TeV Colliders and Cosmology

1: The Density of Dark Matter

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SSI - July 2006

There are many points of connection between particle physics and cosmology:

The elementary particles present at high energies determined the equation of state at very early times.

CP- and baryon-number violation in the early universe determined the cosmic density of atoms.

Phase transitions between vacuum states of quantum field theory led to cosmic inflation. Quantum fluctuations in the course of this process formed the nuclei for cosmic structures.

The small energy density of the present vacuum state is (may be ?) responsible for 'dark energy'.

In these lectures, however, I will concentrate on a differerent topic whose relevance to TeV-energy physics is very striking -- Dark Matter.

Here is an outline of the lecture series:

lecture 1:

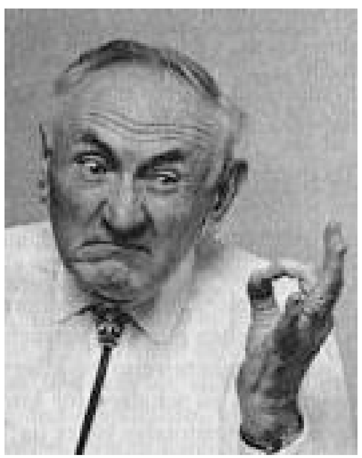
evidence for dark matter
cosmic density of dark matter
WIMP model and the energy scale of dark matter

lecture 2:

direct detection of dark matter,
and the local density of dark matter
gamma-ray detection of dark matter,
and clustering of dark matter in the galaxy
super-WIMP dark matter

The first evidence for dark matter came from the the early period of extragalactic astronomy.

In 1933, Fritz Zwicky measured the mass of the Coma cluster of galaxies.



Fritz Zwicky

O. Lopez-Cruz and I. K. Sheldon - Kitt Peak

By measuring the relative Doppler shifts of galaxies, Zwicky measured the internal kinetic energy of motion of the cluster along the line of sight. Assuming that the the motion is isotropic gives the total kinetic energy. Then, using the virial theorem,

$$\langle V \rangle = -2 \langle T \rangle$$

he could estimate the gravitational potential energy and hence the total mass of the cluster.

The result was 400 times larger than the total mass of the stars in the galaxies of the cluster $(4 \times 10^{11} M_{\odot})$.

Soon after, Smith found a similar result for the Virgo cluster.

Not all of this missing matter is dark matter.

To get an idea of the magnitudes involved, here is the virial relation:

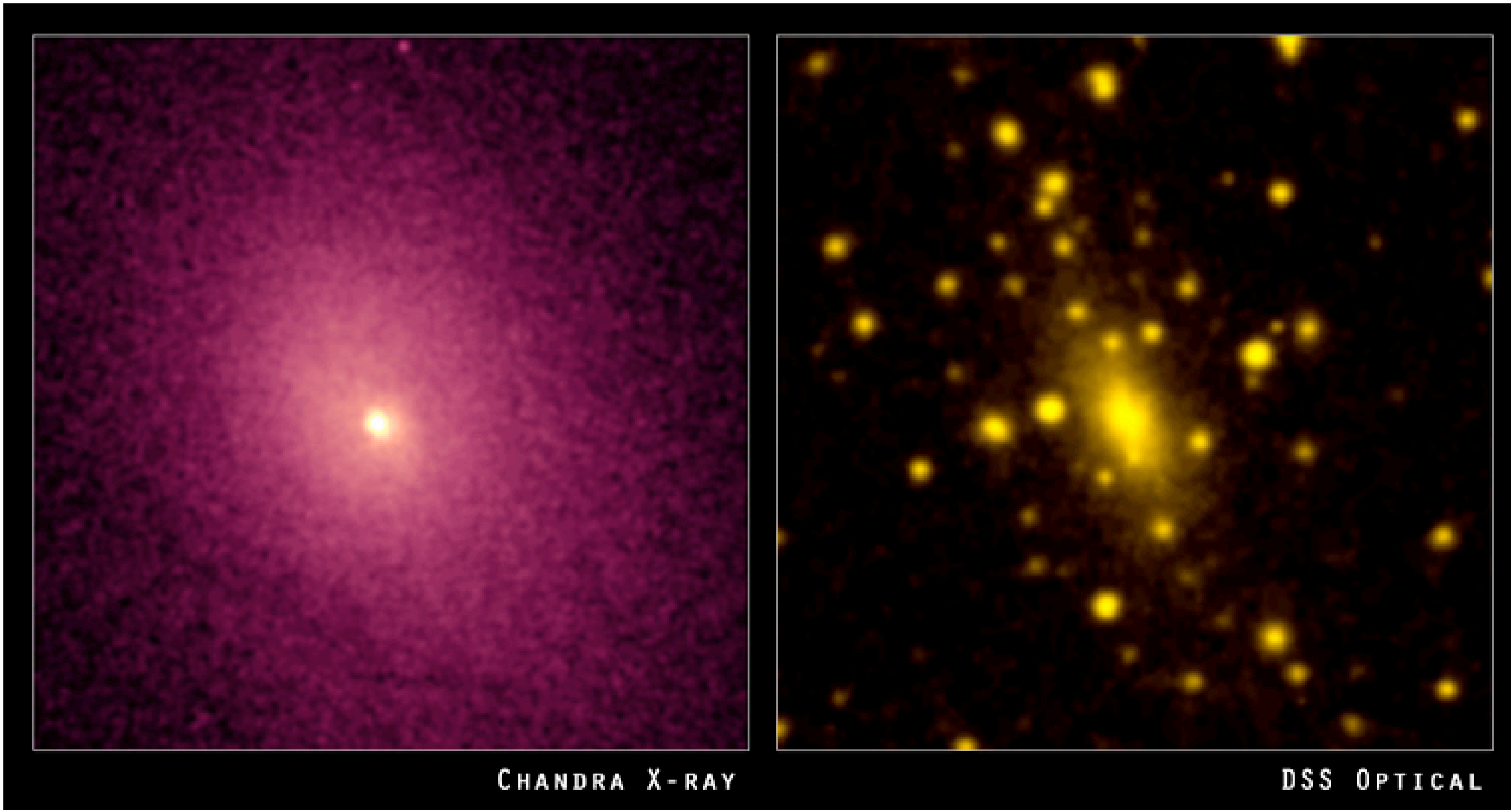
$$M = \frac{Rv^2}{G_N} \sim \left(\frac{R}{1\ \mathrm{Mpc}}\right)\left(\frac{v}{10^3\ \mathrm{km/sec}}\right)^2 \cdot 10^{15} M_\odot$$

Free gas in the cluster will be moving with the same velocity distribution. This implies a temperature

$$kT \sim m_p v^2 \sim 6 \left(\frac{v}{10^3\ \mathrm{km/sec}}\right)^2 \ \mathrm{ke}V$$

This gas radiates in the X-ray. It is visible (to X-ray satellites) and gives a mass about 15% of the total.

However, (a) this is not 100%; (b) this gas would freely stream out unless a much larger gravitating mass were keeping it in place.



Abell 2029

If we see dark matter on extragalactic scales, can we also see dark matter associated with single galaxies ?

Begin with the Milky Way. Estimate the mass of the Milky Way from the orbital velocities of globular clusters.

As a reference, our sun is 8.5 kpc from the center of the galaxy. Most of the visible stars in the galaxy are within 20 kpc of the galactic center.

Mass of the Milky Way, determined from the orbital velocities of globular clusters

distance (kpc)	result (billion solar masses)
17	200
20	30-200
44	890
50-100	500
50-100	200
100	900
100	1000
118 (one cluster)	< 1000
(total)	1000

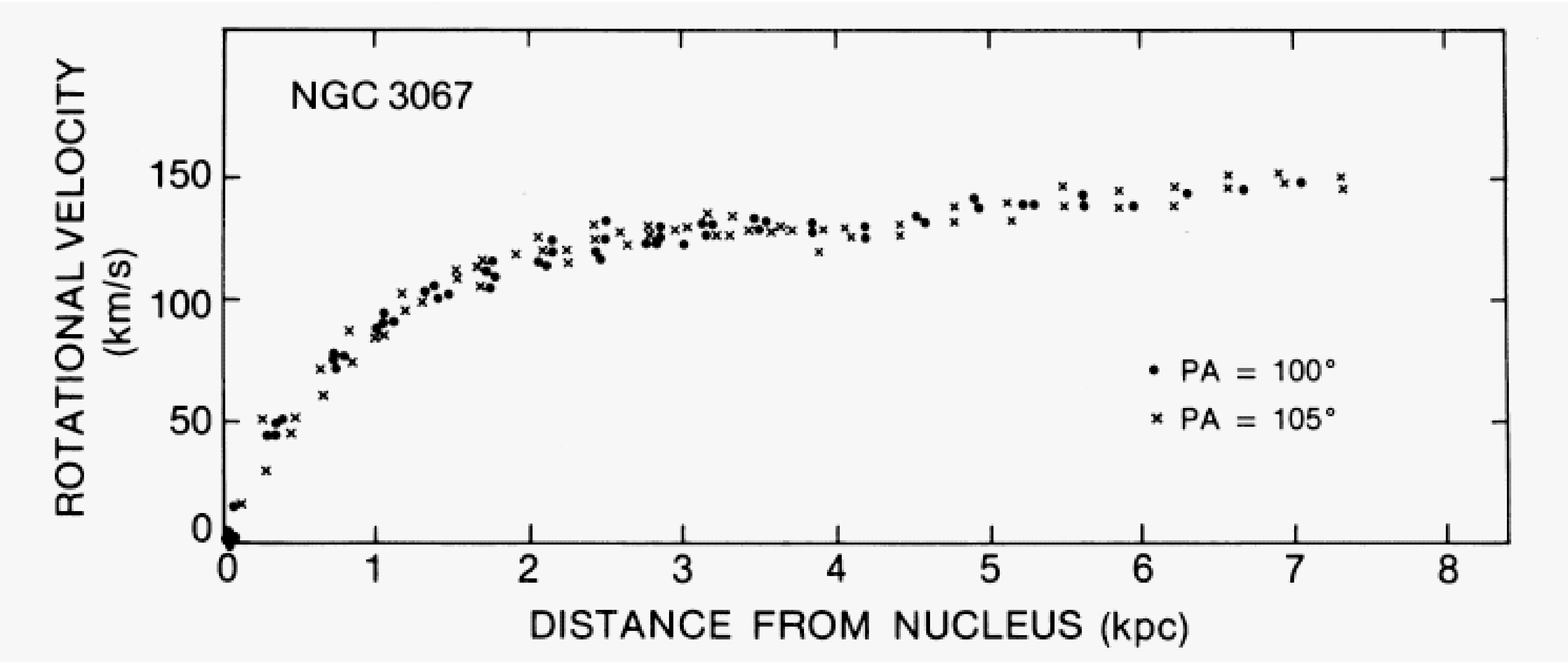
V. Trimble, Ann. Rev. Astro. Astro. (1987)

For other galaxies, it is possible to measure the radial component of the rotation velocities of individual stars and of hydrogen gas cloud (H1 regions).

For objects outside the visible part of the galaxy, the expectation would be Kepler's law:

$$T^2 \sim r^3 \text{ or } v \sim 1/\sqrt{r}$$

What is actually seen ?



Rubin, Thonnard, Ford

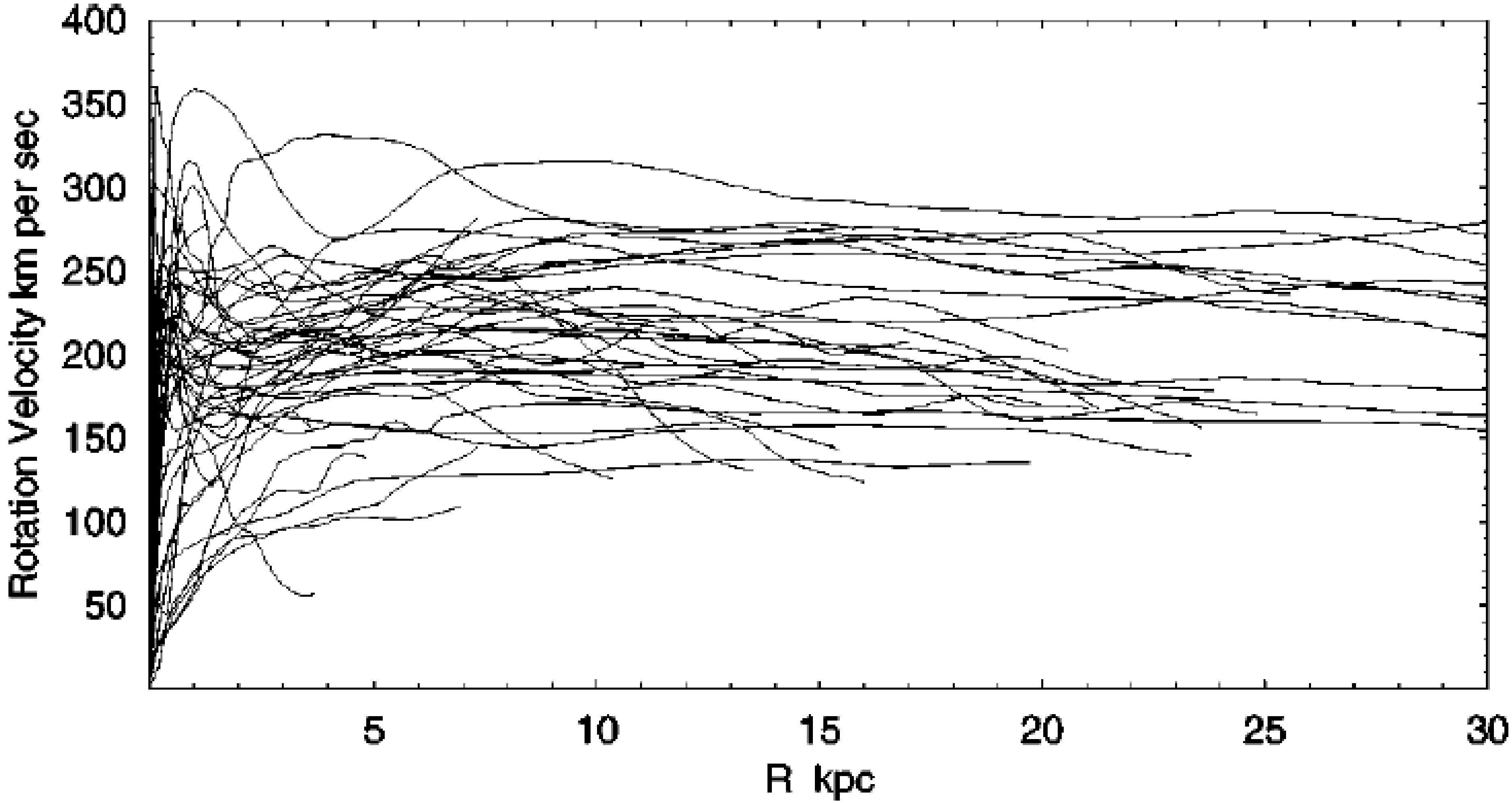
"Such a velocity implies that 94% of the mass is located beyond the optical image; this mass has a ratio M/L greater than 100."

from Sofue and Rubin, Ann. Rev. Astro. Astro (2001)

When Rubin and Ford (1970) published the rotation curve of M31, formed from velocities of 67 HII regions, they noted that the mass continued to rise out to the last-measured region, 24 kpc. They concluded that 'extrapolation beyond that distance is clearly a matter of taste'.

Over the next 10 years, many observations established approximately flat rotation curves as the typical situation.

Here is a sampler of 25 galactic rotation curves, from this review:



Sofue and Rubin

The flat rotation curves of galaxies obey a regularity

$$v_{rot}^4 \sim M$$

equivalent to the Tully-Fisher law. Milgrom interpreted this as a requirement for the acceleration of gravity to take the asymptotic form:

$$a = \left(\frac{G_N M}{R^2} \cdot a_0 \right)^{1/2}$$

The theory is called Modified Newtonian Dynamics (MOND).

This is a somewhat dangerous postulate: It is straightforward to modify dynamics at short distances by adding new, higher-dimensional interactions, but modifying dynamics at large distances requires new nonlocal interactions.

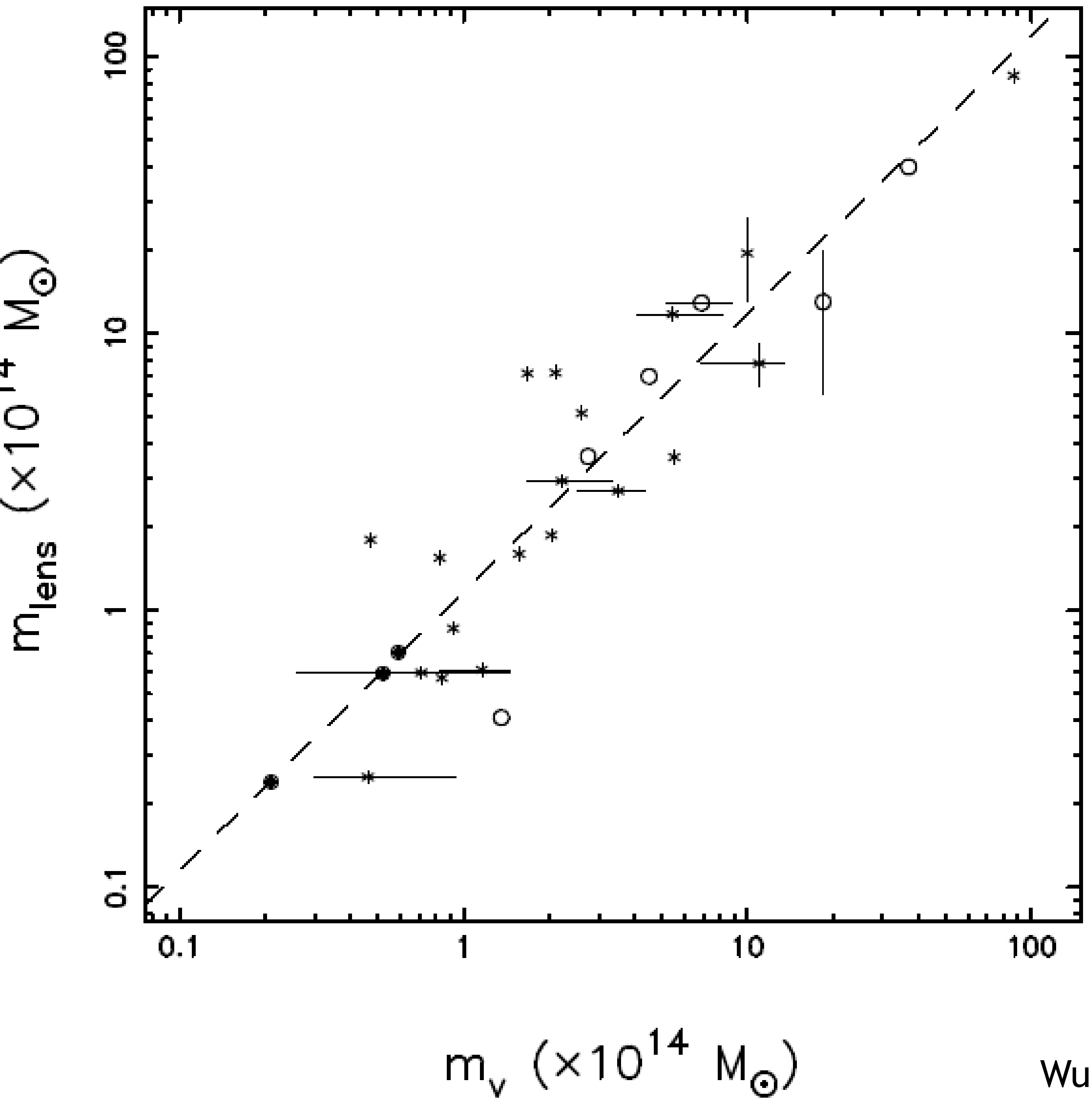
It is not straightforward to make MOND give the correct predictions for cluster size scales (100 x larger than galaxies) or for gravitational bending of light. However, there are generalizations that can fit the data.

It is not clear how else to challenge MOND quantitatively. But recently MOND has been challenged by interesting qualitative observations. This comes from another way to measure the mass distribution in cluster-size objects: gravitational lensing.

Zwicky first proposed this technique in 1937. It has come of age with the Hubble Space Telescope images.

In general, gravitational lensing estimates of cluster mass are in good agreement with virial estimates.

0024+1654 W. N. Colley, E. Turner, J. A. Tyson -- Hubble Space Telescope


100
10
1
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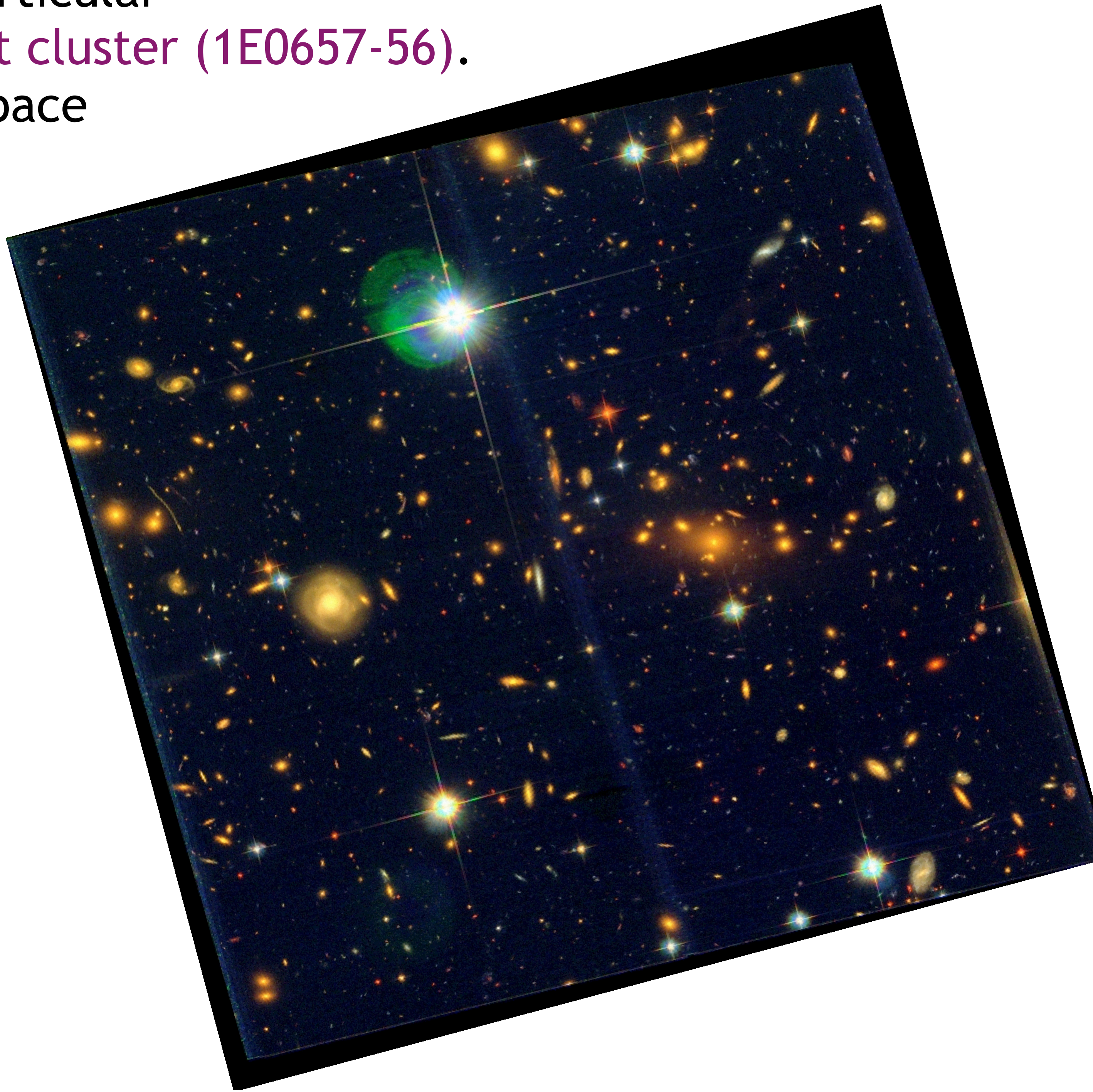
m_{lens} ($\times 10^{14}$ $M_\odot$)
m_v ($\times 10^{14}$ $M_\odot$)

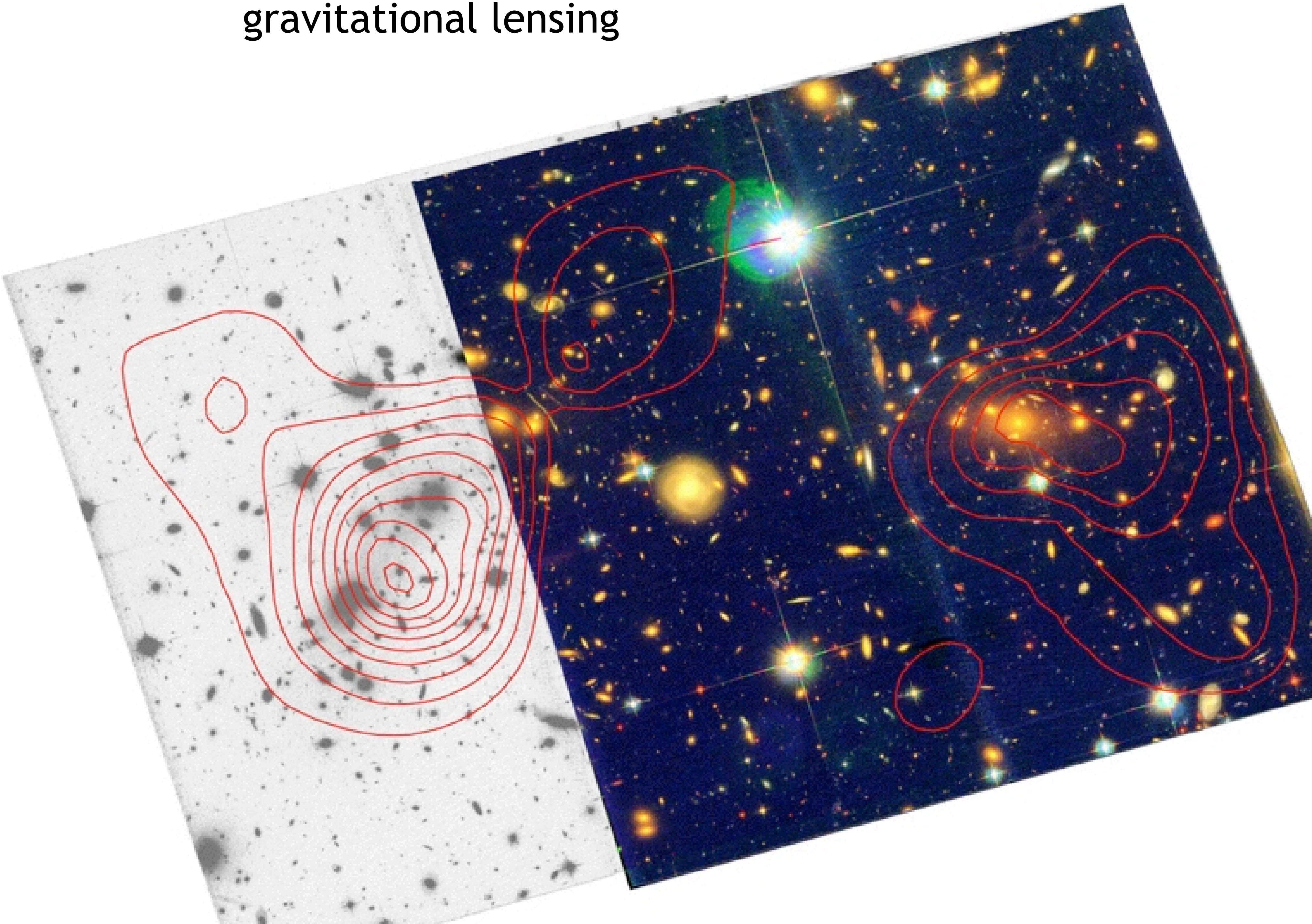
Wu and Fang

Let's examine the particular example of the bullet cluster (1E0657-56). Here is the Hubble Space Telescope Image:

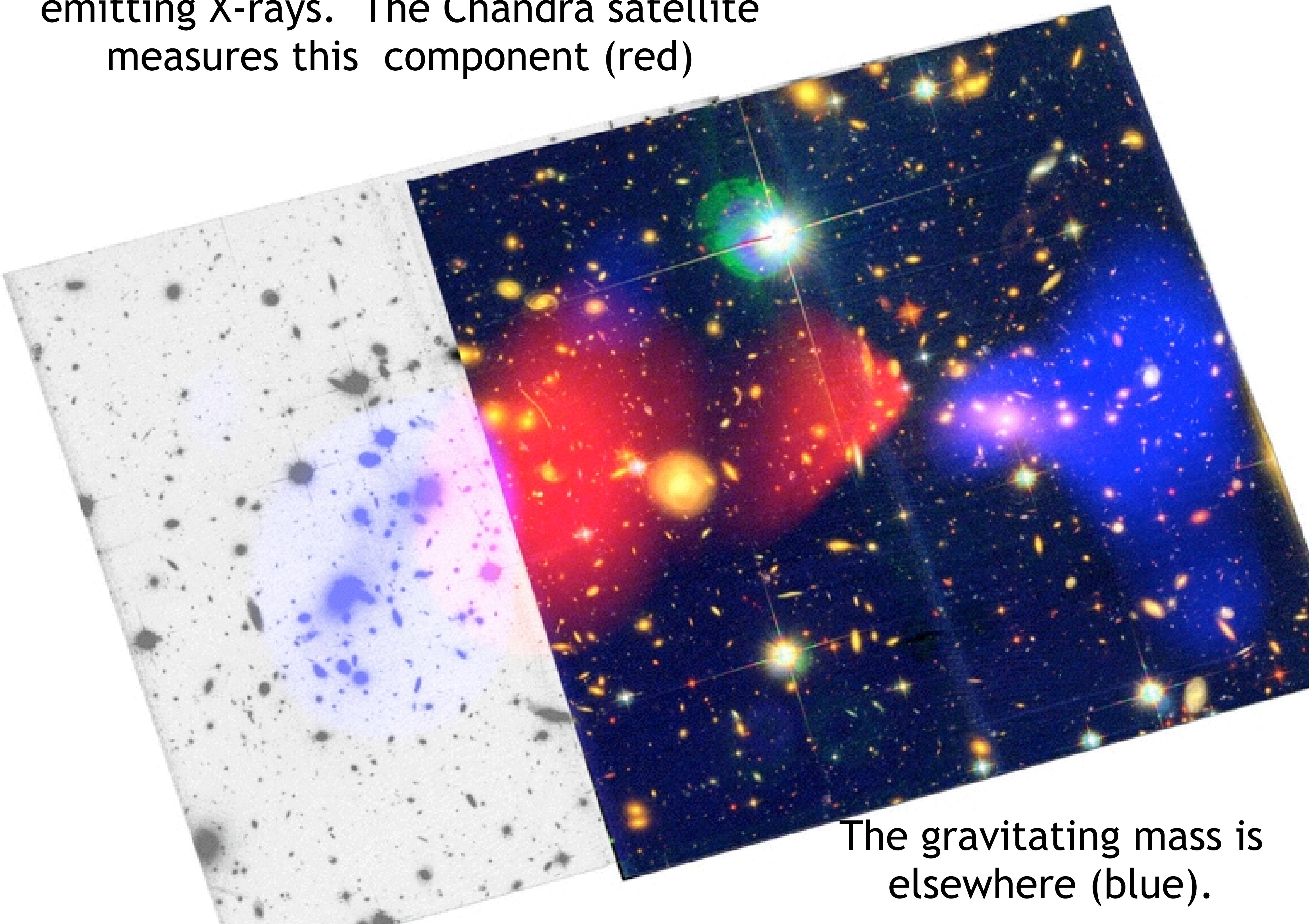


analysis of Bradac, Clowe, Gonzalez, Marshall, Forman, Jones, Markevitch, Randall, and Schrabback

Here is the mass distribution reconstructed from gravitational lensing



The atomic matter is mainly in hot gas, emitting X-rays. The Chandra satellite measures this component (red)



The gravitating mass is elsewhere (blue).

Our best understanding of the cosmic density of dark matter, however, comes from none of these sources, but, rather, from studies of the early universe through the cosmic microwave background.

In the early universe, structures could grow by gravitational collapse only after matter-radiation equality, which occurred at red shift

$$z \sim 2900$$

The radiation from the cosmic microwave background originated at the time of `recombination'

$$z \sim 1300$$

and thus gives evidence of a very early period in the growth of structure.

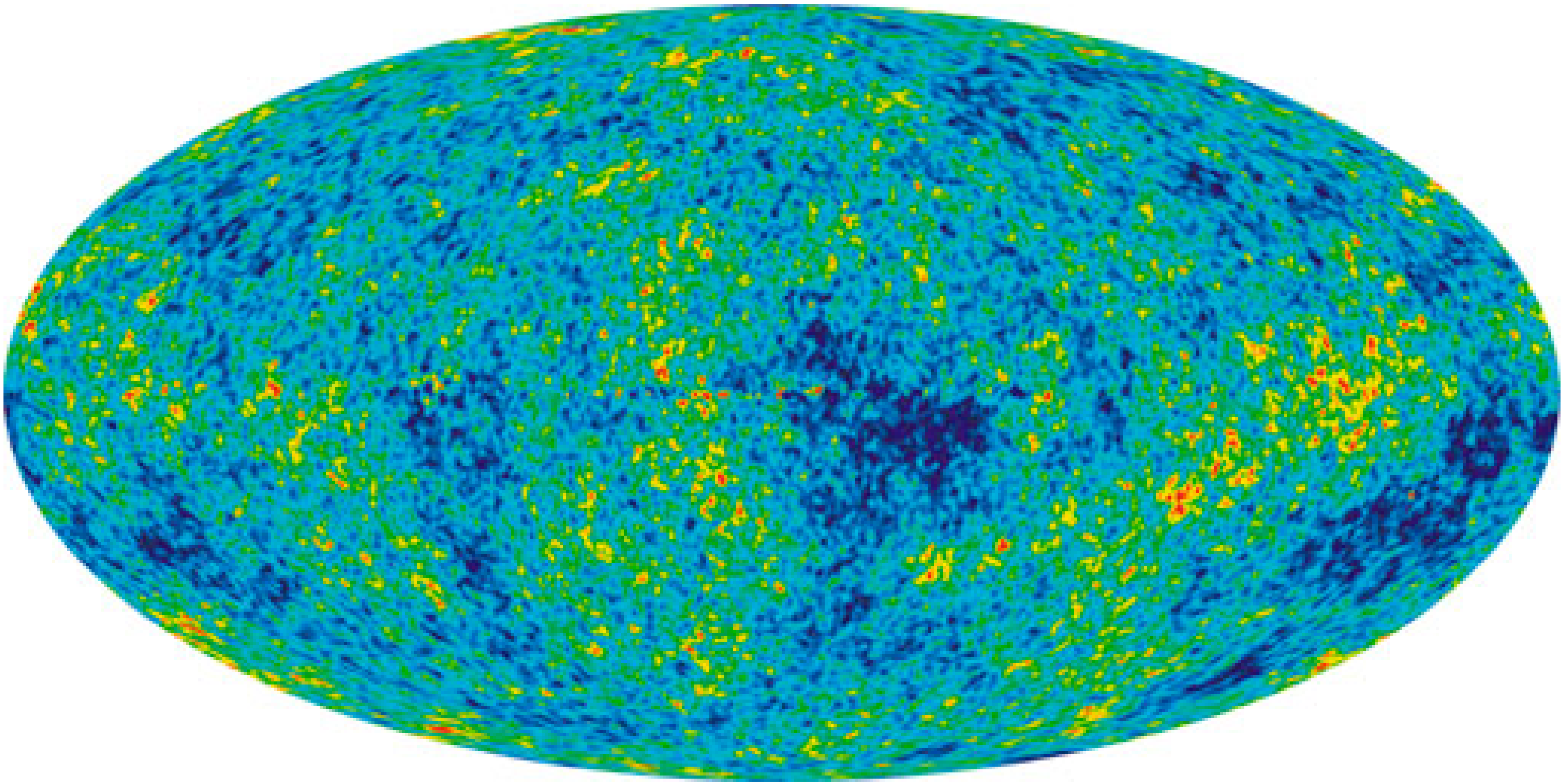
Key features of the CMB radiation are:

- the radiation is approximately thermal black-body radiation. The local fluctuations in T are of the order of 10^{-5}

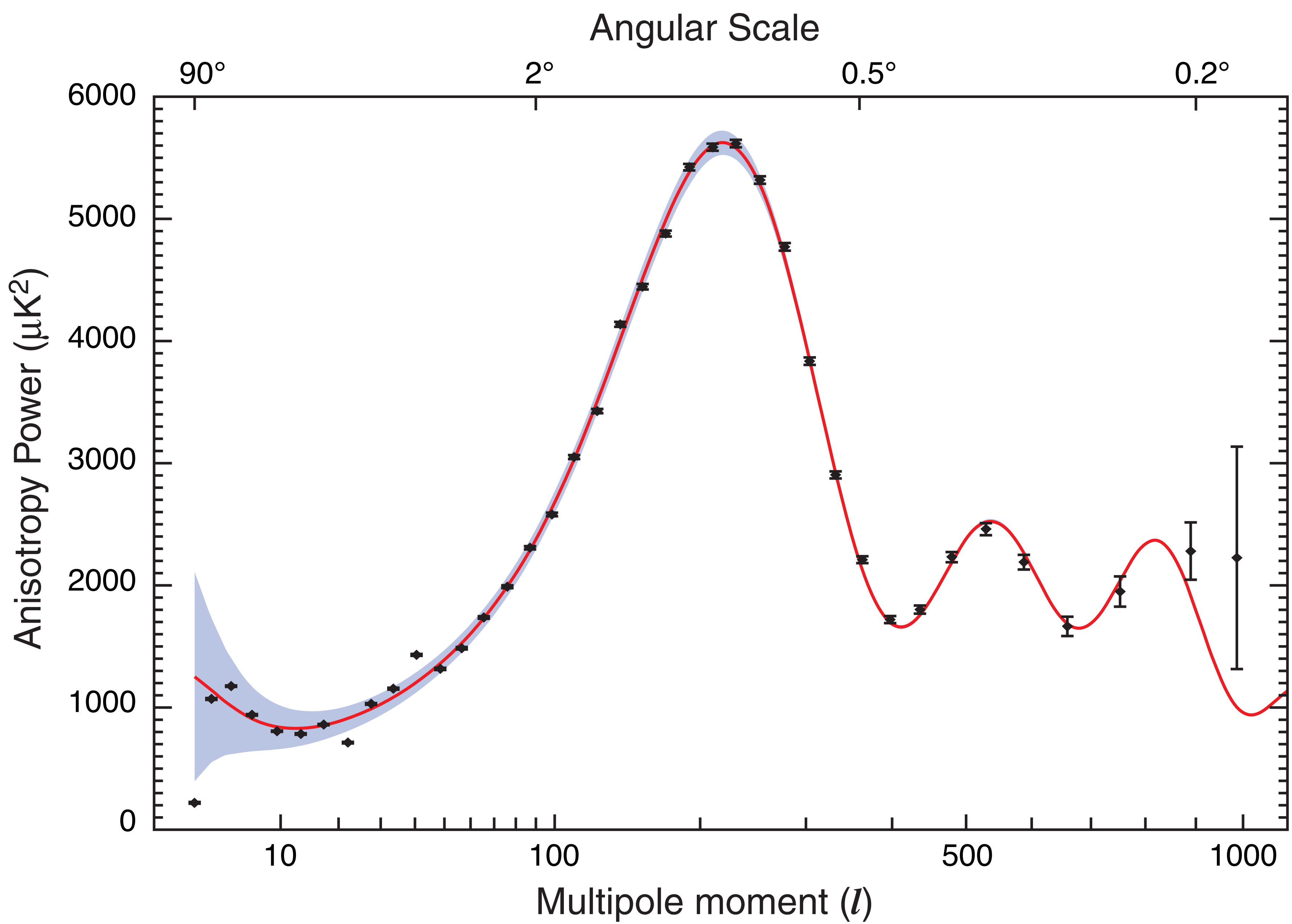
- the fluctuation spectrum has an 'acoustic' peak at angular sizes of 1°. This should correspond to the beginning of the collapse of matter into gravitational potential wells The size of the structure should be approximately the size of the sound horizon at recombination, expanded with the universe:

$$10^5 \text{ l-yr } \cdot z_{rec} = 10^8 \text{ l-yr} \qquad \text{(for a flat universe)}$$

- the fluctuation spectrum has additional peaks corresponding to the overtones of the acoustic oscillations. The relative sizes of these peaks measure the equation of state and the dissipation in the primordial medium.



WMAP science team - 2006



WMAP Science Team - 2006

If one models the CMB as being generated by a medium composed of hydrogen gas plus noninteracting dark matter, it is possible to extract the curvature of the universe and the fraction of each component. With the notation

$$\Omega_i = \rho_i/\rho_c \qquad h = H_0/(100\ \mathrm{km/sec/Mpc})$$

the results are: (WMAP 2006)

1. The universe is flat: $\Omega_{tot} = 0.99 \pm 0.01$

2. The density of matter is much larger than that of baryons

$$\Omega_m h^2 = 0.126 \pm 0.01$$
$$\Omega_b h^2 = 0.0223 \pm 0.0008$$

Using $h^2 = 0.50 \pm 0.06$ from the Hubble space telescope, we find that 80% of the mass and 20% of the total energy density of the universe is in the form of dark matter.

What do we know about dark matter at this stage ?

It is a new species of matter that is stable and has interactions that are negligible in astrophysics (< barn). It is present at a density of 20% of the critical density:

$$\rho_{DM} = 1 \text{ GeV/m}^3$$

A huge range of hypothetical particles fit this description. Some examples are

the axion m = 10^{-5}eV

the WIMPzilla m = 10^{18}GeV

black holes of < $10^{-2}\ M_\odot$

It is a major channel to elementary particle physicists to discover the particle identity of dark matter.

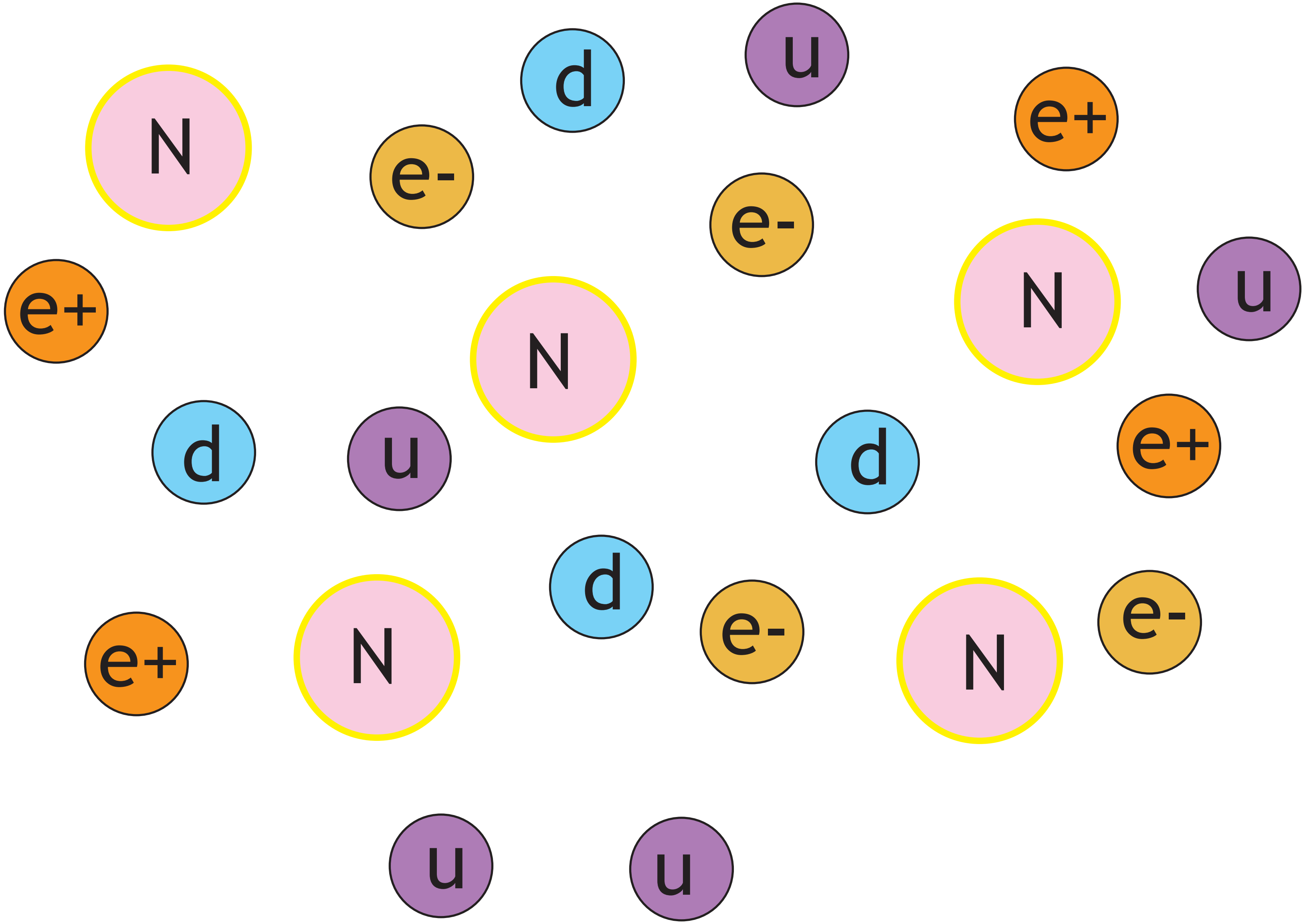
To make progress, we need to add further assumptions. Here is one that I consider weak (although the models on the previous slide are counterexamples):

Dark matter particles were in thermal equilibrium at some time in the early universe.

I will call a neutral, stable, weakly-interacting particle that satisfies this assumption a WIMP. Even if the particle is stable, it can be maintained in equilibrium if it can be created or annihilated in pairs.

This assumption allows us to compute the cosmic density of dark matter:

Start from the initial condition of thermal equilibrium. As the temperature decreases, the density of WIMPs decreases. Eventually, as the universe expands, WIMPs cannot find their partners, and a residual 'relic' density is 'frozen out'.



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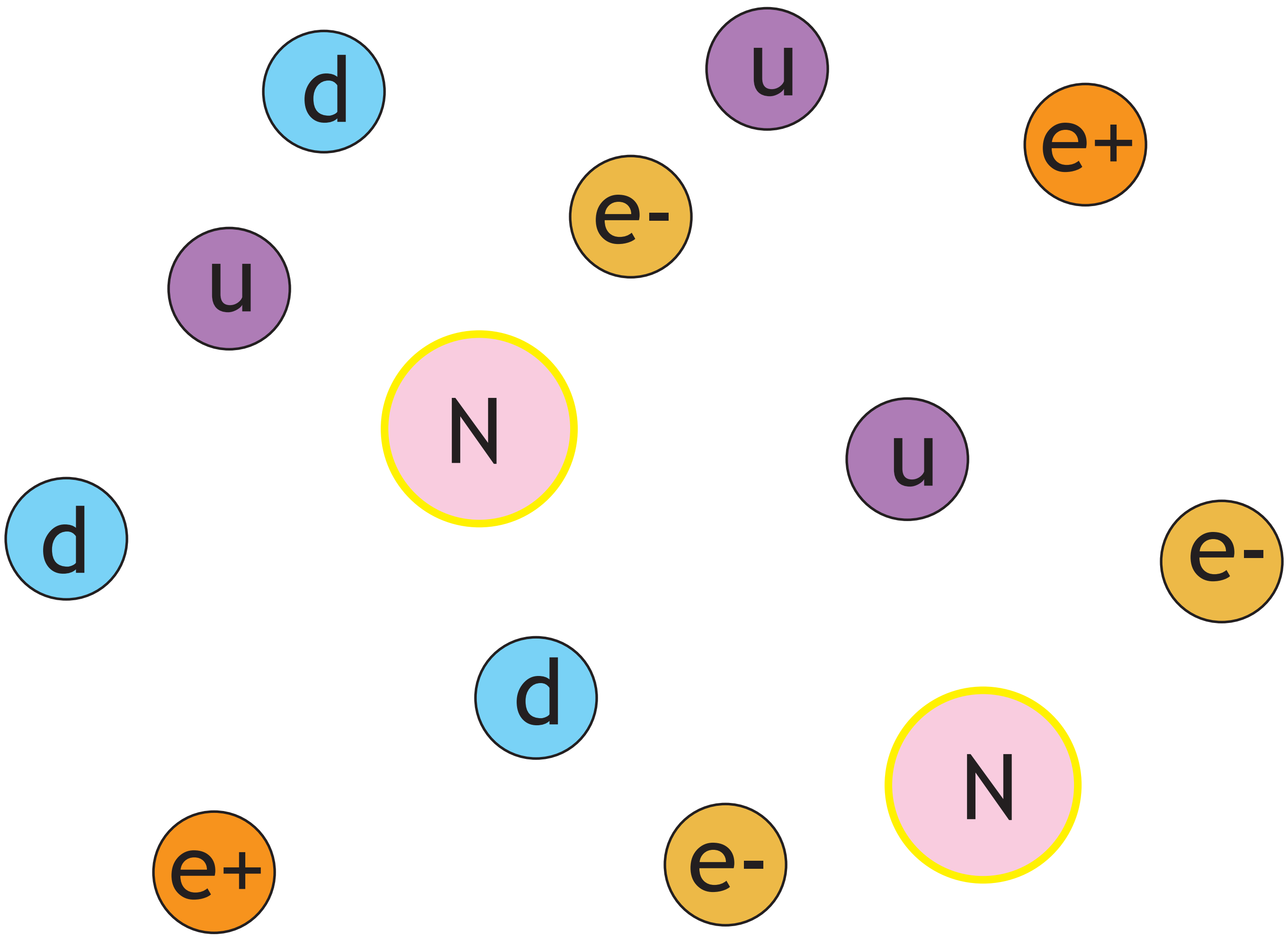
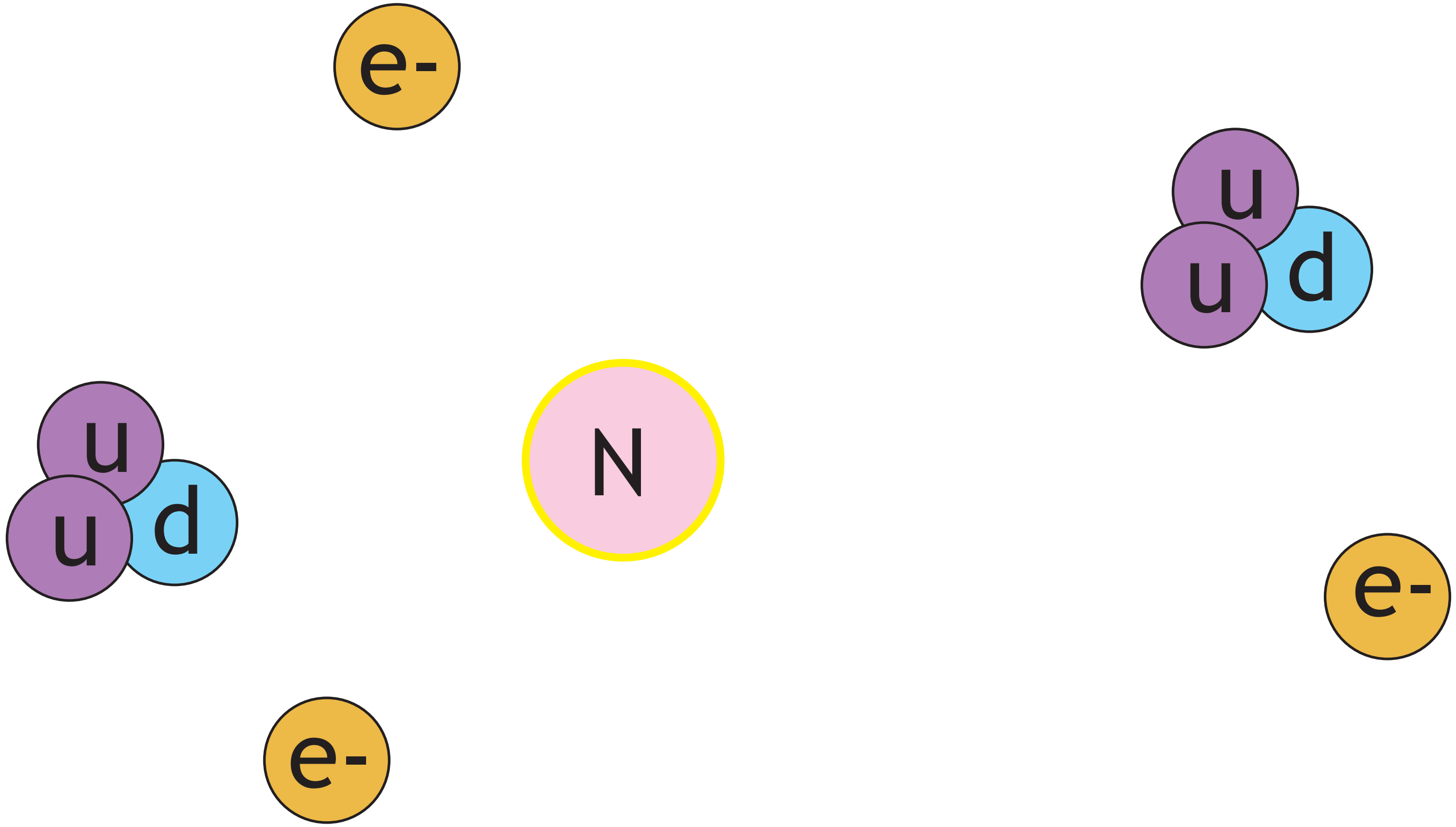
the universe expands and cools ...



until today ...



Discuss this quantitatively:

Pair creation and annihilation and the expansion of the universe are accounted by the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle \left(n^2 - n_{eq}^2\right)$$

where the expansion rate (still radiation-dominated) is

$$H = \frac{1}{2t} = \left[\frac{8\pi^3}{90} g_*\right]^{1/2} \frac{T^2}{m_{\rm Pl}}$$

where, e.g., g_* = 86.25 for the Standard Model at 10 GeV. The annihilation term dominates until the WIMPs become very nonrelativistic and

$$e^{-m_N/T} \sim \frac{T^{1/2}}{m_N^{3/2} m_{\rm Pl} \langle \sigma v \rangle}$$

This is the condition for freeze-out. Numerically, for any electroweak cross section, $\xi_f = T_f/m_N = 1/25$.

Since the universe is expanding so slowly, the expansion is approximately adiabatic, that is, entropy is conserved. Define $Y = n/s$ and use the entropy as a reference point. In a radiation-dominated universe,

$$s = \frac{2\pi^2}{45} g_* T^3$$

so the Boltzmann equation becomes

$$\frac{dY}{dt} = -\frac{2\pi^2}{45} g_* T^3 \langle \sigma v \rangle (Y^2 - Y_{eq}^2)$$

$$\frac{dY}{d\xi} = -C(Y^2 - Y_{eq}^2)$$

where

$$\xi = T/m_N \ , \ C = (\pi g_*/45)^{1/2} m_N m_{\mathrm{Pl}} \langle \sigma v \rangle$$

Now there is a nice approximation (Turner-Scherrer): Assume that Y equals its thermal value until freezeout; after freezeout, drop the second term on the right. This approximation is good to 5-10%.

$$\frac{dY}{Y^2} = -C d\xi$$

Integrate from freezeout to T = 0:

$$\frac{1}{Y(0)} - \frac{1}{Y(\xi_f)} = C\xi_f$$

The second term on the left is small, so we can write, finally,

$$Y(0) = \left(\frac{45}{\pi g_*}\right)^{1/2} \frac{1}{m_N m_{\rm Pl}} \frac{1}{\xi_f \langle \sigma v \rangle}$$

If $\langle \sigma v \rangle$ depends strongly on temperature as $T \to 0$, replace

$$\xi_f \langle \sigma v \rangle \to \int_0^{\xi_f} d\xi \, \langle \sigma v \rangle$$

I'll rewrite the final result in terms of $\Omega_N = \rho_N / \rho_c$, taking the normalization to be set by the current entropy density of the universe. The result becomes

$$\Omega_N = \frac{s_0}{\rho_c} \left(\frac{45}{\pi g_*} \right)^{1/2} \frac{1}{\xi_f m_{\rm Pl}} \frac{1}{\langle \sigma v \rangle}$$

Putting in measured values, we can extract the value of the WIMP annihilation cross section:

$$\langle \sigma v \rangle = 1 \ \mathrm{pb}$$

This is, amazingly, the characteristic size of cross sections at the LHC ! Alternatively, parametrize

$$\langle \sigma v \rangle = \frac{\pi \alpha^2}{8 m^2}$$

Then m = 100 GeV.

Is this a coincidence ? Most astrophysicists think so.

I have just the opposite opinion.   We know that we need new physics at the 100 GeV mass scale to explain electroweak symmetry breaking.   If we want a mechanism for EWSB, we need new interactions, not just one Higgs boson.

So we should be asking, do such theories contain WIMPs ?

Generically, models of EWSB contain new neutral particles. These have weak-interaction cross sections. The only nontrivial question is, are these particles stable ?

The lightest new particle will be stable if there is an exact discrete symmetry P such that this particle, and some or all new particles, carry the discrete quantum number.

Almost every model of EWSB either can contain or must contain such a discrete symmetry:

Supersymmetry: Generically, proton decay is very rapid.

$R = (-1)^{B-L+2S}$ removes the dangerous operators.

Flat extra dimensions: $P_5 \ : \ x^5 \to -x^5$ is naturally present.

Warped extra dimensions: proton decay is again a problem.

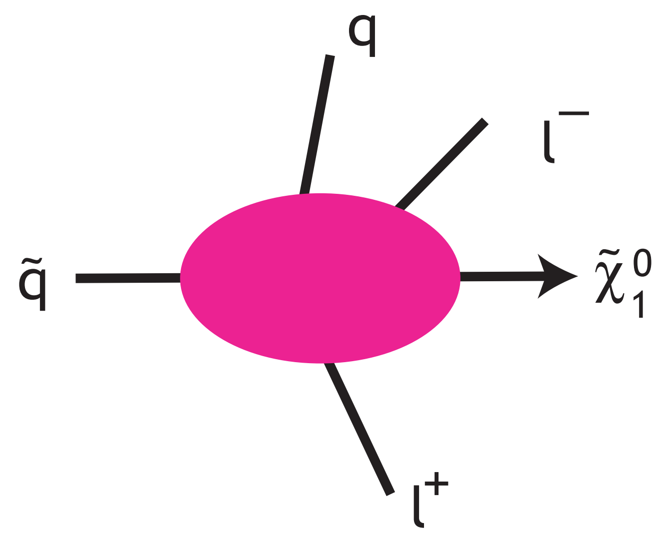
Katz-Nelson advocate applying R-parity
Agashe-Servant advocate a Z_3 parity

Little Higgs: T-parity alleviates problems with precision electroweak measurements

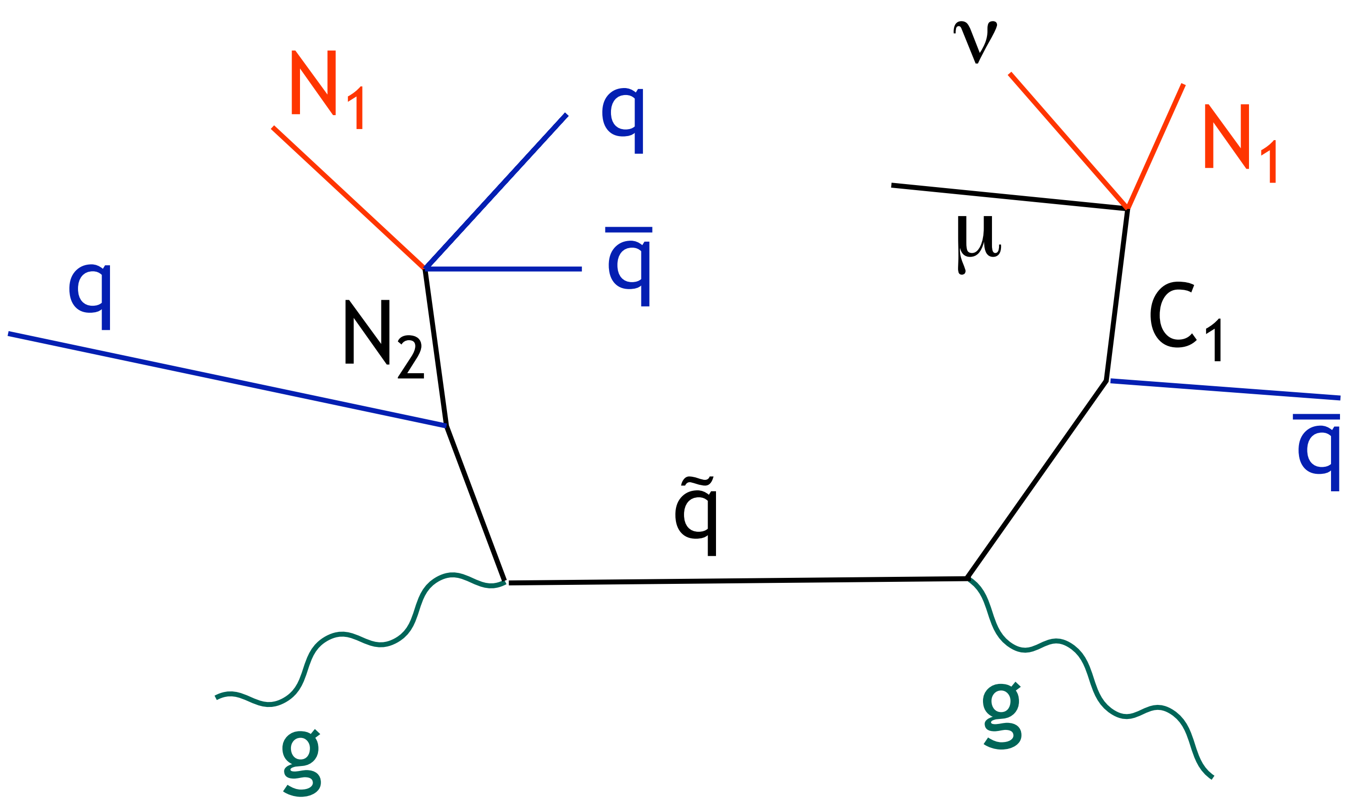
All of the models on the previous page contain one more very interesting feature:

There exist particles with QCD color that carry the conserved discrete quantum number and have masses comparable to the WIMP mass.

Such particles are produced at the LHC with pb cross sections. They decay to WIMPs, together with jets and leptons.



This complicated supersymmetry event is actually typical of events with new physics in models where EWSM leads to dark matter.



The event has 4 large-p_T jets and unbalanced total p_T due to the invisible dark matter particles.

Let me summarize the discussion to this point:

20% of the energy, 80% of the mass in the universe is in the form of dark matter, material composed of a new, weakly-interacting elementary particle

The assumption that dark matter particles were once in thermal equilibrium implies a relation between the density of dark matter particles and the scale of electroweak symmetry breaking.

Models in which dark matter and electroweak symmetry breaking have a common origin predict large cross sections for missing ET + multijet events at LHC.

Tomorrow, we'll go further down this road and find more surprises.