body decays



Go to rest system of intermediate particle *b*:

$$|\vec{p_p}|^2 = |\vec{p_a}|^2 = [m_p^2, m_b^2, m_a^2] \qquad |\vec{p_q}|^2 = |\vec{p_c}|^2 = [m_q^2, m_b^2, m_c^2]$$
(2)

We are interested in the invariant mass of the two visible particles: m_{pq}^2 :

$$m_{pq}^{2} = (E_{p} + E_{q})^{2} - (\vec{p_{q}} + \vec{p_{q}})^{2} = m_{p}^{2} + m_{q}^{2} + 2(E_{p} + E_{q} - |\vec{p_{p}}||\vec{p_{q}}|\cos\theta)$$

 m_{pq} has maximum or minimum value when p or q are back-to-back or collinear in rest frame of b:

$$(m_{pq}^{max})^2 = m_p^2 + m_q^2 + 2(E_p + E_q + |\vec{p_p}||\vec{p_q}|)$$
(3)

Let us specialize to the decay:

By substituting into Equation 3 $p, q \to \ell^+ \ell^-$, $c \to \tilde{\chi}_2^0$, $b \to \tilde{\ell}_R$, $a \to \tilde{\chi}_1^0$, and by treating the leptons as massless, we obtain:

$$(m_{\ell\ell}^{max})^2 = 4|\vec{p}||\vec{q}| = 4\sqrt{[0, m_{\tilde{\ell}_R}^2, m_{\tilde{\chi}_1^0}^2]}\sqrt{[0, m_{\tilde{\ell}_R}^2, m_{\tilde{\chi}_2^0}^2]}$$

By substituting the formula for [x, y, z] we obtain the desired result:

$$(m_{\ell\ell}^{max})^2 = \frac{(m_{\tilde{\chi}^0_2}^2 - m_{\tilde{\ell}_R}^2)(m_{\tilde{\ell}_R}^2 - m_{\tilde{\chi}^0_1}^2)}{m_{\tilde{\ell}_R}^2}$$

Typical algebraic structure: end points determined by differences of masses squared Procedure can be extended to chains of three or more cascade decays. Complete results for $\tilde{q}_L \rightarrow \tilde{\ell}\ell$ decay chain: (Allanach et al. hep-ph/0007009)

$$\begin{split} l^+l^- & \text{edge} \quad (m_{ll}^{\max})^2 \ = \ (\tilde{\xi} - \tilde{l})(\tilde{l} - \tilde{\chi})/\tilde{l} \\ l^+l^-q & \text{edge} \quad (m_{llq}^{\max})^2 \ = \ (\tilde{q} - \tilde{\xi})(\tilde{\xi} - \tilde{\chi})/\tilde{\xi} \\ l^+l^-q & \text{thresh} \quad (m_{llq}^{\min})^2 \ = \ \begin{cases} [\ 2\tilde{l}(\tilde{q} - \tilde{\xi})(\tilde{\xi} - \tilde{\chi}) \\ + (\tilde{q} + \tilde{\xi})(\tilde{\xi} - \tilde{l})(\tilde{l} - \tilde{\chi}) \\ - (\tilde{q} - \tilde{\xi})\sqrt{(\tilde{\xi} + \tilde{l})^2(\tilde{l} + \tilde{\chi})^2 - 16\tilde{\xi}\tilde{l}^2\tilde{\chi}} \\] / (4\tilde{\xi}) \end{cases} \\ l_{\text{near}}^{\pm}q & \text{edge} \quad (m_{\text{lnear}}^{\max})^2 \ = \ (\tilde{q} - \tilde{\xi})(\tilde{\xi} - \tilde{l})/\tilde{\xi} \end{split}$$

 $l^{\pm}_{\rm far} q \, \, {\rm edge} \qquad (m^{\rm max}_{l_{\rm far}q})^2 \ = \ (\tilde{q} - \tilde{\xi}) (\tilde{l} - \tilde{\chi}) / \tilde{l}$

 $\text{With} \quad \tilde{\chi} = m_{\tilde{\chi}_1^0}^2, \qquad \tilde{l} = m_{\tilde{l}_R}^2, \qquad \tilde{\xi} = m_{\tilde{\chi}_2^0}^2, \qquad \tilde{q} = m_{\tilde{q}}^2$

If four measurements of end-points experimentally viable, can solve for four unknown

masses

Example: Point SPS1a

 $m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, A = -100 GeV, $\tan \beta = 10$, $\mu > 0$

Chosen as a point friendly to a 1 TeV linear Collider, with appropriate Dark Matter



Point SPS1a

Total cross-section: \sim 50 pb

Identify long decay chain with clean signature from study of Branching Ratios:

$$\begin{aligned} \mathsf{BR}(\tilde{g} \to \tilde{q}_L q) \sim & \mathsf{25\%} & \mathsf{BR}(\tilde{g} \to \tilde{q}_R q) \sim & \mathsf{40\%} & \mathsf{BR}(\tilde{g} \to \tilde{b}_1 b) \sim & \mathsf{17\%} \\ \mathsf{BR}(\tilde{q}_L \to \tilde{\chi}_2^0 q) \sim & \mathsf{30\%} & \mathsf{BR}(\tilde{q}_L \to \tilde{\chi}^{\pm} q') \sim & \mathsf{60\%} \\ \mathsf{BR}(\tilde{\chi}_2^0 \to \tilde{\ell}_R \ell) = & \mathsf{12.6\%} & \mathsf{BR}(\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau) = & \mathsf{87\%} & \mathsf{BR}(\tilde{\chi}_1^{\pm} \to \tilde{\tau}_1 \nu_\tau) \sim & \mathsf{100\%} \\ \end{aligned}$$

- Measure $m_{\tilde{\chi}^0_1}$, $m_{\tilde{\ell}_R}$, $m_{\tilde{\chi}^0_2}$, $m_{\tilde{q}_L}$ from the $\tilde{q}_L \to \tilde{\ell}\ell$ decay chain
- \bullet Go up the decay chain one step: address $\tilde{g} \rightarrow \tilde{b} b$
- Identify shorter or rarer decay chains: $\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau$, $\tilde{\chi}_4^0 \to \tilde{\ell}\ell$, $\tilde{\ell} \to \ell \tilde{\chi}_1^0$, $\tilde{q}_R \to q \tilde{\chi}_1^0$ and extract masses using measured $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0}$

Go in detail through first step in analysis, providing basic building block for measurement

Isolate SUSY signal by requiring:

- At least four jets: $p_{T,1} > 150 \text{ GeV}, \quad p_{T,2} > 100 \text{ GeV}, \quad p_{T,3} > 50 \text{ GeV}.$
- $M_{\text{eff}} \equiv E_{T,\text{miss}} + p_{T,1} + p_{T,2} + p_{T,3} + p_{T,4} > 600 \text{ GeV}, E_{T,\text{miss}} > \max(100 \text{ GeV}, 0.2M_{\text{eff}})$
- Exactly two opposite-sign same-flavour e, μ (OSSF) with $p_T(l) > 20$ GeV and $p_T(l) > 10$ GeV

W and Z suppressed by jet requirements, and $\overline{t}t$ by hard kinematics Build lepton-lepton invariant mass for selected events



SM background almost negligible SUSY background mostly uncorrelated $\tilde{\chi}_1^{\pm}$ decays Subtract SUSY and SM background using flavour correlation:

$$e^+e^- + \mu^+\mu^- - e^\pm\mu^\mp$$

For 100 pb^{-1} error dominated by 0.1% uncertainty on lepton energy scale



Identification of jets from required chain. SPS1a: $m(\tilde{q}_L) - m(\tilde{\chi}_2^0) \gg m(\tilde{g}) - m(\tilde{q}_L) \Rightarrow$ Consider two leading jets: plot min $(m_{\ell\ell j_1}, m_{\ell\ell j_2})$ (left), max $(m_{\ell\ell j_1}, m_{\ell\ell j_2})$ (right) Shape of falling edge depends on mass hyerarchy, and is modified by experimental cuts, resolutions and backgrounds.

Evaluate statistical uncertainty with simple linear fit. With 100 fb⁻¹ errors at the percent level



Require $m_{\ell\ell}$ below edge, $m_{\ell\ell j} < 600$ GeV, choose jet giving minimum $m_{\ell\ell j}$ Define: $m_{lq(high)} = \max(m_{l^+q}, m_{l^-q})$ $m_{lq(low)} = \min(m_{l^+q}, m_{l^-q})$ Five end-points measured: can solve for sparticle masses

Sparticle mass calculation

Generate sets of edge measurements normal distributed according to statistical errors estimated for 300 fb⁻¹. For each set solve constraints for sparticle masses.

Strong correlation among masses, as kinematic constraints measure mass differences



Probability distributions for reconstructed masses \sim gaussian $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\ell}_R$ masses reconstructed with $\sim 5 \text{ GeV}$, \tilde{q}_L mass with $\sim 9 \text{ GeV}$ (300 fb⁻¹) Statistical and E-scale errors only, systematics should also be considered

Additional measurements build on measured \tilde{q}_L , $\tilde{\ell}_R$, $\tilde{\chi}_2^0$, $\tilde{\chi}_1^0$ masses:

• Measure slepton left direct production

• Use shorter decay chains to measure additional masses: $\tilde{q}_R \rightarrow \tilde{\chi}_1^0 q$, $\tilde{q}_L \rightarrow \tilde{\chi}_4^0 q$, ...

Available measurements for SPS1a (300 fb⁻¹):

		Errors		
Variable	Value (GeV)	Stat. (GeV)	Scale (GeV)	Total
$m_{\ell\ell}^{max}$	77.07	0.03	0.08	0.08
$m_{\ell\ell q}^{max}$	428.5	1.4	4.3	4.5
$m_{\ell q}^{low}$	300.3	0.9	3.0	3.1
$m^{high}_{\ell q}$	378.0	1.0	3.8	3.9
$m_{\ell\ell q}^{min}$	201.9	1.6	2.0	2.6
$m_{\ell\ell b}^{min}$	183.1	3.6	1.8	4.1
$m(\ell_L) - m(\tilde{\chi}_1^0)$	106.1	1.6	0.1	1.6
$m_{\ell\ell}^{max}(ilde{\chi}_4^0)$	280.9	2.3	0.3	2.3
$m_{ au au}^{max}$	80.6	5.0	0.8	5.1
$m(\tilde{g}) - 0.99 \times m(\tilde{\chi}_1^0)$	500.0	2.3	6.0	6.4
$m(\tilde{q}_R) - m(\tilde{\chi}_1^0)$	424.2	10.0	4.2	10.9
$m(\tilde{g}) - m(\tilde{b}_1)$	103.3	1.5	1.0	1.8
$m(\tilde{g}) - m(\tilde{b}_2)$	70.6	2.5	0.7	2.6

Using the measurements: model-based approach

Simplest approach: postulate SUSY breaking model, and verify if any set of the model parameters fits measured quantities. Exercise performed for SPS1a postulating mSUGRA



- m_0 dominated by sleptons ($\Delta m_0 \sim 2\%$)
- $m_{1/2}$ " by light gauginos ($\Delta m_{1/2} \sim 0.6\%$)
- Need \tilde{b}_1 and \tilde{b}_2 for aneta, otherwise long tails
- Trilinear couplings A_0 related to μ , fixed by $ilde{\chi}_4^0$
- Wrong μ sign ruled out by bad fit

Somewhat academic: can only do that once you know it is indeed mSUGRA

Can be used to exclude a given SUSY breaking model, if no good fit to measurement is found for any set of parameters of the model

Using the measurements: agnostic approach

The measurements do not depend a priori on a special choice of the model For instance, we can state that in the data appear the decays:

$$\begin{array}{cccc} a \to & b & q \\ & & & {} & {} \\ & & & {} & c & \ell^{\mp} \\ & & & {} & {} \\ & & {} & {} & d & \ell^{\pm} \end{array}$$

$$\begin{array}{cccc} a \to & b & q \\ & & \stackrel{|}{\longmapsto} & e & \tau \mp \\ & & \stackrel{|}{\longmapsto} & d & \tau^{\pm} \end{array}$$

We know the masses of a, b, c, d, e, we might conjecture that a, b, d appearing in both decays are the same having the same masses So we have a mass hierarchy, some of the decays relating these particles and, perhaps, the relative rates (\Rightarrow ratios of couplings) Having decay chains help restricting the possibilities, if one imposes some conservations, e.g. charges or quantum numbers

Model dependence enters when we try to give a name to the particles, and match them to a template decay chain

Various models proposed to solve the hierarchy problem, some of them provide a full spectrum of new particles, with cascade decays:

- Universal extra-dimensions: first Kaluza Klein excitation of each of the SM fields
- Little Higgs with T parity

Special feature of SUSY: if one identifies the heavy partners through their quantum numbers, the spins of all of them are wrong by 1/2

Worth investigating if exploiting the identified chains one can obtain information on the sparticle spins Sparticle spins in squark decay chain

Consider usual squark decay chain in SPA point (A.Barr)

Three visible particles in final state: 1 jet, two leptons

Spin analyser is the angle between the quark and the lepton from $\tilde{\chi}_2^0$ decay

No dynamic information from angle between two leptons, as $\tilde{\ell}_R$ is spin zero







Invariant mass distribution for visible particles



The angle θ between the two visible particles in rest frame of b related to m_{pq} as:

$$m_{pq}^2 = 2|\vec{p_p}||\vec{p_q}|(1 - \cos\theta)$$
 and $(m_{pq}^{max})^2 = 4|\vec{p_p}||\vec{p_q}|$

for p, q massless

We can thus define the dimensionless variable:

$$\hat{m}^2 = \frac{m_{pq}^2}{(m_{pq}^{max})^2} = \frac{1}{2}(1 - \cos\theta) = \sin^2\frac{\theta}{2}$$

For intermediate particle with spin zero:

$$\frac{dP}{d\cos\theta} = \frac{1}{2} \quad \Rightarrow \frac{dP}{d\hat{m}} = 2\hat{m}$$

Spin 1/2: two cases:

• Lepton same helicity as quark:

 $l^+q, \ l^-\bar{q} \text{ for } \tilde{q}_L, \ \tilde{\ell}_L$ $\frac{dP}{d\cos\theta} = \frac{1}{2}(1 - \cos\theta) \quad \Rightarrow \frac{dP}{d\hat{m}} = 4\hat{m}^3$

• Lepton opposite helicity to quark:

 $l^{-}q, \ l^{+}\bar{q} \text{ for } \tilde{q}_{L}, \ \tilde{\ell}_{R}$ $\frac{dP}{d\cos\theta} = \frac{1}{2}(1 + \cos\theta) \qquad \Rightarrow \frac{dP}{d\hat{m}} = 4\hat{m}(1 - \hat{m}^{2})$

Difference in shape of m_{ℓ^+q} and m_{ℓ^-q} : indication for $\tilde{\chi}_2^0$ spin 1/2



Experimental measurement

 $\ell^{near}q$ shows nice charge asymmetry:

 \Rightarrow Excellent probe of $\tilde{\chi}_2^0$ spin

Experimental problems in measurement:

• Can't tell quark jet from anti-quark

– Both q and \bar{q} appear in decay chain

– pp Collider \rightarrow PDF favour production of squarks over anti-squarks

• Two leptons in the event

- We are only interested in the first lepton (from neutralino decay)

– Plot $\ell^+ q$ and $\ell^- q$, minimal distorsion of asymmetry from ℓ^{far}

Production asymmetry

For squark production in considered model ($m_{\tilde{q}} \sim 600$ GeV), dominant contribution of $x \sim 0.1$



At $x\sim 0.1$ dominant contribution of valence quarks

$\ell^{far}q$ invariant mass

Lepton from slepton decay only: not directly measurable



Small residual asymmetry from boost of slepton in $\tilde{\chi}_2^0$ rest frame

Parton level

We now build at parton level on simulated events the lepton-jet invariant mass, and take the bin-by-bin asymmetry of ℓ^+ and ℓ^- distributions

Experimentally measurable: both q and \bar{q} in plot, both near and far lepton



Shape shows clear deviation from what expected for spin-zero $ilde{\chi}_2^0$

After parametrised detector simulation



Charge asymmetry survives detector simulation

Similar shape for asymmetry as at parton level, but with BG and smearing

Cross-check



- Can switch off spin correlations
 - Distribution for scalar $\tilde{\chi}_2^0$
 - Consistent with flat
 - Not consistent with spin-1/2 $\tilde{\chi}_2^0$ of previous page



Luminosity required?



Further evidence: slepton spin



Scalar particle carrying lepton number

Dilepton invariant mass

- Right-handed slepton
- $\bullet \ \ell^+$ and $\ \ell^-$ are right-handed
- might expect pronounced spin effects
- none beacuse slepton is scalar

Comparison with spin 1

For the SPS1a SUSY model, it can be shown that $\tilde{\chi}_2^0$ is not a scalar In competing models (UED) spin of partner of Z is 1, as in Standard Model Not studied in previous analysis because model not available in MC generator Comparison with spin one performed by theorists (Smillie, Webber) with very rough detector simulation

Same spectrum of sparticle masses as for SPS1a point with two spin assignments: SM-like (solid lines), SUSY (dashed lines)



Two spin assignments:

SM-like (solid lines), SUSY (dashed lines) Excellent discrimination also against spin one case

Conclusions

No statistical problems for the quick discovery of SUSY at the LHC, if $m(SUSY) \sim 1-2 {\rm ~TeV}$

Clear but difficult signatures, long work on understanding detector performance and Standard Model backgrounds

Once convincing signal claimed, try to pin down what kind of SM extension generated deviation

Definition of most effective approach strongly depends on features of observed signal

A few benchmark models studied, and some general techniques developed for mass and spin measurements of SUSY particles

If indeed we do observe the signal many years of great fun, lots of room for young people with bright ideas!