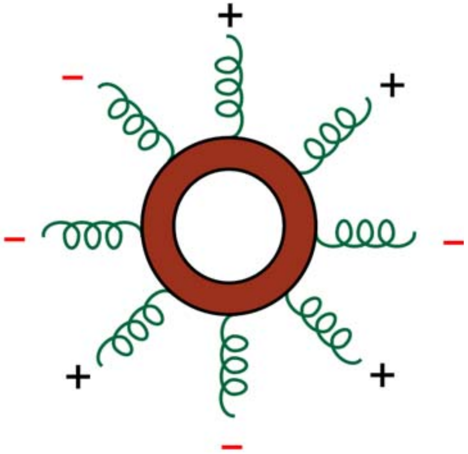
Higher Order QCD
Lecture 2



Lance Dixon, SLAC

SLAC Summer Institute
The Next Frontier: Exploring with the LHC
July 21, 2006

Lecture 2 Outline

- NLO
- NNLO

What is the state of the art?

Example of pure jet production

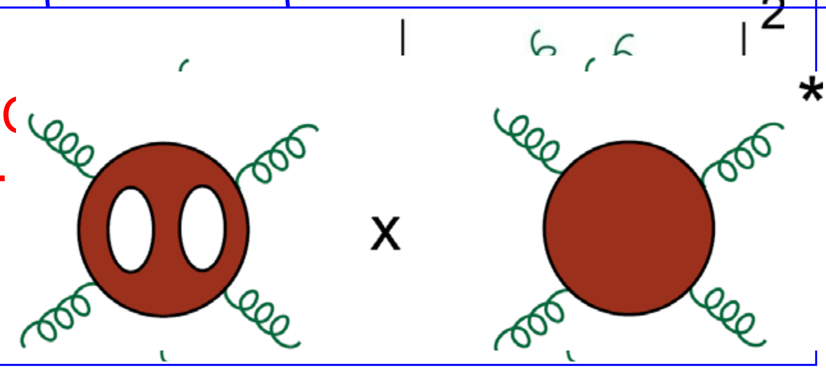
- For W/Z/Higgs + jets, situation is similar or worse, if you count the W/Z/Higgs as a jet

$$\sigma(\text{n jets}) = [\alpha_s(\mu)]^n \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu) + \cdots\right]$$

NNLONLO=2-loop x tree* + … …

LO

n=2 n=3



NLO improves normalizations

Recent example (Higgs background)

$p\bar{p} \to W b\bar{b}$ at Tevatron, $m_b \neq 0$

Febres Cordero, Reina, Wackeroth, hep-ph/0606102

$m_b = 0$ case: Ellis, Veseli, hep-ph/9810489; Campbell, Ellis, hep-ph/0202176

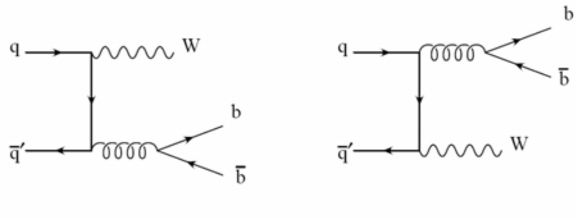
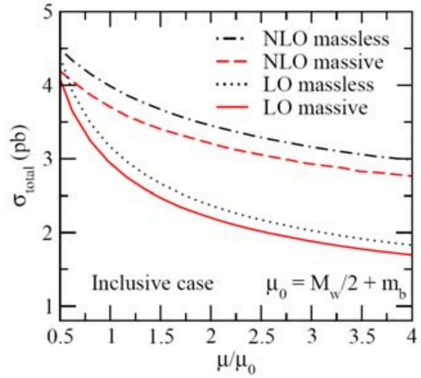
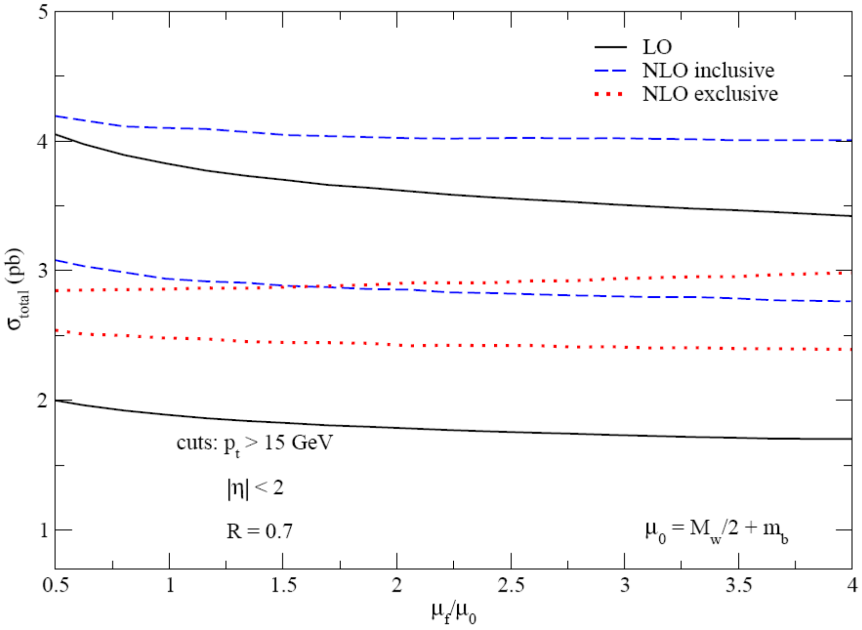


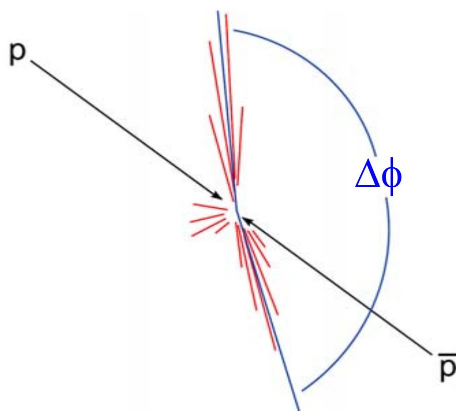
FIG. 1: Tree level Feynman diagrams for $q\bar{q}' \to b\bar{b}W$.

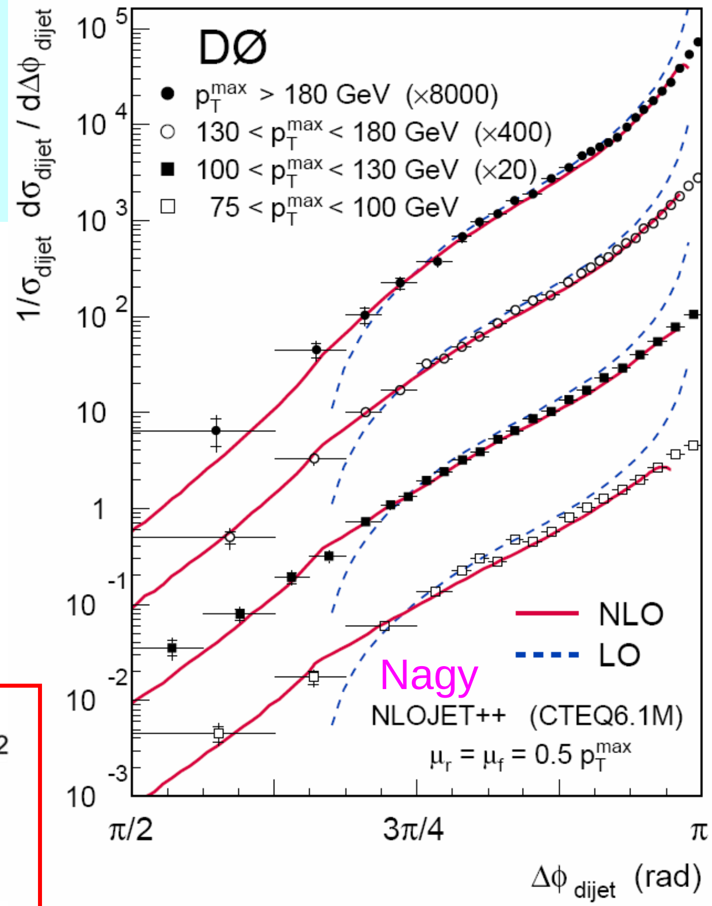




NLO also improves distributions

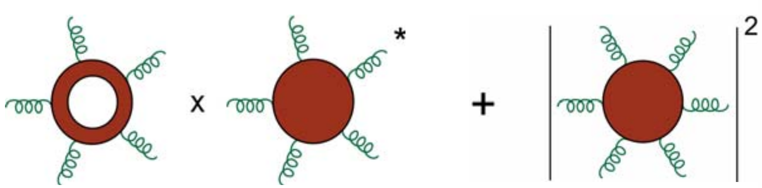
Tevatron Run II example:
Azimuthal decorrelation of di-jets at D0
due to additional radiation





NLO

n=3



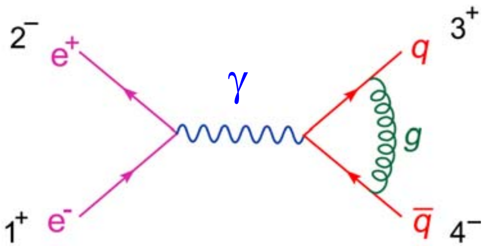
NLO challenges

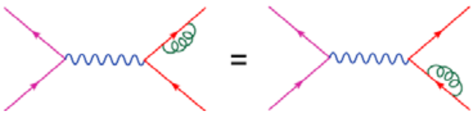
1. Virtual 1-loop amplitudes with many external legs are very complicated; several new approaches being pursued, some more numerical, some more analytical
   Ellis, Giele, Zanderighi (2005-6); Binoth, Guillet, Heinrich, Pilon, Schubert (2005); Kramer, Nagy, Soper (2002-3); Berger, Bern, LD, Forde, Kosower (2005-6); …
2. Subtraction terms – necessary to render finite the phase-space integration of real corrections – are very intricate to construct; integrations are time-consuming to perform to good numerical accuracy (though general formalisms have existed for a while)

   Giele, Glover (1992); Giele, Glover, Kosower, hep-ph/9302225;
   Frixione, Kunszt, Signer, hep-ph/9512328;
   Catani, Seymour, hep-ph/9605323; Kosower, hep-ph/9710213

Virtual Corrections

Simplest case – virtual corrections for DY, DIS, or e^+e^- annihiliation:





cancellation of UV & IR divergences!

overlap of soft & collinear IR divergences

$$\mathcal{A}_4^{1-\mathrm{loop}} = \mathcal{A}_4^{\mathrm{tree}} \frac{\alpha_s}{4\pi} \exp[\epsilon(\ln(4\pi) - \gamma_E)] \times 2C_F \left(\frac{\mu^2}{-s_{12}}\right)^\epsilon \left[-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - \frac{7}{2} - \frac{\delta_R}{2} + \frac{\pi^2}{12}\right]$$

So simple because there is only one relevant scale, s_{12}

For multi-particle kinematics, even after color/helicity disentangling, there are enough kinematic variables to build very complicated results!

More complicated examples

$gg g g g$

$+++++$

$$A(+++++) = \frac{i}{96\pi^2}\frac{s_{12}s_{23}+s_{23}s_{34}+s_{34}s_{45}+s_{45}s_{51}+s_{51}s_{12}+\varepsilon(1,2,3,4)}{\langle 1 2\rangle\langle 2 3\rangle\langle 3 4\rangle\langle 4 5\rangle\langle 5 1\rangle},$$

$-++++$

$$A(-++++) = \frac{i}{48\pi^2}\frac{1}{[1 2]\langle 2 3\rangle\langle 3 4\rangle\langle 4 5\rangle[5 1]}\Big[(s_{23}+s_{34}+s_{45})[2 5]^2 - [2 4]\langle 4 3\rangle[3 5][2 5] - \frac{[1 2][1 5]}{\langle 1 2\rangle\langle 1 5\rangle}\Big(\langle 1 2\rangle^2\langle 1 3\rangle^2\frac{[2 3]}{\langle 2 3\rangle} + \langle 1 3\rangle^2\langle 1 4\rangle^2\frac{[3 4]}{\langle 3 4\rangle} + \langle 1 4\rangle^2\langle 1 5\rangle^2\frac{[4 5]}{\langle 4 5\rangle}\Big)\Big]$$

$$V^g = -\frac{1}{\epsilon^2}\sum_{j=1}^{5}\left(\frac{\mu^2}{-s_{j,j+1}}\right)^{\epsilon} + \sum_{j=1}^{5}\ln\left(\frac{-s_{j,j+1}}{-s_{j+1,j+2}}\right)\ln\left(\frac{-s_{j+2,j-2}}{-s_{j-2,j-1}}\right) + \frac{5}{6}\pi^2 - \frac{\delta_R}{3}$$

the following functions for the $(1^-,2^-,3^+,4^+,5^+)$ helicity configuration,

~ 1 page

$--+++$

$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{23}}\right) + \ln\left(\frac{\mu^2}{-s_{51}}\right)\right] - 2, \qquad V^s = -\frac{1}{3}V^f + \frac{2}{9}$$

$$F^f = -\frac{1}{2}\frac{\langle 1 2\rangle^2(\langle 2 3\rangle[3 4]\langle 4 1\rangle + \langle 2 4\rangle[4 5]\langle 5 1\rangle)}{\langle 2 3\rangle\langle 3 4\rangle\langle 4 5\rangle\langle 5 1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{1}{3}\frac{[3 4]\langle 4 1\rangle\langle 2 4\rangle[4 5](\langle 2 3\rangle[3 4]\langle 4 1\rangle + \langle 2 4\rangle[4 5]\langle 5 1\rangle)}{\langle 3 4\rangle\langle 4 5\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3} - \frac{1}{3}F^f - \frac{1}{3}\frac{\langle 3 5\rangle[3 5]^3}{[1 2][2 3]\langle 3 4\rangle\langle 4 5\rangle[5 1]} + \frac{1}{3}\frac{\langle 1 2\rangle[3 5]^2}{[2 3]\langle 3 4\rangle\langle 4 5\rangle[5 1]} + \frac{1}{6}\frac{\langle 1 2\rangle[3 4]\langle 4 1\rangle\langle 2 4\rangle[4 5]}{s_{23}\langle 3 4\rangle\langle 4 5\rangle s_{51}}$$

and the corresponding ones for the $(1^-,2^+,3^-,4^+,5^+)$ helicity configuration,

$-+-++$

$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{34}}\right) + \ln\left(\frac{\mu^2}{-s_{51}}\right)\right] - 2, \qquad V^s = -\frac{1}{3}V^f + \frac{2}{9}$$

$$F^f = -\frac{\langle 1 3\rangle^2\langle 4 1\rangle[2 4]^2}{\langle 4 5\rangle\langle 5 1\rangle}\frac{\mathrm{Ls}_1\left(\frac{-s_{23}}{-s_{51}},\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^2} + \frac{\langle 1 3\rangle^2\langle 5 3\rangle[2 5]^2}{\langle 3 4\rangle\langle 4 5\rangle}\frac{\mathrm{Ls}_1\left(\frac{-s_{12}}{-s_{34}},\frac{-s_{51}}{-s_{34}}\right)}{s_{34}^2} - \frac{1}{2}\frac{\langle 1 3\rangle^3(\langle 1 5\rangle[5 2]\langle 2 3\rangle - \langle 3 4\rangle[4 2]\langle 2 1\rangle)}{\langle 1 2\rangle\langle 2 3\rangle\langle 3 4\rangle\langle 4 5\rangle\langle 5 1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}}$$

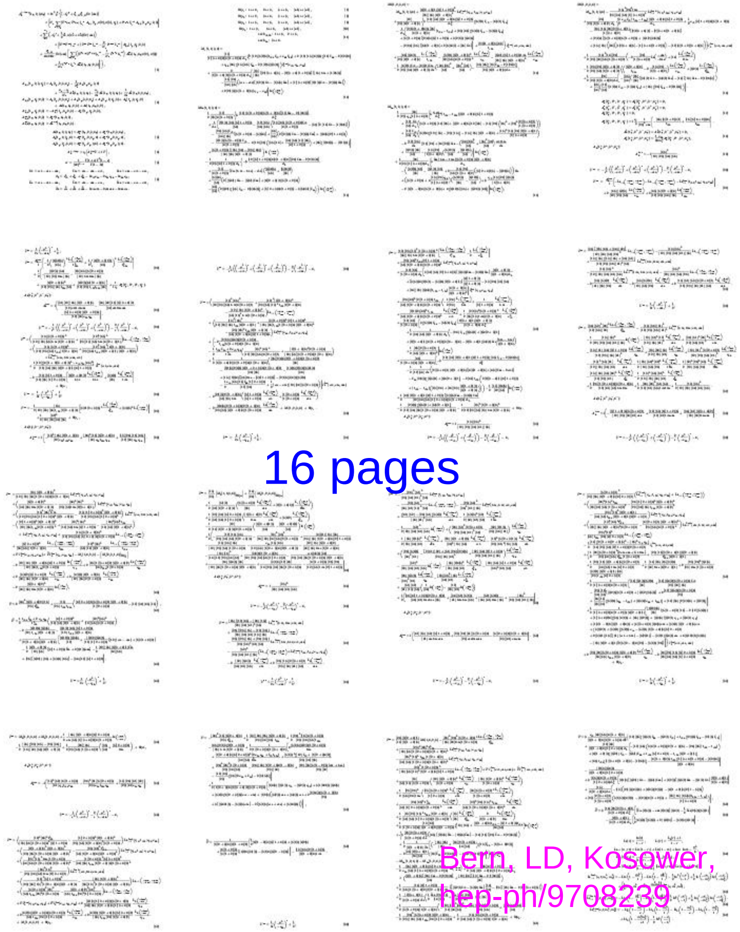
$$\begin{aligned}F^s = &-\frac{\langle 1 2\rangle\langle 2 3\rangle\langle 3 4\rangle\langle 4 1\rangle^2[2 4]^2}{\langle 4 5\rangle\langle 5 1\rangle\langle 2 4\rangle^2}\frac{2\,\mathrm{Ls}_1\left(\frac{-s_{23}}{-s_{51}},\frac{-s_{34}}{-s_{51}}\right) + \mathrm{L}_1\left(\frac{-s_{23}}{-s_{51}}\right) + \mathrm{L}_1\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^2}\\ &+ \frac{\langle 3 2\rangle\langle 2 1\rangle\langle 1 5\rangle\langle 5 3\rangle^2[2 5]^2}{\langle 5 4\rangle\langle 4 3\rangle\langle 2 5\rangle^2}\frac{2\,\mathrm{Ls}_1\left(\frac{-s_{12}}{-s_{34}},\frac{-s_{51}}{-s_{34}}\right) + \mathrm{L}_1\left(\frac{-s_{12}}{-s_{34}}\right) + \mathrm{L}_1\left(\frac{-s_{51}}{-s_{34}}\right)}{s_{34}^2}\\ &+ \frac{2}{3}\frac{\langle 2 3\rangle^2\langle 4 1\rangle^3[2 4]^3}{\langle 4 5\rangle\langle 5 1\rangle\langle 2 4\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3} - \frac{2}{3}\frac{\langle 2 1\rangle^2\langle 5 3\rangle^3[2 5]^3}{\langle 5 4\rangle\langle 4 3\rangle\langle 2 5\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{12}}{-s_{34}}\right)}{s_{34}^3}\\ &+ \frac{\mathrm{L}_2\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^3}\Big(\frac{1}{3}\frac{\langle 1 3\rangle[2 4][2 5](\langle 1 5\rangle[5 2]\langle 2 3\rangle - \langle 3 4\rangle[4 2]\langle 2 1\rangle)}{\langle 4 5\rangle}\\ &+ \frac{2}{3}\frac{\langle 1 2\rangle^2\langle 3 4\rangle^2\langle 4 1\rangle[2 4]^3}{\langle 4 5\rangle\langle 5 1\rangle\langle 2 4\rangle} - \frac{2}{3}\frac{\langle 3 2\rangle^2\langle 1 5\rangle^2\langle 5 3\rangle[2 5]^3}{\langle 5 4\rangle\langle 4 3\rangle\langle 2 5\rangle}\Big)\\ &+ \frac{1}{6}\frac{\langle 1 3\rangle^3(\langle 1 5\rangle[5 2]\langle 2 3\rangle - \langle 3 4\rangle[4 2]\langle 2 1\rangle)}{\langle 1 2\rangle\langle 2 3\rangle\langle 3 4\rangle\langle 4 5\rangle\langle 5 1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}} + \frac{1}{3}\frac{[2 4]^2[2 5]^2}{[1 2][2 3][3 4]\langle 4 5\rangle[5 1]}\\ &- \frac{1}{3}\frac{\langle 1 2\rangle\langle 4 1\rangle^2[2 4]^3}{\langle 4 5\rangle\langle 5 1\rangle\langle 2 4\rangle[2 3][3 4]s_{51}} + \frac{1}{3}\frac{\langle 3 2\rangle\langle 5 3\rangle^2[2 5]^3}{\langle 5 4\rangle\langle 4 3\rangle\langle 2 5\rangle[2 1][1 5]s_{34}} + \frac{1}{6}\frac{\langle 1 3\rangle^2[2 4][2 5]}{s_{34}\langle 4 5\rangle s_{51}}\end{aligned}$$

Bern, LD, Kosower, hep-ph/9302280

More legs, or massive legs, rapidly increases complexity!

Some helicity config's more complex than others

$V q\bar{q} g g \qquad V = W, Z, \gamma^*$



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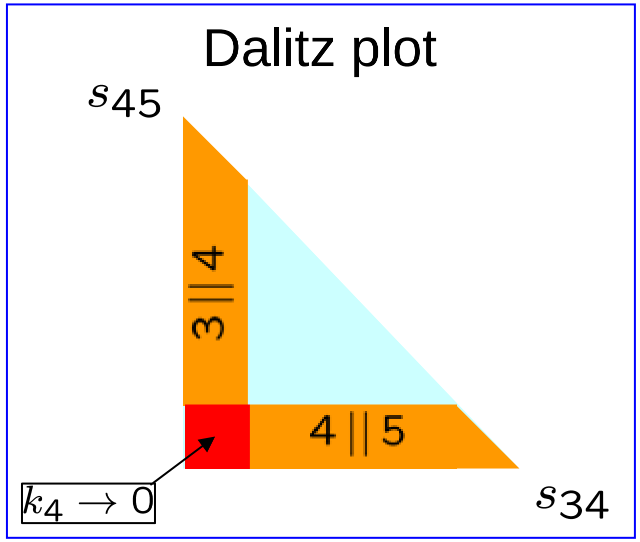
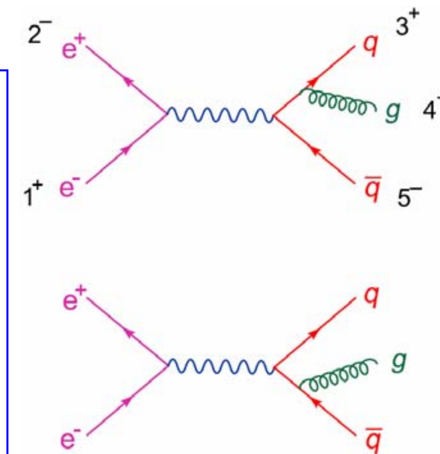
Bern, LD, Kosower, hep-ph/9708239

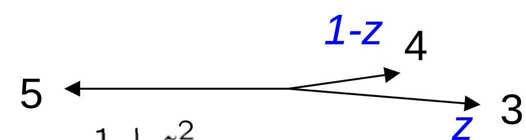
Real radiation prototype: infrared cancellations in e+e-

$$s_{34} + s_{45} + s_{35} = s_{12}$$

Or can use as variables:
- 1 invariant mass, say s_{34}
- CM polar angle for "decay" of (3,4) pair, cosθ, equivalent to z







$$\sigma^{\rm tot}_{\rm real} = \mu^{2\epsilon}\int d^{4-2\epsilon}{\rm LIPS}_{3,4,5}\,|\mathcal{A}_5|^2 \;\propto\; \left(\frac{\mu^2}{s_{12}}\right)^{\epsilon} |\mathcal{A}_4|^2 \frac{\alpha_s}{2\pi}\int_0 \frac{ds_{34}}{s_{34}^{1+\epsilon}}\int_0^1 dz\,(1-z)^{-\epsilon} C_F \frac{1+z^2}{1-z}$$

$$= |\mathcal{A}_4|^2 \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{s_{12}}\right)^{\epsilon}\left(-\frac{1}{\epsilon}\right)\int_0^1 dz\left[\frac{2}{(1-z)^{1+\epsilon}} - 1 - z\right]$$

$$= |\mathcal{A}_4|^2 \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{s_{12}}\right)^{\epsilon}\left(-\frac{1}{\epsilon}\right)\left[-\frac{2}{\epsilon} - \frac{3}{2}\right]$$

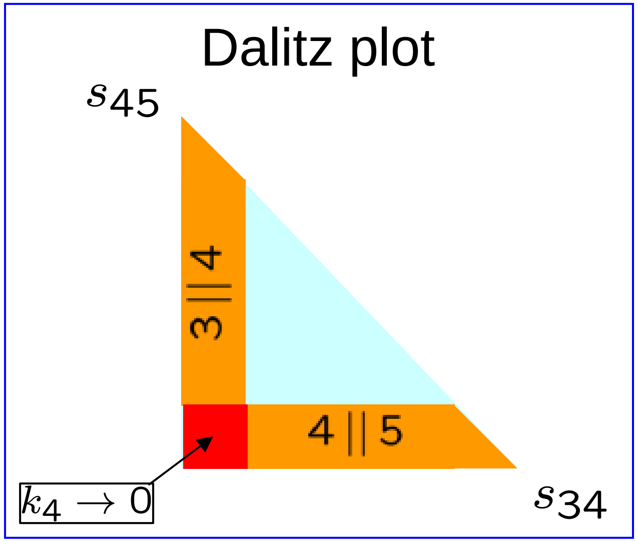
$$= |\mathcal{A}_4|^2 \frac{\alpha_s}{\pi} C_F \left(\frac{\mu^2}{s_{12}}\right)^{\epsilon}\left[\frac{1}{\epsilon^2} + 2\frac{3}{4\epsilon} + \text{finite}\right]$$

after doubling $z < 1$ term to account for 4 || 5 region

Real radiation in general case

Cannot perform the phase-space integral analytically in $D=4-2\varepsilon$, especially not for generic experimental cuts

Also can't do it numerically, because of $1/\varepsilon^2$ poles



2 solutions:

1. Slice out singular regions of phase-space, with (thin) width s_{min} Perform integral there approximately. Rest of integral done numerically. Check cancellation of s_{min} dependence.
2. Subtract a function that mimics the soft/collinear behavior of the radiative cross section, and which you can integrate (analytically). Integral of the difference can be done numerically.

Dipole formalism

Catani, Seymour, hep-ph/9602277, hep-ph/9605323

Popular (stable) version of the subtraction method

Build dipole subtraction function $D_{ij,k}$ for each pair of partons i,j that can get singular, and for each “spectator” parton k

$$\mathcal{D}_{ij,k}(p_1,...,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} \cdot {}_m\langle 1,..,\widetilde{ij},..,\widetilde{k},..,m+1| \frac{\boldsymbol{T}_k\cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2} \boldsymbol{V}_{ij,k} |1,..,\widetilde{ij},..,\widetilde{k},..,m+1\rangle_m$$

The $D_{ij,k}$ multiply the LO cross section, at a boosted phase-space point:

$$\tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}\, p_k^\mu \quad , \quad \tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}\, p_k^\mu$$

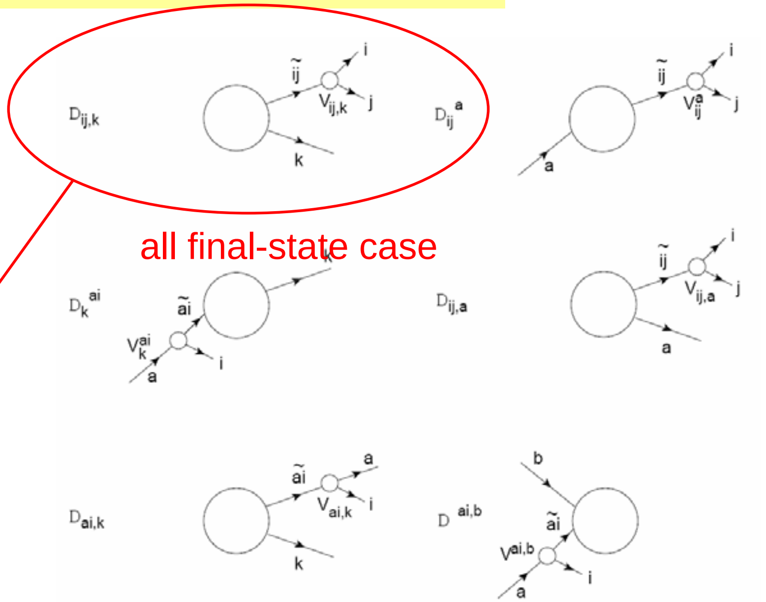


Figure 3: Effective diagrams for the different dipole formulae introduced in Sect. 5. The blobs denote the m-parton matrix element. Incoming and outgoing lines respectively stand for initial-state and final-state partons.

All dipole integrals can be done analytically

Hundreds of dipoles for NLO pp → 3 jets

The NNLO Frontier

NNLO QCD required for high precision at LHC:

- parton distributions
  - evolution (NNLO DGLAP kernels)
  - fits to DIS, Drell-Yan, and jet data
- LHC production of single *W*s and *Z*s
  - "partonic" luminosity monitor
  - precision m_W
- Higgs production via gluon fusion and extraction of Higgs couplings

- NNLO progress historically in terms of number of scales: 0,1,2, ∞
- More scales tougher, but more flexible applications

No-scale, inclusive problems

$R(e^+e^- \to \text{hadrons})$ & $R(\tau \to \nu_\tau + \text{hadrons})$

Gorishnii, Kataev, Larin; Surguladze, Samuel (1990)

DIS sum rules: $\int_0^1 dx\, F_n(x)$

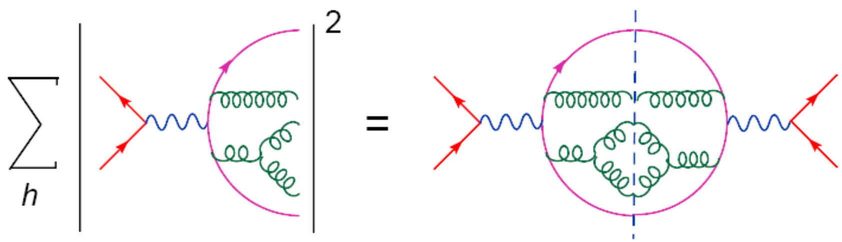
Bjorken $(\bar{\nu}p - \nu p)$ — Larin, Tkachov, Vermaseren (1990)

Bjorken $(\vec{e}p)$ & Gross-Llewellyn-Smith $(\nu p + \bar{\nu} p)$ — Larin, Vermaseren (1991)

Use unitarity of *S*-matrix to relate real production of hadrons to imaginary part of virtual-photon forward scattering:

$$S = 1 + iT$$
$$1 = S^\dagger S = (1 - iT^\dagger)(1 + iT)$$
$$2\ \text{Im}\, T = T^\dagger T$$

$\Rightarrow$



$T^\dagger T$ $\qquad$ $2\ \text{Im}\, T$ ($S \to$)

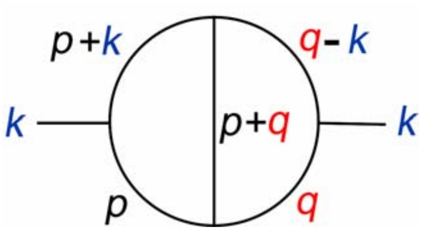
cut imposes $\delta(p_i^2 - m_i^2)$

Transforms inclusive phase-space integrals into loop integrals for virtual photon propagator. *s* (or Q^2) factors out by dim. analysis

No-scale problems (cont.)

- Multi-loop integral technology: Integration by parts (IBP)

Chetyrkin, Tkachov (1981)



$$0 = \int d^D p\, d^D q \ldots \frac{\partial}{\partial q^\mu} \frac{k^\mu}{p^2 q^2 (p+q)^2 \ldots}$$

- Reduces problem to system of linear equations, solved recursively by MINCER, in terms of few "master integrals"

Gorishnii, Larin, Surguladze, Tkachov (1989)

No-scale $\Rightarrow$ analytic simplicity – pure numbers

$$\begin{aligned}\frac{R_{e^+e^-}}{R^{(0)}} = 1 &+ \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-11\zeta(3) + \frac{365}{24} + n_f\left(\frac{2}{3}\zeta(3) - \frac{11}{12}\right)\right] \\ &+ \left(\frac{\alpha_s}{\pi}\right)^3 \left[\frac{275}{6}\zeta(5) - \frac{1103}{4}\zeta(3) - \frac{121}{8}\zeta(2) + \frac{87029}{288}\right. \\ &+ n_f\left(-\frac{25}{9}\zeta(5) + \frac{262}{9}\zeta(3) + \frac{11}{6}\zeta(2) - \frac{7847}{216}\right) \\ &\left. + n_f^2\left(-\frac{19}{27}\zeta(3) - \frac{1}{18}\zeta(2) + \frac{151}{162}\right)\right]\end{aligned}$$

1-scale, inclusive problems

Drell-Yan, W, Z total cross section

$\sigma^{\rm tot}(pp \to V + X)$ Hamberg, van Neerven, Matsuura (1990)

Higgs total cross section $(m_t \to \infty)$

$\sigma^{\rm tot}(pp \to H + X)$ Harlander, Kilgore; Anastasiou, Melnikov (2002); Ravindran, Smith, van Neerven (2003)

$\hat{\sigma}$ depends on $z = M_{V,H}^2/\hat{s}$

DIS coefficient functions $C_i(z)$ Van Neerven, Zijlstra (1991)

F_L -- Moch, Vermaseren, Vogt (2004)

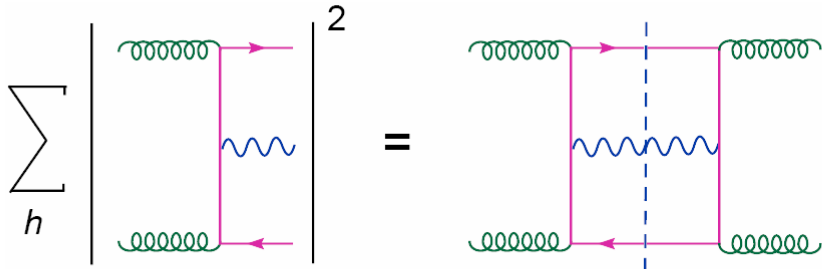
Leading-twist anomalous dimensions

DGLAP kernels $P_{ij}(x)$ Moch, Vermaseren, Vogt (2004)

1-scale problems (cont.)

- Can apply unitarity and multi-loop integral technology to DY/Higgs production too: Anastasiou, Melnikov (2002)
  forward 2 → 2 scattering instead of propagator

$$\hat{\sigma}(gg \to V q\bar{q}) \quad = \quad 2\,\mathrm{Im}\,\mathcal{A}(gg \to gg)\big|_{Vq\bar{q}}$$



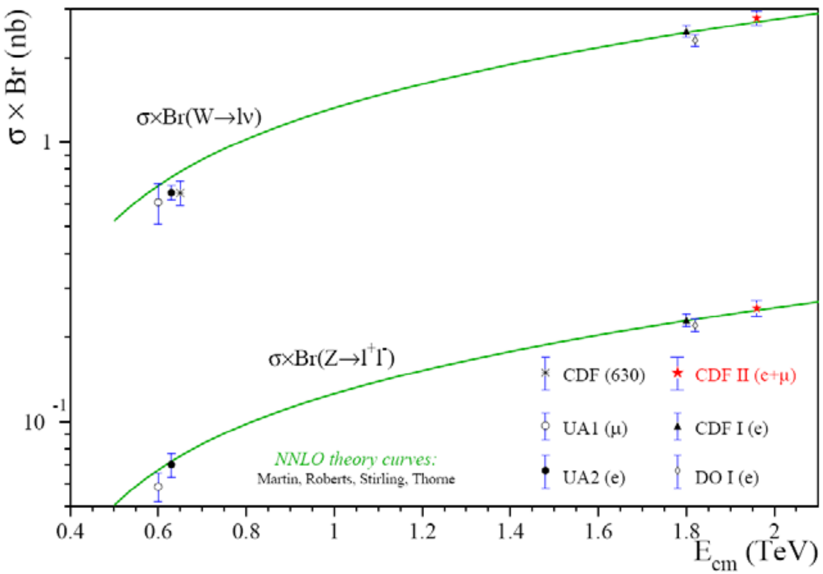
$T^\dagger T$ 2 Im T

cut imposes $\delta(p_i^2 - m_i^2)$, which includes $\delta(m_V^2 - z\hat{s})$

analytic structure of 1-scale integrals: (harmonic) polylogarithms $\mathrm{Li}_n(z)$

1-scale applications

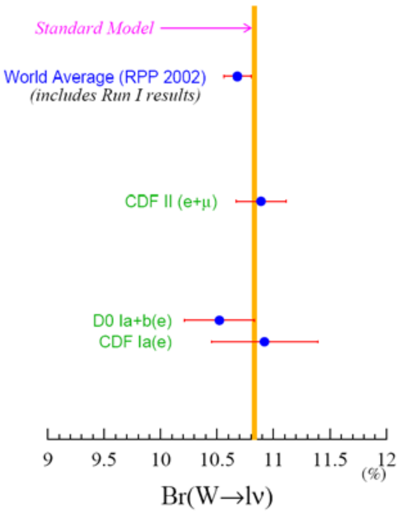
- Precise prediction of total cross sections σ_W and σ_Z at Tevatron (and LHC) – use ratio to measure Br($W \rightarrow l\nu$)



Hamberg, van Neerven, Matsuura (1991); Harlander, Kilgore (2002)

$$R \equiv \frac{\sigma_W}{\sigma_Z}\frac{\text{Br}(W\to\ell\nu)}{\text{Br}(Z\to\ell\ell)} = 10.92 \pm 0.15 \pm 0.14$$

→



CDF, hep-ex/0508029

NNLO theory

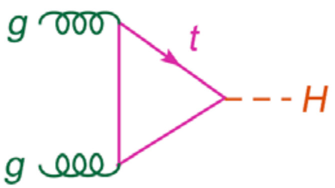
LEP

1-scale applications (cont.)

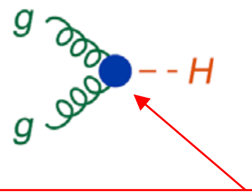
- Reduction of uncertainty on $\sigma(gg \to H+X)$ at LHC

Critical for extracting quantitative Higgs couplings (to gg, $\gamma\gamma$, …) from a Higgs signal, ~ σ x Br

To make NNLO computation tractable – reduce number of loops by 1 – work in large m_t limit (OK at NLO): Djouadi, Spira, Zerwas, hep-ph/9504378



$\Rightarrow$

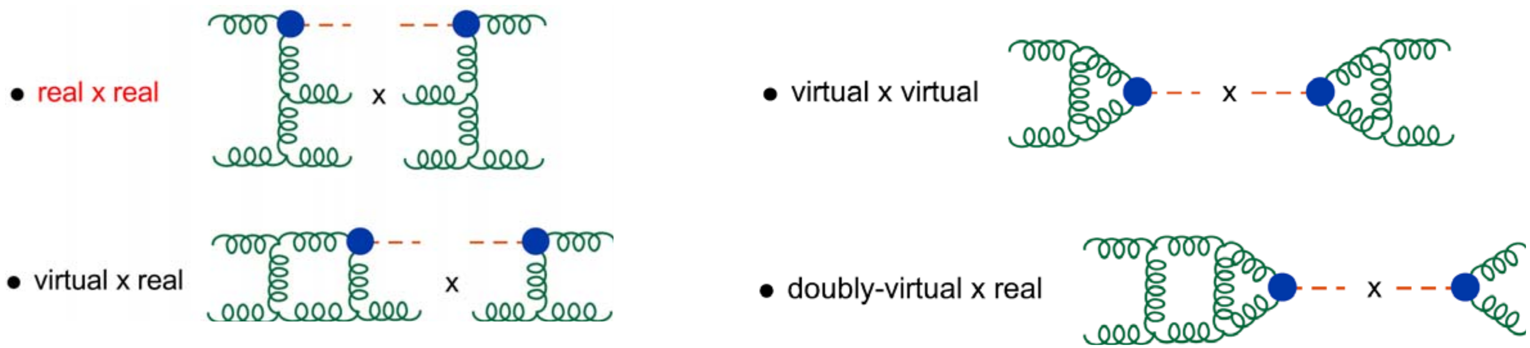


1-loop → tree-like effective coupling $C H G^a_{\mu\nu} G^{a\,\mu\nu}$

$$C = \frac{\alpha_s}{6\pi v}\left[1 + \frac{11\alpha_s}{4\pi} + \cdots\right] \qquad v = 246\ \text{GeV}$$

$\sigma(pp \to H + X)$ at NNLO

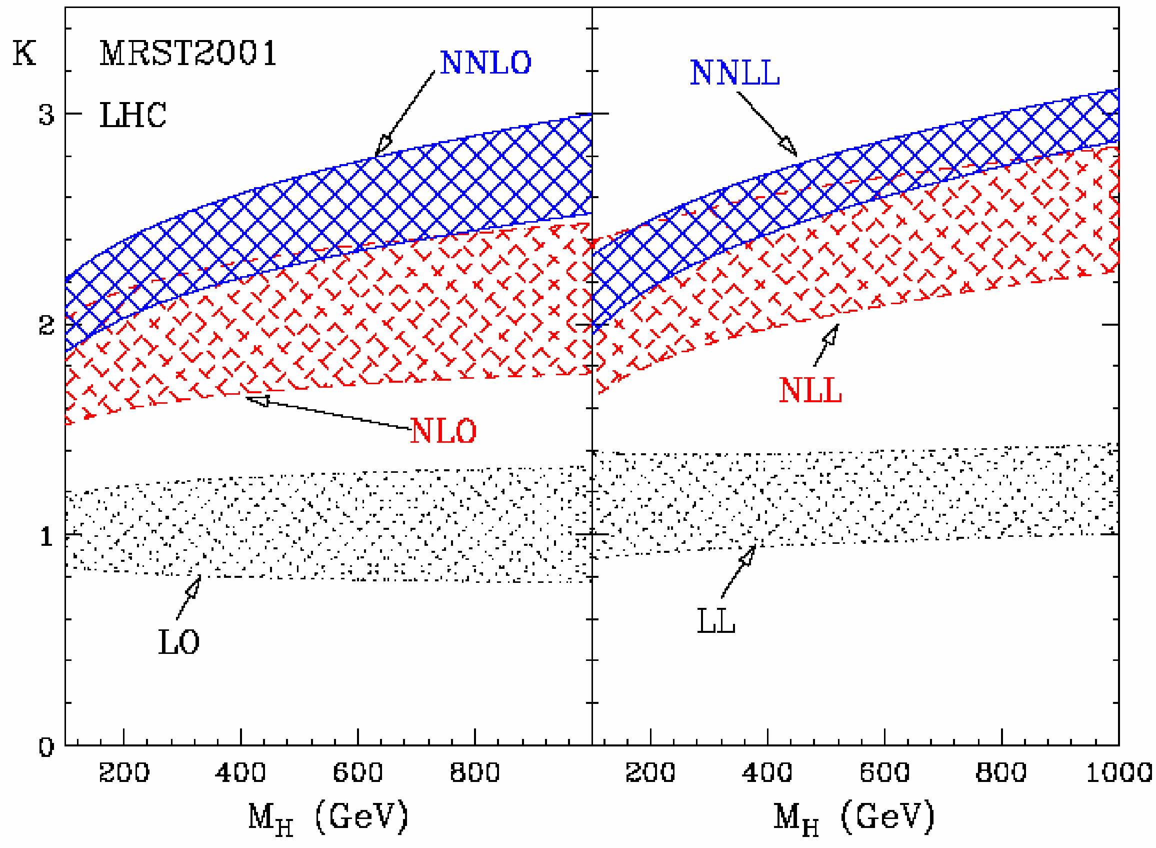
Still many amplitude interferences, with different numbers of final state gluons (or quarks). Each diverges; only sum is finite.



- Kinematics equivalent to *W/Z/γ** production
- Can apply unitarity + multi-loop integral technology here too (first done here, in fact) Anastasiou, Melnikov (2002)

$\sigma(pp \rightarrow H + X)$ at NNLO

- K factor = $\sigma^X(\mu)/\sigma^{LO}(\mu=1)$ for $gg \rightarrow H+X$ at LHC



Harlander, Kilgore;
Anastasiou, Melnikov;
Ravindran, Smith, van Neerven;
Catani, De Florian, Grazzini, Nason;
Grazzini, hep-ph/0209302

1-scale applications (cont.)

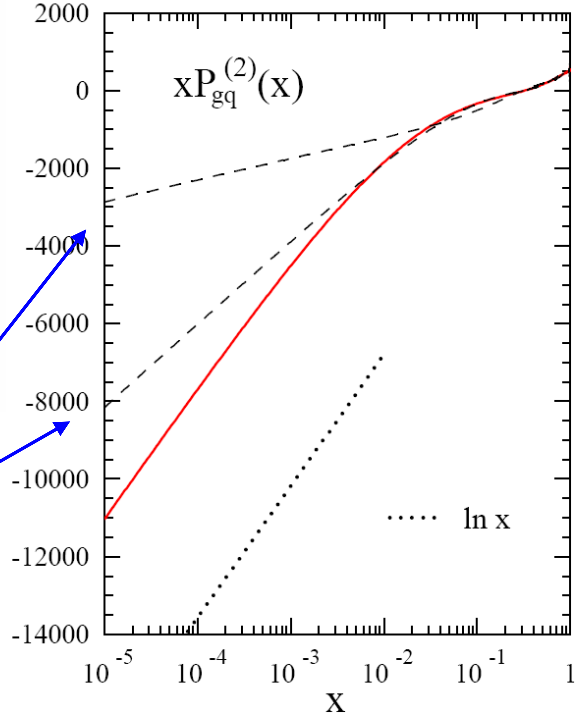
NNLO DGLAP kernels

Moch, Vermaseren, Vogt, hep-ph/0403192, hep-ph/0404111

	tree	1-loop	2-loop	3-loop
$q\gamma$	1	3	25	359
$g\gamma$		2	17	345
$h\gamma$			2	56
qW	1	3	32	589
$q\phi$		1	23	696
$g\phi$	1	8	218	6378
$h\phi$		1	33	1184
sum	3	18	350	9607

Table 1. The number of diagrams employed in our calculation of the three-loop splitting functions.

for example, $g \rightarrow q$



prior estimate of NNLO kernel
Van Neerven, Vogt, hep-ph/0007362
based on moments 2,…,12
Larin et al., hep-ph/9605317;
Retey, Vermaseren, hep-ph/0007294

1-scale applications (cont.)

- NNLO DGLAP kernels and DIS coefficient functions permit
  - NNLO DIS pdf fits   Alekhin, hep-ph/0211096; hep-ph/0508248
  - ~NNLO global pdf fits   Martin, Roberts, Stirling, Thorne, hep-ph/0410230

2-scale, semi-inclusive problem

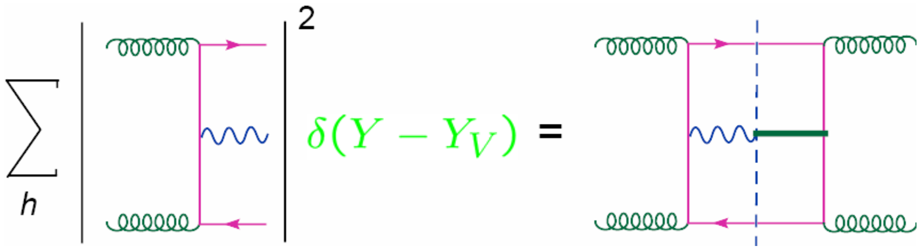
Drell-Yan, W, Z rapidity distribution $\quad Y_V = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$

$$\frac{d\sigma(pp\to V+X)}{dY_V}$$

Anastasiou, LD, Melnikov, Petriello (2003)

$\hat{\sigma}$ depends on $z = M^2_{V,H}/\hat{s}$ and Y_V

- Unitarity + multi-loop IBP method still works – one more "propagator" to implement rapidity δ function



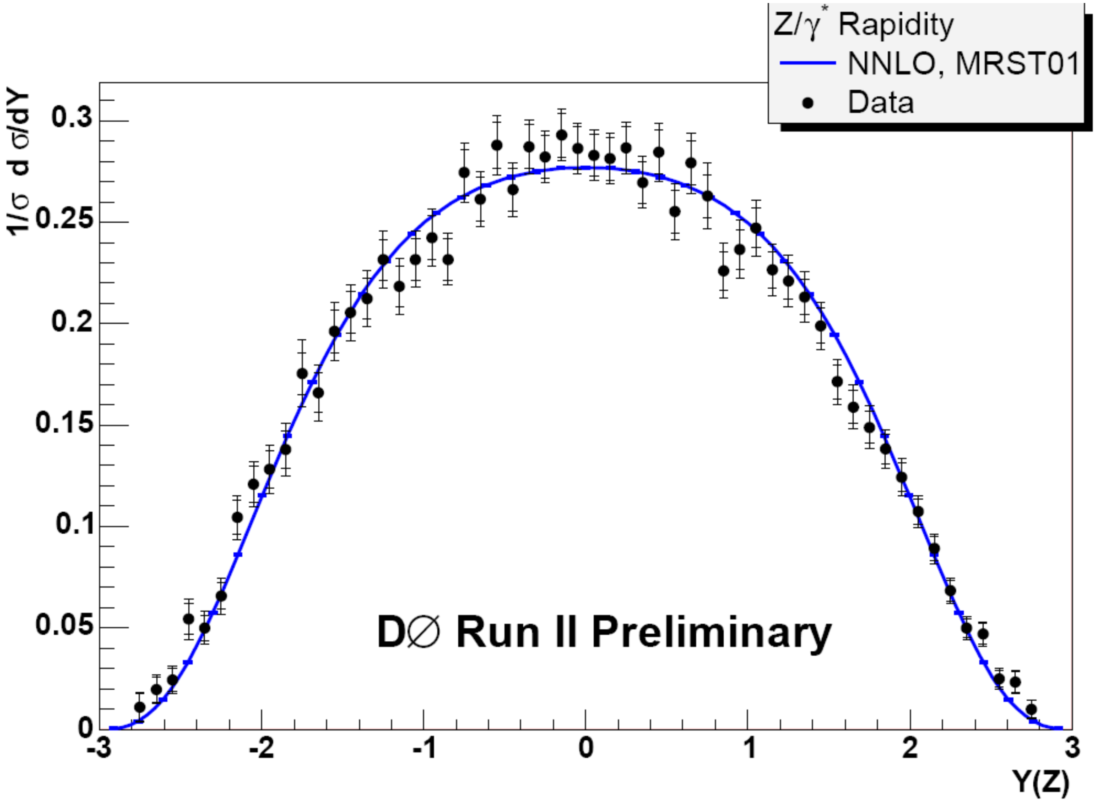
- Complicated analytic structure:

$$\mathrm{Li}_2\left(\frac{u-1-i\sqrt{(4u^2-z(1+u)^2)/z}}{2u}\right)$$

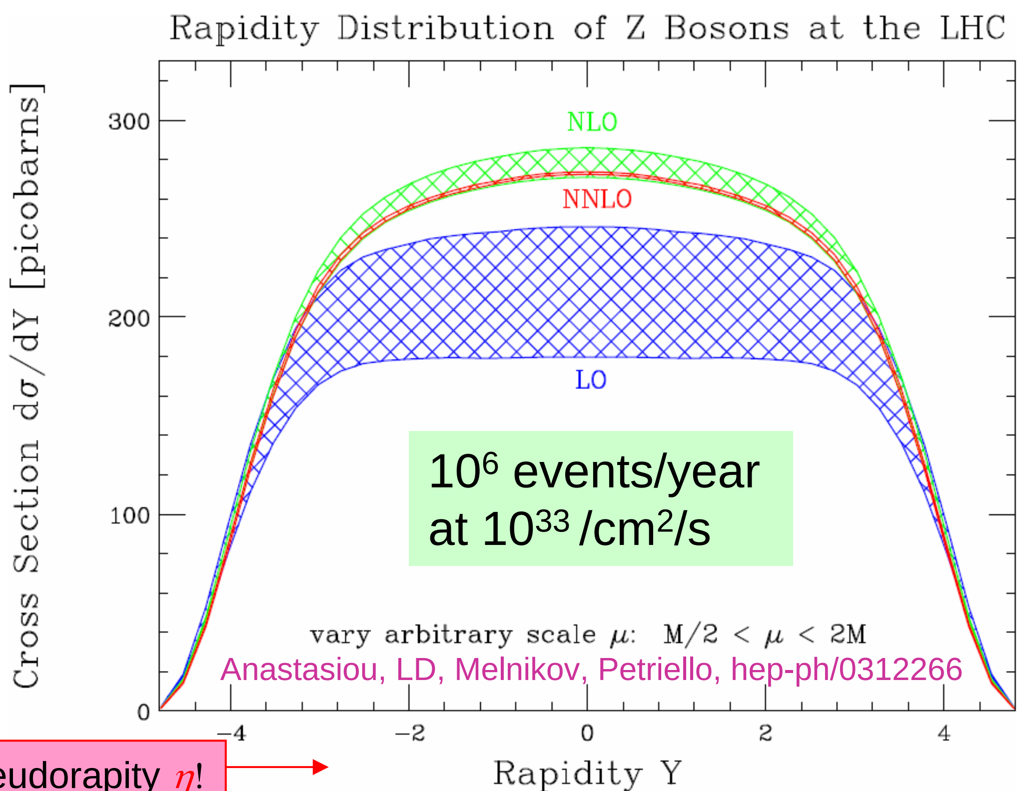
$$u \equiv \frac{x_1}{x_2}\exp(-2Y_V)$$

2-scale applications

$$\frac{d\sigma(pp \to V + X)}{dY_V}$$ at Tevatron



2-scale applications (cont.)



10^6 events/year at 10^{33} /cm^2/s

Anastasiou, LD, Melnikov, Petriello, hep-ph/0312266

not pseudorapity η!

- Uncertainty due to omitted higher-order terms very small

- Permits use of d$\sigma_{W,Z}$/dY as "partonic luminosity monitor": use it to normalize other cross sections – pp luminosity, some detector efficiencies, drop out of ratio

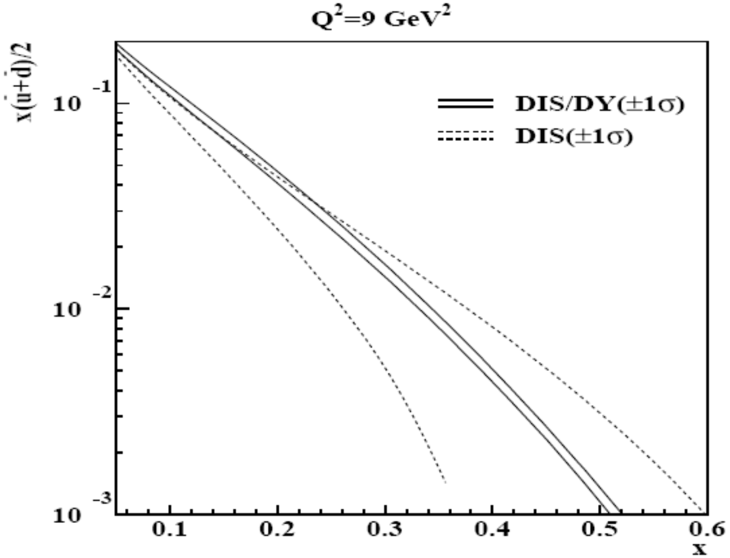
$$\frac{d\sigma}{dY} \sim \sum_q [q_1(x_1)\bar{q}_2(x_2) + \bar{q}_1(x_1)q_2(x_2)]$$

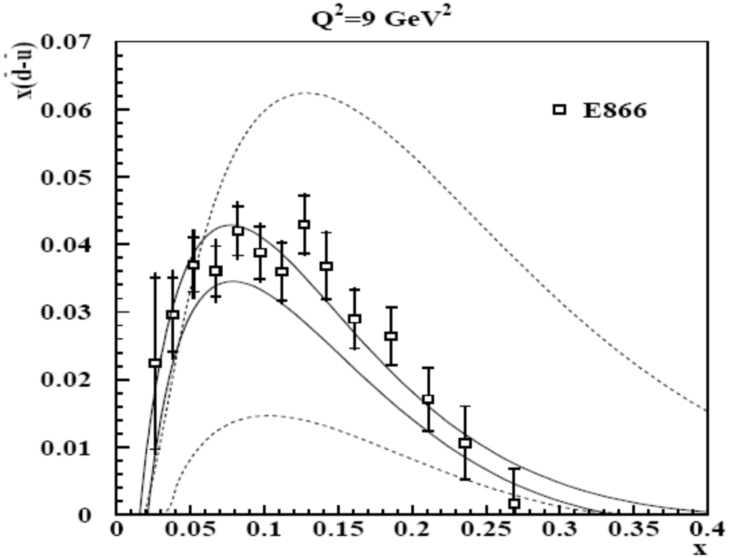
$$x_1 = \frac{M_V}{\sqrt{s}} e^{Y} \qquad x_2 = \frac{M_V}{\sqrt{s}} e^{-Y}$$

Dittmar, Pauss, Zurcher, hep-ex/9705004

2-scale applications (cont.)

- Improve NNLO pdf fits using Drell-Yan data at NNLO
- Sensitive to large x sea quark distributions
- And, thereby, to large x gluon
- NNLO corrections ~ 10% at large $x_F = x_1 - x_2$ due to large logs





NNLO DIS/DY fit: Alekhin, Melnikov, Petriello, hep-ph/0606237

2-scale applications (cont.)

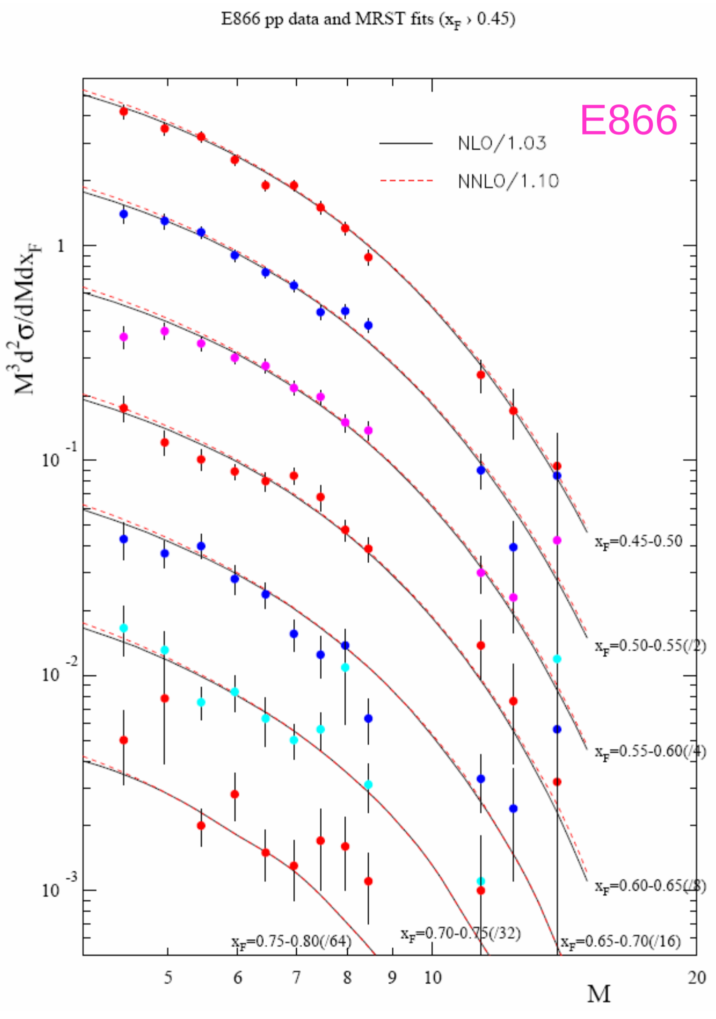
• Impact of DY data @ NNLO on global NNLO pdf fits:

Martin, Roberts, Stirling, Thorne, hep-ph/0606244

• Hard to see on log plot, but global NNLO fit has improved χ^2, vs. global NLO, a little larger $\alpha_s(M_Z) = 0.119$

Although fit gets more tension as DY@NLO → DY@NNLO (less sea → less gluon at high x) vs. Tevatron high E_T jets (more gluon at high x) Stirling

Jet cross section still NLO!



∞-scale problem

- "Holy grail": Flexible method for arbitrary (infrared-safe) observable at NNLO. Include isolation, p_T, rapidity cuts, jet algorithm dependence, …

Partial wish list:

- e^+e^- event-shape observables
- pp or ep $\rightarrow$ inclusive jets, dijets, multijets
- $pp \rightarrow (W, Z, H) + X$ with parton-level cuts
- $pp \rightarrow (W, Z) +$ jets

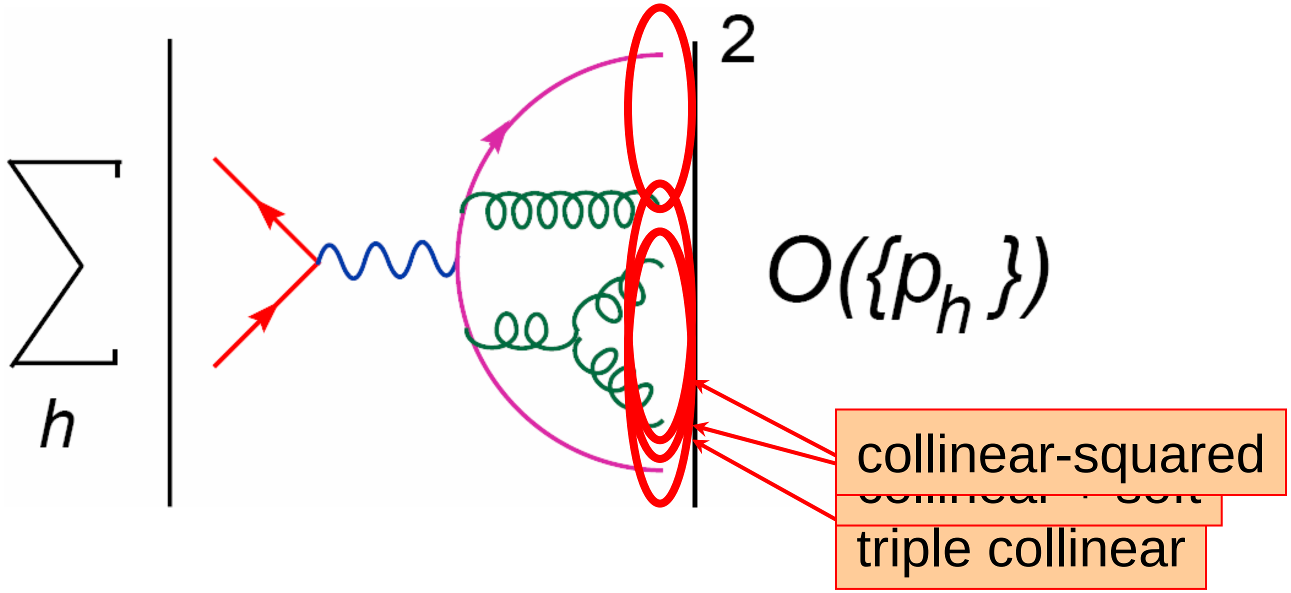
- Amplitudes known for many of these processes
- Phase-space integration is the stumbling block

- Analytic structure too complicated; go numerical

Numerical phase-space integration

- Integration has to be done in *D=4-2ε* due to severe infrared divergences ($1/\varepsilon^4$)

Example: e^+e^- event-shapes



Phase-space integration (cont.)

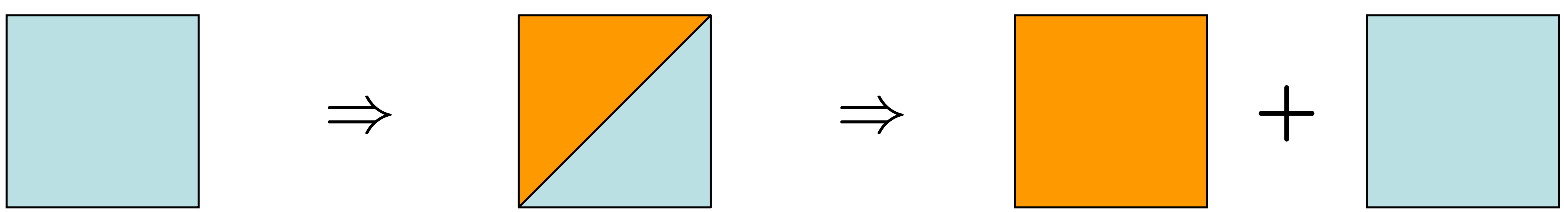
Two basic approaches at present:

Method 1. Iterated sector decomposition

Partition integration region and remap to make divergences "1-dimensional", let computer find subtraction terms

Binoth, Heinrich; Anastasiou, Melnikov, Petriello (2003,2004)

Simple example: $I = \int_0^1 \int_0^1 dx\,dy \frac{x^\epsilon y^\epsilon}{(x+y)^2}$



Phase-space integration (cont.)

Method 2. Use known factorization properties of amplitudes to build subtraction terms for general processes, and integrate them, a la NLO

Many authors (~1997-2003)

Kosower; Weinzierl; Gehrmann, Gehrmann-de Ridder, Heinrich (2003)
Frixione, Grazzini (2004); Gehrmann, Gehrmann-de Ridder, Glover (2004,2005);
Del Duca, Somogyi, Trocsanyi (2005)

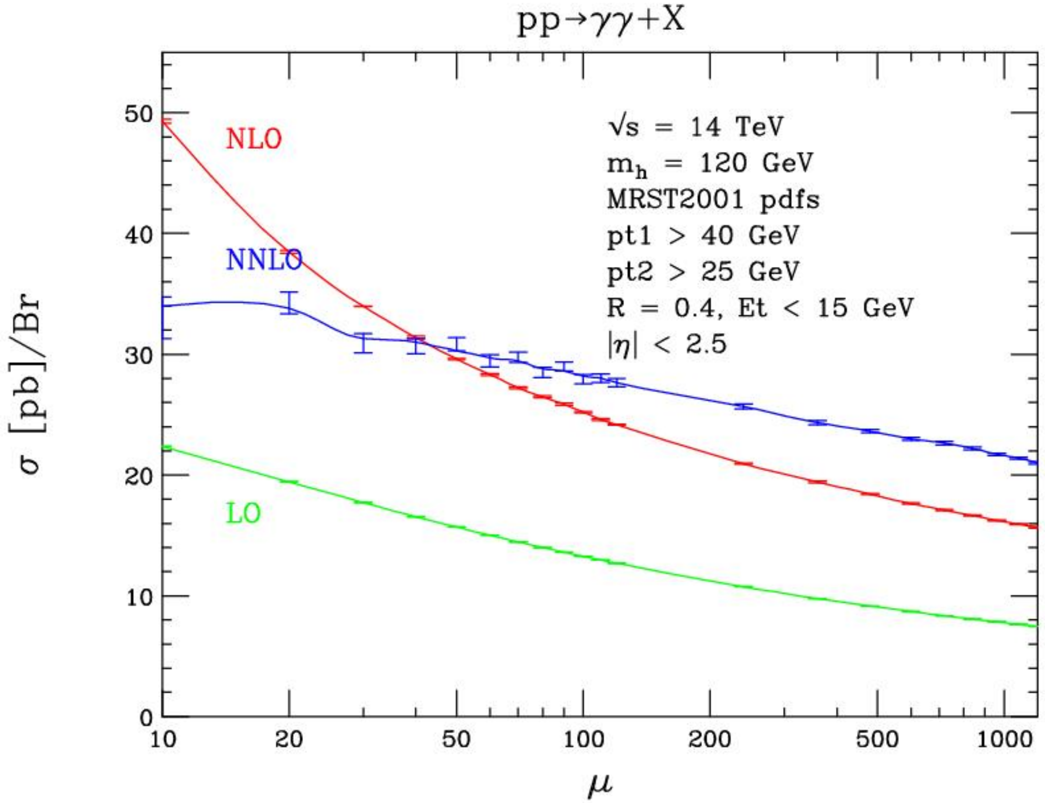
- construct the 2-unresolved-parton counterterm using the IR currents

$$
\begin{aligned}
\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r \sum_{s\neq r} \Bigg\{ \sum_{i\neq r,s} \Bigg[& \frac{1}{6}\mathbf{C}_{irs} + \sum_{j\neq i,r,s} \frac{1}{8}\mathbf{C}_{ir;js} + \frac{1}{2}\mathbf{S}_{rs} \\
& + \frac{1}{2}\left(\mathbf{CS}_{ir;s} - \mathbf{C}_{irs}\mathbf{CS}_{ir;s} - \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\mathbf{CS}_{ir;s} \right) \Bigg] \\
& - \sum_{i\neq r,s} \left[\mathbf{CS}_{ir;s}\mathbf{S}_{rs} + \mathbf{C}_{irs}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \right. \\
& \left. + \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \right] \Bigg\} |\mathcal{M}^{(0)}_{m+2}|^2
\end{aligned}
$$

LHC applications

- So far, just from method 1

- $pp \rightarrow H + X \rightarrow \gamma\gamma + X$ with parton-level cuts



Anastasiou, Melnikov, Petriello, hep-ph/0409088; hep-ph/0501130

LHC applications (cont.)

- $pp \to W + X \to \ell\nu + X$ with parton-level cuts

Important to constrain observed lepton rapidity (not W rapidity) and to impose realistic cuts on lepton p_T and missing E_T

Melnikov, Petriello, hep-ph/0603182

$p_\perp^e > p_\perp^{e,\min}, \quad |\eta^e| < 2.5, \quad E_\perp^{\rm miss} > 20$ GeV

$p_\perp^{e,\min}$	LO	NLO	NNLO
Inc	11.70,13.74,15.65	16.31,16.82,17.30	16.31, 16.40, 16.50
20	5.85,6.96,8.01	7.94,8.21,8.46	8.10,8.07,8.10
30	4.305, 5.12,5.89	6.18,6.36,6.54	6.18,6.17,6.22
40	0.628,0.746,0.859	2.07,2.10,2.11	2.62,2.54,2.50
50	0,0,0	0.509,0.497,0.480	0.697,0.651,0.639

TABLE I: The lepton invariant mass distribution $d\sigma/dM^2$, $M = m_W$, for on-shell W production in the reaction $pp \to W^- X \to e^- \bar{\nu} W$, in pb/GeV^2, for various choices of $p_\perp^{e,\min}$, GeV and $\mu = m_W/2, m_W, 2m_W$.

As in more inclusive computations, scale uncertainties highly reduced at NNLO

Conclusions

- Higher order QCD for the LHC is an extremely rich field – we only scratched the surface of it here
- At hadron colliders, the physics **is** QCD – up to small, electroweak corrections!
- To uncover new physics of electroweak strength, we will need to understand QCD at colliders quite well!
- Fortunately, there are lots of energetic young researchers
- But still, lots of work to be done before, and as, the data roll in from the energy frontier!

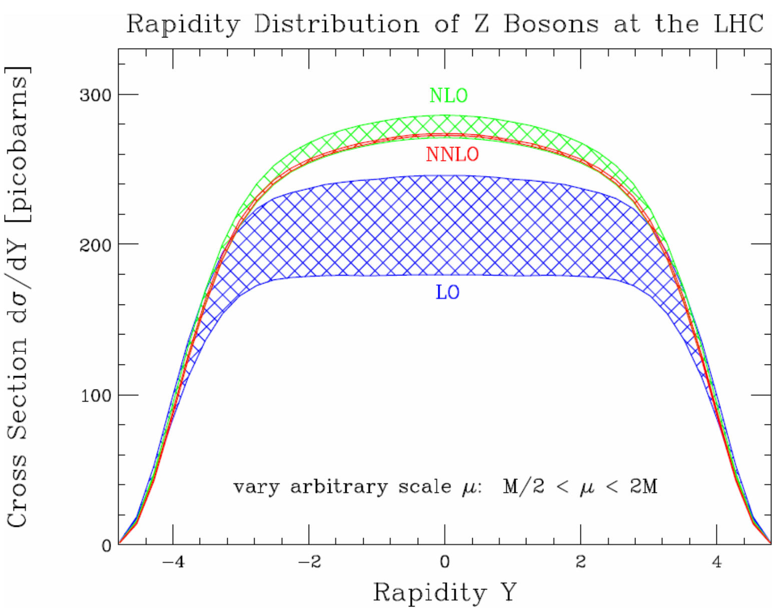
Extra Slides

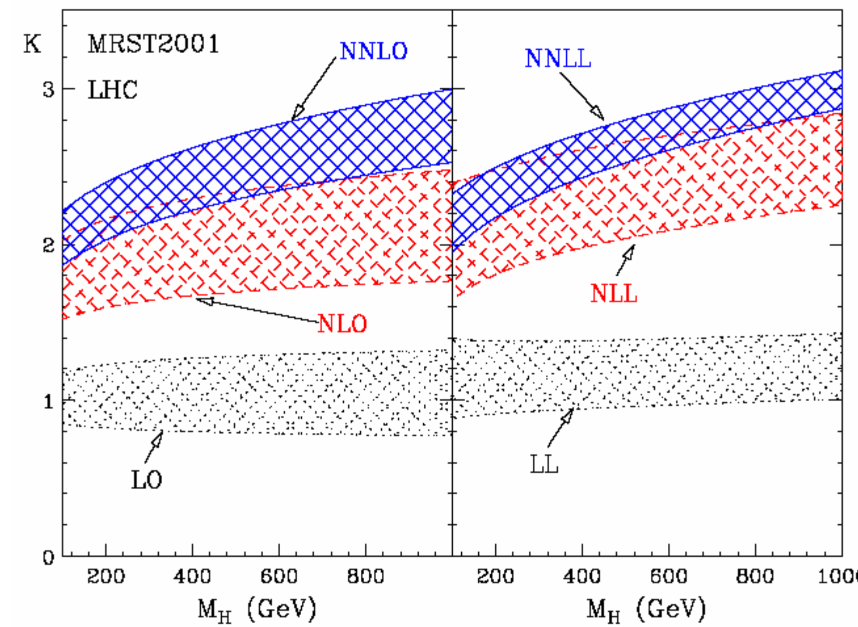
Why are (N)NLO corrections large?

+ 30% typical for quark-initiated (*W/Z*, …)

+ 80-100% for some

gluon-initiated ($gg \to \text{Higgs} + X$)

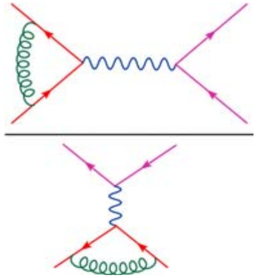




This is much bigger than $R_{e^+e^-} = 1 + \frac{\alpha_s}{\pi} \approx 1 + \frac{0.1}{\pi} \approx 1 + 0.03$!!

Some answers (not all for all processes)

1. LO parton distribution fits not very reliable due to large theory uncertainties
2. New processes can open up at NLO. In *W/Z* production at Tevatron or LHC, *qg -> γ*q* opens up, and *g(x)* is very large – but correction is negative!
3. Large π^2 from analytic continuation from space-like region where pdfs are measured (DIS) to time-like region (Drell-Yan/W/Z):

2 Re 

$$= 1 + \frac{\alpha_s}{\pi} C_F \left(-\frac{1}{\epsilon^2}\right) \text{Re}\left[\left(\frac{\mu^2}{-Q^2}\right)^\epsilon - \left(\frac{\mu^2}{+Q^2}\right)^\epsilon\right]$$

$$= 1 + \frac{\alpha_s}{\pi} C_F \left(-\frac{1}{\epsilon^2}\right) \text{Re}\left[\exp(i\pi\epsilon) - 1\right] = 1 + \frac{\alpha_s}{\pi} C_F \frac{\pi^2}{2}$$

4. Soft-gluon/Sudakov resummation

- A prevalent theme in QCD whenever one is at an edge of phase space.
- Infrared-safe but sensitive to a second, smaller scale
- Same physics as in (high-energy) QED: $e^+e^- \;=\; e^+e^-(\gamma)$
- What is prob. of no γ with $E > \Delta E$, $\theta > \Delta\,\theta$?

$$P = 1 - \frac{\alpha}{\pi}\int_{\Delta E}\frac{dE}{E}\int_{\Delta\theta}\frac{d\theta}{\theta} + \cdots \;=\; 1 - \frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta + \cdots$$
$$= \exp\left(-\frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta\right) + \cdots$$

soft

collinear

leading double logarithms -- in contrast to single logs of renormalization group, DGLAP equations.

exponentiation because soft emissions are independent

Hadron collider examples

$p_T(Z)$, important application to $p_T(W)$, m_W measurement at Tevatron

Production of heavy states, like
• top quark at the Tevatron (W and Z production less so),
• even a light Higgs boson at the LHC, via $gg \to H$
Called threshold resummation or $x \longrightarrow 1$ limit,
where $x = M^2/s$.
Can be important for $x \ll 1$ though.
For m_H = 120 GeV at 14 TeV LHC, $x = 10^{-4}$!
Radiation is being suppressed because you are
running out of phase space – parton distributions are falling fast.

Threshold Resummation

Can see the first threshold log in the NLO corrections to Drell-Yan/*W/Z* production:

$$C_F D_q(z,\mu_F) \;=\; 4C_F(1+z^2)\left(\frac{\ln(1-z)+\ln(M/\mu_F)}{1-z}\right)_+ \\ -2\frac{1+z^2}{1-z}\ln z + \delta(1-z)\left(\frac{2}{3}\pi^2-8\right)$$

It is a double-log expansion:

$$D_q^{(n)}(z,\mu_F) \;\propto\; (C_F\alpha_s)^n\left[\left(\frac{\ln^{2n+1}(1-z)}{1-z}\right)_+ + \cdots\right]$$

For *gg* -> *H*, same leading behavior at large *z*.
Except color factor is much bigger: $C_A = 3$, not $C_F = 4/3$

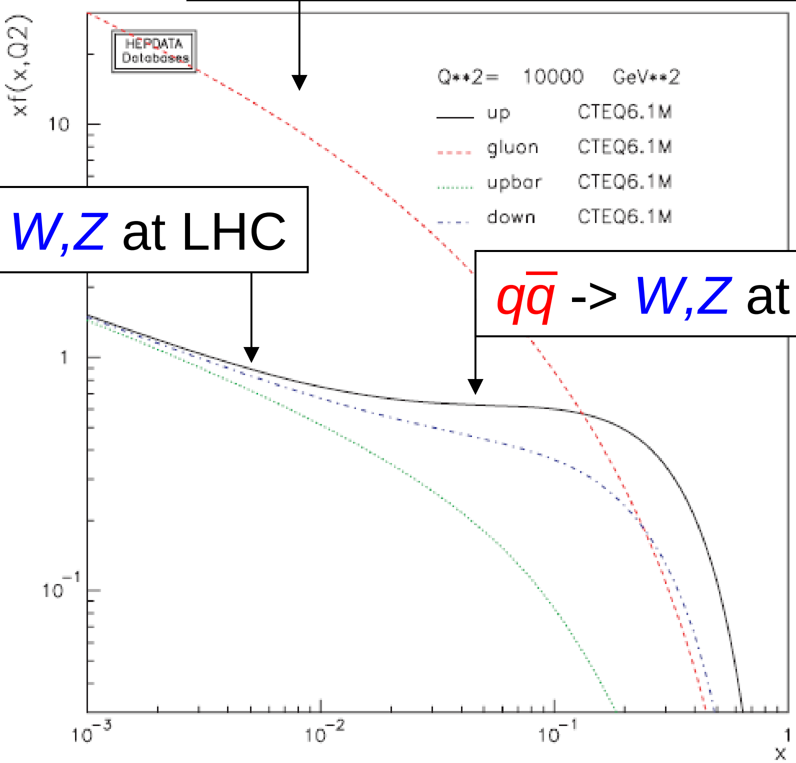
$$D_{gg\to H}^{(n)}(z,\mu_F) \;\propto\; (C_A\alpha_s)^n\left[\left(\frac{\ln^{2n+1}(1-z)}{1-z}\right)_+ + \cdots\right]$$

Fast falling pdfs -- worse for gluons

gg -> *light Higgs* at LHC

$q\bar{q}$ -> *W,Z* at LHC

$q\bar{q}$ -> *W,Z* at Tevatron



Q: If it is called Sudakov suppression, why does it increase the cross section?

A. Because the same suppression happens in the DIS process used to measure the pdfs. Both parton distributions "bigger than you thought": 2 – 1 > 0.

pdfs → 2

partonic cross section → 1