Threshold resummation and finite-order expansions through NNNLO

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- **Soft-gluon corrections**
- **Threshold resummation**
- **NLO and NNLO corrections**
- **NNNLO soft-gluon corrections**
- **Applications to top quark production and W production at large Q_T**

Soft-gluon corrections

Factorized cross section for $h_1 + h_2 \rightarrow F(p) + X$

$$\sigma = \sum_f \int \left[\prod_i dx_i \, \phi_{f/h_i}(x_i, \mu_F^2)\right] \hat{\sigma}(s, t_i, \mu_F, \mu_R)$$

Hard-scattering cross section - perturbatively calculable

Near threshold for production of the system there is restricted phase space for real gluon emission

Incomplete cancellation of infrared divergences between real and virtual graphs $\rightarrow$ large logarithms

Soft and collinear corrections $\longrightarrow$ plus distributions

For $f_1(p_1) + f_2(p_2) \rightarrow F(p) + X$ define $s = (p_1 + p_2)^2$, $t_1 = (p_1 - p)^2$, $t_2 = (p_2 - p)^2$, $s_4 = s + t_1 + t_2 - \sum m^2$ At threshold $s_4 \rightarrow 0$

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/M^2)}{s_4}\right]_+$$

with $l \leq 2n - 1$ for the n-th order corrections

Define moments of the cross section

$$\hat{\sigma}(N) = \int_0^\infty ds_4 \, e^{-N s_4/M^2} \, \hat{\sigma}(s_4)$$

Soft corrections

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/M^2)}{s_4}\right]_+ \rightarrow \frac{(-1)^{l+1}}{l+1} \ln^{l+1} N + \cdots$$

We can formally resum these logarithms to all orders in α_s:

factorize soft gluons from the hard scattering

Invert back to momentum space

Resummation prescriptions to deal with Landau singularity

theoretical ambiguities and differences between prescriptions can be numerically bigger than higher-order terms

Also, kinematics in resummation is often approximately treated in these approaches beyond LL

Alternatively, expand to finite order:

- No prescription necessary
- Kinematics treated fully with no further approximation

At NLO, $\mathcal{D}_1(s_4)$ and $\mathcal{D}_0(s_4)$ terms

LL NLL

At NNLO, $\mathcal{D}_3(s_4)$, $\mathcal{D}_2(s_4)$, $\mathcal{D}_1(s_4)$, and $\mathcal{D}_0(s_4)$ terms

LL NLL NNLL NNNLL

At NNNLO, $\mathcal{D}_5(s_4)$, $\mathcal{D}_4(s_4)$, $\mathcal{D}_3(s_4)$, $\mathcal{D}_2(s_4)$, $\mathcal{D}_1(s_4)$, and $\mathcal{D}_0(s_4)$ terms

LL NLL NNLL ...

N. Kidonakis, hep-ph/0509079, Phys. Rev. D 73, 034001 (2006)

A N^{k}LO-N^{l}LL calculation means that we include the N^{l}LL terms in the α_s^k corrections (e.g. NNLO-NNLL).

Threshold resummation formalism has been applied to:

- Top quark pair hadroproduction
- Beauty and charm production
- Jet production
- Direct photon production
- Large-Q_T W production
- FCNC top production
- Charged Higgs production

Numerical results:

Soft corrections a good approximation of full NLO result

Higher-order corrections are sizable

Dramatic decrease of scale dependence

Threshold resummation

A unified approach

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E^{f_i}(N_i)\right] \exp\left[\sum_j E'^{f_j}(N_j)\right] \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}\left(\alpha_s(\mu)\right)\right] \exp\left[2d_{\alpha_s}\int_{\mu_R}^{\sqrt{s}} \frac{d\mu}{\mu}\beta\left(\alpha_s(\mu)\right)\right]$$
$$\times \mathrm{Tr}\left\{H^{f_i f_j}\left(\alpha_s(\mu_R)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu}{\mu}\Gamma_S^{\dagger\, f_i f_j}\left(\alpha_s(\mu)\right)\right] \tilde{S}^{f_i f_j}\left(\alpha_s(\sqrt{s}/\tilde{N}_j)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu}{\mu}\Gamma_S^{f_i f_j}\left(\alpha_s(\mu)\right)\right]\right\}$$

In the $\overline{\mathrm{MS}}$ scheme

$$E^{f_i}(N_i) = -\int_0^1 dz \frac{z^{N_i-1}-1}{1-z}\left\{\int_{(1-z)^2}^1 \frac{d\lambda}{\lambda} A_i\left(\alpha_s(\lambda s)\right) + \nu_i\left[\alpha_s\left((1-z)^2 s\right)\right]\right\}$$

with $A_i(\alpha_s) = C_i\left[\alpha_s/\pi + (\alpha_s/\pi)^2 K/2\right] + A_i^{(3)} + \cdots$ and $\nu_i = (\alpha_s/\pi)C_i + (\alpha_s/\pi)^2\nu_i^{(2)} + (\alpha_s/\pi)^3\nu_i^{(3)} + \cdots$
and (for any massless final-state partons at lowest order)

$$E'^{f_j}(N_j) = \int_0^1 dz \frac{z^{N_j-1}-1}{1-z}\left\{\int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_j\left(\alpha_s(\lambda s)\right) - B_j\left[\alpha_s\left((1-z)s\right)\right] - \nu_j\left[\alpha_s\left((1-z)^2 s\right)\right]\right\}$$

where $B_j = (\alpha_s/\pi)B_j^{(1)} + (\alpha_s/\pi)^2 B_j^{(2)} + (\alpha_s/\pi)^3 B_j^{(3)}$ with $B_q^{(1)} = 3C_F/4$ and $B_g^{(1)} = \beta_0/4$
H are hard scattering matrices; S are soft matrices (noncollinear soft-gluon emission);
Γ_S are soft anomalous dimension matrices

NLO master formula

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R^2)}{\pi} \{c_3 \, \mathcal{D}_1(s_4) + c_2 \, \mathcal{D}_0(s_4) + c_1 \, \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c \, \mathcal{D}_0(s_4) + T_1^c \, \delta(s_4)]$$

Here $c_3 = \sum_i 2\, C_i - \sum_j C_j$ **For quarks** $C_F = (N_c^2 - 1)/(2N_c)$ **For gluons** $C_A = N_c$

and $c_2 = c_2^{\mu} + T_2$ **with** $c_2^{\mu} = -\sum_i C_i \ln\left(\mu_F^2/M^2\right)$ **and**

$$T_2 = -\sum_i \left[C_i + 2\, C_i \, \ln\left(\frac{-t_i}{M^2}\right) + C_i \, \ln\left(\frac{M^2}{s}\right)\right] - \sum_j \left[B_j^{(1)} + C_j + C_j \, \ln\left(\frac{M^2}{s}\right)\right]$$

For quarks $B_q^{(1)} = \gamma_q^{(1)} = 3C_F/4$ **For gluons** $B_g^{(1)} = \gamma_g^{(1)} = \beta_0/4$

Also $c_1 = c_1^{\mu} + T_1$**, with**

$$c_1^{\mu} = \sum_i \left[C_i \, \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right)$$

Matrix term $A^c = \mathrm{tr}\left(H^{(0)} {\Gamma'}_S^{(1)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} {\Gamma'}_S^{(1)}\right)$

NNLO master formula

$$\hat{\sigma}^{(2)} = \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \frac{1}{2} c_3^2 \mathcal{D}_3(s_4) + \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j C_j \frac{\beta_0}{8} \right\} \mathcal{D}_2(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \frac{3}{2} c_3 A^c \mathcal{D}_2(s_4)$$

$$+ \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} C_{D_1}^{(2)} \mathcal{D}_1(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \left\{ \left(2 c_2 - \frac{\beta_0}{2}\right) A^c + c_3 T_1^c + F^c \right\} \mathcal{D}_1(s_4)$$

$$+ \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} C_{D_0}^{(2)} \mathcal{D}_0(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \left\{ \left[c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right) + \frac{\beta_0}{4} \ln\left(\frac{M^2}{s}\right) \right] A^c \right.$$

$$\left. + \left(c_2 - \frac{\beta_0}{2}\right) T_1^c + F^c \ln\left(\frac{M^2}{s}\right) + G^c \right\} \mathcal{D}_0(s_4)$$

Here $C_{D_1}^{(2)} = c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + c_3 \frac{K}{2} - \sum_j \frac{\beta_0}{4} B_j^{(1)}$

$$C_{D_0}^{(2)} = c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 - \frac{\beta_0}{2} T_1 + \frac{\beta_0}{4} c_2 \ln\left(\frac{\mu_R^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0^2}{8} \ln\left(\frac{M^2}{s}\right) - \sum_i \nu_i^{(2)} - \frac{\beta_0}{2} \sum_i \gamma_i^{(1)} \ln\left(\frac{M^2}{s}\right)$$

$$+ \sum_i C_i \frac{\beta_0}{8} \left[\ln^2\left(\frac{\mu_F^2}{M^2}\right) - \ln^2\left(\frac{M^2}{s}\right) - 2 \ln\left(\frac{M^2}{s}\right) \right] - \sum_i C_i \frac{K}{2} \left[\ln\left(\frac{\mu_F^2}{M^2}\right) + 2 \ln\left(\frac{-t_i}{M^2}\right) + \ln\left(\frac{M^2}{s}\right) \right]$$

$$- \sum_j \left(B_j^{(2)} + \nu_j^{(2)}\right) + \sum_j C_j \left[-\frac{\beta_0}{8} \ln^2\left(\frac{M^2}{s}\right) - \frac{\beta_0}{4} \ln\left(\frac{M^2}{s}\right) - \frac{K}{2} \ln\left(\frac{M^2}{s}\right) \right] - \sum_j \frac{\beta_0}{2} B_j^{(1)} \ln\left(\frac{M^2}{s}\right)$$

$$F^c = \mathrm{tr}\left[H^{(0)} \left(\Gamma_S^{(1)\dagger}\right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)}\right)^2 + 2 H^{(0)} \Gamma_S^{(1)\dagger} S^{(0)} \Gamma_S^{(1)} \right]$$

$$G^c = \mathrm{tr}\left[H^{(1)} \Gamma_S^{(1)\dagger} S^{(0)} + H^{(1)} S^{(0)} \Gamma_S^{(1)} + H^{(0)} \Gamma_S^{(1)\dagger} S^{(1)} + H^{(0)} S^{(1)} \Gamma_S^{(1)} + H^{(0)} \Gamma_S^{(2)\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(2)} \right]$$

NNNLO master formula

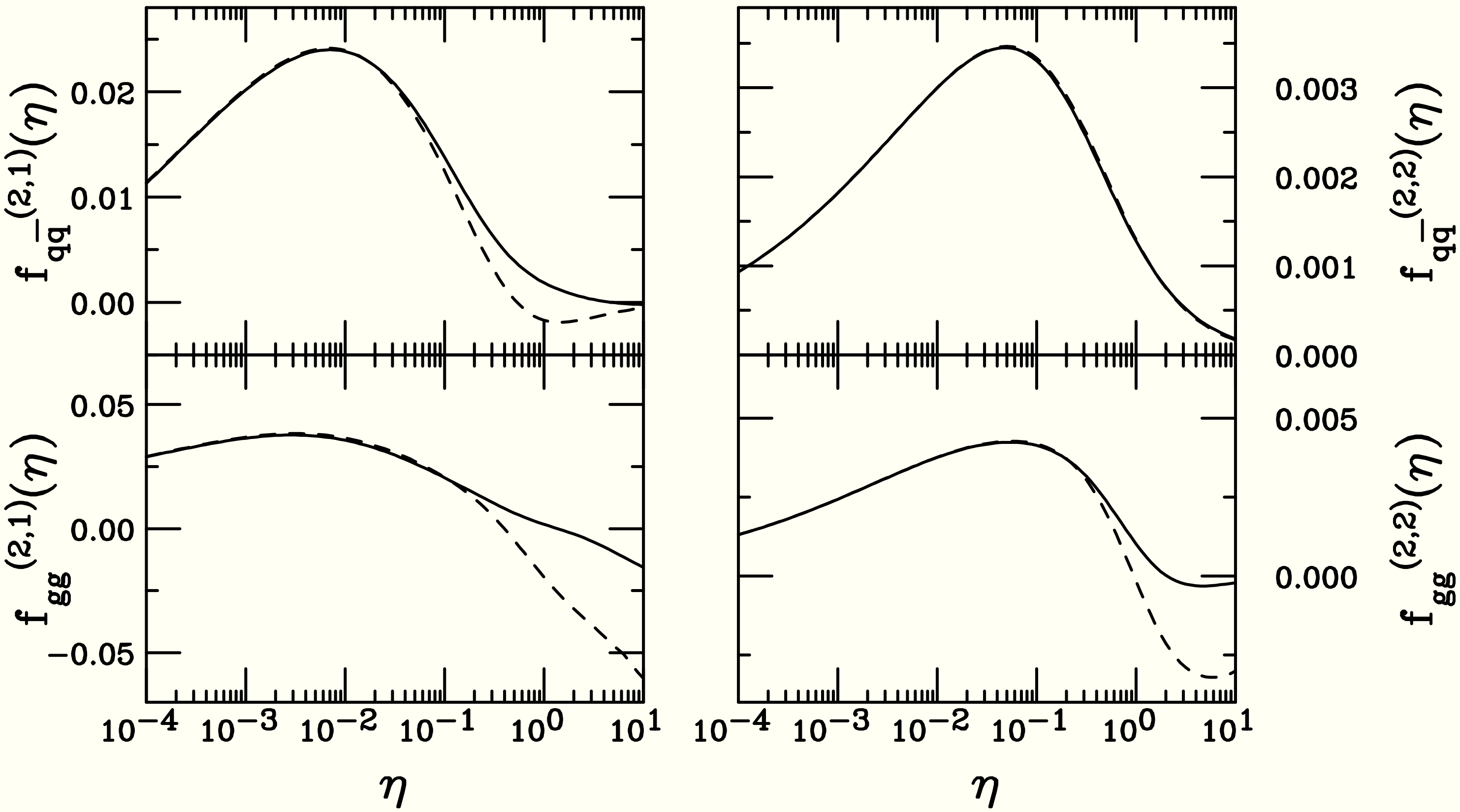
$$
\begin{aligned}
\hat{\sigma}^{(3)} = \; & \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \, \frac{1}{8} \, c_3^3 \, \mathcal{D}_5(s_4) + \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \, \left\{ \frac{5}{8} \, c_3^2 \, c_2 - \frac{5}{2} \, c_3 \, X_3 \right\} \mathcal{D}_4(s_4) + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \, \frac{5}{8} \, c_3^2 \, A^c \, \mathcal{D}_4(s_4) \\
& + \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ c_3 \, c_2^2 + \frac{c_3^2}{2} \, c_1 - \zeta_2 \, c_3^3 + (\beta_0 - 4c_2) \, X_3 + 2c_3 \, X_2 - \sum_j C_j \, \frac{\beta_0^2}{48} \right\} \mathcal{D}_3(s_4) \\
& + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \, \left\{ \frac{1}{2} \, c_3^2 \, T_1^c + \left[2 \, c_3 \, c_2 - \frac{\beta_0}{2} \, c_3 - 4 \, X_3 \right] \, A^c + c_3 \, F^c \right\} \, \mathcal{D}_3(s_4) \\
& + \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \, \left\{ \frac{3}{2} \, c_3 \, c_2 \, c_1 + \frac{1}{2} \, c_2^3 - 3 \, \zeta_2 \, c_3^2 \, c_2 + \frac{5}{2} \, \zeta_3 \, c_3^3 + \left(-3 \, c_1 + \frac{27}{2} \, \zeta_2 \, c_3 \right) \, X_3 + (3 \, c_2 - \beta_0) \, X_2 - \frac{3}{2} \, c_3 \, X_1 \right. \\
& \qquad \left. - \sum_i C_i \, \frac{\beta_1}{8} + \sum_j \frac{\beta_0^2}{16} \, B'^{(1)}_j + \sum_j \frac{3}{32} \, C_j \, \beta_1 + \sum_j C_j \, \frac{\beta_0}{16} \, \left[\beta_0 \, \ln\left(\frac{\mu_R^2}{M^2} \right) + 2 \, K \right] \right\} \, \mathcal{D}_2(s_4) \\
& + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \, \left\{ \left(\frac{3}{2} \, c_3 \, c_2 - 3 \, X_3 \right) \, T_1^c + \frac{3}{2} \, \left[c_2 + c_3 \, \ln\left(\frac{M^2}{s} \right) \right] \, F^c + \frac{3}{2} \, c_3 \, G^c + \frac{1}{2} \, K_3^c \right. \\
& \qquad \left. + \left[\frac{3}{2} \, c_2^2 + \frac{3}{2} \, c_3 \, c_1 - 3 \, \zeta_2 \, c_3^2 + 3 \, X_2 + \frac{\beta_0^2}{4} - \frac{3}{4} \, \beta_0 \, \left(c_2 - \frac{c_3}{2} \, \ln\left(\frac{\mu_R^2}{M^2} \right) \right) - \frac{3\beta_0}{8} \, c_3 \, \ln\left(\frac{M^2}{s} \right) \right] \, A^c \right\} \, \mathcal{D}_2(s_4) \\
& + \cdots
\end{aligned}
$$

Here $X_3 = (\beta_0/12) c_3 - \sum_j C_j \beta_0/24$ $\quad$ $X_2 = -(\beta_0/4) T_2 + (\beta_0/8) c_3 \ln(\mu_R^2/M^2) + c_3 K/4 - \sum_j (\beta_0/8) B_j^{(1)}$

$X_1 = c_2 \, c_1 - \zeta_2 \, c_3 \, c_2 + \zeta_3 \, c_3^2 + (\beta_0/4) \, \zeta_2 \, c_3 - \sum_j C_j (\beta_0/8) \zeta_2 - C_{D_0}^{(2)}$

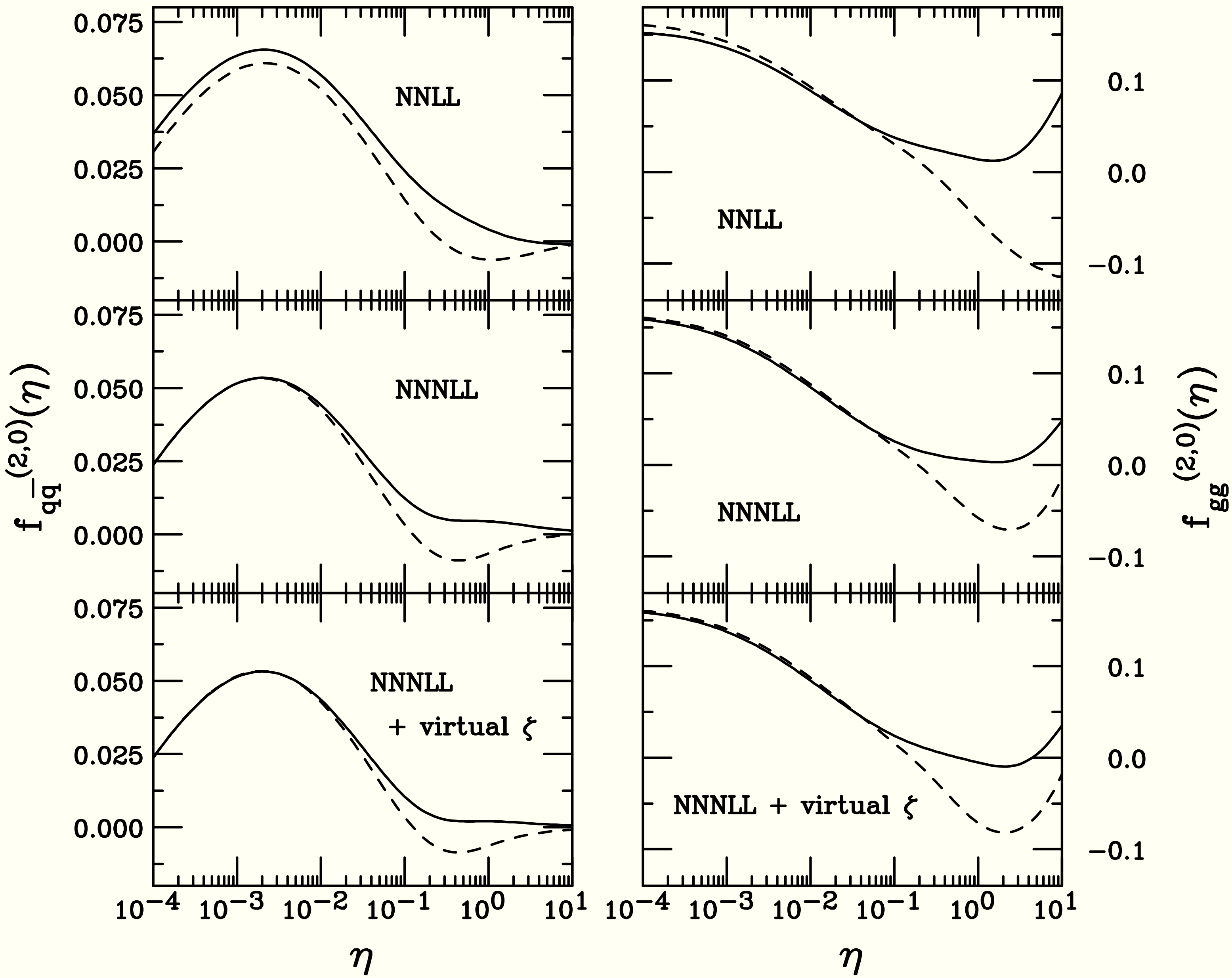
$$
K_3^c = \mathrm{tr} \left[H^{(0)} \left(\Gamma_S^{(1)\dagger} \right)^3 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)} \right)^3 + 3 \, H^{(0)} \left(\Gamma_S^{(1)\dagger} \right)^2 S^{(0)} \Gamma_S^{(1)} + 3 \, H^{(0)} \Gamma_S^{(1)\dagger} S^{(0)} \left(\Gamma_S^{(1)} \right)^2 \right]
$$

NNLO soft-gluon corrections for top quark production



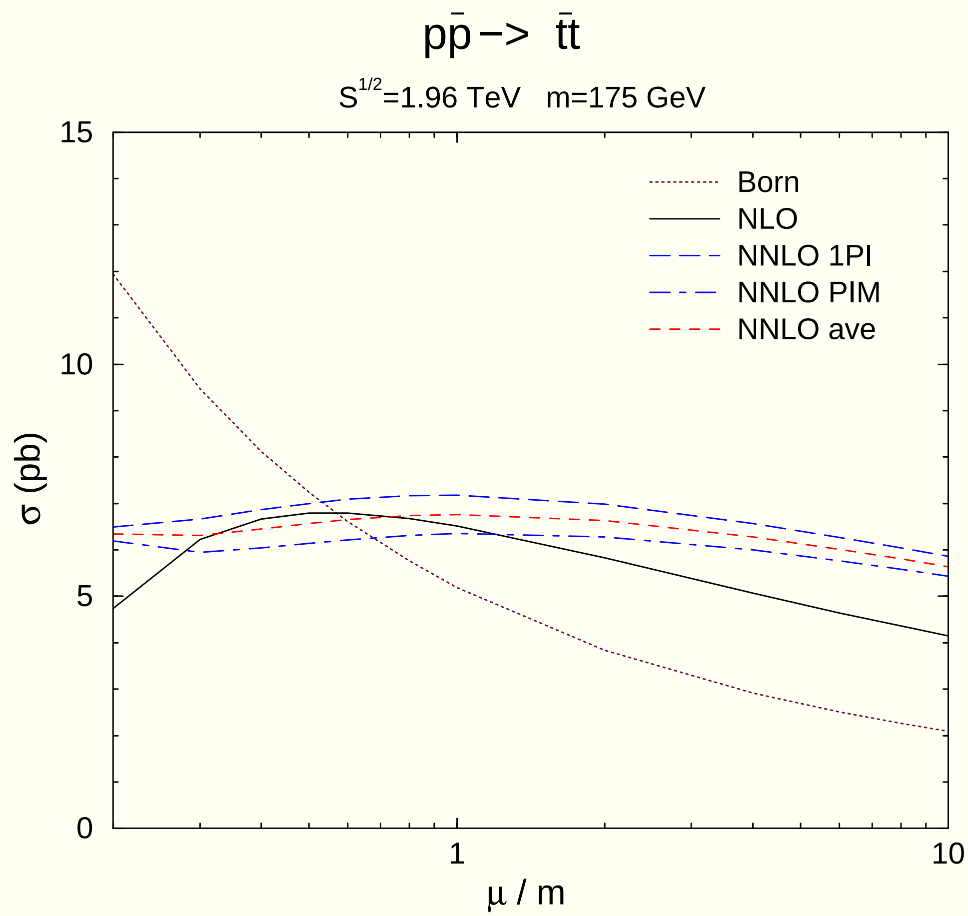
Scaling functions $f(\eta)$: 1PI (solid), PIM (dashed) $\quad \eta = s/4m^2 - 1$

$$\sigma_{ij}(s,m^2,\mu^2) = \frac{\alpha_s^2(\mu)}{m^2} \sum_{k=0}^{\infty} \left(4\pi\alpha_s(\mu)\right)^k \sum_{r=0}^{k} f_{ij}^{(k,r)}(\eta) \ \ln^r\left(\frac{\mu^2}{m^2}\right)$$



1PI (solid), PIM (dashed) Approximate NNNLL corrections work well

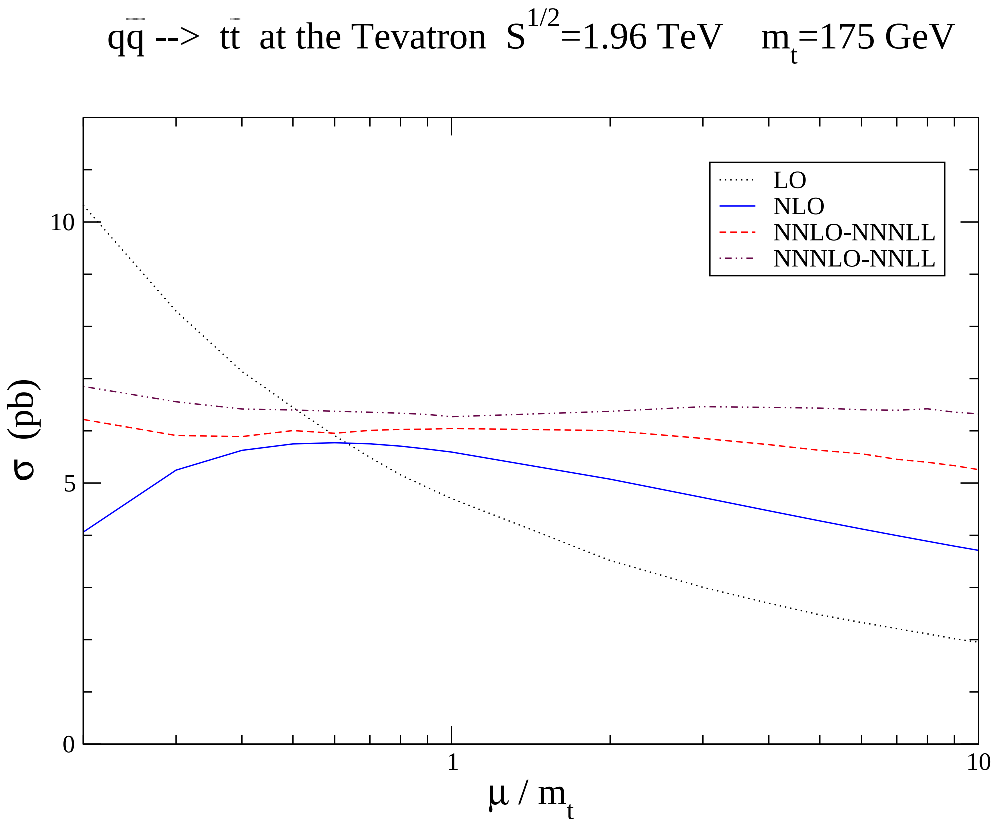
Top quark cross section at the Tevatron



Reduced scale dependence when soft-gluon corrections are added

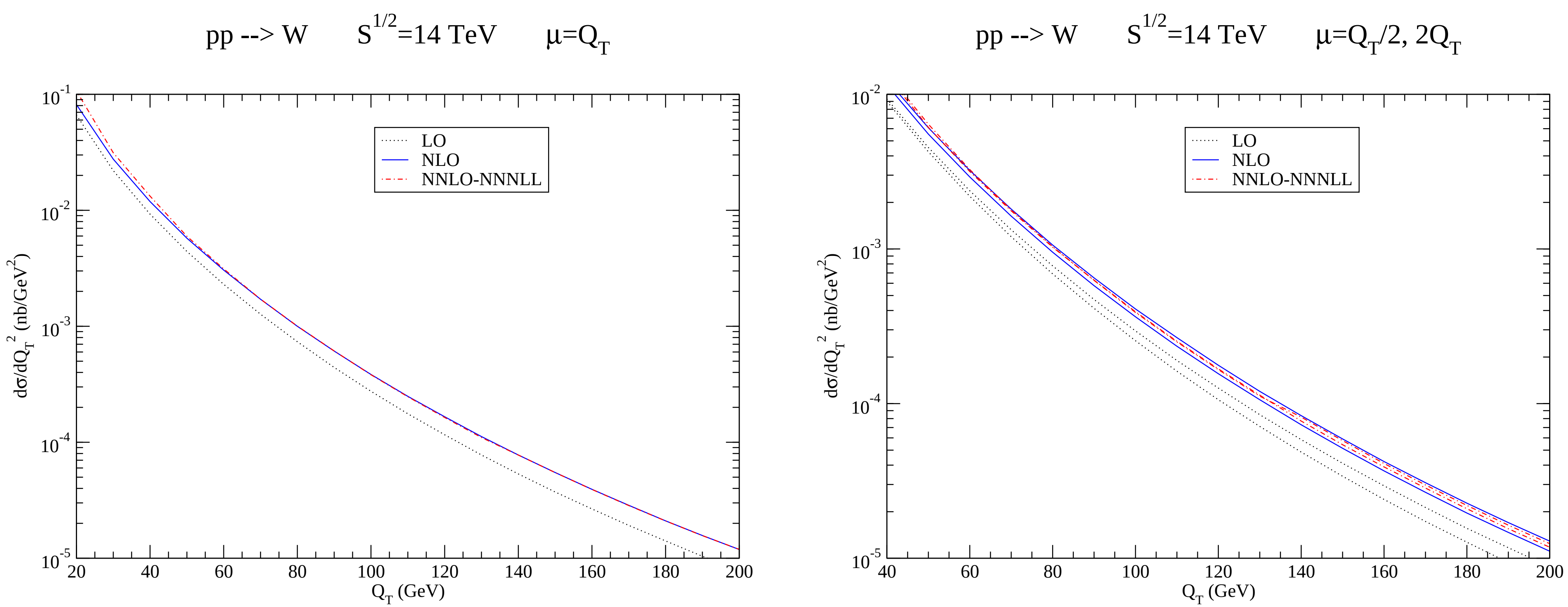
N. Kidonakis and R. Vogt, hep-ph/0308222, Phys. Rev. D 68, 114014 (2003)

NNNLO corrections for top quark production in the $q\bar{q}$ channel at the Tevatron in 1PI kinematics



Higher-order corrections greatly reduce the scale dependence

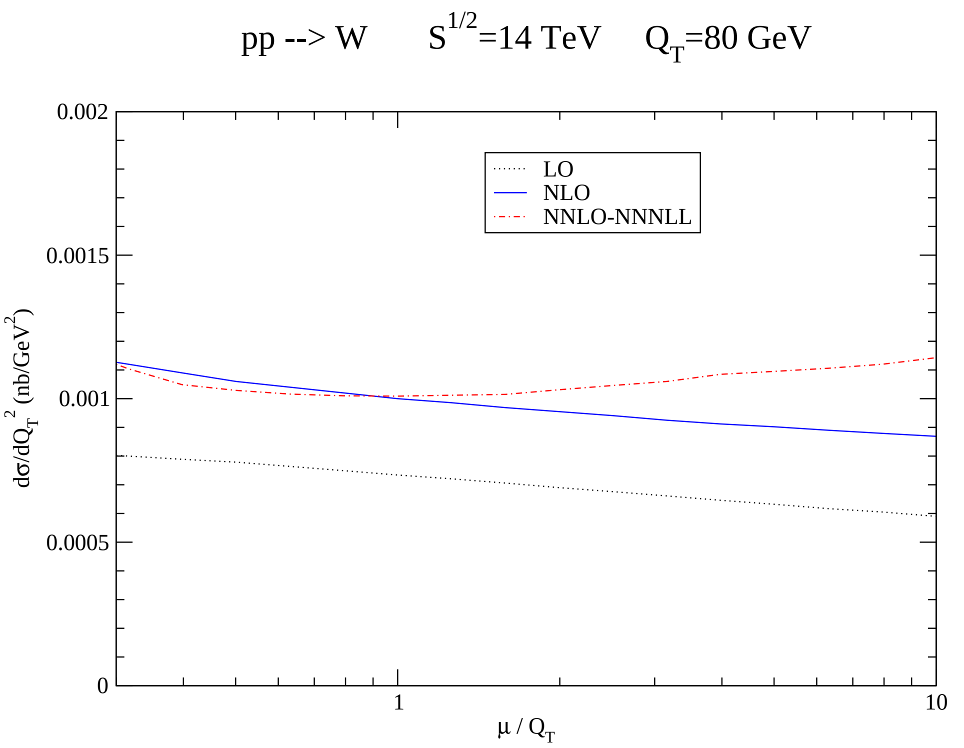
W production with large Q_T at the LHC

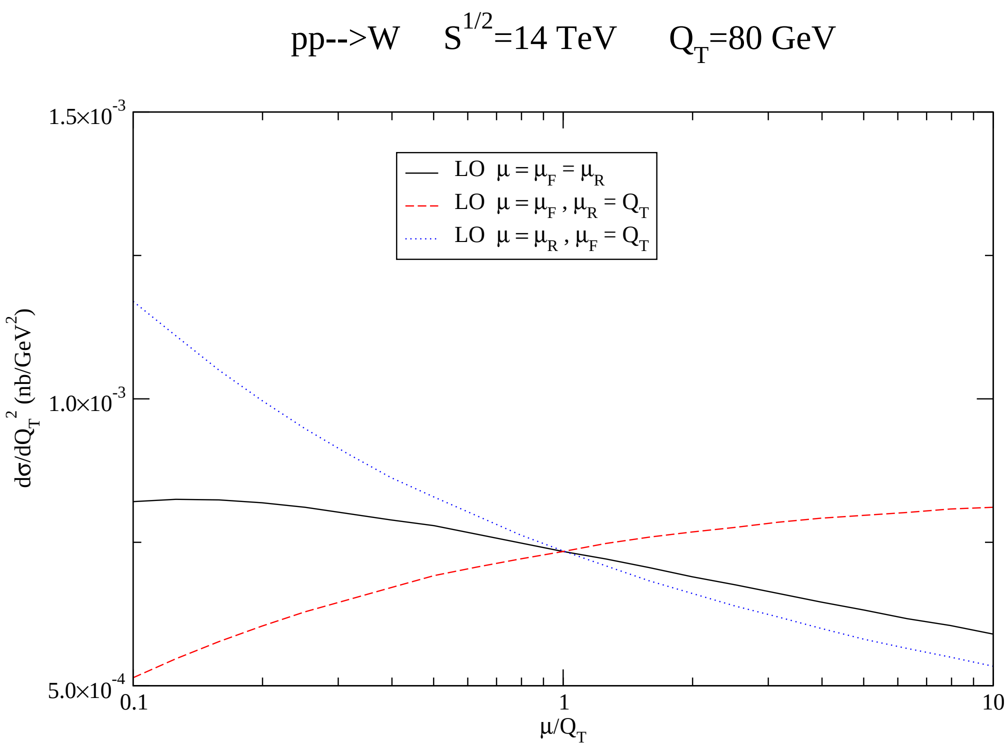


R.J. Gonsalves, N. Kidonakis, A. Sabio Vera, hep-ph/0507317, Phys. Rev. Lett. 95, 222001 (2005)

NLO corrections are large; NNLO soft-gluon corrections small for $\mu = Q_T$

W production with large Q_T at the LHC





At LO μ_F and μ_R dependence largely cancel each other

gluon-initiated process $qg \to Wq$ dominant

Summary

- **Soft-gluon resummation**
- **Soft-gluon threshold corrections are sizable**
- **NLO, NNLO, and NNNLO soft-gluon expansions**
- **Full treatment of kinematics dependence**
- **Important for greater theoretical accuracy**
- **Significant contributions to K-factors**
- **Dramatic decrease of the scale dependence**