Fully exclusive NNLO QCD computations

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Outline

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- Technology
- Higgs boson production
- W and Z production
- Conclusion

Introduction: challenges

- In about a year, LHC begins its first physics run offering unprecedented opportunities to high-energy physics community.
- Two distinct features: high luminosity and high energy.
- Enormous rates for SM processes:
  - study of the Old Physics will be only limited by systematics;
  - Old Physics has to be understood to disantangle New Physics.
- Different processes require different level of sophistication for their description:
  - generic multi-jet processes – LO, NLO;
  - calibration processes – NLO, NNLO;
  - discovery processes with large pert. corrections– NLO, NNLO.

Introduction: challenges

- LO and NLO computations for hadron colliders are performed at a fully differential level. The same is required from NNLO computations.
- Many attempts to develop NNLO subtraction schemes in the past few years. [Campbell, Glover, Weinzierl, Gehrmann-de Ridder, Gehrmann, Kilgore, Grazzini, Frixione].
- Our method is based on the so-called sector decompozition:
  - automated extraction of IR and collinear singularities;
  - numerical cancellation of divergences;
  - delivers results.
- Available fully differential NNLO QCD results:
  - $gg \to H$; Anastasiou, Petriello, K.M.
  - $pp(\bar{p}) \to W, Z \to l_1 + \bar{l}_2$. Petriello, K.M.
- Realistic NNLO QCD phenomenology for the LHC.

Method: what do we want

- Fully automated, numerical method for extracting and cancelling the infra-red singularities.
  - The NNLO cross-section:

$$\mathrm{d}\sigma_{\mathrm{NNLO}} = \mathrm{d}\sigma_{VV} + \mathrm{d}\sigma_{RV} + \mathrm{d}\sigma_{RR}.$$

  - For each component, obtain an expansion:

$$\mathrm{d}\sigma_{AB} = \sum_{j=0}^{j=4} \frac{M_j^{\mathrm{AB}}}{\epsilon^j},$$

    where M_j^{AB} are ϵ-independent and integrable throughout the phase-space.
  - M_j^{AB} can be computed numerically. Poles in ϵ cancel, when $\mathrm{d}\sigma_{AB}$ are combined.
- The method deals with the differential cross-sections $\Rightarrow$ arbitrary cuts are allowed.

Method: the sketch of the algorithm

- The method applies to VV, RV and RR, with minimal modifications. I focus on RR.
- The algorithm:
  - map the differential phase-space onto the unit hypercube:

$$\int \prod_i \frac{\mathrm{d}^{d-1} p_i}{2p_i^0} \delta^d \left(P_{\mathrm{in}} - \sum p_i \right) .... \Rightarrow \int_0^1 \prod_j \mathrm{d}x_j x_j^{-a_j \epsilon} (1-x_j)^{-b_j \epsilon} ...$$

  - use the "sector decomposition" to disentangle overlapping singularities;

Binoth, Heinrich , Denner, Roth.

  - use "plus"-distribution expansion for book-keeping:

$$\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon}\delta(x) + \left[\frac{1}{x}\right]_+ - a\epsilon \left[\frac{\ln x}{x}\right]_+ + ...$$

- The outcome: all singularities from RR diagrams are extracted without a single integration.

Method: the phase space parameterization

- Convenient phase-space parameterization is crucial for the efficiency.
- Different parameterizations for different classes of diagrams.
- The “energy” parameterization $(z = m_h^2/s_{\rm part})$:

$$N \int_0^1 \{{\rm d}\lambda_i\}[\lambda_1(1-\lambda_1)]^{1-2\epsilon}[\lambda_2(1-\lambda_2)]^{-\epsilon}[\lambda_3(1-\lambda_3)]^{-\epsilon}\times[\lambda_4(1-\lambda_4)]^{-\epsilon-1/2}D^{2-d};$$

$$\begin{aligned} N &= \Omega_{d-2}\Omega_{d-3}(1-z)^{3-4\epsilon}/2^{4+2\epsilon}, \\ D &= 1-(1-z)\lambda_1\left(1-\boldsymbol{n}_1\cdot\boldsymbol{n}_2\right)/2 \; > 0, \\ 1-\boldsymbol{n}_1\cdot\boldsymbol{n}_2 &= 2\left[\lambda_2+\lambda_3-2\lambda_2\lambda_3+2(1-2\lambda_4)\sqrt{\lambda_2(1-\lambda_2)\lambda_3(1-\lambda_3)}\right]. \end{aligned}$$

- Expressions for invariant masses may look complicated; the guiding principle is the simplicity of the singular limits.

$$s_{13} = -(1-z)\lambda_1(1-\lambda_2), \quad s_{23} = -(1-z)\lambda_1\lambda_2,$$
$$s_{34} = (1-z)^2\lambda_1(1-\lambda_1)\left(1-\boldsymbol{n}_1\cdot\boldsymbol{n}_2\right)/2/D,$$

Method: the structure of singularities

- Usually, singularities are classified in terms of their physical origin (infra-red, collinear, UV). For our purposes this is not very relevant.
- Mathematical structure of singularities of the matrix elements is important:
  - factorized singularities: $\frac{1}{\lambda_1 \lambda_2}$;
  - line singularities: $\frac{1}{|\lambda_1 - f(\lambda_i)|}$;
  - entangled singularities: $\frac{1}{(\lambda_1+\lambda_3)(\lambda_1+\lambda_2)}$;
- Factorized singularities are straightforward.
- Line singularities are difficult but can be avoided by a variable transformation:

$$\frac{1}{|\lambda_1 - f(\lambda_i)|} \to \frac{1}{|\lambda_1 - \lambda_2'|}.$$

- Entangled singularities are disentangled by using the sector decomposition.

Method: general comments

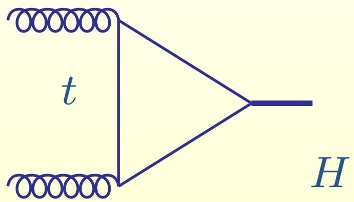
- This is a working method.
- The method is general. The major problem is efficiency. Careful organization of the calculation becomes an important issue.
- Parallel computing.
- The method is "topological":

$$\mathcal{M}^2 \sim \frac{\mathrm{Num}(s_{ij}, F_J(s_{ij}))}{\prod s_{ab}}.$$

  Denominators are sector decomposed, while numerators are treated as arbitrary finite functions $\Rightarrow$ solution valid for all $2 \to 1$ processes.

Higgs production: preliminaries

- $gg \longrightarrow H$ is the dominant Higgs production mechanism at the LHC.



- Large perturbative corrections:
  - $K_{\mathrm{NLO}} \sim 1.7$; Dawson, Djouadi, Spria, Zerwas
  - $K_{\mathrm{NNLO}} \sim 2$

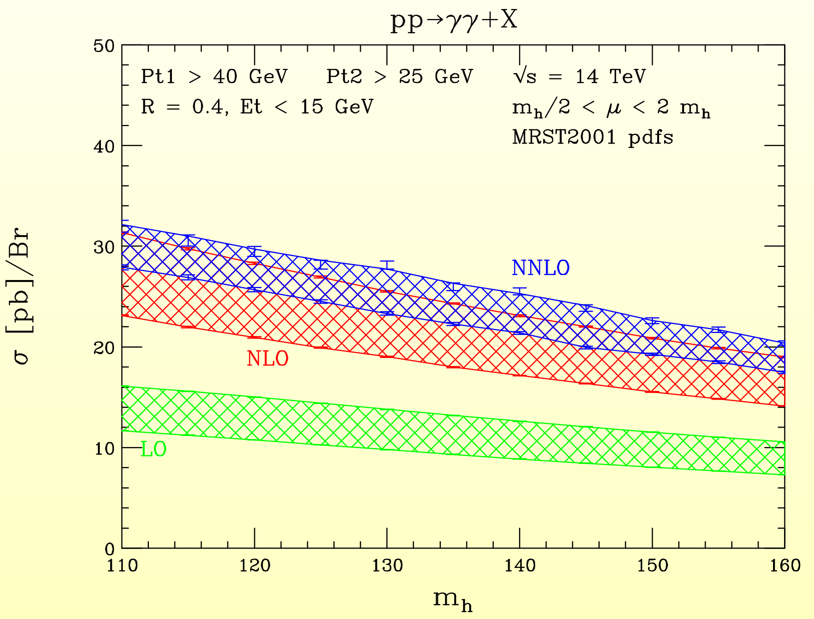
  Harlander and Kilgore
  Anastasiou and Melnikov
  van Neerven, Ravindran, Smith

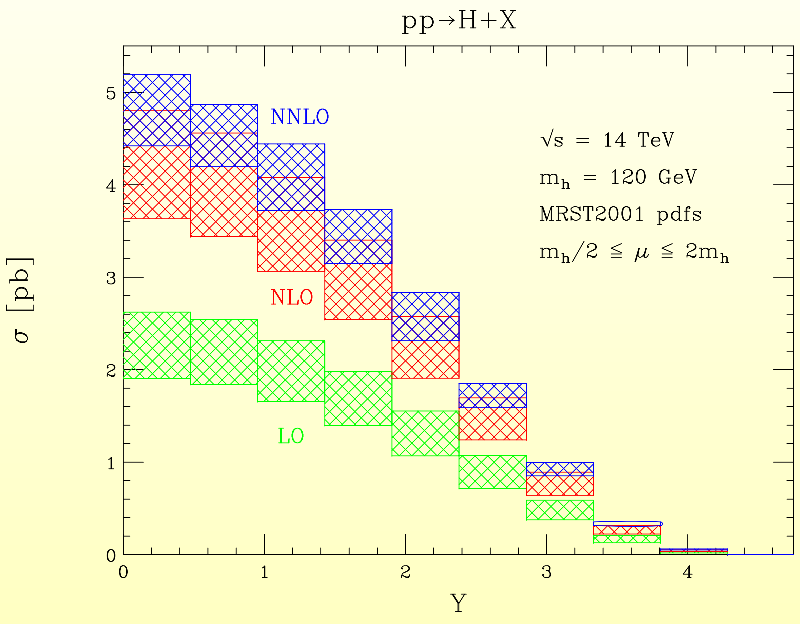
- NNLO result has improved scale stability. NNLO cross-sections match well to "threshold resummed" results Catani, Grazziani, de Florian
- Leading $\mathrm{N}^3\mathrm{LO}$ contribution further stabilizes the cross-section. Moch, Vermaseren, Vogt

Higgs production: preliminaries

- Inclusive Higgs production cross-section is not a realistic observable.
- For $H \to \gamma\gamma$, the following cuts on the final photons are imposed (ATLAS,CMS):
  - $p_\perp^{(1)} \geq 25\ \mathrm{GeV}$, $p_\perp^{(2)} \geq 40\ \mathrm{GeV}$.
  - $|\eta_{1,2}| \leq 2.5$.
  - Isolation cuts, e.g. $E_{\mathrm{T,hadr}} \leq 15\ \mathrm{GeV}$, $\delta R = \sqrt{\delta\eta^2 + \delta\phi^2} < 0.4$.
- For $H \to W^+W^- \to 2l + E_\perp^{\mathrm{miss}}$, shapes of lepton distributions are essential for the discovery.
- Do the conclusions based on inclusive calculations change when those cuts are imposed?
- Since the technology for exclusive NNLO computations exists, we can give a quantitative answer to this question.
- Note: because the Higgs boson is typically produced with $E_h \sim m_h$, and $|p_\perp| \ll m_h$, inclusive calculations should be fairly accurate.

Results: realistic di-photon cross-sections





- $p_\perp^{\gamma,1} > 40$ GeV and $p_\perp^{\gamma,2} > 25$ GeV; $|\eta^{\gamma,1(2)} < 2.5$.
- Isolation cut: $E_\perp^{\mathrm{hadr}} < 15$ GeV for $R < 0.4$.
- NNLO corrections important, but things do look convergent;
- Improved stability w.r.t. scale variations;
- Insignificant kinematic dependence of the K-factor.

Higgs production: decay distributions

- The access to Higgs kinematics can be used to describe Higgs decay products kinematics.
- Fully differential fixed order results are not valid close to kinematic boundaries.
- To extend fixed order computations, we need to combine them with either resummations or shower event generators.
- MC@NNLO is beyond present capabilities.
- Poor man's solution: reweighting shower event generators.

MC event generators: re-weighting to NNLO

- Choose a particular observable $\mathcal{O}$. Require:

$$\sigma(\mathrm{O}) = \sum_i \int \mathrm{d}\Pi_i K(\{p_i\}) w_i^{\mathrm{MC}}(\{p_i\}, \mathrm{O}(\{p_i\})),$$

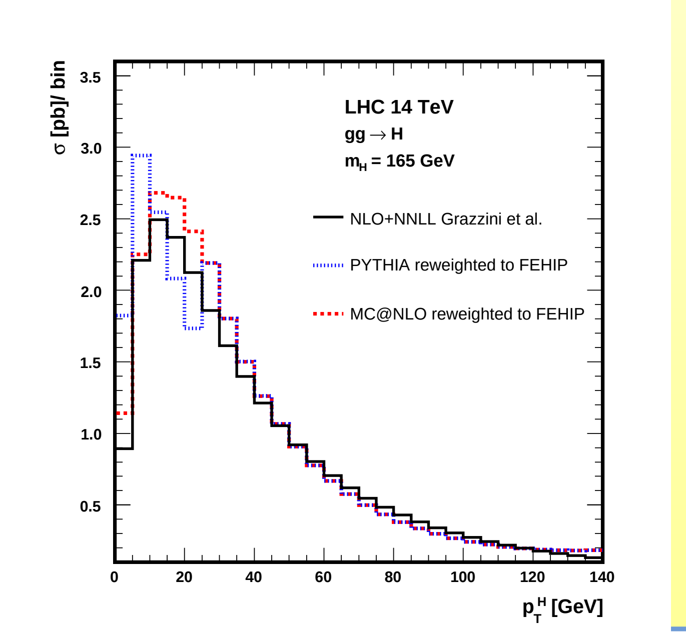
  Use this equation to find the re-weighting coefficients K_i.

- Simplest example: match inclusive cross-sections with a constant K-factor

$$\sigma^{\mathrm{rMC}} = \sigma^{\mathrm{pert}}, \quad \text{but} \quad \mathrm{d}\sigma^{\mathrm{rMC}} \neq \mathrm{d}\sigma^{\mathrm{pert}}.$$

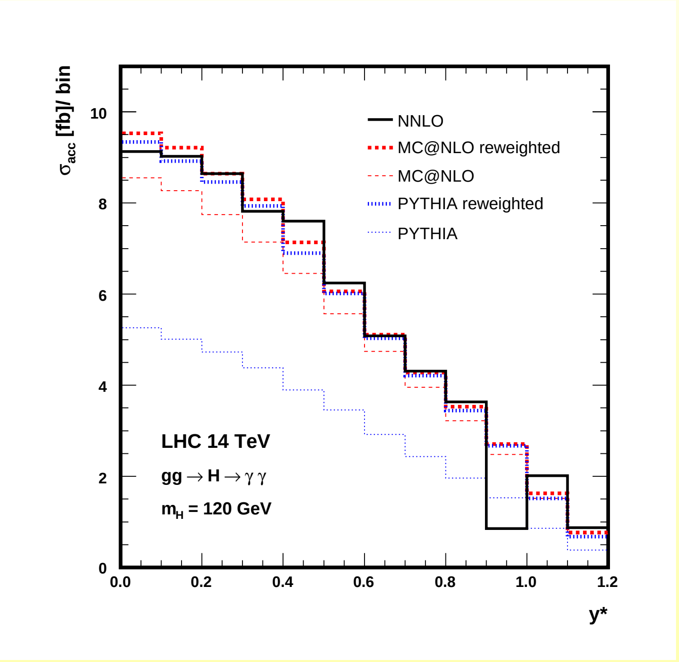
A more complicated example: Re-weighting MC@NLO and PYTHIA to NNLO double differential distribution in Higgs $p_\perp$ and rapidity. [Davatz et al.]

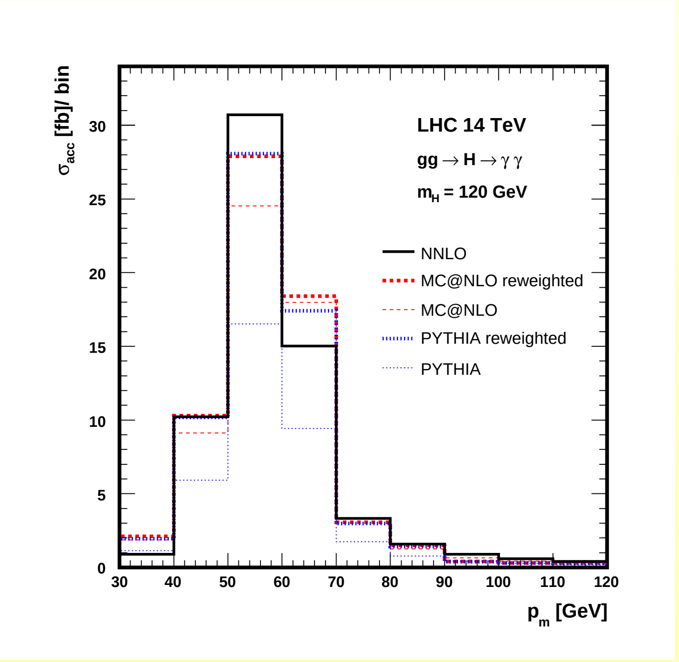
$p_\perp^H \in [0,\ 20]$ GeV: keep the shape from MC; constant K-factor.



Results: di-photon distributions

- Rapidity and $p_\perp$ distributions of the photon can be used as additional discriminators between the signal and the background.

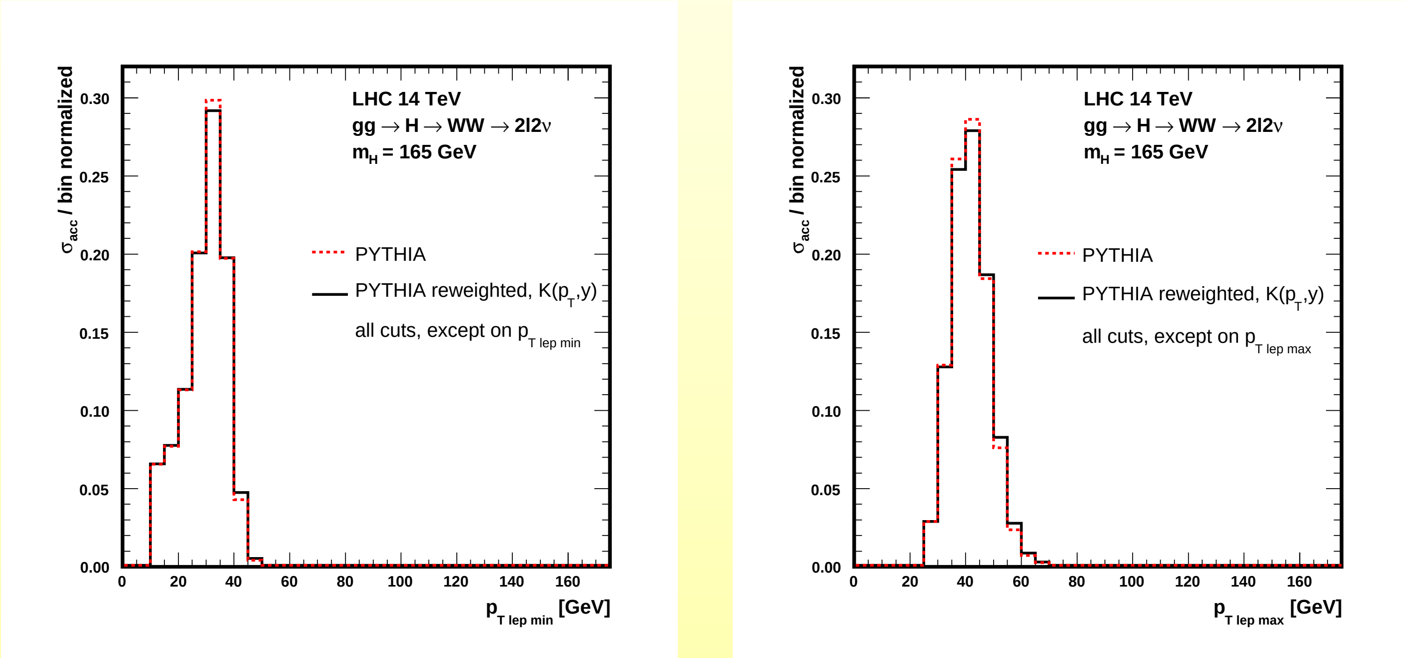




- $Y_s = |\eta^{1,\gamma} - \eta^{2,\gamma}|/2; \quad p_t = (p_\perp^{1,\gamma} + p_\perp^{2,\gamma})/2.$
- Y_s distribution of the background is almost flat Bern, Dixon, Schmidt.
- p_t distribution of the two photons is saturated around $m_h/2 \pm 10$ GeV. The shape is stable against NNLO corrections.

Results: $H \to W^+W^-$

- For $H \to W^+W^- \to 2l + E_\perp^{\rm miss}$ dilepton distributions are important for discriminating against the background.



- No large effects on the shapes from NNLO effects.

Z and W production: preliminaries

- Z and W production processes are very important for the LHC:
  - lepton energy calibration;
  - PDFs and luminosity monitoring;
  - W mass;
  - W width;
  - the Weinberg angle;
  - Z', W'
- Very accurate perturbative predictions exist:
  - the total cross-section;

    van Neerven, Matsuura, Kilgore, Harlander
  - W, Z, γ^* rapidity distribution.

    Anastasiou, Dixon, K.M., Petriello
- For many applications, fully differential NNLO computations for $pp \to (W, Z) \to l_1 + l_2$ are required mainly because access to lepton kinematics with spin correlations is needed.

Z and W rapidity distributions

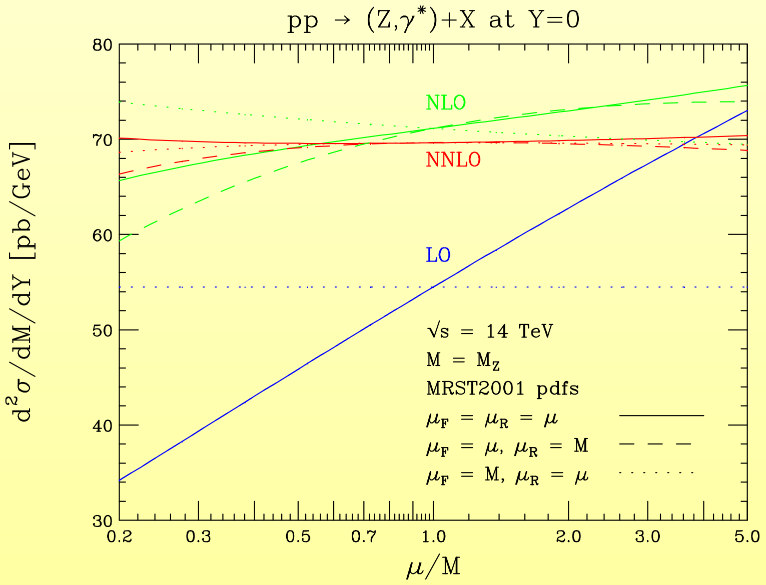
- Use the Z, W production to measure partonic luminosities.

Dittmar et al.

- Partonic luminosities $\leftrightarrow$ rapidity distribution of gauge bosons

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M\mathrm{d}Y} \sim q_1(x_1)q_2(x_2), \quad x_{1,2} = \frac{M}{\sqrt{S}}e^{\pm Y}.$$

- NNLO results: the scale stability and the sensitivity to PDFs.



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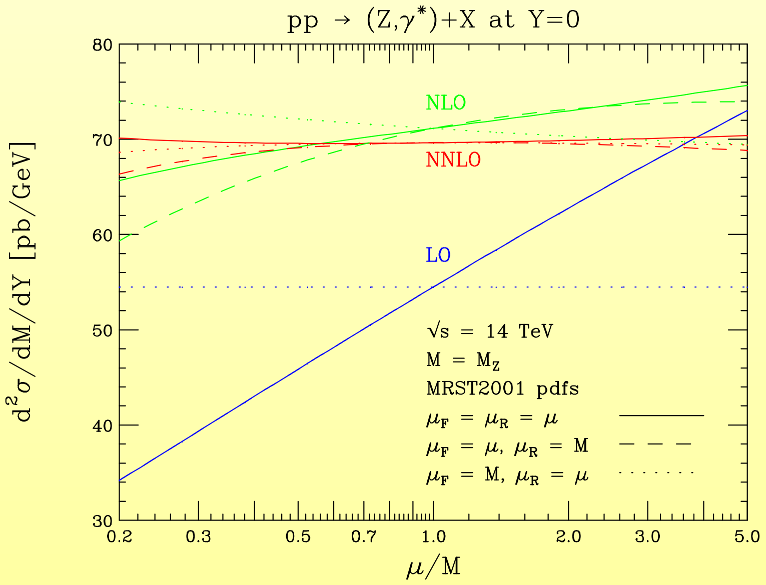

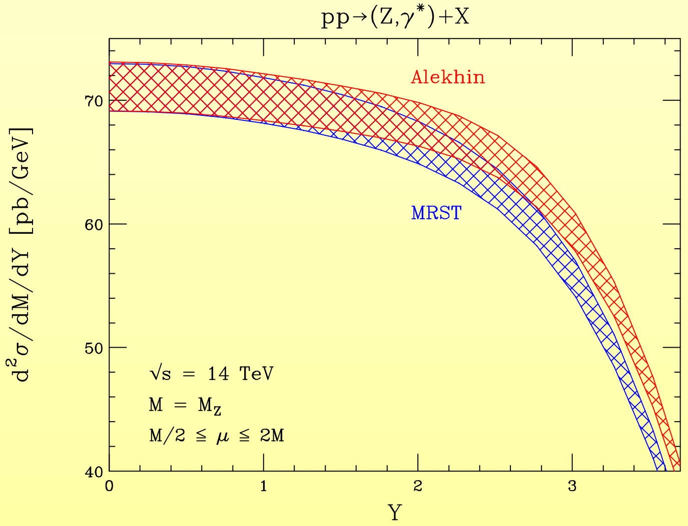

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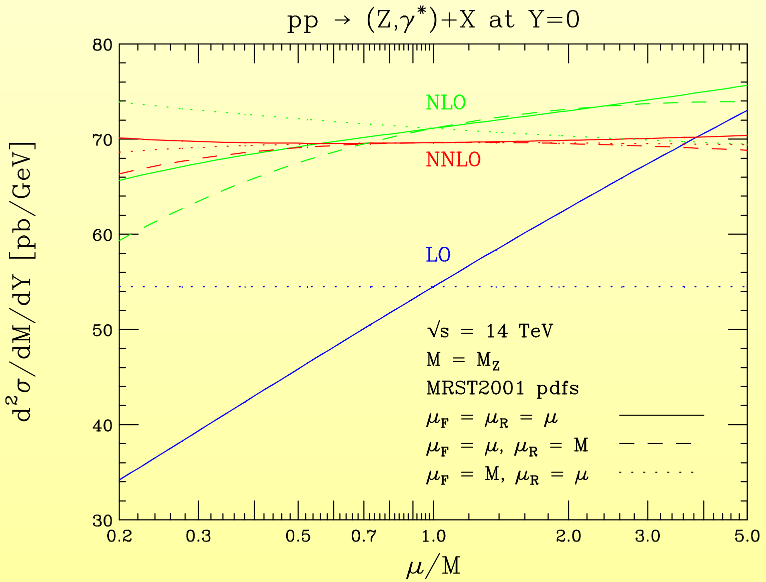

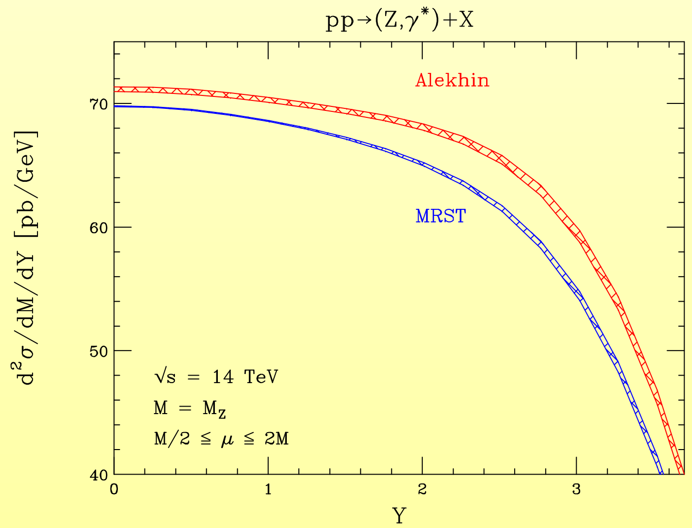

Anastasiou, Dixon, Petriello, K.M.

W^- acceptances

- The knowledge of rapidity distributions of Z, W bosons is insufficient for deriving lepton distributions because of spin correlations.
- The fully differential NNLO QCD calculation for $pp \to e + \bar{\nu} + X$ is now available. Cuts of the form (ATLAS, CMS)

$$\text{Cut1} \quad p_\perp^e > 20 \text{ GeV}, \quad |\eta_e| < 2.5, \quad E_{\text{miss}} > 20 \text{ GeV}$$
$$\text{Cut2} \quad p_\perp^e > 40 \text{ GeV}, \quad |\eta_e| < 2.5, \quad E_{\text{miss}} > 40 \text{ GeV}$$

LHC	A(MC@NLO)	$\frac{\sigma_{\text{MC@NLO}}}{\sigma_{\text{NLO}}}$	A(NNLO)	$\frac{\sigma_{\text{NNLO}}}{\sigma_{\text{NLO}}}$
Cut1	0.485	1.02	0.492	0.983
Cut2	0.133	1.03	0.155	1.21

- $1-2$ percent NNLO effects for $p_\perp^{e,\min} > 20-30$ GeV;
  $10-20$ percent NNLO effects for $p_\perp^{e,\min} > 40-50$ GeV. Petriello, K.M.
- For Cut2, MC@NLO gets the acceptance wrong since second hard emission is important.

Conclusions

- The technology for fully differential NNLO computations for $2 \longrightarrow 1$ processes exists:
  - complete control over the kinematics of the final state;
  - arbitrary cuts;
  - spin correlations.
- Realistic NNLO phenomenology is starting to emerge.
- Further progress requires:
  - extending the method to $2 \longrightarrow 2$ processes (high $p_\perp$ jets; heavy quarks);
  - merging NNLO computations with shower event generators or resummations.