Measurements of $|V_{cb}|$, $|V_{ub}|$ and $\phi_{3/\gamma}$

M. Różańska

Institute of Nuclear Physics PAN, Kraków, ul. Radzikowskiego 152, Poland

An overview of experimental status of the CKM matrix elements $V_{cb}$ and $V_{ub}$ is presented. Measurements of the magnitudes of $V_{cb}$ and $V_{ub}$ from inclusive and exclusive B decays and of the phase $\phi_{3/\gamma}$ from charmed B decays are reviewed. Determination of these elements has a strong impact on constraining the unitarity triangle.

1. INTRODUCTION

Over-constraining the Unitarity Triangle (UT) provides stringent tests of the flavor sector of the Standard Model (SM). Currently much experimental and theoretical effort is focused on measurements of the magnitudes of the CKM matrix elements $V_{ub}$ and $V_{cb}$ and their relative phase, $\phi_{3/\gamma}$. The magnitudes of $V_{cb}$ and $V_{ub}$ can be extracted from decays which involve only tree-level diagrams, providing constraints on the apex of UT without pollution by loop induced processes.

2. DETERMINATION OF $|V_{cb}|$ AND $|V_{ub}|$

The primary tools to extract magnitudes of $V_{cb}$ and $V_{ub}$ elements are semi-leptonic $B$ decays which are reasonably well understood theoretically and accessible experimentally. There are two complementary ways to measure $|V_{cb}|$ and $|V_{ub}|$ which use either inclusive or exclusive modes. The theoretical and experimental tools employed in these two approaches are basically independent enabling important cross-checks.

At the parton level the semi-leptonic decay of a free $b$ quark is simply related to the relevant CKM element:

$$\Gamma_b \equiv \Gamma(b \to ql\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} m_b^5 \Phi$$

where $\Phi$ is a phase space factor and $m_b$ denotes the $b$ quark mass.

At the hadron level the above expression has to be modified to take into account the bound-state effects. These corrections can be reliably calculated due to the fact that $m_b$ is large compared to the $\Lambda_{QCD}$ scale that determines low-energy hadronic physics.

2.1. Measurements of $|V_{cb}|$

2.1.1. $|V_{cb}|$ from inclusive decays

The rate for inclusive decay $B \to X_s \ell \bar{\nu}$ can be obtained within Operator Product Expansion (OPE) which provides theoretical framework to calculate systematically corrections to the free quark rate (1). The resulting Heavy Quark Expansion (HQE) [1,2] expresses the total semi-leptonic rate as expansions in powers of $\alpha_s$ and $1/m_b$ which can be written schematically:

$$\Gamma_{incl} = \Gamma_0 \times (1 + A_{\text{EW}} \alpha_s) A_{\text{pert}}(\alpha_s) A_{\text{non-pert}}(m_b, \lambda) \quad (2)$$

$A_{\text{pert}}$ and $A_{\text{non-pert}}$ represent electroweak and QCD perturbative corrections respectively. The non-perturbative QCD part, $A_{\text{non-pert}}$, is expanded in terms of inverse powers of the heavy quark mass $1/m_b$. The non-perturbative effects are contained in coefficients $\lambda$ which represent expectation values of heavy quark operators and describe universal properties of $B$ mesons. The specific implementations depend on the renormalization scale and the chosen renormalization scheme.

The HQE also provides expansions in $\alpha_s$ and $1/m_b$, to calculate moments of lepton energy $E_\ell$ and the invariant mass of the hadronic final state $M_X$ in terms of the non-perturbative parameters $\alpha_s$. Measurements of the inclusive semi-leptonic decay rate and the spectral moments allow a simultaneous extraction of $|V_{cb}|$ together with the non-perturbative heavy quark parameters and quark mass. The parameter set can be overconstrained with large number of measured moments in different portions of phase space providing cross-checks of the procedure.

The $E_\ell$ and $M_X$ moments have been measured by BaBar [3], Belle [4], CLEO [5] and Delphi [6]. Hadronic mass spectrum has been also measured by CDF [7]. The experiments use different experimental techniques aiming at lowering the lepton energy cut-off values in the $B$ meson rest frame to reduce experimental and theoretical uncertainties. BaBar and Belle, using events tagged with fully reconstructed $B$ meson and with a well identified lepton on the signal side, lowered the lepton energy-cutoff down to 0.7 GeV (Belle) and 0.9 GeV (BaBar) for hadronic mass moments and to 0.4 GeV (Belle), 0.6 GeV (BaBar) for lepton energy moments. The large boost of $B$ mesons at LEP allowed Delphi to determine the moments without a cut on the lepton energy.

Additional information on the heavy quark parameters can be obtained from photon spectra in inclusive radiative decays $B \to X_s \gamma$. Smearing of the photon energy (which is monochromatic at the parton level), probes the internal structure of the $B$ meson. Photon energy spectra have been measured by experiments operating at $\Upsilon(4S)$ applying the same techniques as in semileptonic decays [8-11].

---

1 Charge conjugation is implied throughout the paper unless explicitly stated otherwise.
The measurements from semi-leptonic and radiative decays can be combined to extract the $|V_{ub}|$ and heavy quark parameters. A global fit to the data set, including the most recent measurements, has been performed in the kinetic scheme [12] with an excellent overall agreement ($\chi^2$/dof=19.3/44). The resulting $|V_{ub}|$ is determined with $\sim 2\%$ uncertainty [13]:

$$|V_{ub}| = (4.96 \pm 0.23 \pm 0.35 \pm 0.59) \times 10^{-3}$$

The first error in the above expression comes from experimental uncertainties. The second one reflects theoretical uncertainties due to the perturbative QCD and $1/m_b$ series truncation. The third error corresponds to the estimated accuracy of the HQE for the total semi-leptonic rate. The accuracy of $|V_{ub}|$ determination is currently limited by theoretical uncertainties.

The same fit yields

$$m_b = (4.59 \pm 0.04) GeV, \mu^2_q = (0.4 \pm 0.04) GeV^2.$$ 

Precise determination of these parameters is essential in extracting the $|V_{ub}|$ from inclusive semi-leptonic charmless B decays.

2.1.2. $|V_{cb}|$ from exclusive decays

Exclusive semi-leptonic B decays offer another alternative to determine $|V_{cb}|$. Semi-leptonic B decays into the ground state charmed mesons $D$ and $D^*$ are most useful. In exclusive methods, the $|V_{cb}|$ is extracted from differential decay rates in the $w$ variable corresponding to the boost of $D^{(*)}$ meson in the $B$ rest frame:

$$\frac{d\Gamma(B \to D^{(*)} l \bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} \times \frac{1}{(\alpha(m_{\gamma}, m_{\nu}, \mu^2_{\nu}))^2} (F(w))^2.$$  (3)

Here $\alpha$ contains known kinematic factors and the non-perturbative effects are contained in the form factor $F(w)$ which, up to the heavy quark symmetry breaking terms, coincides with the Isgur-Wise function [14]. In the heavy quark limit $F(1)=1$ and lattice QCD can be used to compute effects due to finite quark masses. The calculations in the quenched approximation give for $B \to D^* l \bar{\nu}$ and $B \to D l \bar{\nu}$ the values given above and $F(1)=1.074 \pm 0.018 \pm 0.016$ [16] respectively.

Experimental analyses extract the product $|V_{cb}| F(1)$ by measuring differential decay rate $d\Gamma/dw$ and extrapolating it to the zero recoil limit $w=1$. Using the average values of $|V_{cb}| F(1)$ and the values given above for $F(1)$ one obtains [17]:

$$|V_{cb}| = (40.9 \pm 1.0 \pm 1.0) \times 10^{-3}$$

from $B \to D^* l \bar{\nu}$ and:

$$|V_{cb}| = (39.4 \pm 3.4 \exp{+1.3}) \times 10^{-3}$$

from $B \to D l \bar{\nu}$ modes. The results are consistent but not competitive yet with inclusive measurements. While there is not much room to improve the form factor calculation for $B \to D^* l \bar{\nu}$, it is expected that the theoretical accuracy for $B \to D l \bar{\nu}$ can reach $\sim 1\%$ level [18].

The BaBar collaboration has recently released results of a new analysis of $B \to D^{(*)} e \bar{\nu}$ decay [19]. The values of the slope $\rho^2$ and form factor ratios $R_l$ and $R_2$ (nearly independent of $w$) are determined from a fit to the four-fold differential rate in terms of $w$ and three angles, which completely describe the decay kinematics. The uncertainties of extracted quantities are by factor 5 better than in previous measurements. This new input will reduce the systematic uncertainty in $|V_{ub}|F(1)$ determination.

2.2. Measurements of $|V_{ub}|$

2.2.1. $|V_{ub}|$ from inclusive decays

$|V_{ub}|$ is related to the inclusive \( B \to X_c l \bar{\nu} \) full rate by an expression analogous to (2) with a theory uncertainty of $\sim 5\%$ [20]. In practice, experiments measure only partial decay rates selecting the phase space regions where the background from CKM-favored $B \to X_c e \bar{\nu}$ decays is at acceptable level. The kinematical cuts exploit the mass difference between $u$ and $c$ quark. The commonly used variables are the lepton energy $E_l$, four-momentum transfer to the lepton pair $q^2$, invariant mass of the hadronic system $M_X$ and the light-cone momentum component $P_\perp E_l$. Experimental analyses have to compromise between statistical power of selected data samples and robustness against systematic uncertainties. In the regions where background is highly suppressed the spectra are sensitive to the momentum distribution of the $b$-quark inside the $B$ meson and non-perturbative shape function (SF) of unknown form [21,22] is introduced to resum non-perturbative contributions. This is a severe complication compared to the $|V_{ub}|$ determination, where hadronic effects are contained in a set of numbers. There are two approaches to overcome this problem. The information on SF at the leading order can be extracted from photon spectra in radiative $B \to X_u \gamma$ decays. Alternatively, the dependence on the SF can be reduced by extending the measurements of $B \to X_c e l \bar{\nu}$ into the region where HQE calculations are valid. With the large data samples recorded at B-factories, the better understanding of the charm background allows to extend analyses to the regions with lower signal to background (S/B) ratio but with smaller sensitivity to non-perturbative effects. In the very first analyses, CLEO and ARGUS selected $B \to X_c l \bar{\nu}$ events requiring lepton energy $E_l > 2.3$ GeV, i.e. beyond the kinematical limit of $B \to X_c l \bar{\nu}$ decays. With the better charm background knowledge it became possible to relax this cut down to 2.0 GeV (CLEO[23], BaBar[24]) and 1.9 GeV (Belle[25]).

The earlier mentioned method of studying semi-leptonic B decays in the recoil of a fully reconstructed another $B$ meson, enables reconstruction of kinematical variables $q^2$, $M_X$ and $P_\perp$. These variables, in particular $P_\perp$, provide the best tools to separate
signal from background in phase space regions where SF effects are expected to be small. Such measurements have been performed by BaBar [26] and Belle [27] using data samples consisting of \(210 \times 10^6 \ BB\) and \(275 \times 10^6 \ BB\) respectively. An example is given in Figure 1 which shows background subtracted distributions of \(M_X\), \(q^2\) and first measurement of \(P_s\) by Belle.

\[\text{Figure 1: Background subtracted distributions of } M_X\text{ (left), } q^2\text{ (middle) and } P_s\text{ (right) measured by Belle. Points are the data. Histograms represent the fitted } B \rightarrow X_u l \nu \text{ contribution} [27].\]

In order to extract \(|V_{ub}|\) from measured branching fractions one uses the relation:

\[|V_{ub}| = \sqrt{\frac{\text{AB}(B \rightarrow X_u l \nu)}{\tau_B R_{th} (\Delta \Phi)}}\]  \hspace{1cm} (4)

where \(\tau_B\) is the average lifetime of \(B\) and \(R_{th}\) is predicted decay rate in the selected phase space \(\Delta \Phi\). \(R_{th}\) can be calculated within different theoretical frameworks (e.g. BLNP[28], DGE[29]). A unified determination of \(|V_{ub}|\) from variety of measurements is performed by the HFAG[17]. Results presented in Table 1 are obtained within the BNLP framework using HQE parameters from [13]. The latest average gives:

\[|V_{ub}| = (4.40 \pm 0.20_{\text{exp}} \pm 0.27_{\text{th}}) \times 10^{-3}\]

Theoretical uncertainties in extracting \(|V_{ub}|\) from the above measurements are dominated by shape function effects. Recently BaBar presented results using alternative approach [30], based on a relation between partial decay rate \(\Gamma(B \rightarrow X_u l \nu)\) and the photon energy spectrum in \(B \rightarrow s \gamma\) decays [31,32]:

\[\Delta \Gamma(B \rightarrow X_u l \nu) = \left| \frac{V_{ub}^2}{V_{ub}} \right|^2 \int W(E_\gamma) \frac{d\Gamma(b \rightarrow s \gamma)}{dE_\gamma} dE_\gamma\]  \hspace{1cm} (5)

The integration is performed over an appropriate phase space region. The weight function \(W(E_\gamma)\) is predicted by theory with reasonable accuracy. The BaBar analysis was performed on the data sample selected from \(88 \times 10^6 \ BB\) pairs, using fully reconstructed \(B\) tags. Figure 2 shows the extracted \(|V_{ub}|\) as a function of \(\zeta\), the maximum mass of the hadronic system, up to which the \(\Delta \Gamma(B \rightarrow X_u l \nu)\) is measured. Theoretical uncertainties decrease with increasing \(\zeta\) but at the same time the experimental errors increase due to charm background subtraction. The best sensitivity is achieved at \(\zeta=1.67 \text{ GeV}/c^2\). The \(|V_{ub}|\) value extracted from the spectrum up to \(\zeta=2.5 \text{ GeV}/c^2\), comprising about 96% of the total rate, is also shown.

A summary of \(|V_{ub}|\) results from inclusive methods is given in Table 1. The total uncertainty on \(|V_{ub}|\) from inclusive measurements is 7.5%, dominated by theory. The uncertainty due to limited knowledge of the shape function and \(m_b\) is about 4.5%. Other theoretical uncertainties are at the 5% level.

\[\text{Table 1: } |V_{ub}|\text{ results from inclusive measurements within BNLP framework. } f_{PS}\text{ denotes the fraction of accepted phase space. The variable } s^{\text{max}}\text{ is the maximum kinematically allowed } M_X^2 \text{ at given } E_l\text{ and } q^2.\text{The quoted errors represent experimental (first) and theoretical (second) uncertainties.}\]

FRIPL01
2.2.2. $|V_{ub}|$ from exclusive decays

The aim of the ongoing programme of the exclusive measurements of $|V_{ub}|$ is to achieve a precision comparable to that from inclusive methods. The exclusive determination of $|V_{ub}|$ requires knowledge of the relevant heavy-to-light meson form factors. The simplest are processes where the final state meson is spinless and the mass of the charged lepton can be neglected, since in such case only one form factor is needed. For $B \rightarrow \pi l \bar{\nu}$ the differential decay rate can be expressed as:

$$\frac{d\Gamma(B \rightarrow \pi l \bar{\nu})}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 p_\pi^2 f_+(q^2)$$  \hspace{1cm} (6)

where $p_\pi$ denotes the pion momentum and $f_+(q^2)$ is the form factor.

In $B$ decays to light mesons the application of heavy quark symmetry is much more limited than in the $B \rightarrow D^0\bar{\nu}$ case and the form factors are not normalized at any kinematic point. There are several theoretical approaches to compute them. Lattice QCD calculations provide form factor values for the high $q^2$ region (>16 GeV$^2$/c$^2$). Recently unquenched simulations have became available for $B \rightarrow \pi l \bar{\nu}$ mode [35,36]. Light cone sum rules (LCSR) are used to calculate the form factors in the low momentum transfer region (<14 GeV$^2$/c$^2$) [37].

Experimentally, the exclusive charmless semi-leptonic $B$ decays provide smaller event yields than inclusive modes, however it is partially offset by higher purity of the selected data samples. There are three basic methods of signal extraction employed by experiments operating at $\Upsilon(4S)$.

a) Untagged events, in which the four-momentum of escaping neutrino is inferred from the difference between the four-momentum of the colliding beam particles and the sum of the four-momenta of all detected particles. The major advantage of this method is relatively high efficiency, of the order of few percent.

b) Tagging with semi-leptonic $B$ decays to charm. This methods exploits kinematic constraints on two missing neutrinos in the event. This method provides better background reduction than the previous one, but with substantially reduced efficiency.

c) Tagging with fully reconstructed hadronic $B$ decays. This method provides the highest purity of data samples and the best neutrino four-momentum resolution but at the expense of a very low efficiency (a fraction of a percent).

Measurements of several exclusive semi-leptonic $B$ decays to light mesons are available from CLEO, BaBar and Belle. The final states include $\pi^0, \pi^+, \rho^0, \rho^+$, $\omega$ and $\eta$. The $|V_{ub}|$ determination is presently based on the $B \rightarrow \pi l \bar{\nu}$ mode which is the best controlled both experimentally and theoretically. Summary of the current measurements of the branching fraction for $B \rightarrow \pi l \bar{\nu}$ decays is presented in Table 2. As can be seen, the best experimental precision is presently achieved with the untagged method. The average branching fraction for $B \rightarrow \pi l \bar{\nu}$ quoted by HFAG is $(1.34 \pm 0.08 \pm 0.08) \times 10^{-4}$, with the experimental precision around 8% [17].

Table 2. Measurements of branching fractions of exclusive $B \rightarrow \pi l \bar{\nu}$ decays. The quoted errors are statistical (first), experimental systematic (second), and due to form factor uncertainties (third). The fourth error (if quoted) represents uncertainties due to form factor uncertainties from cross-feed modes. Symbols U, S and F denote untagged, semi-leptonic and full reconstruction tagging methods respectively.

<table>
<thead>
<tr>
<th>Experiment Method</th>
<th>Mode</th>
<th>Branching Fraction [$10^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO [38]</td>
<td>$B^0 \rightarrow \pi l \bar{\nu}$</td>
<td>$1.33\pm0.18\pm0.12\pm0.01\pm0.07$</td>
</tr>
<tr>
<td>BaBar [39]</td>
<td>$B^0 \rightarrow \pi l \bar{\nu}$</td>
<td>$1.38\pm0.10\pm0.16\pm0.08$</td>
</tr>
<tr>
<td>Belle [40]</td>
<td>$B^0 \rightarrow \pi l \bar{\nu}$</td>
<td>$1.38\pm0.19\pm0.14\pm0.03$</td>
</tr>
<tr>
<td>BaBar [40]</td>
<td>$B^+ \rightarrow \pi l \bar{\nu}$</td>
<td>$0.77\pm0.13\pm0.04\pm0.00$</td>
</tr>
<tr>
<td>BaBar [41]</td>
<td>$B^0 \rightarrow \pi l \bar{\nu}$</td>
<td>$1.03\pm0.25\pm0.13$</td>
</tr>
<tr>
<td>BaBar [42]</td>
<td>$B^+ \rightarrow \pi l \bar{\nu}$</td>
<td>$1.80\pm0.37\pm0.23$</td>
</tr>
<tr>
<td>BaBar [43]</td>
<td>$B^0 \rightarrow \pi l \bar{\nu}$</td>
<td>$1.14\pm0.27\pm0.17$</td>
</tr>
<tr>
<td>BaBar [43]</td>
<td>$B^+ \rightarrow \pi l \bar{\nu}$</td>
<td>$0.86\pm0.22\pm0.11$</td>
</tr>
</tbody>
</table>

Most of the experimental analyses provide now branching fractions measured in several $q^2$ intervals. Figure 3 shows the distributions in $q^2$ of $B^0 \rightarrow \pi l \bar{\nu}$ was measured by Belle (left) [40] and BaBar (right)[39] using respectively the semi-leptonic and hadronic decays of the tagging $B$. Theoretical predictions are also shown. The measurements of $q^2$ dependence of the branching fractions are not used yet in $|V_{ub}|$ determination.

![Figure 3: The $q^2$ dependence of the partial branching fraction for $B^0 \rightarrow \pi l \bar{\nu}$ as measured by Belle (left) and BaBar (right)]. The multiple data points in Belle results illustrate the effect of the form factor model on the extracted branching fractions.

In the procedure adopted by HFAG to extract $|V_{ub}|$ from exclusive $B \rightarrow \pi l \bar{\nu}$ measurements, the average branching fractions are evaluated in $q^2$ ranges which are appropriate for applying LCSR or Lattice QCD calculations. The $|V_{ub}|$ is then determined in the $q^2 > 16$
GeV/c^2 range using Lattice QCD calculations of HPOQCD [35] and FNAL [36] collaborations and for \( q^2 < 16 \text{ GeV}^2/c^2 \) using LCSR results [37]. The resulting |\( W_{ab} | values are listed below:

\[
|W_{ab}| = (3.25 \pm 0.17 \text{ GeV}) \times 10^{-3} \quad \text{LCSR} \quad q^2 < 16 \text{ GeV}^2/c^2
\]

\[
|W_{ab}| = (4.44 \pm 0.08 \text{ GeV}) \times 10^{-3} \quad \text{HPQCD} \quad q^2 > 16 \text{ GeV}^2/c^2
\]

\[
|W_{ab}| = (3.76 \pm 0.5 \text{ GeV}) \times 10^{-3} \quad \text{FNAL} \quad q^2 > 16 \text{ GeV}^2/c^2.
\]

The first error is a combination of the experimental, statistical and systematic uncertainties and the second one represents theory uncertainties. The errors on extracted values of |\( q^2 \) are dominated by the theory which contributes >10%. The errors in agreement with the |\( W_{ab} \) extracted from inclusive decays, however the exclusive measurements tend to show slightly lower values. Better knowledge of the \( q^2 \) dependence of the decay rates will reduce in future theory uncertainties in exclusive |\( W_{ab} \) determination.

3. MEASUREMENTS OF \( \phi_3/\gamma \)

The angle \( \phi_3/\gamma = \arg(W_{ab}/V_{cb}/V_{ub}) \) can be measured by studying interference effects between \( b \rightarrow c \) and \( b \rightarrow u \) transitions in \( B \) decays. Such measurements involve only tree level amplitudes, which are unlikely to be affected by non SM contributions.

3.1. Measurements of \( \phi_3/\gamma \) from \( B \rightarrow D K \) modes

Theoretically clean modes to measure angle \( \phi_3/\gamma \) as pointed out first by I. Bigi and A. Sanda [44], are strangeness-changing \( B \) decays to neutral charm mesons \( B \rightarrow D K^0 \), where \( D \) stands for \( D^0 \) or \( \bar{D}^0 \) as well as their excited states. Example diagrams for such decays are shown in Figure 4.

- \( D \rightarrow D^{(*)+} K^0 \)
- \( B \rightarrow D K^0 \)

Figure 4: Feynman diagrams for \( B \rightarrow D^{(*)+} K^0 \)

The decays \( B \rightarrow D^{(*)+} K^0 \) are driven by \( b \rightarrow c \) transition and \( B \rightarrow D^{(*)+} K^0 \) by \( b \rightarrow u \) transition. In the Wolfenstein parametrization of the CKM matrix, the weak parts of the amplitudes that contribute to the decay \( B \rightarrow D K \) are given by \( V_{cb} V_{ub}^\ast \) (for \( D^0 K \) final state) and \( V_{cb} V_{ub}^\ast \) (for \( D^{0} \bar{K} \) final state) and are both of the order \( A_3 \). When \( D^{(*)+} \) and \( D^{(*)-} \) decay to a common final state, the two amplitudes interfere, leading to direct CP violation (CPV):

\[ A_{CP} = \frac{\Gamma(D \rightarrow f^+) \Gamma(B \rightarrow f) - \Gamma(B \rightarrow f) \Gamma(D \rightarrow f^+)}{\Gamma(B \rightarrow f) \Gamma(B \rightarrow f^+)}. \tag{7} \]

As can be seen from (7), the asymmetry \( A_{CP} \) depends on the weak phase \( \phi \) and two additional parameters: the magnitude of the ratio of the interfering amplitudes \( r_2 \) and their relative strong phase \( \delta \). While a number of methods have been proposed to extract \( \phi/\gamma \) together with \( r_2 \) and \( \delta \) from data, all of them present a challenge for experiments. The critical issue is the sensitivity of CPV to the \( r_2 \) value, which is typically around 0.1-0.2 due to the color suppression of the amplitude with the \( b \rightarrow u \) transition. Another problem is small product branching fraction and background from more abundant non-strange modes like \( B \rightarrow D \pi \).

3.1.1. \( \phi_3/\gamma \) from Dalitz plot analysis

The best present determination of \( \phi_3/\gamma \) comes from the method proposed by Giri, Grossman, Soffer and Zupan (GGSZ) [45]. In this method the \( B \rightarrow D^{0/\bar{D}^0} K^\pm \) decay is followed by \( D^{(*)}\) decay to a three-body self-conjugate final state. Studying interference effects in observed Dalitz distributions allows to extract \( \phi_3 \), \( r_2 \) and \( \delta \).

Among the possible \( D^0 \) three-body decay modes, the \( K^0_S \pi^+ \pi^- \) channel has been suggested [45,46] as the most promising one to extract \( \phi_3 \). It combines the relatively large branching fraction and low background level. Large strong phases due to the rich resonant structure can enhance the CPV effects.

Assuming no CPV in \( D^{(*)} \) decay, the Dalitz plot density of \( D^0 \) from \( B^0 \rightarrow D^{(*)} K^\mp \) can be written, as:

\[ d\sigma = \frac{\Gamma(D^0 \rightarrow f K^\mp)}{\Gamma(B^0 \rightarrow f)} \text{d}m_1 \text{d}m_2 \text{d}m_3 \equiv \frac{\Gamma(D^0 \rightarrow f K^\mp)}{\Gamma(B^0 \rightarrow f)} \text{d}m_1 \text{d}m_2 \text{d}m_3 \cos^2(\phi - \phi_3), \tag{9} \]

where \( A_0 \) is the Dalitz plot amplitude of the \( D^0 \rightarrow K^0_S \pi^+ \pi^- \) decay, and \( m_1 \), \( m_2 \), and \( m_3 \) denote the invariant mass of the \( (K^0_S \pi^+) \) and \( (K^0_S \pi^-) \) system respectively. If the \( A_0 \) amplitude is known, the values of \( \phi_3 \), \( r_2 \) and \( \delta \) can be obtained from simultaneous fit to the Dalitz plot densities of \( K^0_S \pi^+ \pi^- \) from \( B^+ \) and \( B^0 \) decays.

Both Belle and BaBar perform Dalitz plot analysis of \( D^0 \rightarrow K^0_S \pi^+ \pi^- \) using \( B^0 \rightarrow D^{(*)} K^\mp \). \( A_D \) is described by a coherent sum of 18 (Belle [47]) or 16 (BaBar [48]) two-body decay amplitudes and one non-resonant component. The models include two-body final states consisting of \( \pi \) and \( K \) resonances (both Cabibbo-allowed and Cabibbo-suppressed) and of \( K^0_S \) and \( \pi^+ \pi^- \) resonances.

The Dalitz distributions of the \( B^+ \) and \( B^0 \) samples are fitted separately, using Cartesian parameters \( x_1 = r_x \cos(\pm \phi + \delta) \) and \( y_2 = r_y \sin(\pm \phi + \delta) \). The indices “+”
and \( \pm \) refer to \( B^+ \) and \( B^0 \) decays, respectively. By way of illustration, Figure 5 shows results of the Belle analysis [47], based on 357 fb\(^{-1}\) data sample. The plots show the constraints on the parameters \( x \) and \( y \) from \( D^0 \) and \( D^+ \bar{K}^0 \) modes.

![Figure 5: Results of Dalitz distributions fits with \( x \) and \( y \) free parameters for \( B^0 \rightarrow D K^\pm \) (left) and \( B^0 \rightarrow D^* K^\pm \) samples. Contours indicate integer multiples of the standard deviation [47].](image)

From the combined analysis of the \( B^0 \rightarrow D K^\pm \), \( D^0 \bar{K}^\pm \) and \( D K^* \) modes Belle obtains:

\[
\phi_3 = 53.3^{+14.8}_{-17.3} \pm 2.5^\circ \pm 8.7^\circ
\]

where the first error is statistical, the second one is the experimental systematic uncertainty and the third is due to \( D \) decay modeling.

The similar analysis performed by BaBar using 227\times10^6 \( B \bar{B} \) sample gives [48]:

\[
\phi_3 = 67.0^\circ \pm 28^\circ \pm 13^\circ \pm 11^\circ.
\]

In both analyses the statistical errors dominate, however, with increasing data samples, the uncertainty introduced by modeling \( A_\rho \) amplitude contributes at comparable level.

### 3.1.2. Results from other methods

There are several alternative approaches to measure \( \phi_3 \) in \( B \rightarrow D K \) decays.

In the GLW [49] method the \( D^0 \) and \( D^* \) decay to a CP eigenstate. The parameters \( \phi_3 \), \( r_\rho \) and \( \delta \) are extracted from charge-averaged partial rates and partial rates asymmetries measured for both CP-even and CP-odd final states. The GLW method is theoretically clean, but has low sensitivity to \( \phi_3 \) because of the low values of \( r_\rho \).

In the ADS [50] method, in order to overcome the smallness of \( r_\rho \), \( D^0 \) decays to flavor specific states are used. The color allowed and color suppressed transition are followed by Cabibbo-suppressed and Cabibbo-allowed \( D \) decay respectively. In this method the two interfering amplitudes are comparable in magnitude, but the effective branching fractions are very low, of the order of \( 10^{-7} \).

The GLW and ADS methods alone are not able yet to provide significant limits on \( \phi_3 \), however they contribute to constraining the upper limit on \( r_\rho \). The recent Belle and BaBar results for these methods can be found in [51-57].

The CKMfitter group, combining all available measurements, finds [58]:

\[
\phi_3 = 62^\circ^{+35^\circ}_{-25^\circ}.
\]

### 3.2. Measurements of \( \sin(2\phi_3+\phi_i) \)

Measurements of the time-dependent decay rates of the type \( B^0 \rightarrow D^{(*)} h^+ \), where \( h \) denotes a light meson (e.g. \( \pi, \rho, a_i \)) provide another, theoretically clean way of extracting \( \phi_3 \) from \( \sin(2\phi_3+\phi_i) \) measurement [59].

As shown in Figure 6, both \( B^0 \) and \( B^\pm \) can decay to \( D^{(*)} h^+ \) (with amplitudes driven by \( b \rightarrow c \) and \( b \rightarrow u \) transitions respectively), and interference between decays with \( (B^0 \rightarrow B^\pm \rightarrow D^{(*)} h^+) \) and without \( (B^0 \rightarrow D^{(*)} h^+) \) mixing occurs.

![Figure 6: Feynman diagrams for \( B^0 \rightarrow D^{(*)} h^+ \) and \( B^\pm \rightarrow D^{(*)} h^+ \) decays driven by \( b \rightarrow c \) and \( b \rightarrow u \) transitions respectively.](image)

The time dependent-decay rates:

\[
P(B^0 \rightarrow D^{(*)} h^+, \Delta t) \propto 1 \pm C \cos(\Delta m \Delta t) - S^\pm \sin(\Delta m \Delta t)
\]

\[
P(B^\pm \rightarrow D^{(*)} h^+, \Delta t) \propto 1 \pm C \cos(\Delta m \Delta t) + S^\pm \sin(\Delta m \Delta t)
\]

where

\[
C = \frac{1- r^2}{1+ r^2} S^\pm = \frac{2(-1)^i \sin(2\phi_3 + \phi_i) \pm \delta}{1+ r^2}
\]

are sensitive to \( \sin(2\phi_3+\phi_i) \delta \) because the phase difference between decay amplitudes is \( \phi_i \pm \delta \). In the above expressions \( r \) denotes the ratio of the magnitudes of the two decay amplitudes and \( \delta \) is their relative strong phase. \( L \) is the orbital momentum of the final state. The value of \( r \) is predicted to be about 0.02. The very small CPV effects in this method are expected to be offset by copious statistics due to the relatively large branching fractions (~0.5%).

The most recent analyses by BaBar [60,61] and Belle [62] of \( B \rightarrow D^{(*)} \pi \) and \( B \rightarrow D^{(*)} \rho \) modes are based on the statistics of 232\times10^6 \( B \bar{B} \) and 380\times10^6 \( B \bar{B} \) respectively.

The Belle results indicate CP violation in \( B^0 \rightarrow D^+ \pi^- \) and \( B^0 \rightarrow D^+ \pi^0 \) decays at 2.5 \( \sigma \) and 2.2 \( \sigma \) levels respectively. The 68% (95%) confidence level lower limits on \( \sin(2\phi_3+\phi_i) \) from the Belle analysis are 0.44 (0.13) from \( D^+ \pi^- \) mode and 0.52 (0.07) from \( D^+ \pi^0 \). The lower limits from combined BaBar measurements are set at 0.64 (0.40) with 68% (90%) confidence level.

With the present accuracy, the measurements of \( \sin(2\phi_3+\phi_i) \) help to exclude the large values of \( \phi_3 \).

### 4. SUMMARY AND OUTLOOK

There is a continuous progress in measurements of the CKM matrix elements and in over-constraining the Unitarity Triangle. The \( |V_{cb},| \) determination from
inclusive decays reached 2% precision with all hadronic inputs simultaneously determined from data. The precision of 7% has been achieved in inclusive measurements of $|V_{ub}|$. The exclusive determination of $|V_{ub}|$ currently has a precision poorer than 10%, however with the steady progress on experimental and theoretical sides, it can provide an alternative to inclusive measurements in a near future. The $\phi/\gamma$ remains the most difficult measurement. A substantial increase of data samples is required to reduce the statistical error and the uncertainties due to the $D$ decay modeling.

The constraints on the Unitarity Triangle from processes mediated by tree-level diagrams are, within errors, consistent with other measurements.

References

[19] B. Aubert et al. (BaBar Collab.), hep-ex/0602023.
[27] B. Aubert et al., (BaBar Collab.), hep-ex/0507001.
[52] B. Aubert et al. (BaBar Collab.), Phys. Rev. D 73, 051105(R) 2005).
[53] B. Aubert et al. (BaBar Collab.), Phys. Rev. D 72, 071103(R) 2005).
[56] B. Aubert et al. (BaBar Collab.), Phys. Rev. D 72, 071104 2005).
[57] K. Abe et al. (Belle Collab.), hep-ex/0508048.
[61] B. Aubert et al. (BaBar Collab.), Phys. Rev. D 73 :11101, 2006