TeV Colliders and Cosmology

2: The Distribution of Dark Matter

M. E. Peskin
SSI - July 2006

In the previous lecture, I explained that most of the mass in the universe is composed of a new type of elementary particle.

I argued that (under a specific assumption) this particle is very likely to have a mass of order 100 GeV.

If so, (under one additional assumption) this particle will be produced in complex events that appear at the LHC with pb cross sections.

Let’s suppose that this actually happens !

What next ?

If a WIMP is discovered at the LHC, this would be at best a candidate for the dark matter particle.

‘Not all candidates get elected.’ -- M. Goldhaber

To establish this WIMP as the dark matter particle, we need to measure its properties and show that these agree with the values required by astrophysics.

The particularly important properties are:

the mass m_N

the pair annihilation cross section $\sigma v\big|_{NN}$

the WIMP-nucleon cross section σ_{Np}

We need the cross sections at or near threshold.

It would be wonderful if - after WIMPs are discovered at the LHC - we could engineer WIMP beams (like neutrino beams) to use in measuring WIMP cross sections. Such a beam could be produced by 1000 TeV fixed target collisions. That will not be possible anytime soon.

Instead, we must obtain the information from collider data.
A strategy is:

1. Figure out which EWSB theory is actually present in Nature.

2. Measure the Lagrangian parameters of that theory.

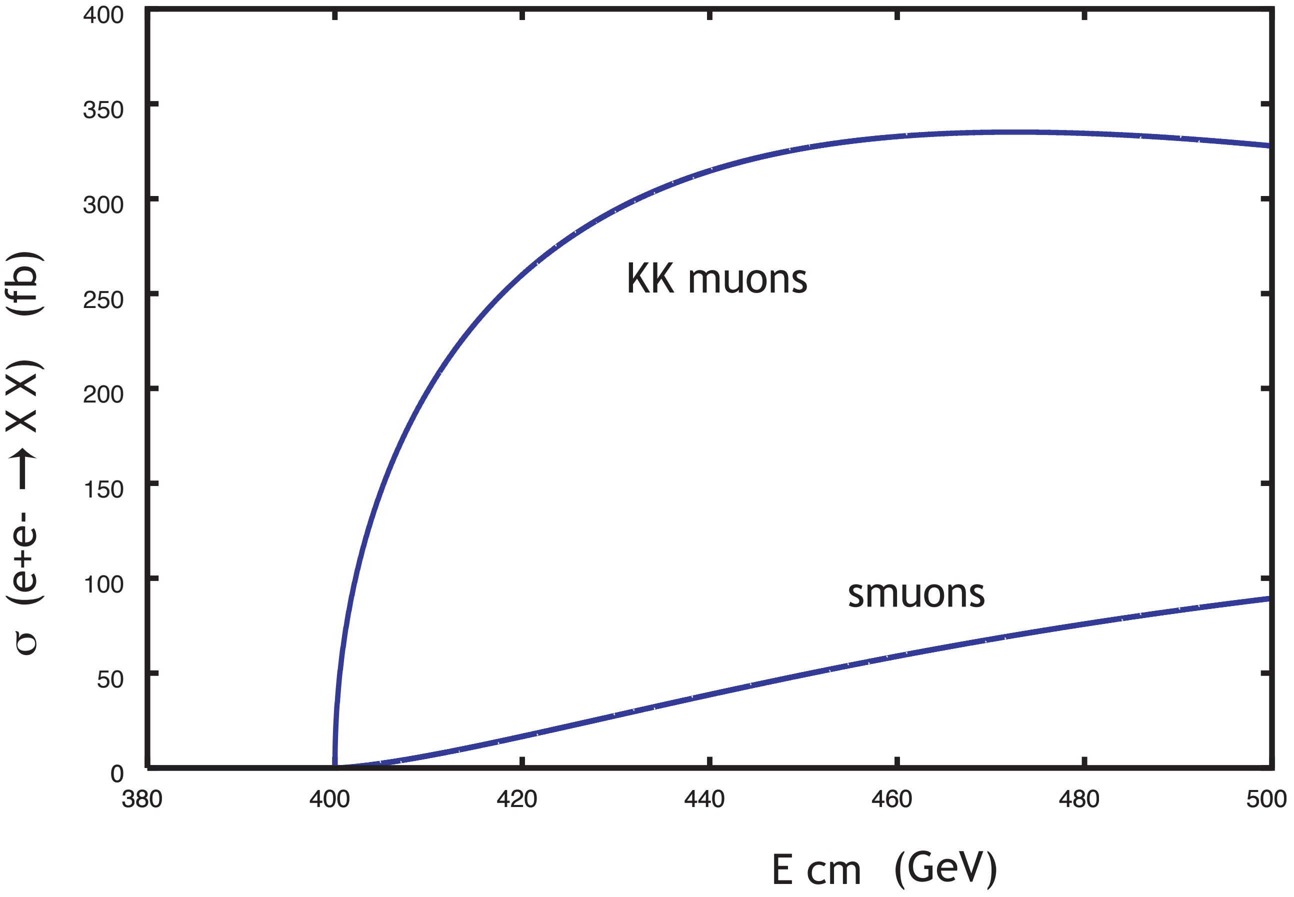
3. Evaluate the needed cross sections.

This is quite ambitious. It might be possible if the cross sections that we need depend only on a few parameters which are more accessible (e.g., masses of the lightest new particles).

The program nicely illustrates the capabilities of LHC and ILC.

Step 1 is completely non-trivial for the LHC. It depends on our ability to measure the spins and quantum numbers of the colored particles that carry the WIMP quantum number. Later speakers will discuss some strategies to do this.

Fortunately, such spin and quantum number measurements can be made straightforwardly from $e^+e^- \to X\overline{X}$ at ILC.



In the rest of this lecture, I will assume that the underlying theory of EWSB is known.

For convenience, I will assume that this theory is supersymmetry, with the WIMP being the neutralino. For supersymmetry, the WIMP cross sections have been worked out in great detail as a function of the parameters. The results are available in public codes:

DarkSUSY	Gondolo, Baltz, Bergstrom, Edsjo, Schelke, Ullio
MicrOMEGAs	Belanger, Boudjema, Pukhov, Semenov
ISATOOLS	Baer, Belyaev, Balazs, Brhlik, Tata

We will need some technical results from supersymmetry. I will try to minimize this, but some details of this lecture will only be explained in the lectures of the 2nd week of the school.

Using this theory, we can address questions like:

How accurate a prediction of the WIMP relic density can we make from collider data ?

WMAP already gives Ω_N to 7%; Planck will reach 1% accuracy. It would be interesting to see if our microscopic theory would give the same answer.

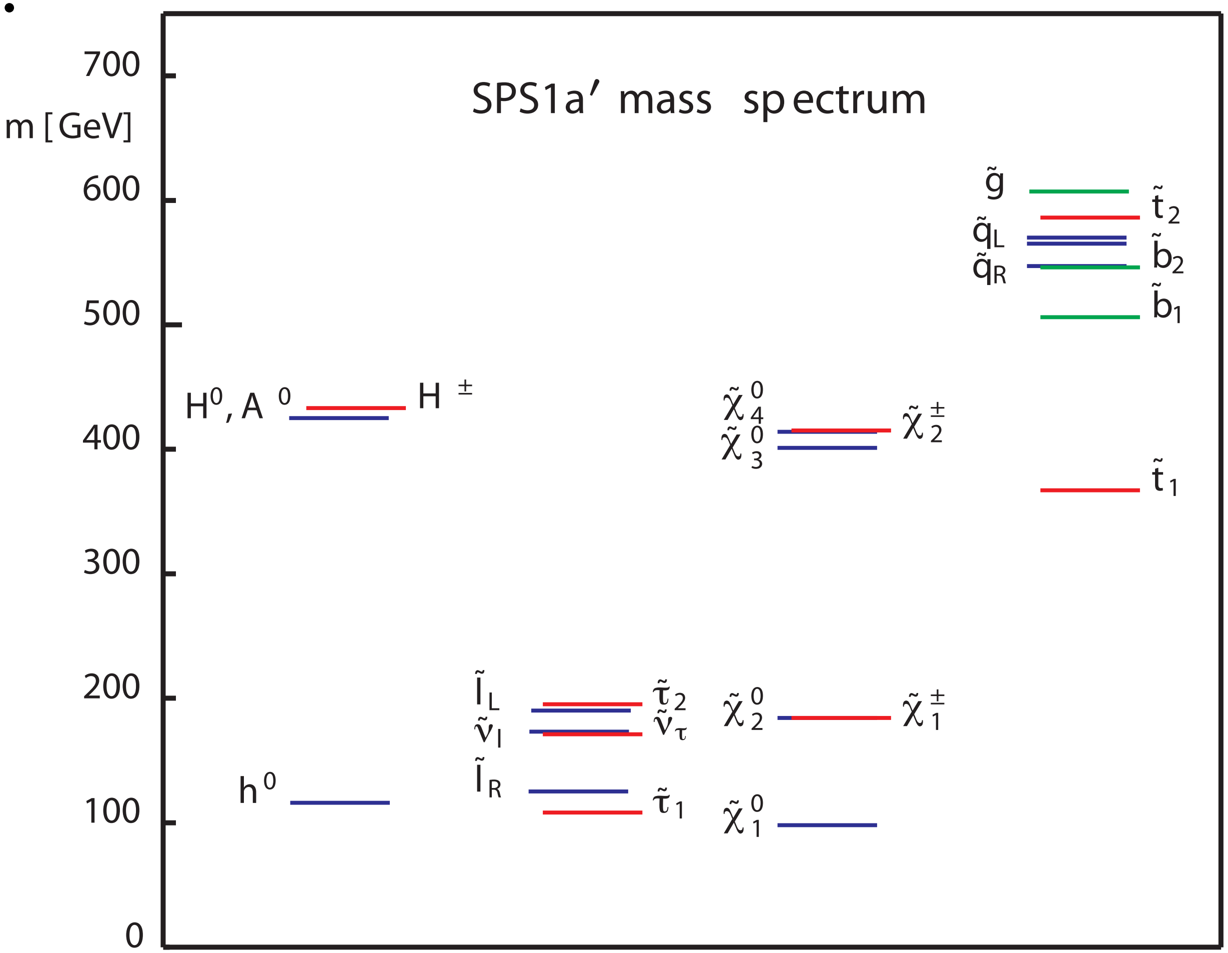
We ask a similar question in discussing Big Bang Nucleosynthesis.

If the measured dark matter density is larger, maybe there are other components of dark matter.

If the measured dark matter density is smaller, maybe the WIMPs are diluted by late decays or other entropy-producing processes.

Or, it could be just right !

Nojiri, Polesello, and Tovey showed that the LHC can give an accurate value of Ω_N at least at one point in the parameter space of SUSY. This point, called SPS1a’, has the new particle spectrum:



This spectrum has several good features for the determination of SUSY particle masses.

The most important annihilation reactions contribution to the relic density are

$$\widetilde{N}_1^0 \widetilde{N}_1^0 \rightarrow \ell^+ \ell^-$$

$$\widetilde{N}_1^0 \widetilde{N}_1^0 \rightarrow \tau^+ \tau^-$$

$$\widetilde{N}_1^0 \ \ \widetilde{\tau} \rightarrow \gamma \ \tau$$

a coannihilation reaction, present because $m(\widetilde{\tau}_1) - m(\widetilde{N}_1^0) < T_f$

To evaluate the cross sections for these reactions, we need to know the SUSY parameters

$$m_1, \ m_2, \ m(\widetilde{\ell}), \ m(\widetilde{\tau}_1), \ \theta_\tau$$

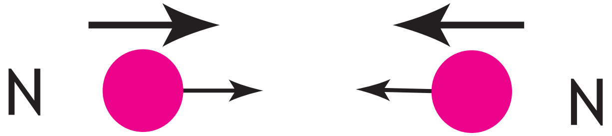
and we need to know that there is only small mixing between the SUSY partners of gauge boson and Higgs bosons.

The parameters are determined by precision measurement of the masses of $\widetilde{N}_1^0, \ \widetilde{N}_2^0, \ \widetilde{\ell}, \ \widetilde{\tau}$

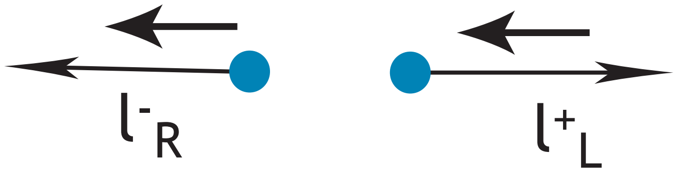
and of $BR(\widetilde{N}_2^0 \rightarrow \tau\widetilde{\tau})/BR(\widetilde{N}_2^0 \rightarrow \ell\widetilde{\ell})$

We need a complex of annihilations because the ordinary annihilation process $\widetilde{N}^0_1 \widetilde{N}^0_1 \to \ell^+ \ell^-$ is not strong enough to produce a small enough Ω_N.

Freeze-out occurs when the WIMPs are nonrelativistic, so annihilations in the S-wave should dominate. But this reaction is suppressed in the S-wave. Neutralinos are Majorana fermions, so they can only annihilate in the S-wave in a total spin-0 state



But the final state should conserve helicity;



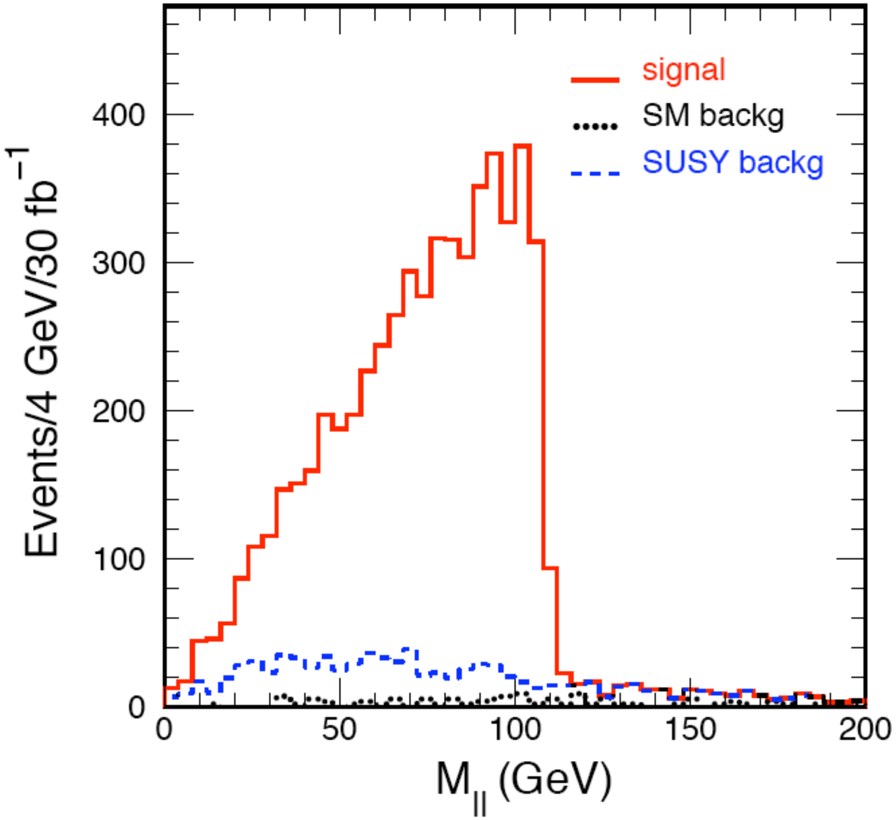
the amplitude that conserves J is suppressed by $(m_\ell / m_N)^2$

The extra process $\widetilde{N}^0_1 \, \widetilde{\tau} \to \gamma \, \tau$ does go in the S-wave.

In this model, the scalar partners of left-handed quarks decay through the long decay chain:

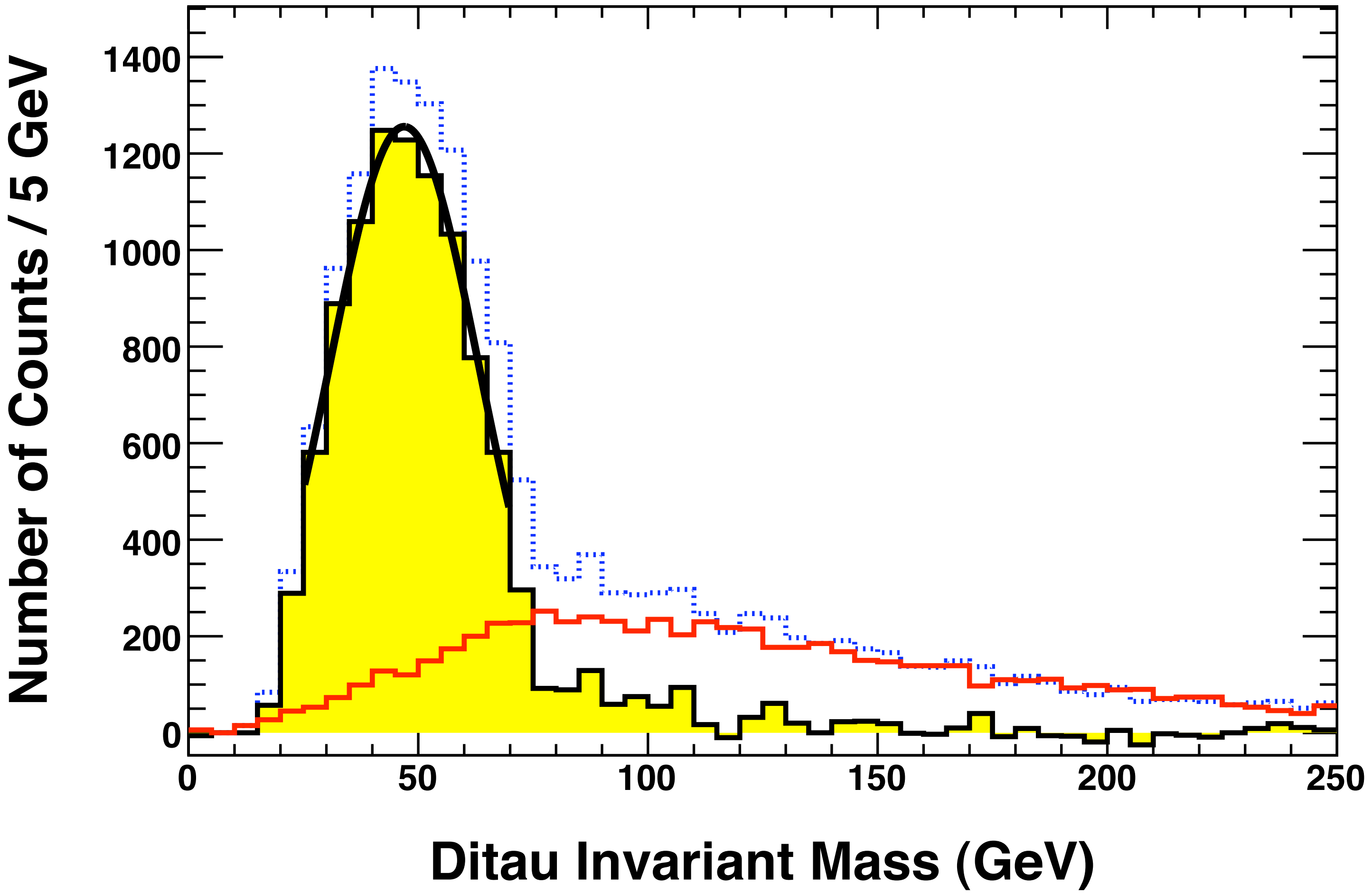
$$\widetilde{q}_L \rightarrow q\widetilde{N}_2^0 \rightarrow q\ell\widetilde{\ell} \rightarrow q\ell\ell\widetilde{N}_1^0$$

There are 4 unknown masses, so if there are 4 or more precisely measured kinematic endpoints of $\ell^+\ell^-, \quad q\ell^\pm, \quad q\ell^+\ell^-$ invariant mass distributions, the masses can be determined. NPT claim that all mass differences except those with $\widetilde{\tau}$ can be measured to 1% and the overall scale to 5%.



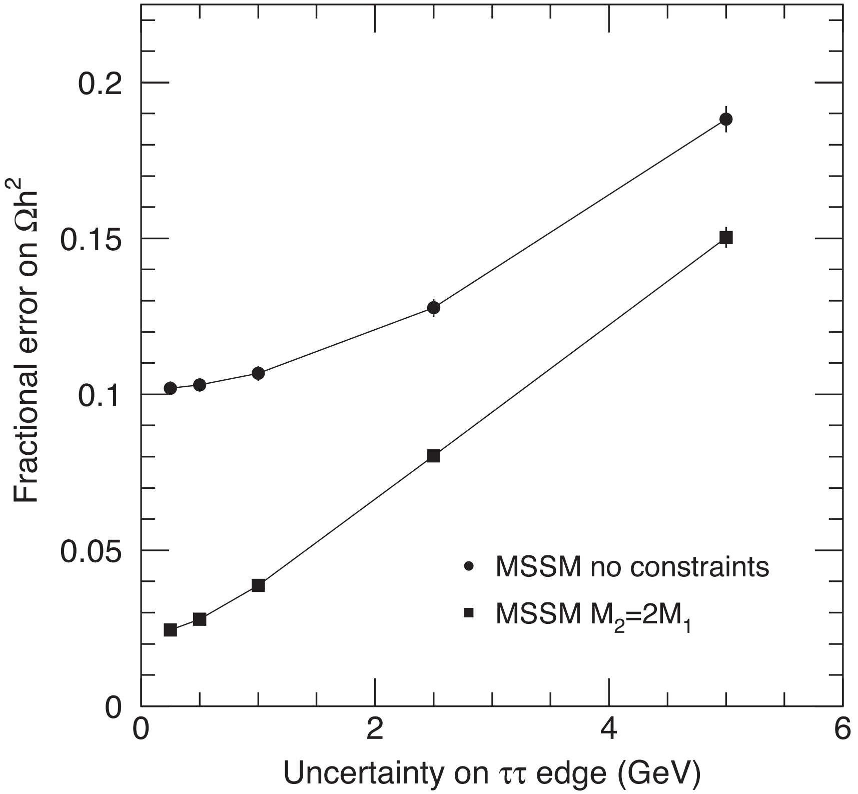
The mass $m(\widetilde{\tau}_1)$ is fixed by the endpoint position in $\widetilde{N}_2^0 \to \tau^+ \tau^- N_1^0$

Here is the di-tau mass spectrum at a similar point studied by Arnowitt et al. It will be a challenge to measure this endpoint to better than 5 GeV.



Arnowitt, Dutta, Kamon, Kolev, Toback

Here is the final NPT prediction, as a function of the endpoint measurement accuracy:



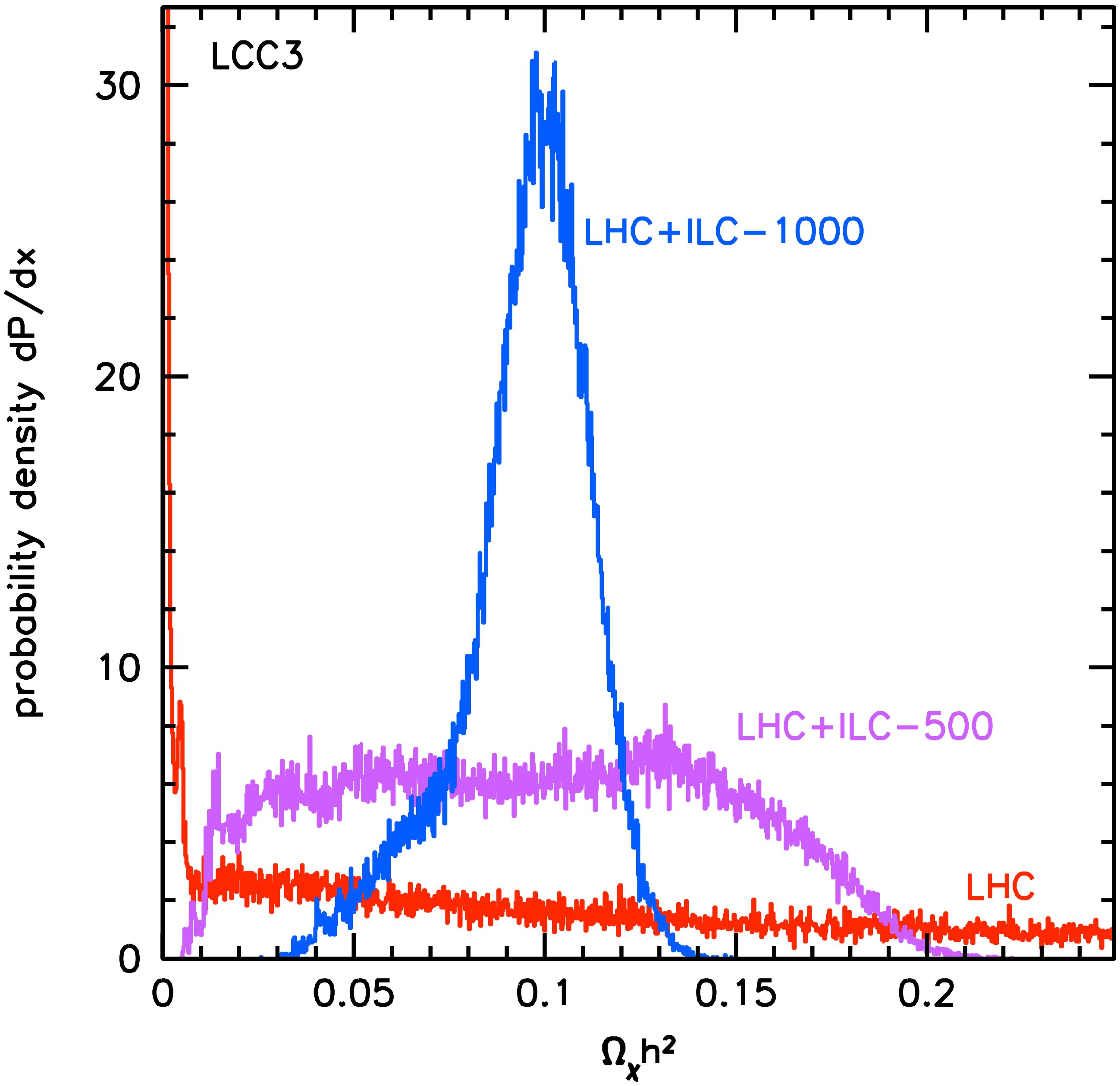
In addition, there are special circumstances (e.g. $m_A \approx 2m_N$), in which a much lower value of Ω_N is predicted.

The higher-precision ILC measurements should improve the accuracy of this prediction to about 1%.

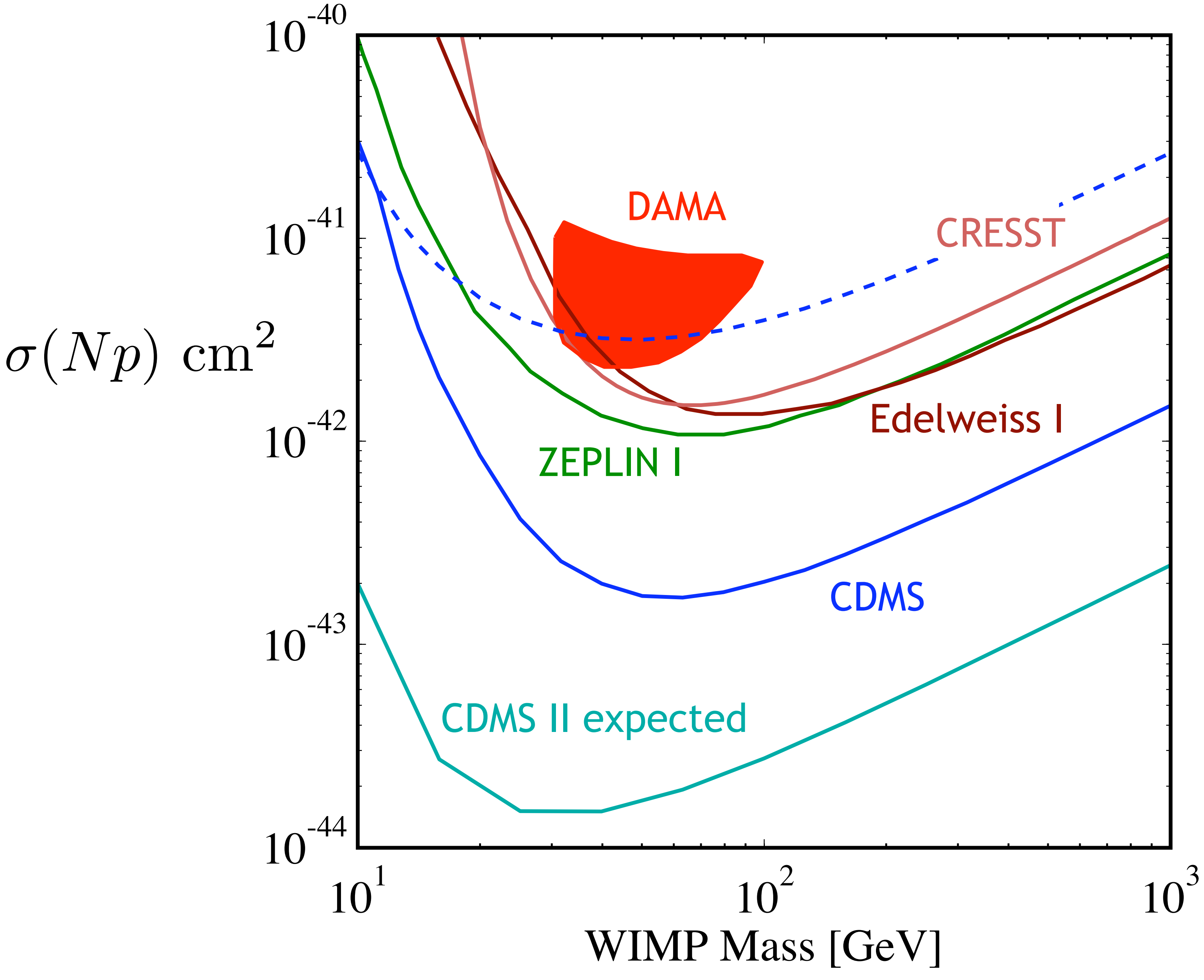
At a more generic point, less precision information will be available from the LHC. Later, though, the ILC will supply precision mass and mixing angle measurements.

Baltz, Battaglia, Wizansky, and I studied the effect of these measurements at a few more points in the SUSY parameter space.

Here is our result for the point studied by Arnowitt et al.



Next, discuss direct detection of WIMPs. The current situation is the exclusion plot



from DMtools - Gaitskell & Mandic

The next generation of detectors

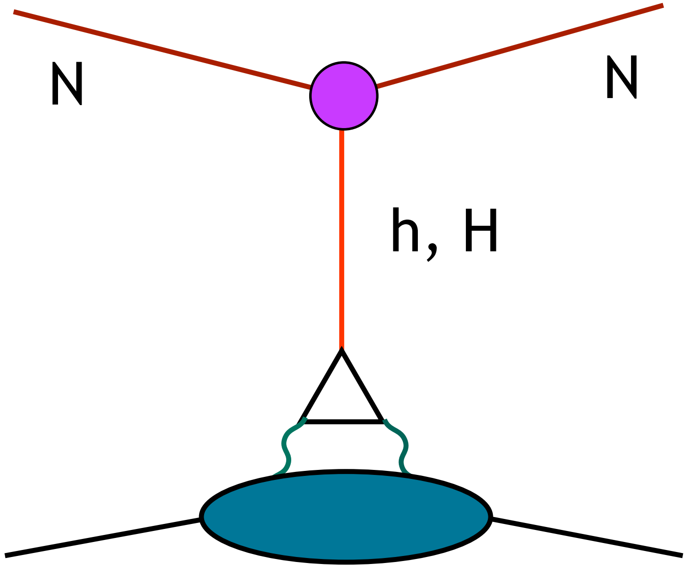
superCDMS (Germanium), Xenon, WARP (Liquid Argon)

and others should reach a sensitivity of

$$\sigma(Np) \sim 10^{-45}\ \mathrm{cm}^2 = 1\ \mathrm{zeptobarn}$$

If (when) a detection is observed, how will we analyze it ?

The full expression for the Np cross section in SUSY is quite complicated. However, if squarks are sufficiently heavy, this cross section is typically dominated by the diagram:

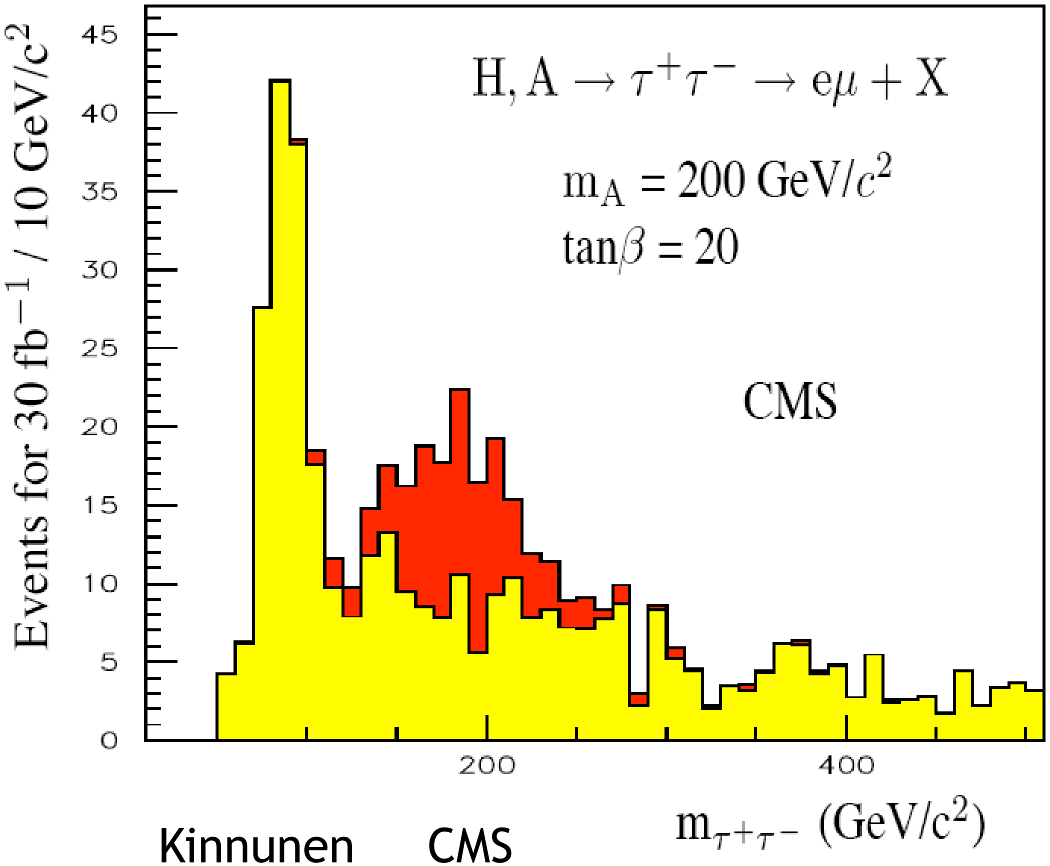


To evaluate the cross section, we need to know the mass and couplings of the Higgs bosons h and H.

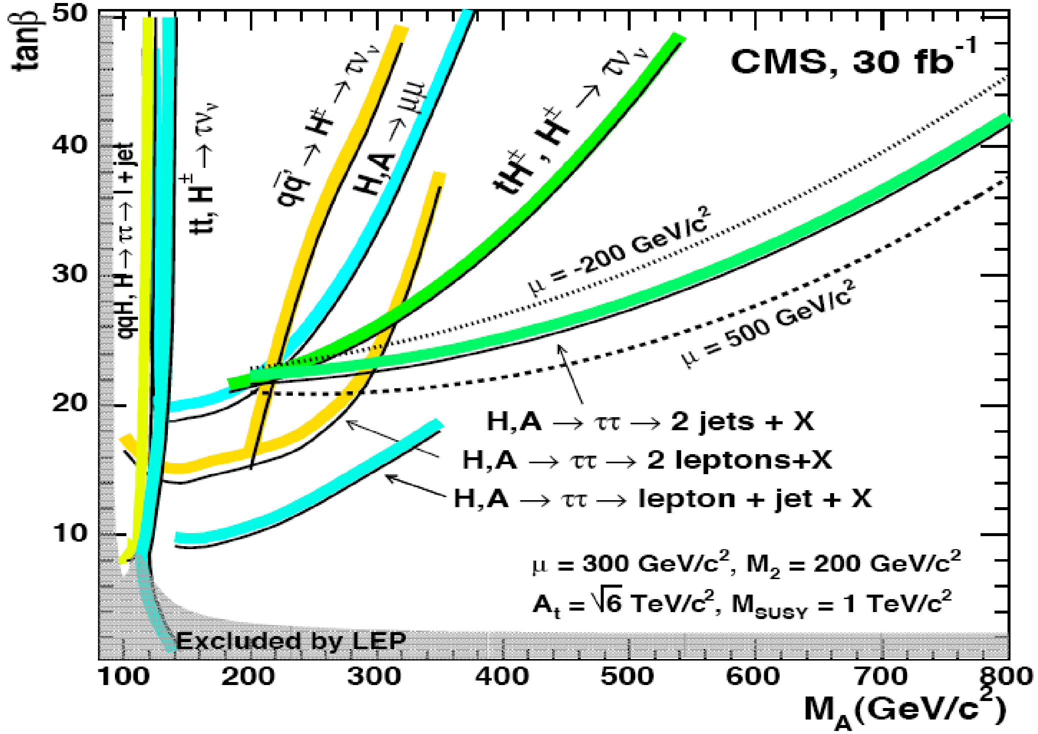
In SUSY models, it is almost assured that the Higgs boson h can be discovered at the LHC.

For large enough $\tan\beta$ the heavy MSSM bosons H and A can also be discovered at the LHC in

$$b\bar{b} \rightarrow H, A \rightarrow \tau^+\tau^-$$

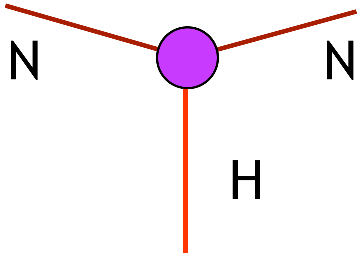


Kinnunen CMS



This gives the mass to a few %, but with a large uncertainty in $\tan\beta$

The neutralino-Higgs coupling



$$= \frac{ie}{2}\left(\frac{V_{11}}{c_w} - \frac{V_{21}}{s_w}\right)(V_{31}\cos\alpha - V_{41}\sin\alpha) + (i \leftrightarrow j)$$

depends both on $\tan\beta$ and on the neutralino mixing angles.

Neutralino mixing can be measured from the spectrum, but it is beautifully measured from polarized e+e- cross section,

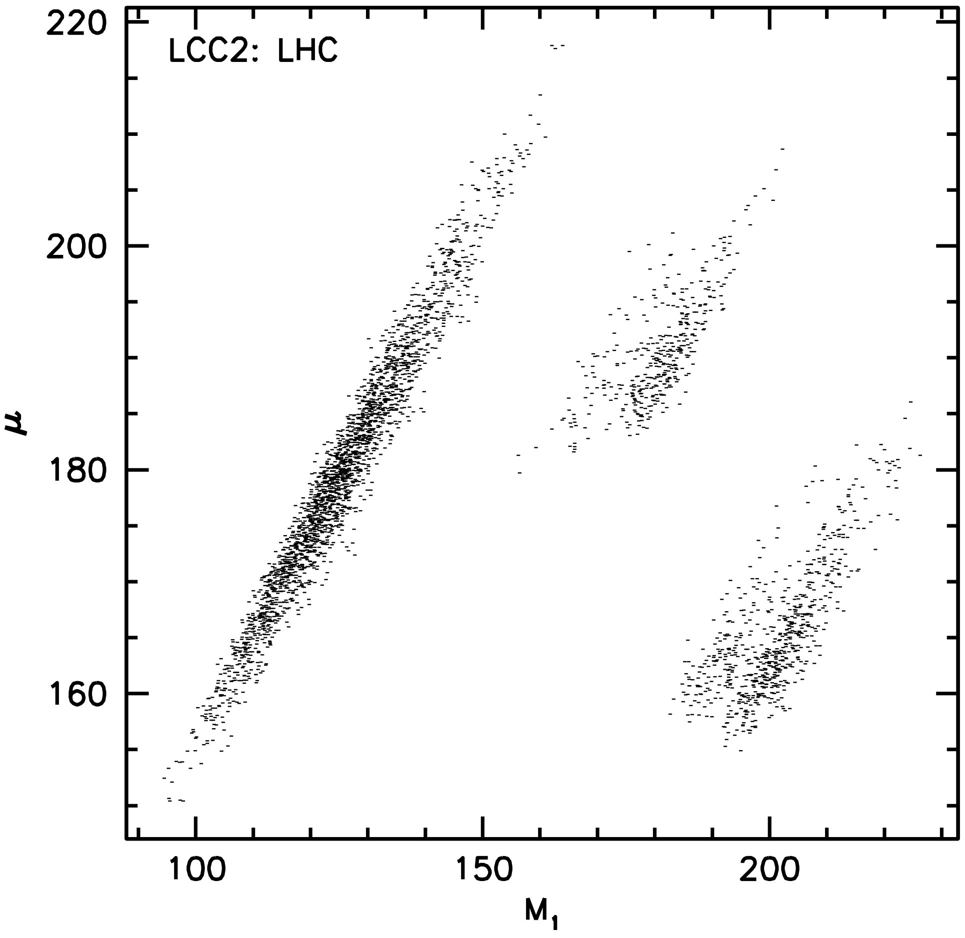
e.g. $$e^+e^- \rightarrow C_1^+C_1^-$$

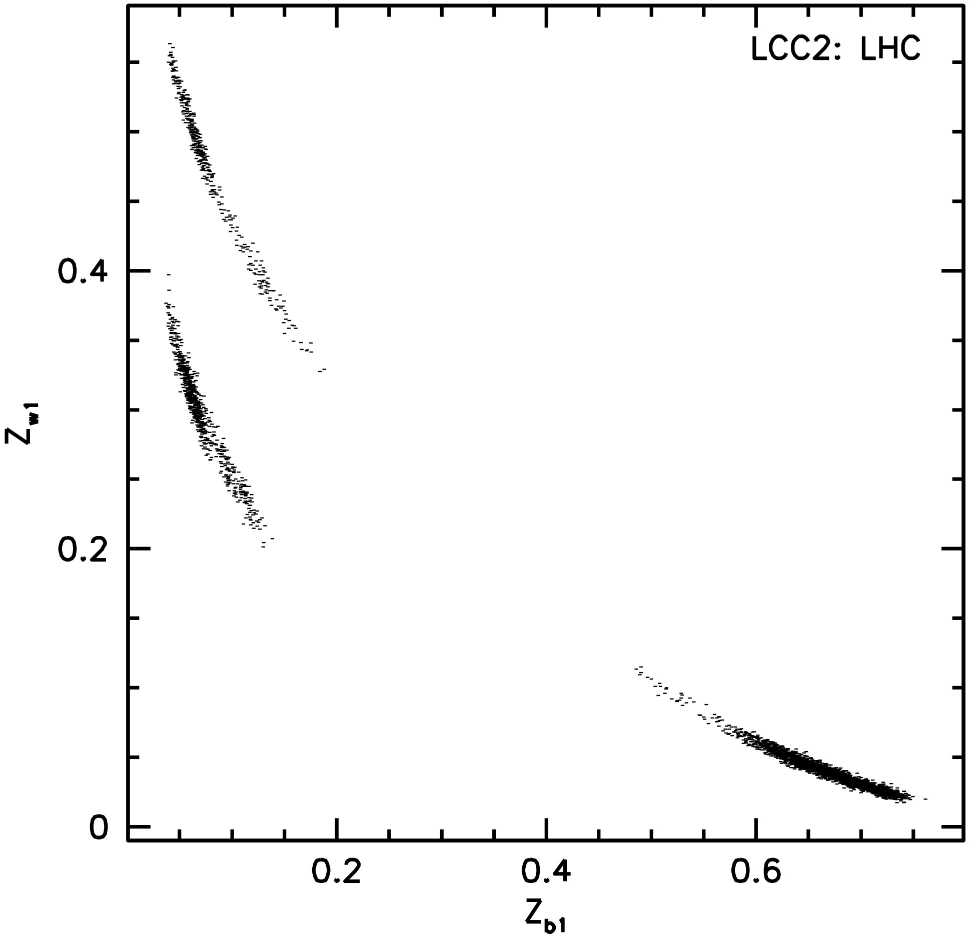
from right-handed polarized e- beams.

ILC will measure this and similar cross-sections at the few-% level.

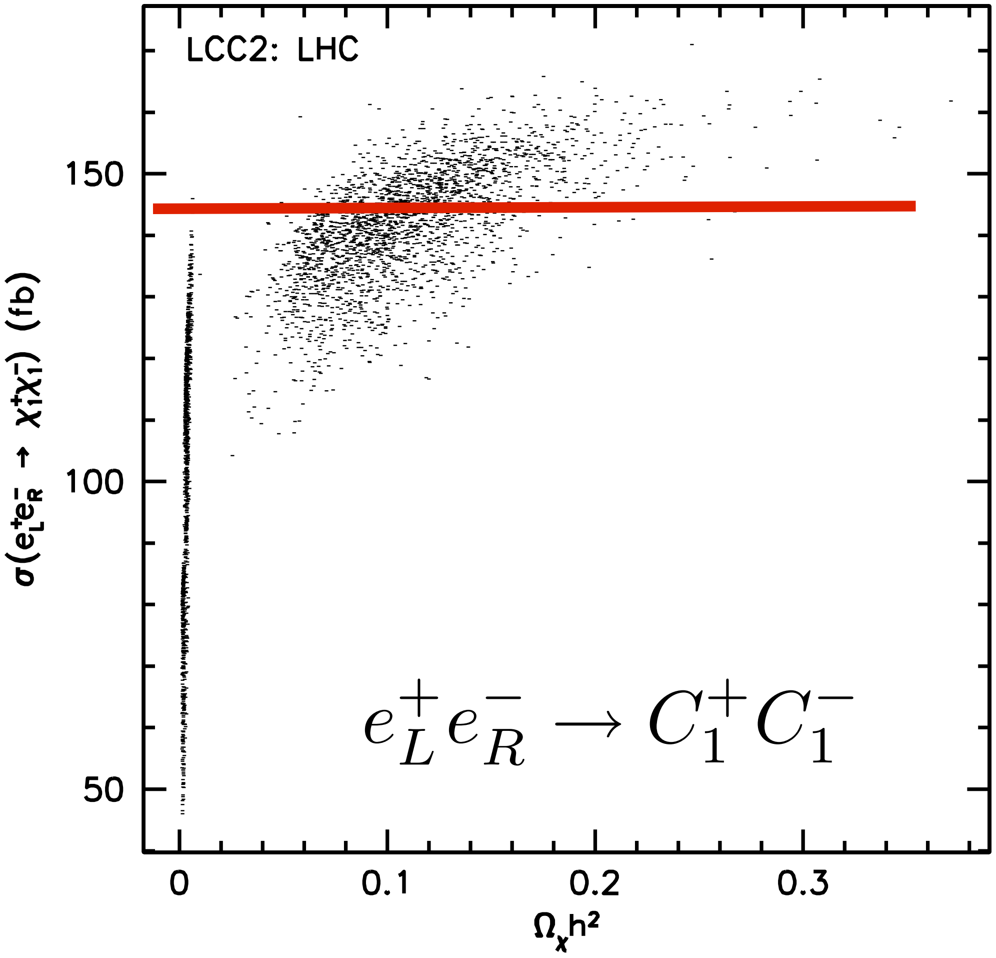
$\tan\beta$ can be measured directly from the H,A width and branching ratios, another measurement probably left for ILC.

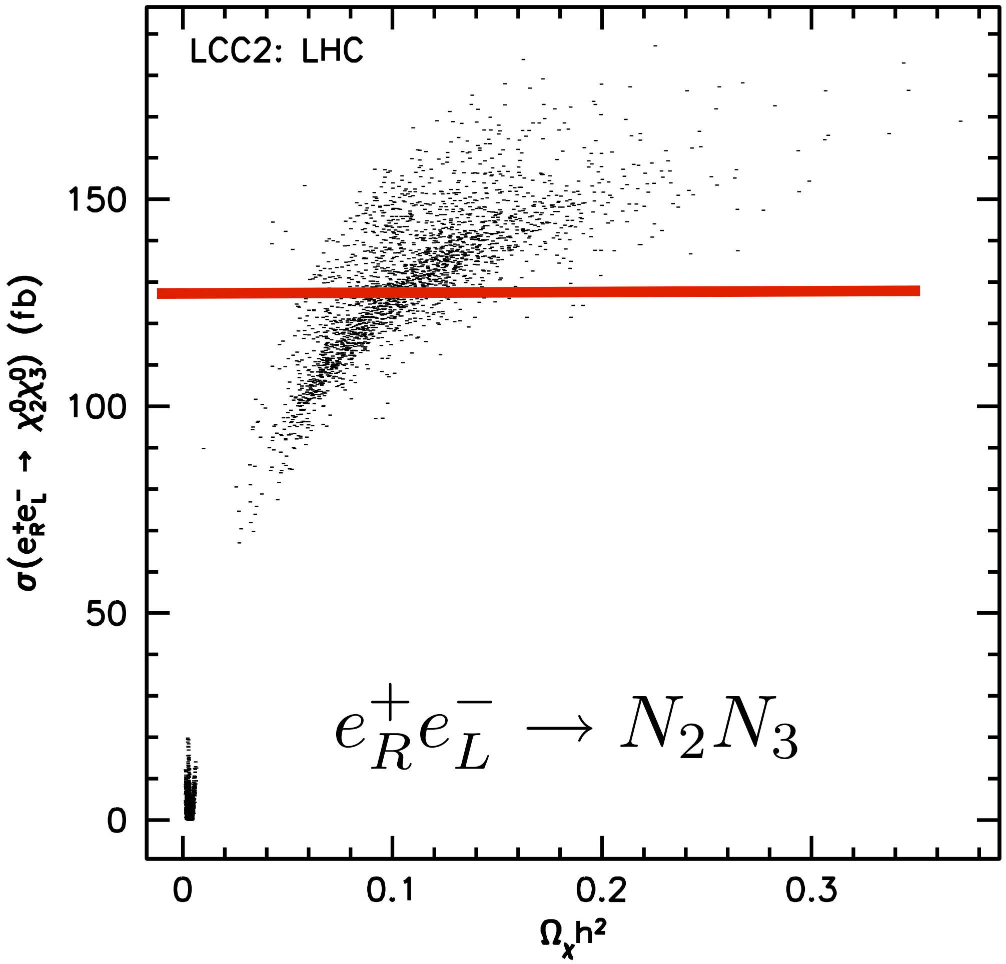
Here are some SUSY parameters sets consistent with spectrum data from the LHC, at a point (LCC2) in the study of Baltz et al.



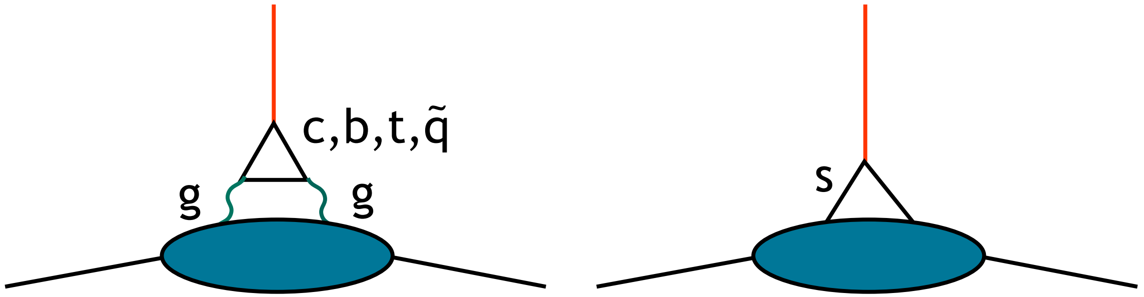


Here are the same points, scattered in the plane of relic density vs. ILC cross sections:





It should be emphasized that we do not yet know the value of the Higgs-proton coupling. This comes from two sources:



leading to the formula (large $\tan\beta$).

$$g_{Hpp} = \frac{m_p}{250\ \mathrm{GeV}}\left[\frac{2}{27} + \frac{25}{27} f_{Ts}\right]\tan\beta + \cdots$$

where $$f_{Ts} = \frac{\langle p|\, m_s \bar{s}s\, |p\rangle}{m_p}$$

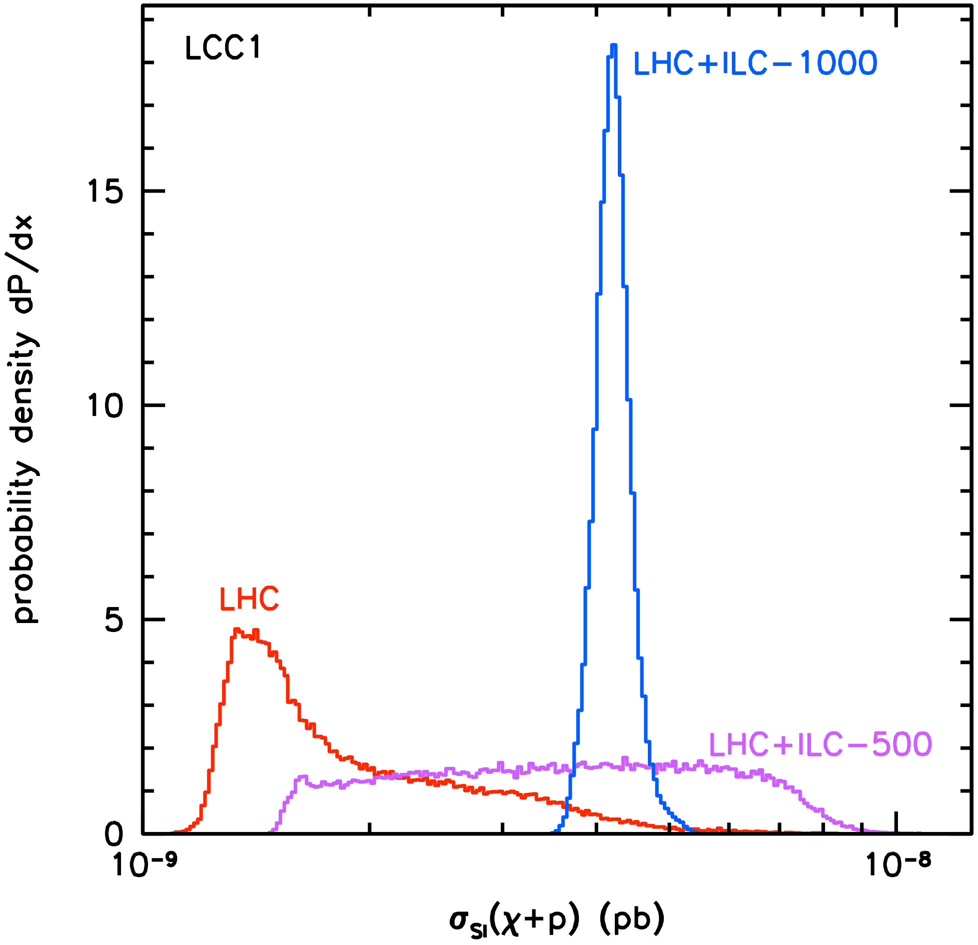
Unfortunately, f_{Ts} is poorly known; estimates in the literature range from 0.14 to 0.49 and the range has not decreased in 20 years. The Np cross section depends on g_{Hpp}^2.

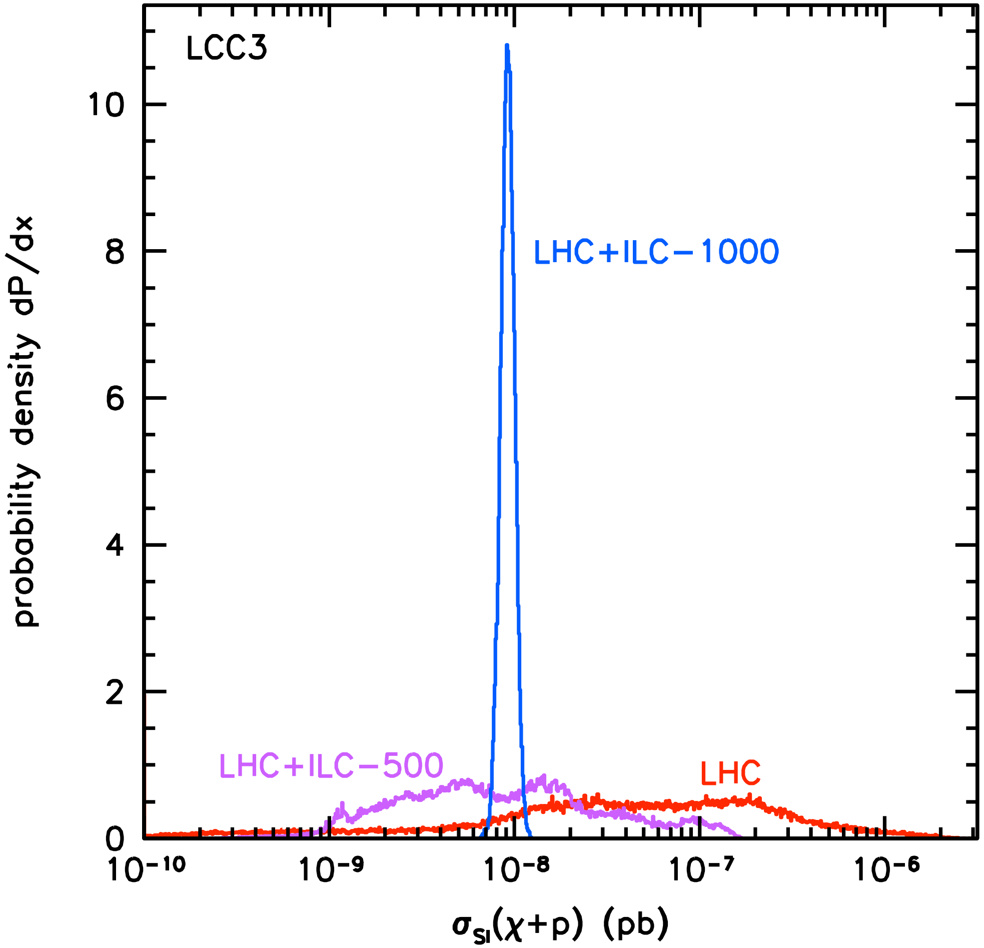
Potentially, lattice gauge theory can resolve this problem. At the moment, though, this is beyond the state of the art. The best current lattice estimate is $f_{Ts} = -0.20 \pm 0.23$ (UKQCD 2001).

Here are estimates of the accuracy with which colliders will determine the Np cross section, from Baltz et al. (ignoring the f_{Ts} uncertainty)

SPS1a:
the H boson cannot be discovered at LHC; this needs the ILC-1000

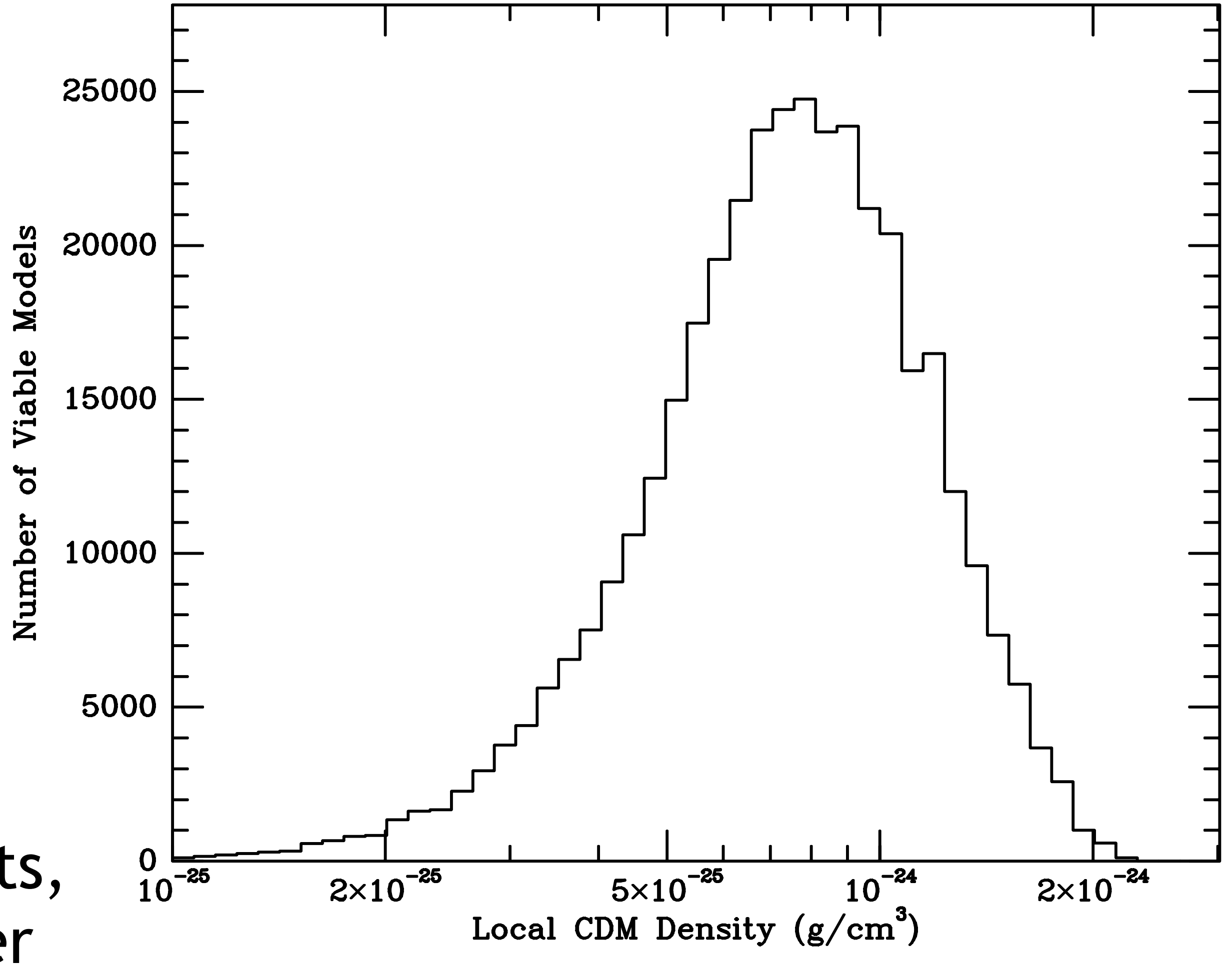
Arnowitt et al point:
the H boson should be discovered at LHC, but mixing angles need the ILC-1000





The direct detection rate also depends on the local density of WIMP dark matter. More specifically, the direct detection rate is proportional to the local flux of WIMPs.

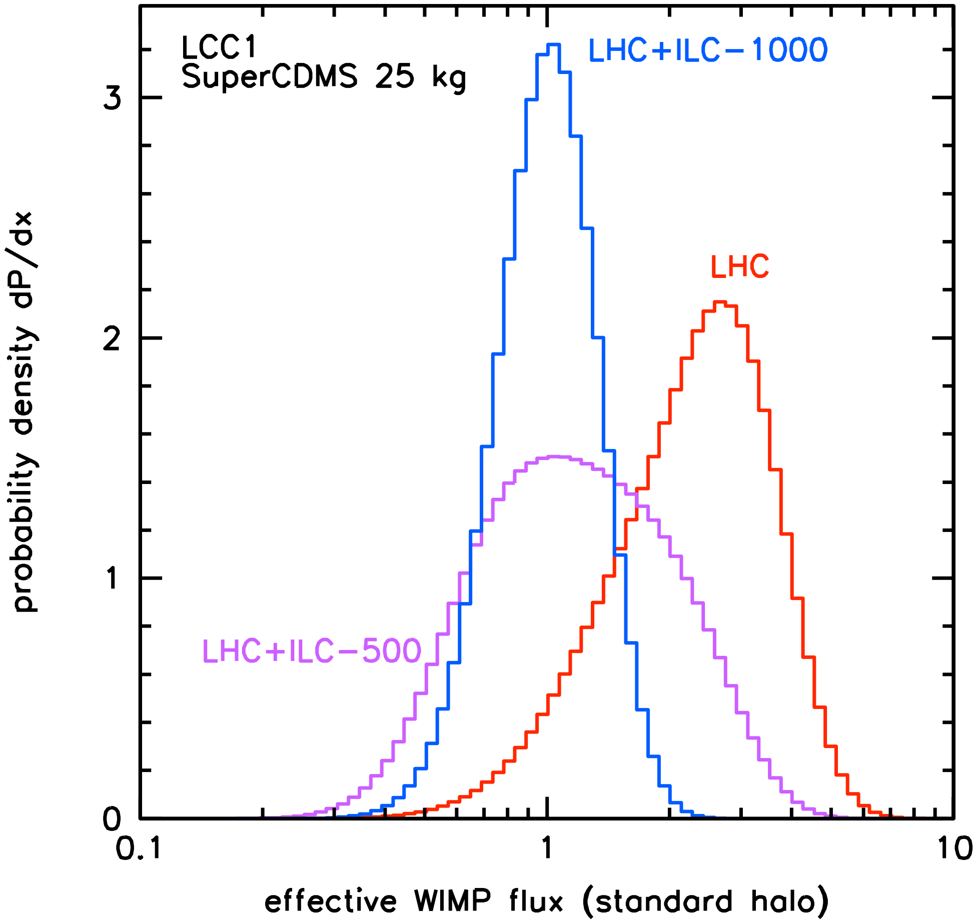
Gates, Gyuk, and Turner constructed a large number of models consistent with the galactic rotation curve. Here is the distribution of the local WIMP density:



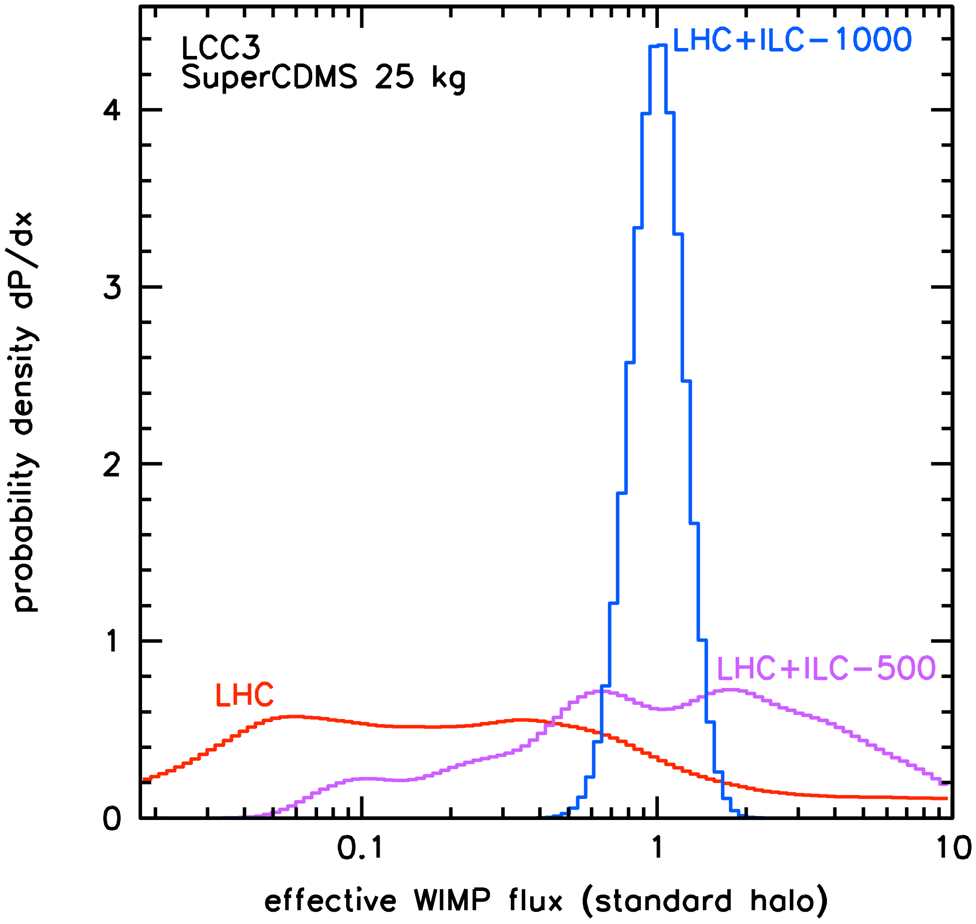
Other models of the galactic halo predict mutliple WIMP components, some of which have higher velocity than the standard one.

If the LHC and ILC measurements would determine $\sigma(Np)$, we could turn the styory around: measure the detection rate, divide by the known cross section, and learn the value of the local WIMP flux. Here is how well it should be done, combining the previous estimates with the counting rates from super-CDMS:

LCC1: 16 events



LCC3: 29 events



There is a second strategy for detecting dark matter particles.

In the WIMP model, WIMPs are still annihilating at a slow rate. Look for high-energy particles in cosmic rays that could be WIMP annihilation products:

gammas, positrons, antiprotons, neutrinos

Positrons and antiprotons are smeared in direction and energy by galactic matter and magnetic fields.

Neutrinos require that WIMPS be collected and concentrated by the earth or the sun to obtain a large enough signal.

Gammas are relatively straightforward to analyze, so I will concentrate on this case to discuss the implications of collider data. Gammas fly to us directly from the annihilation point, so in principle they map the WIMP distribution in the galaxy.

The energy and angular distribution of gammas from WIMP annihilation are given by the formula (1/2 for Majorana WIMPs)

$$\frac{dN_\gamma}{dE_\gamma d\Omega} = \frac{1}{2}\frac{d\sigma v}{dE_\gamma} \cdot \frac{1}{4\pi m_N^2} \cdot \int dz \rho_N^2$$

This factorizes neatly into a particle physics factor and an astrophysical factor.

$$J = \int dz \rho^2 \ / \ r_0 \rho_0^2$$

$$r_0 = 8.5 \text{ kpc} \qquad \rho_0 = 5 \times 10^{-25} \text{g/cm}^3$$

m_N can be found from LHC or from the endpoint of the WIMP gamma spectrum.

The cross section is in principle given by colliders.

The factor J depends on the distribution and clustering of WIMP dark matter. This depends on ρ_N^2 so it is very sensitive to clumping of WIMPs.

The distribution of dark matter is not expected to be smooth.

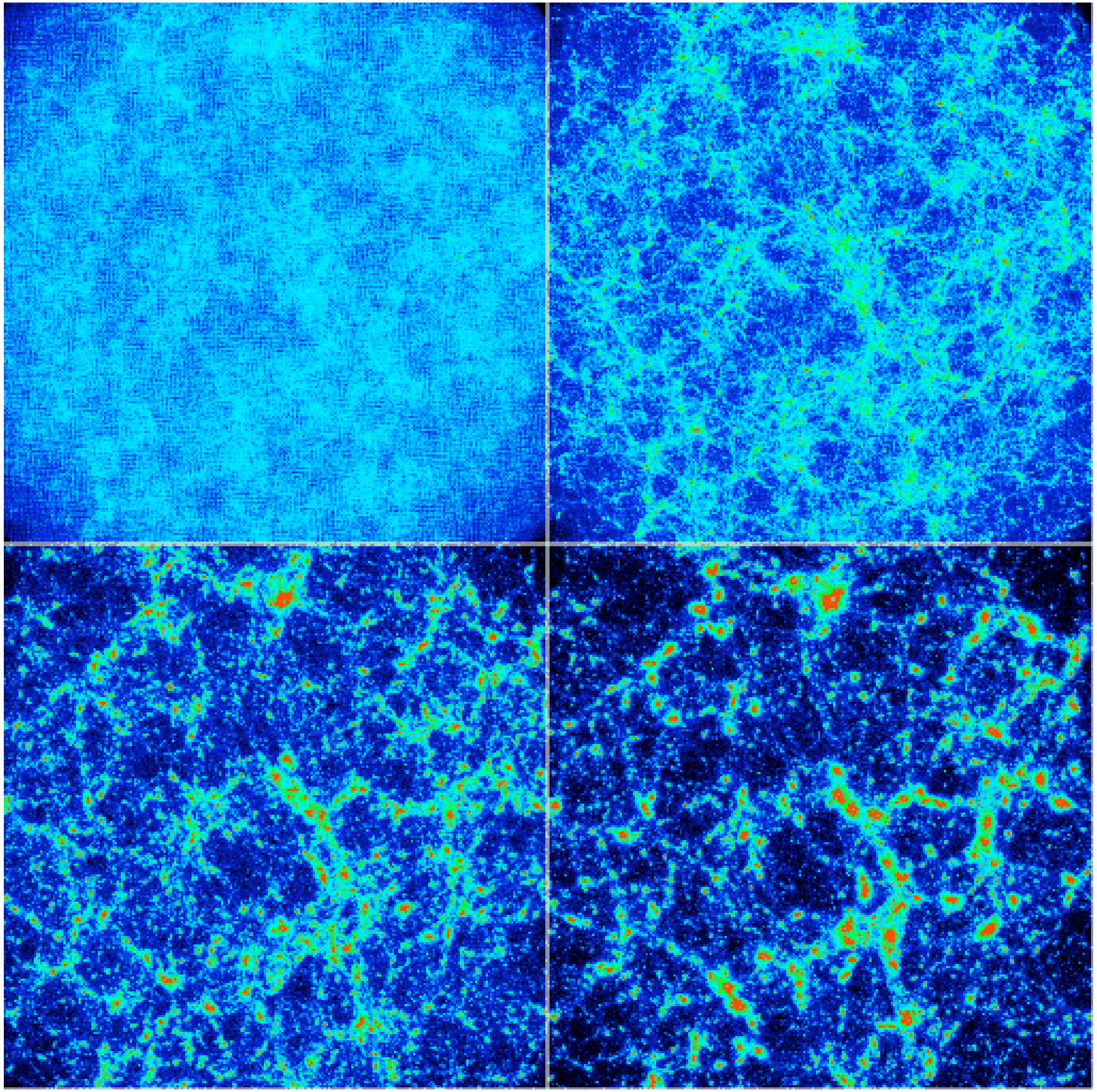
A success of the dark matter model is that density perturbations in a distribution of nonrelativistic dark matter grow in such a way as to produce structures similar to those actually seen in the universe.

The picture requires that dark matter be nonrelativistic (‘cold’). Neutrinos need not apply.

In this picture, large galaxies like the Milky Way arise by merger and coalescence of smaller clusters of dark matter. This leads to hierarchial structure within the dark matter halo.

Large galaxies should be accompanied by many small companions. For the Milky Way, there are possibility too few minor galaxies in the local group. The smaller galaxies should have larger fractions of dark matter. This seems to be correct.

growth of structure in CDM cosmological simulations

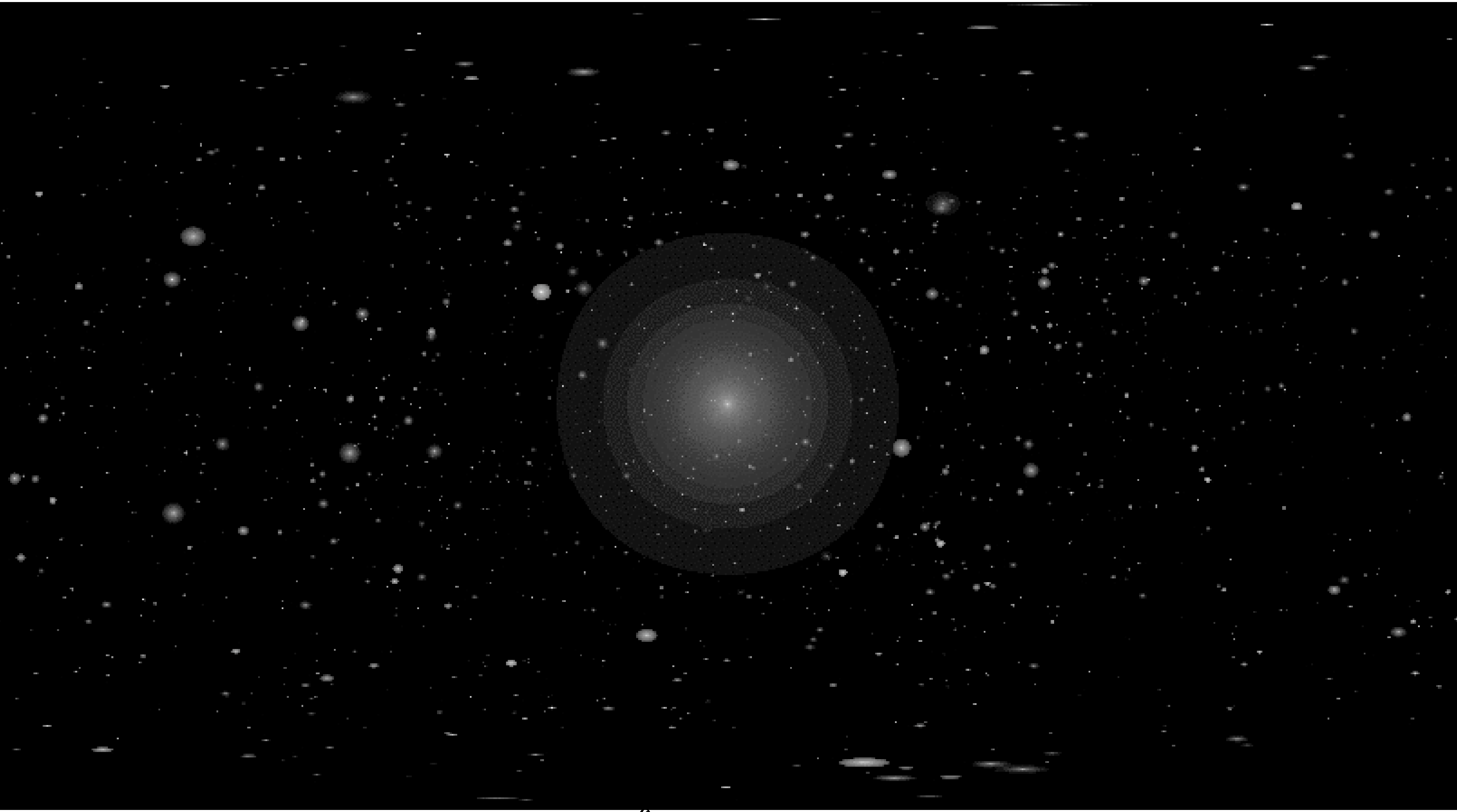


M. S. Warran et al. (Los Alamos NL)

two of the larger minor galaxies in the Local Group,

the Large and Small Magellanic Clouds

Dark matter structure of a model galaxy, with hierarchial clustering, from simulations of Taylor and Babul



visualization of $J \sim \int dz\ \rho_N^2$ by Baltz.

What is the shape of a dark matter cluster ? Here too, there are interesting theoretical pictures, with unresolved questions.

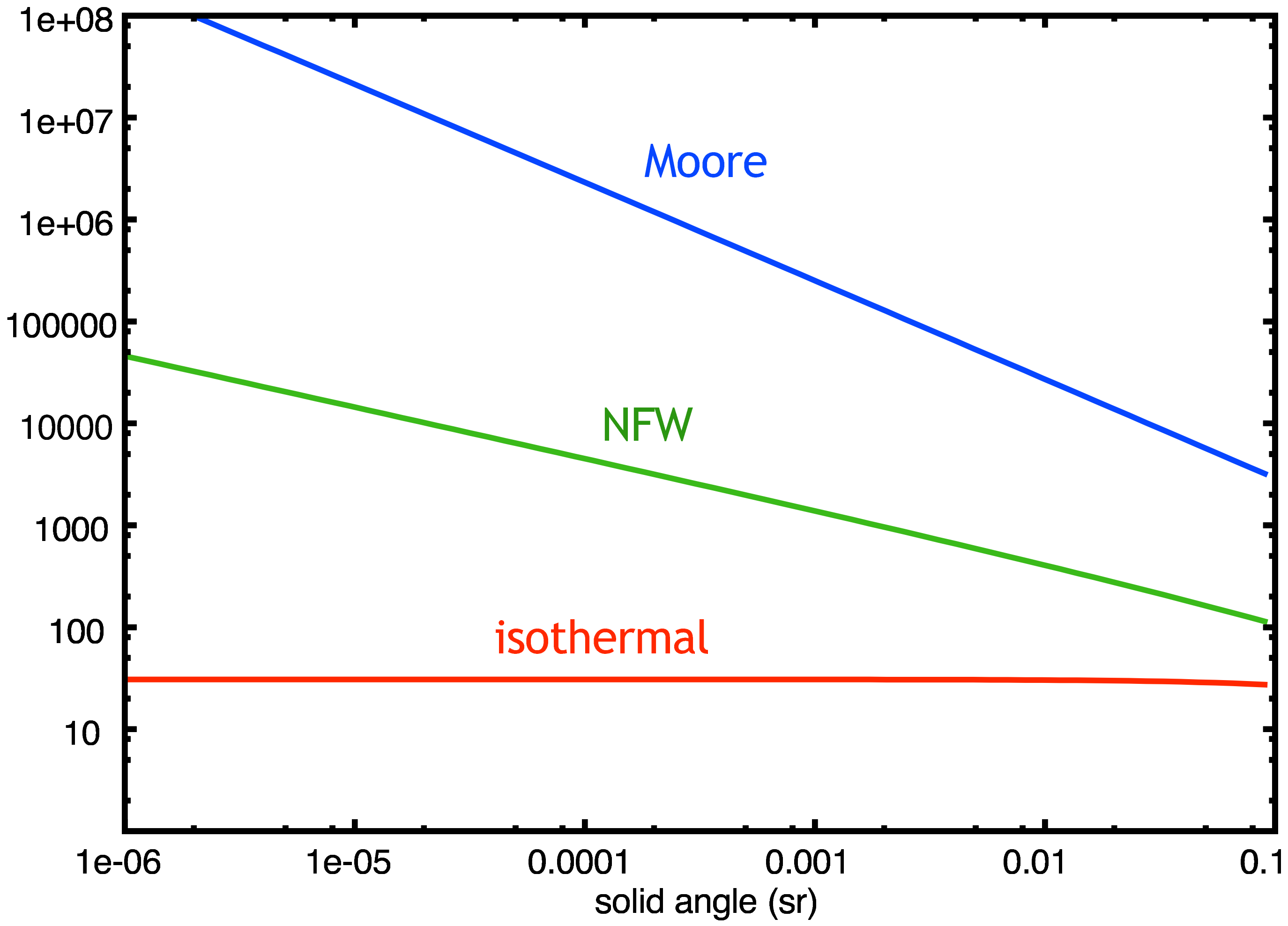
Based on large-scale, but pure dark matter, simulations, Navarro, Frenk, and White proposed that concentrations of dark matter have the form $\rho(r) = \rho_s (r/r_s)^{-1} (1 + r/r_s)^{-2}$

Moore, Governato, Quinn, Stadel, and Lake argued for an even steeper profile, proportional to $r^{-3/2}$.

Primack, Bullock, and Wechsler, and Kravtsov have shown that the variety of clusters in a simulation can span this range of behaviors, depending on the histories of the individual clusters.

These differences in the suggested forms for dark matter clustering make a large difference in the value of the factor J that gives the rate of gamma-ray signals of dark matter.

Here are the results for <J> over a solid angle when these forms are applied to the clustering of dark matter at the galactic center.



To resove this situation, it would be good to directly visualize the galactic dark matter density.

If we could know the WIMP annihilation cross section, we could measure the density of the dark matter peaks in the galaxy from gamma ray observations by GLAST and ground-based telescopes.

How well can we do ?

The same collider measurements that predict the relic density also predict the anniliation cross section, and to a similar accuracy.

However, we can take a short-cut. In the discussion of the relic density, I argued that, if WIMPs account for 100% of the dark matter, their annihilation cross section must be

$$\langle \sigma v \rangle = 1 \text{ pb}$$

Can we just put in this value to analyze GLAST data ?

For the two models discussed earlier, this trick does not work. For SPS1a and for the Arnowitt et al point, the annihilation cross sections relevant to gamma ray observations are

$$\sigma v = 0.012 \text{ pb (SPS1a)} \ , \quad 0.11 \text{ pb (LCC3)}$$

These small values make it very difficult for GLAST.

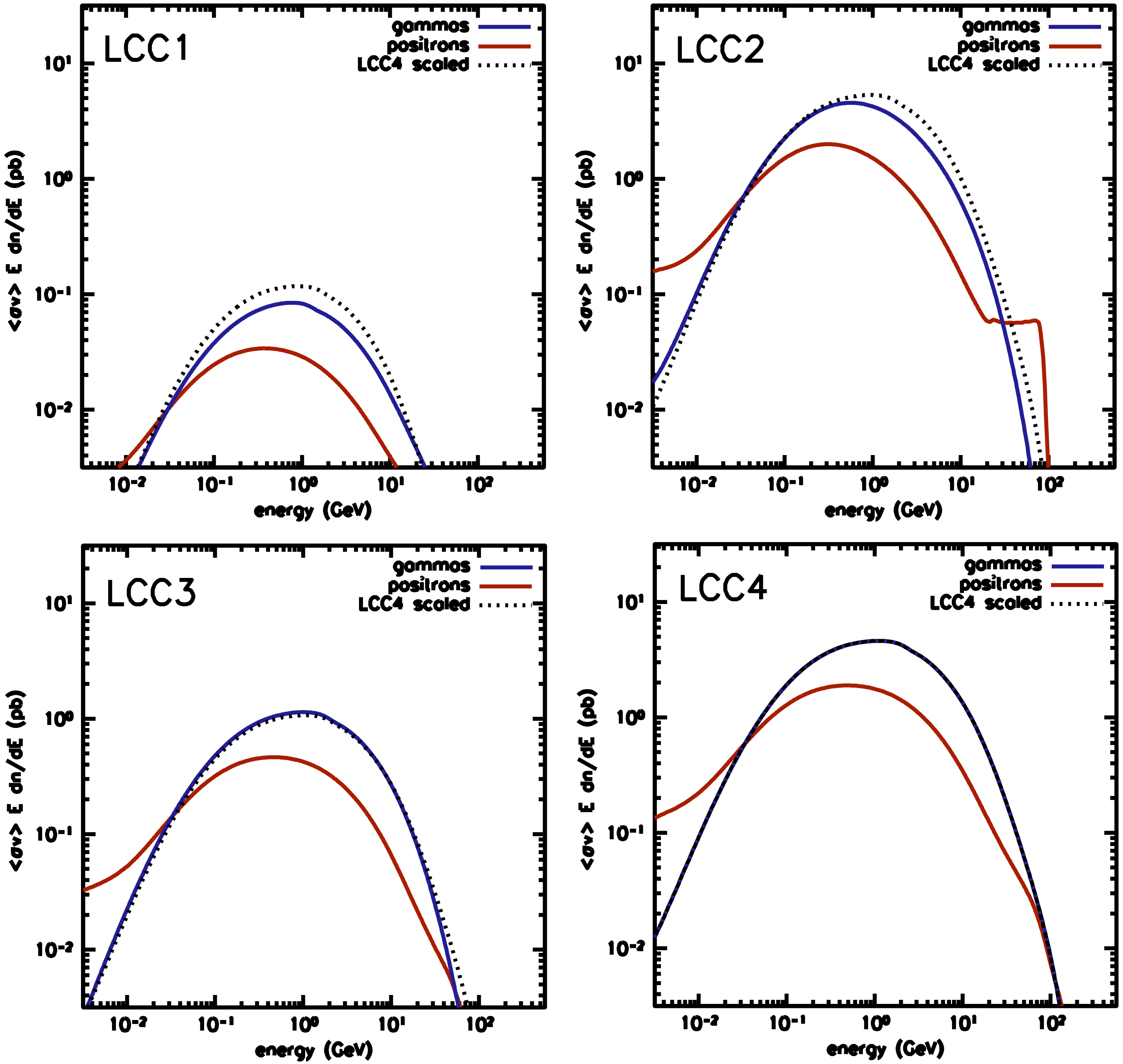
At the two other points studied by Baltz et al., the relevant cross sections are approximately 0.5 pb, a much nicer value.

What makes the difference ? For the relic density, we need the annihilation cross section at the freezeout temperature T_f. For gamma-ray observations, what matters is the annihilation cross section at threshold. These can be very different if the annihilation is dominantly P-wave (SPS1a) or if coannihilation is important at freezeout (LCC3).

The other 2 points have large S-wave annihilation cross sections via $NN \to W^+W^-, Z^0Z^0 \ , \quad NN \to A^0 \to b\bar{b}$

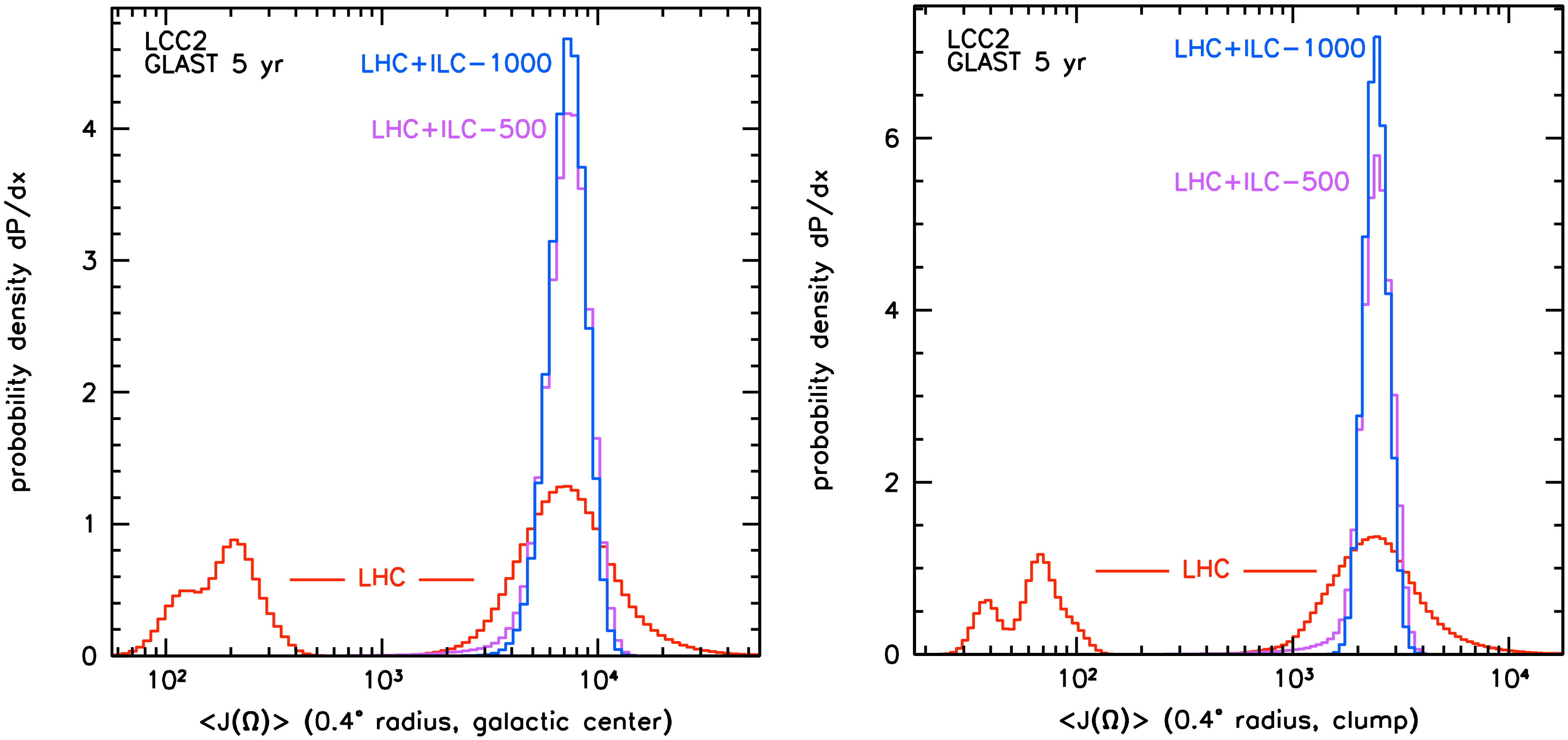
It is interesting to look at the gamma-ray energy spectra in the four cases (as generated by PYTHIA). There are almost indentical in shape, differing only in normalization.

The gammas arise in all cases from hadronic jets fragmenting to π^0 s.


LCC1
LCC2
LCC3
LCC4
gammas
positrons
LCC4 scaled
<σv> E dn/dE (pb)
energy (GeV)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

Here is our estimate - for the favorable SUSY point LCC2 - of the accuracy with which GLAST can measure <J> for the galactic center and for a $10^6 M_\odot$ subhalo clump, assuming in both cases NFW clustering. The σv determination is based entirely on the collider data, with no cosmological assumptions.

galactic center LCC2 sub-halo clump



The distributions have a sharp endpoint, which can give a model-independent WIMP mass determination. This mass can be compared to the WIMP mass from LHC.

The WIMP mass might also be measurable in direct detection. The recoil energy spectrum in elastic scattering of a WIMP from a target nucleus T has the approximate form

$$\langle E_R \rangle = \frac{2v^2 m_T}{(1 + m_T/m_N)^2}$$

So for a 100 GeV WIMP we might expect a 20% error on the mass.

It would be amazing if these three masses were found to be in good agreement. This could happen within the next 5 years.

Before concluding this talk, I would like to describe one more variant of dark matter models with interesting implications for colliders.

It is possible that the final stable particle in a model of EWSB is not the lightest particle with electroweak interations but rather a particle with even weaker interactions.

For example, in SUSY, the lightest Standard Model superpartner could decay to the SUSY partner of the graviton.

In general, we call this particle a 'super-WIMP'.

This gives a smaller relic density than that predicted from collider data by the argument given earlier. If m_{SN} is the super-WIMP mass, this model predicts

$$\Omega h^2 = \Omega_N h^2 \cdot \frac{m_{SN}}{m_N}$$

Feng, Takayama, Rajaraman, and Smith studied the case of super-WIMPs in supersymmetry. The option

$$\widetilde{\chi}^0 \to \gamma + \widetilde{G}$$

is excluded by its late energy release after BBN. The option

$$\widetilde{\ell} \to \ell + \widetilde{G}$$

is acceptable, and gives roughly $\tau_{\widetilde{\ell}}$ ~ 1 yr.

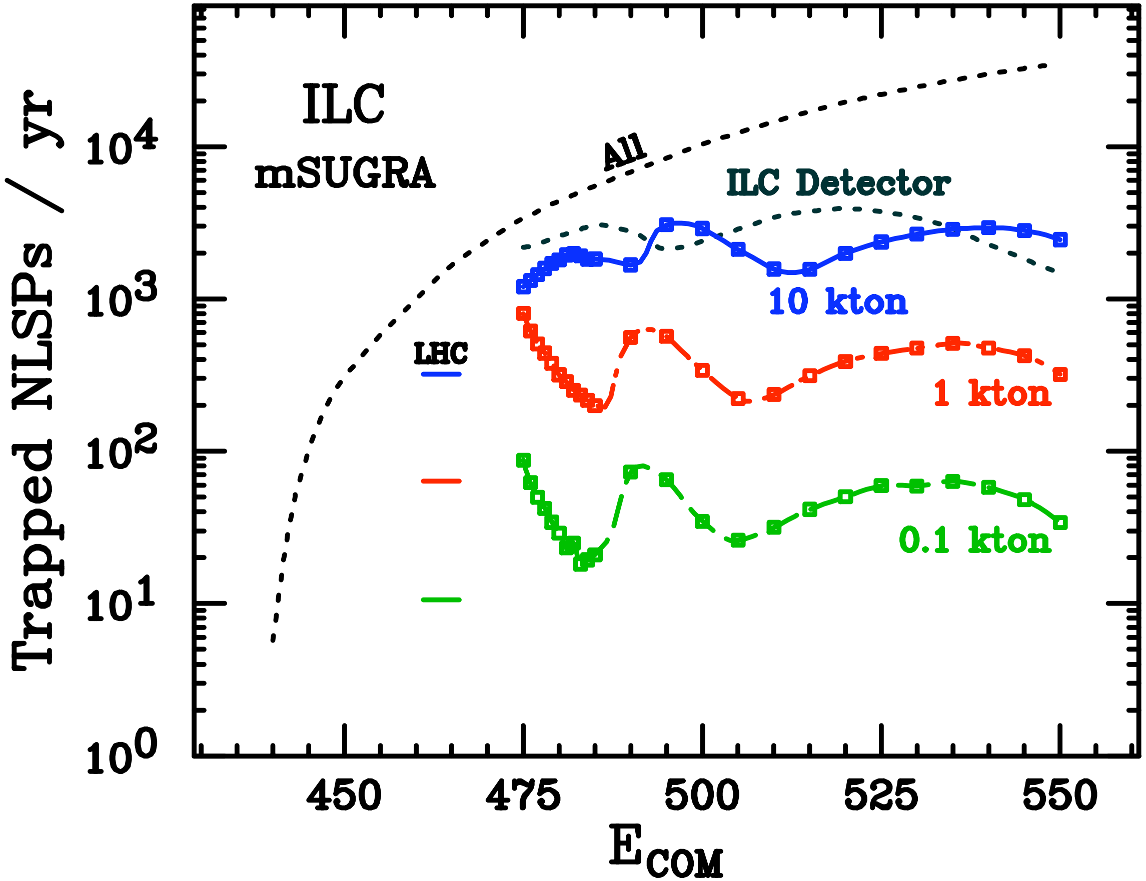
In this scenario, all astrophysical searches for dark matter fail.

However, it is possible to observe $\widetilde{\ell}$ at colliders as a stable massive charged particle.

A Linear Collider operating near threshold can collect $\widetilde{\ell}$'s in a water tank and measure $\tau_{\widetilde{\ell}}$ and $m(\widetilde{\ell}) - m(\widetilde{G})$.

Spontaneously broken supersymmetry makes a precise prediction, which can be checked:

$$\tau = \frac{6}{G_N} \frac{m_{\widetilde{G}}^2}{m_{\tilde{\ell}}^5} \left(1 - \frac{m_{\widetilde{G}}^2}{m_{\tilde{\ell}}^2}\right)^{-4}$$



Feng and Smith

Hamaguchi, Kuno, Nakaya, and Nojiri

In this model, every SUSY production event at the LHC contains two heavy stable charged particles !

This could provide a way to do hadron collider experiments at luminosity $\mathcal{L} = 10^{36}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}$.

Write to tape every event with stable heavy particles, ignore the rest, use the computer time to separate the SUSY production from the 1000 underlying pp collisions.

It is clearly time to summarize.

In the previous lecture, I argued that LHC and ILC are very likely to produce dark matter particles in the laboratory.

In this lecture, I have argued that the data from these colliders can in fact make accurate microscopic determinations of the cross sections that are important for dark matter detection.

These determinations could give evidence that the observed particle does in fact make up the dark matter. But they could also be useful to astrophysicists in determining the local density of dark matter and in mapping its distribution in the galaxy.

The results would thus advance our understanding of the universe on both the smallest and the largest distance scales.