Scott Thomas

Lecture 2

Softly Broken
Supersymmetric Standard Model

and

Indications for Supersymmetry

- $m_B \neq m_F$ So SUSY is Broken (Spontaneously) in Nature

- What Form for SUSY Expect?

1. ϕ Partners of ψ_R, ψ_L $m \gtrsim 100$ GeV.

   λ^a " $\gamma, W^\pm, Z, g$ "

   $\frac{1}{2} m^2 \phi^* \phi$ (Doesn't Require EWSB)

   $\frac{1}{2} m \lambda\lambda$ (- - -)

So Superpartner masses Require SUSY but not EWSB - In Contrast to ψ's, $\underline{A^\mu\text{'s}}$.

↳ Good in $M_{SUSY} \gtrsim M_W$ That's why Haven't Seen Any yet

And No Analog of χ-Sym.
(At least for Scalars) - Only SUSY Protects.

— Possible Skip —

2. SUSY Couplings Should not be Modified – otherwise Reintroduce Divergences:

Couplings must be Related

{ Say Succinctly: in Context of Asym Free Renormalizable Theory

(Required to Cancel ∞'s)

SUSY Theory – Counter Terms for Renormalizable Couplings

Theory with Spontaneous SUSY: Should be No New (SUSY) Counter Terms for Renormalizable Couplings.

↳ otherwise Loose SUSY Relations Among Couplings, (Quad). Divergences introduced, becomes Non-SUSY Theory

↳ How Possible?

L Must be SUSY terms which Don't Require Counter Terms:

Then UV Structure of Theory (SUSY) Unaffected

Terminology:

Super Renormalizable Op	- No Counter Terms	Relevant Op.
Renormalizable Op	- Finite # of Counter Terms	Marginal Op.
Non-Renormalizable Op	- ∞ # of Counter Terms	Irrelevant Op.

Why Don't Relevant Op Require Counter Terms?

Scale Transformations $x \to e^{-\sigma} x$ $\quad \sigma \to \begin{matrix} +\infty & \text{IR} \\ -\infty & \text{UV} \end{matrix}$

by Def $\mathcal{O} \to e^{D\sigma} \mathcal{O}$ $\quad$ D = Dimension

$$\int d^4y\ \partial_\mu\phi\ \partial_\mu\phi$$

σ: -4 $\quad +1$ $\quad 1$ $\quad +1$ $\quad 1$ $\qquad D[\phi]=1$

Look at $\int d^4x\, \frac{1}{2} m^2 \phi^2$ $\qquad D[m^2]=2$

-4 $\quad 2$ $\quad 2$

└ More important in IR (Relevant)

(Irrelevant in W)

But Relevant Op Less important in W

Since Irrelevant in W Doesn't (Need Counter Term)

More Terminology: Form of Sym

Soft Breaking Relevant Ops. Viol. Sym

Hard Breaking Marginal or Irrelevant Op. Viol. Sym

→ Destroys W Sym (Effectively not There)

So we expect to See Soft SUSY in Low Energy Theory if SUSY Spontaneous.

With Soft SUSY Breaking

Since (No SUSY Counter Terms) Mass Scale of Soft SUSY is Stable Against Q-Corrections!

$m^2 \phi^\dagger \phi$ Does not Recieve Div. Corrections!

↳ Stable! ...

c.f. Standard model $m^2 \phi^\dagger \phi$ Does Receive Div. Corrections

↳ Not Stable! ...

Already Saw Explicitly: Calculate Corrections to m_H^2
Above m_ϕ^2 Scale SUSY
Cancels Divergences! --->

** So Soft SUSY Masses are Protected by SUSY

When a Lagrangian Parameter is Protected
by a Symmetry which is Restored when
Parameter Vanishes

↳ Small Value for Parameter
is Said to be Technically
Natural

** →

- Parameter itself Determines The Magnitude of Breaking

Def) Technical Naturalness: Small parameter ε Protected by a Sym which is Restored for $\varepsilon \to 0$.

(c.f Small Yukawa Couplings in SM) Sym is χ-Sym

- So SUSY SM + Soft SUSY Gives Technically Natural Sol. Hierarchy Prob.

(Natural Sol. would explain why $m^2 \ll M_p^2$ .. Next Lecture).

- Most General
Soft (Relevant / Superrenormalizable) Susy Terms:

$L_{soft} \supset -\frac{1}{2} m^2_{ij} \phi_i^* \phi_j$ $\phi_i = \{Q_i, \bar{u}_i, \bar{d}_i, L_i, \bar{e}_i\}$

$- \frac{1}{2} m^2 \lambda^a \lambda^a$ 3-2-1 Gauginos

$+ A^u_{ij} Q_i H_u \bar{u}_j + A^d_{ij} Q_i H_d \bar{d}_j + A^e_{ij} Q_i H_d \bar{e}_j$

$+ A'^u_{ij} Q_i H_d^* \bar{u}_j + A'^d_{ij} Q_i H_u^* d_j + A'^e_{ij} Q_i H_u^* \bar{e}_j$

+ --- A-term Analogs R-parity --

I Assume Forbidden by Exact Discrete Sym At least Some of Them Otherwise p Decay Problem

$+ m^2_{ud} H_u H_d$

Scalars

Mass Terms, Scalar Trilinear A-terms:
$H_u H_d$ mass^2 parameter $m^2_{ud} = B\mu$

(Renormalizable) SUSY SM with (Accidental)
R-Parity Had $U(1)_B$, $U(1)_{L_i}$ Accidental Sym

But in Soft terms only $\underline{\underline{U(1)_B}}$ $\underline{\underline{U(1)_L}}$ Remain

$$G_{flavor} = \prod_{a=Q,U,d,L,e} SU(3)_a$$

Spurions That Viol. Sym →

$$m_{ij}^2 \in (8+1)_a$$

(Only Sym That forbids is Flavor Sym itself)

$$A_{ij} \in (3_a, \bar{3}_b)$$

Viol. Same X-Sym as Yukawas...

1. Loose Accidental Lepton flavor Consv.
2. Additional Source of Quark Flavor Viol.

↳ SUSY Flavor Problem.

- SUSY Flavor <u>Problem</u>:

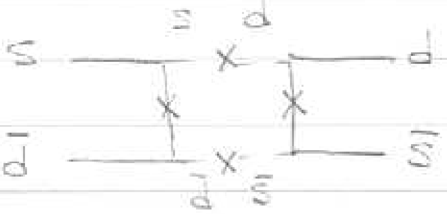
1) $\mu \to e\gamma$ Forbidden Renorm <u>SM</u>:
Allowed Softly Broken SUSY SM

$Br(\mu \to e\gamma) \lesssim 10^{-11}$.

a) $\frac{\delta m^2_{e_L\mu_L}}{m^2} \lesssim 10^{-4}$ $\frac{\delta m^2_{\tau\tau}}{m^2}$ no <u>Bound</u>.

b) $m \gtrsim 50$ TeV (Ruins

2) $K - \bar{K}$ Oscil<u>lation</u> <u>$\Delta S = 2$</u>



$m^2 \gtrsim 50$ TeV.

$\frac{\delta m^2_{sd}}{m^2} \lesssim 10^{-2}$

↳ Worst <u>Ones</u>!! Other m^2_{ij} not So Constrained

- SUSY CP Problem: (Flavor Consv)

$\Downarrow$

Relative CP Phases: $\mathrm{Arg}(m_\lambda A^*) = \theta$
$\mathrm{Arg}(m_\lambda m_d^2 \mu^*) = \theta'$

(Dozens if include Flavor Viol.)

1) EDM:

a) $\theta \lesssim 10^{-2}$
b) $m \gtrsim$ few TeV.

2) Im K-$\bar{K}$ oscillations

a) $\theta \lesssim 10^{-2}$

b) $m_\phi \gtrsim 500$ TeV.

- Pretty Severe Problems: → (So Should take This as teaching us Something About SUSY Sector).

But Depends on Theory of SUSY:

And ∃ Theories Automatically No Problem

How That Aired Dirty Laundry:

** Indications Pointing Towards SUSY **

0. Hierarchy Problem

* 1. Gauge Coupling Unification

* 2. Top Quark Mass IR fixed pt

* 3. EWSB Q-Phase Transition - Heavy Top

* 4 Light Higgs - Precision EW.

5. b·τ Unification

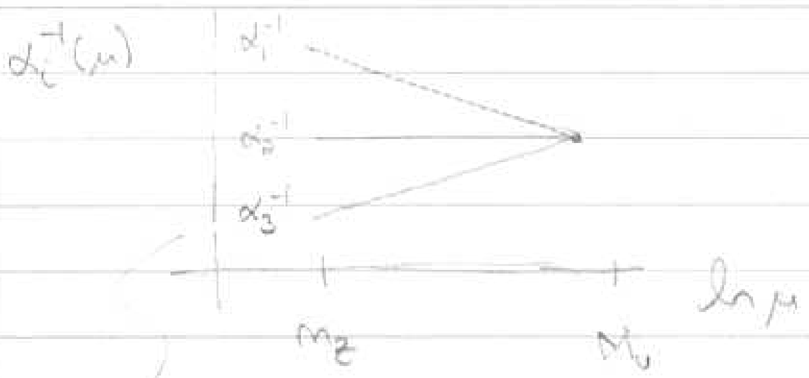
6. Dark Matter Candidate
if Accidental R-parity:

1. Gauge Coupling Unification

$g_i = g_i(\mu)$ Scale Dependent

Conditions under which Expect Couplings to Unify 1) GUT , 2) Perturbative String. (Level 1)

But if Assume Unification



$$\beta_{g_i} = -\frac{b_i}{16\pi^2} g_i^3 \qquad \text{1-loop}$$

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(m_Z) + \frac{b_i}{2\pi}\ln(\mu/m_Z)$$

If Assume Unification $\alpha_i(M_U) = \alpha(M_U)$

$$\alpha_3^{-1}(m_Z) = (1+B)\,\alpha_2^{-1}(m_Z) - B\,\alpha_1^{-1}(m_Z)$$

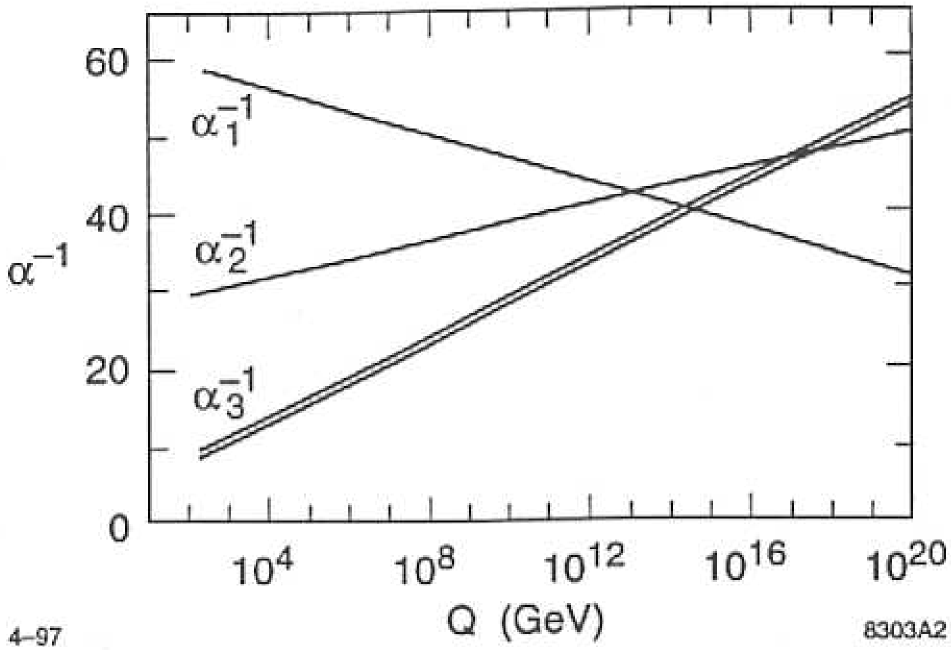
$$B = \frac{b_3 - b_2}{b_2 - b_1}$$

Exp: $B = 0.719 \pm .009$

SM $B = 0.528$ Not Close

MSSM $B = \frac{5}{7} = 0.714$ Very Close

Impressive Prediction



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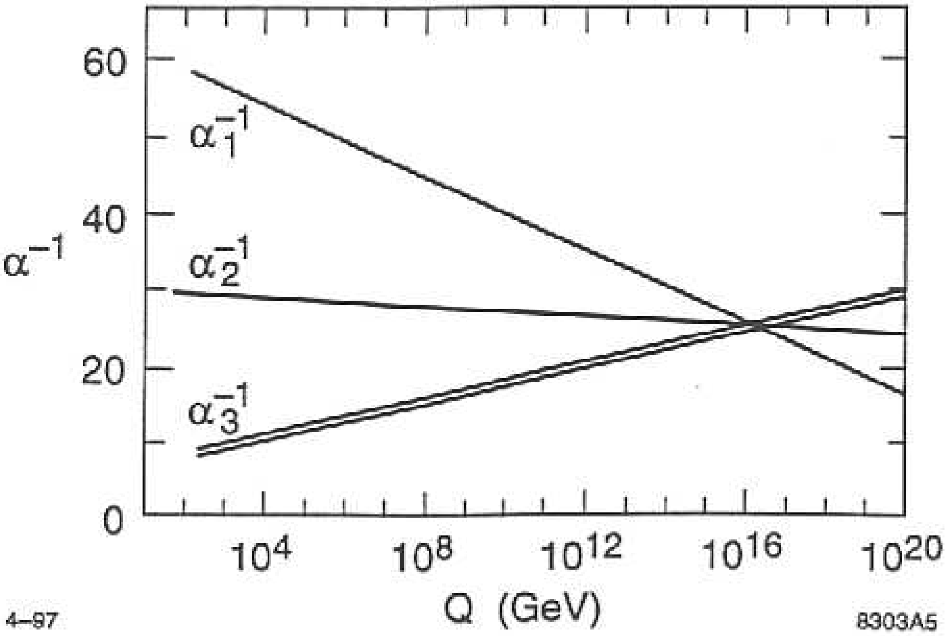

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2. Top Quark Infrared Quasi-Fixed Point

Yukawas are Scale Dependent

↑

Super Pot. Terms

$W(\phi)$ Not Renormalized at Any Order in Pert. Theory: $N=1$ SUSY

Doesn't Mean Couplings Don't Run
Just Means No Corrections to 1PI Action
$\int d^4x \, d^2\theta \, W$

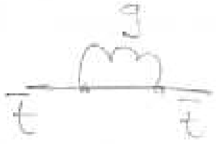
↳ Still Wave fcn Corrections to Canonically Normalized Fields

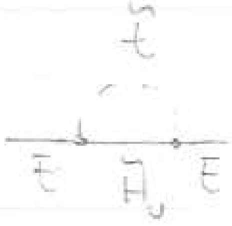
Diagrammatically:

General $\lambda \sigma = \lambda \prod_i \Phi_i$

$$\beta_\lambda = \mu \frac{\partial \lambda}{\partial \mu} = \underbrace{\sum_i \gamma_i}_{\text{wave fcn}} + \underbrace{\overset{\to 0}{\gamma_\sigma}}_{\text{vertex}} \overset{W}{\overset{\downarrow}{=}} \sum_i \gamma_i$$

Top Yukawa:





$$\beta_{\lambda_t} = \frac{\lambda_t}{16\pi^2}\left(-\frac{16}{3} g_3^2 + 6\lambda_t^2 \right)$$

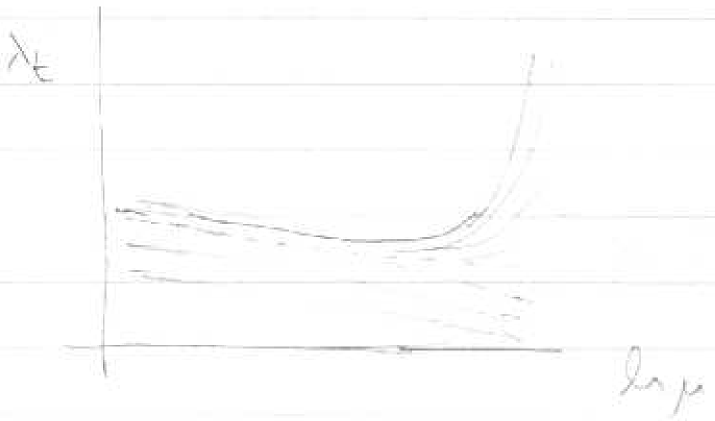
$\uparrow$ Drives Up $\qquad$ $\uparrow$ Drifts Down as $\to$ IR

$$\lambda_t^* = \frac{8}{9} g_3$$

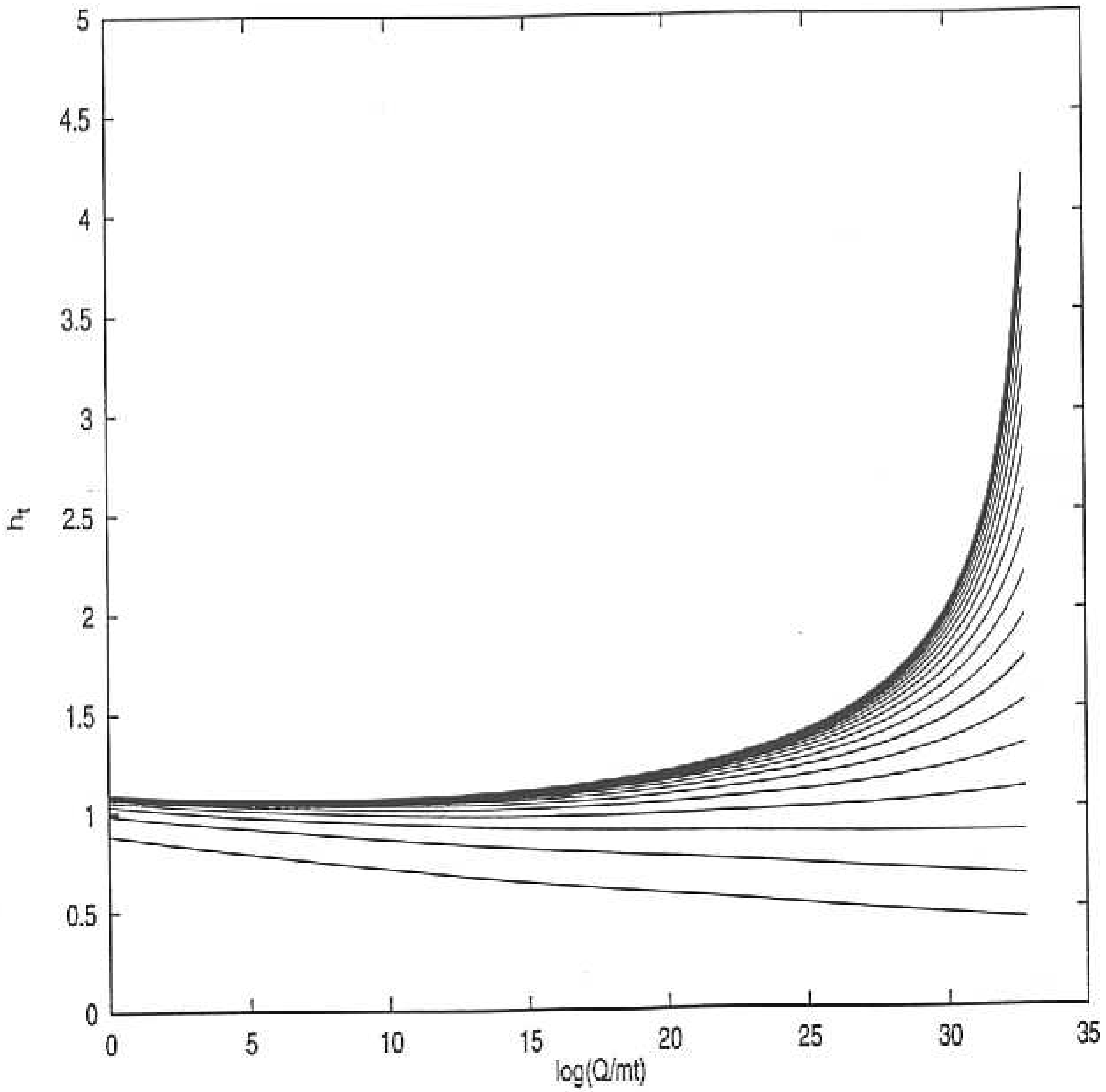
Infrared (Quasi)-Fixed point Attractive

Unstable

Large λ_t Strongly Attracting from Above
Small λ_t Weakly - - from Below



So Natural for Any λ to be At or Below λ^*
but Unnatural for it to be Above



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
h_t
0
5
10
15
20
25
30
35
log(Q/mt)

- Including Finite Corrections

$$m_t^{pole} = m_t^{\overline{DR}}\left(1 + \frac{4}{3\pi}\alpha_3(m_t)\right)$$

$$m_{t,\,SUSY}^{pole,*} = \frac{188\text{ GeV}}{\sin\beta} \qquad \tan\beta = \frac{\langle H_u\rangle}{\langle H_d\rangle}$$

$$m_{t,SM}^{pole} \simeq \frac{250\text{ GeV}}{\sin\beta}$$

$$m_t^{pole} \simeq 175 \pm 5\text{ GeV}$$ (Just as it Should be)

Other Yukawas Started Small / Remained Small

– If $m_t \gg m_{t,SUSY}^{*}$ Would Have been a Real Problem for High Scale SUSY

Just Kinematics (No Additional Assumptions about Flavor etc.)

3. EWSB

MSSM $\subset$ SQuarks, Slepton, Higgs:

$\langle H \rangle$ $SU(2)_L \times U(1)_Y \to U(1)_Q$ Good

$\langle Q \rangle$ $SU(3)_c \times U(1)_Q \to 1$ $\Big\}$ Unacceptable

$\langle L \rangle$ $U(1)_Q \to 1$

$\frac{1}{2} m^2 \phi^\dagger \phi$ $m_H^2 < 0$ Higgs Required to be Tachyonic $\langle H \rangle \neq 0$.

$m^2_{Q,L} > 0$ Can't be Tachyonic

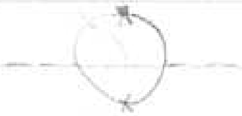
Many / Most Theories of SUSY Have All $m_\phi^2 > 0$:

1. Good to Avoid $SU(3)_c \times U(1)_Q$ Breaking.
2. But Need Higgs Tachyonic

Radiative EWSB

$m^2 = m^2(\mu)$ Scale Dep.

$m_j^2\lambda^2$



$g^2 m_\lambda^2$

Consider λ_t, g_3 Only: Largest Couplings

$W = \lambda_t \, t H_u \bar{t}$

$V \supset \lambda_t^2 \left[|tH_u|^2 + |H_u\bar{t}|^2 + |t\bar{t}|^2 \right]$

N_c Doublets

$-\frac{3\lambda_t^2}{16\pi^2}(m_t^2 + m_{\bar{t}}^2) \quad -\frac{2\lambda_t^2}{16\pi^2}(m_{\bar{t}}^2 + m_{H_u}^2) \quad -\frac{1\lambda_t^2}{16\pi^2}(m_{H_u}^2 + m_t^2)$

$\times \ln\mu \quad \times \ln\mu \quad \times \ln\mu$

$+\frac{32}{3}\frac{g_3^2 M_3^2}{16\pi^2}\ln\mu \qquad + \frac{32 g_3^2 M_3^2}{16\pi^2}\ln\mu$

$$\left\{ \begin{array}{ll} \text{Yukawas by RG mixing Drive } m^2 & \text{Down.} \\ \text{Gauge Couplings} & \text{Up} \end{array} \right.$$

Most Dangerous Tachyonic Mode
under RG Evol is H_u !!

If All $m^2 > 0$ Comparable Large Scale

$m^2_{H_u}(m) < 0$ Tachyonic at Some Scale

$$\downarrow$$

$$\langle H \rangle \neq 0$$

$\langle H \rangle \sim -\frac{m^2_{H_u}}{\lambda^2_{H_u}}$ So Running Stops at This Scale.

- So Pattern of $SU(3) \times SU(2) \times U(1) \to U(1)$ ✓
is a Prediction of SUSY with Large λ_t

(See Plot) →

{ Example of Q-Phase Transition:
Driven by Vac Fluct. of Stop Squarks }

REWSB

- Large λ_t - (only)
- $\langle H_2\rangle$ Accident on Matter Reps - -
- $m_W \sim m_{SUSY}$

- So Dynamics of EWSB in SUSY is a Q-phase Transition

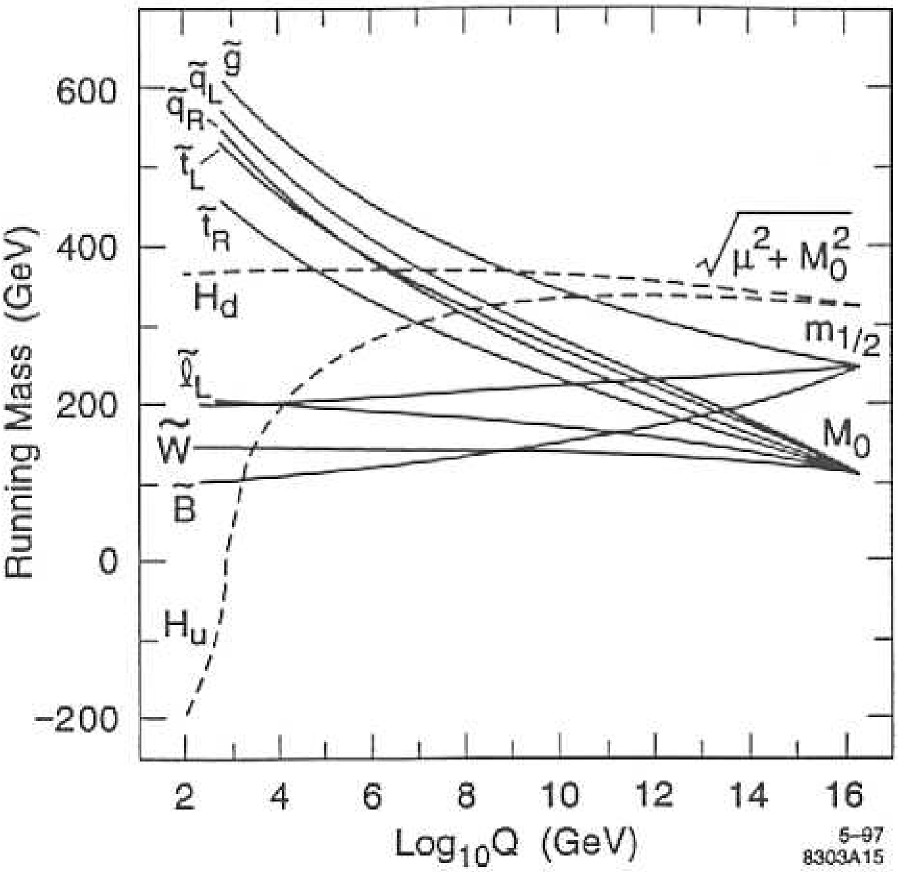
Beautiful Mechanism

** And it Arose Accidentally as Result of Matter Reps

7

(If it Hadn't – would be Problem for SUSY)

** { And Scale EWSB Directly tied to Soft SUSY Scale.


$\tilde{g}$
$\tilde{q}_L$
$\tilde{q}_R$
$\tilde{t}_L$
$\tilde{t}_R$
$\sqrt{\mu^2 + M_0^2}$
H_d
$m_{1/2}$
$\tilde{\ell}_L$
$\tilde{W}$
M_0
$\tilde{B}$
H_u
600
400
200
0
-200
2
4
6
8
10
12
14
16
Running Mass (GeV)
$\text{Log}_{10}Q$ (GeV)
5-97
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4. Higgs Masses:

∃ excitations of Higgs order Parameter:
masses Depend on V(H)

Note

$$V(H) = (m_{H_u}^2 + \mu^2) H_u^* H_u + (m_{H_d}^2 + \mu^2) H_d^* H_d + m_{ud}^2 H_u H_d$$

$$+ \frac{1}{8}(g^2 + g'^2)(H_u^* H_u - H_d^* H_d)^2 + \frac{1}{2} g^2 |H_u^* H_d|^2$$

↳ Related by SUSY to gauge Couplings.

↳ Not Arbitrary:

Look at $\langle H_u \rangle$ only:

tachyonic from REWSB

↳ often Good App. ---

m_{ud}^2 tadpole:

$$m_h^2 = m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$$

Other Higgs in H_d : Entire Doublet $\left\{ \begin{array}{l} H^0, A^0 \\ H^\pm \end{array} \right.$

So Tree level – Lightest Higgs $m_h \le m_Z$

Because $\left\{ \begin{array}{l} 1. \text{ Quartic Couplings} - g^2 \\ 2. \ m_Z \text{ determined by } g^2 \end{array} \right\}$ –

- So SUSY Predicts Light Higgs:

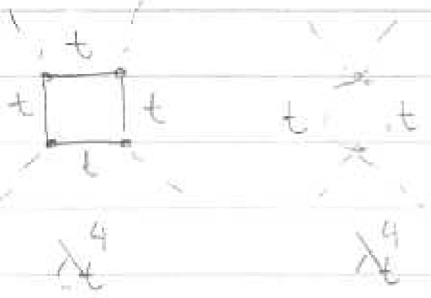
Why LEP Searches Exciting:

↳ However: ∃ Q- Corrections to Quartic Couplings. Modify:

- Above Stop Scale: No SUSY Counter Terms

So Quartic Couplings Related to g^2 ..
No Corrections: High Energy Theory

- Below Stop Scale: SUSY Lost And Corrections Can Arise:



Cancel Above $m_{\tilde{t}}$
Below Only Top Diag

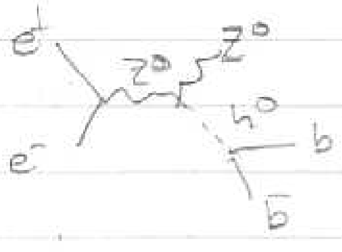
$$\delta V = (H_u^\dagger H_u)^2 (+) \frac{3\lambda_t^2}{16\pi^2} \ln(m_{\tilde{t}}^2/m_t^2)$$

Increase Higgs Masses (How much depends on $m_{\tilde{t}}$).

Other Corrections: trilinear terms:

(See Plot)

Current Bound LEP : $m_h \gtrsim 114$ GeV



(modulo Some Corners of Parameter Space)

(Hints of ~115 GeV Signal)

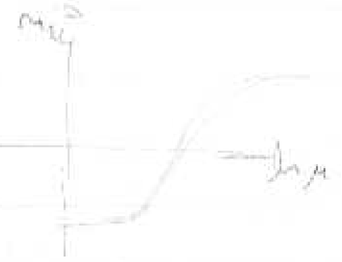
- Can be Additional Contributions to Quartic Couplings in Non-minimal Models ?

↳ Increase m_h

I expect/Hope $m_h \lesssim 120$ GeV

- MSSM: $m_{\tilde{t}} \approx 1.5$ TeV if $A_t = 0$

Mild Tuning of EWSB.



$$\beta_{m_{H_u}^2} = \frac{6\lambda^2}{16\pi^2}\left(m_{H_u}^2 + m_{\tilde{t}}^2 + m_{\tilde{t}}^2 + |A_t|^2\right)$$

Quasi-fixed point $m_{H_u}^2 \simeq -\left(m_{\tilde{t}}^2 + m_{\tilde{t}}^2\right)$

$m_Z^2 \simeq m_{H_u}^2 + \mu^2$ from potential

$\quad\downarrow \qquad \downarrow$

$\mathrm{TeV}^2 \quad (100)^2$ % Level Tuning (At worse)

m_h pulled up $\ln m_{\tilde{t}}$

$-m_{H_u}$ - - - $m_{\tilde{t}}$ } tuning increases exp' $m_{\tilde{t}}$. We are in the exp regime

Few GeV significant !!

- Precision EW

Radiative Corrections to EW observables
Sensitive to "New Physics/Particles"
Can in principle "Discover" ⇒ Through
Virtual Effects.

Most important the oblique Corrections:

Induces Triplet = $H \otimes H \in 3+1$
Contribution to m_W, m_Z

→ Mixing Doesn't Exist at
Tree-Level.

$W^\mu = SU(2)_L$
$B^\mu = U(1)_Y$

SM $g^2=0$

Very Sensitive to m_t

but with log m_t

ok

$$\frac{g^2}{16\pi^2}\int dk\,\frac{K^3}{K\cdot K} \sim \frac{g^2 m_t^2}{16\pi^2}$$

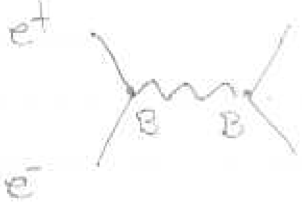
$$\frac{g^2 v^2}{16\pi^2}\int^{m_h} dk\,\frac{K^3}{K^2K^2} \sim \frac{g^2v^2}{16\pi^2}\ln(m_h/m_W)$$

$$\int dk\,\frac{K^3}{K^4}$$

ln Sensitive to m_h

Observables:

$m_W, m_Z, \Gamma_Z, A_{fb}$



$+ \; - - -$

1. Modifies Γ_Z Well Measured at LEP

2. A_{FB} or Polarized σ ← SLAC

or

Can Choose which Component of Z Couple to: by initial Spin Pol since

in Lab $E \gg m \Rightarrow$ Helicity = Chirality

Prior to top Direct Discovery of top at Fermilab

↳ m_t was Deduced by These virtual effects. (± ? GeV)

Now know $m_t = 175 \pm 5$ GeV

↳ So Can Use to Look for More Particles Through virtual Effects!

SM. Higgs - Only $\ln(m_h)$ ← Central Value Actually ≲ 100 GeV.

Combined Data $\Rightarrow$ $m_h \lesssim 170$ GeV 95% CL

250 GeV would be Strongly inconsistent with Data

↳ $\Rightarrow$ Higgs is light!

SUSY: Superpartners $\sim \mathcal{O}\, m_{\tilde{t}, \tilde{w}, \ldots}$

↳ Mass from SUSY and EWSB

Corrections to EW Prop. Suppressed by $(m_Z^2/\tilde{m}^2)$ Decouple Rapidly!

In fact Given Current Direct Bounds on M_{SUSY}
Largest Correction in MSSM is from h^0 Higgs!!

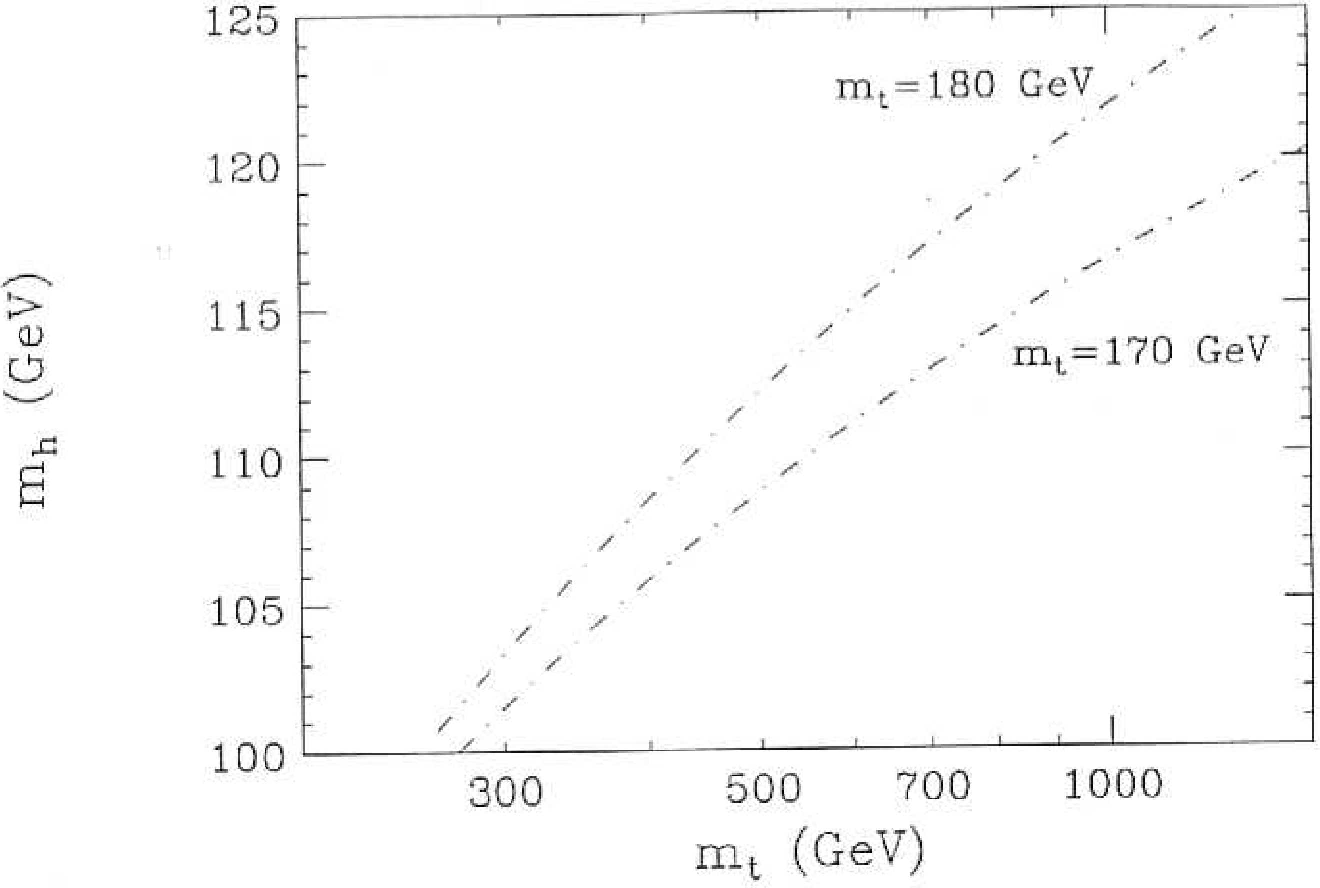
So SUSY Predicts Precision EW Should be
Consistent with SM with Light Higgs!

- In technicolor Generally Large Corrections
↓
Can't Calculate: Strongly Coupled Theory

But if Scale φ QCD → TQCD

Too Large!

Technicolor $N_t = 2$ Ruled out..
Might be possible to Avoid
in Complicated Theories
But most Natural Expectation
would be for Quite Large
Corrections! $\simeq 100$


125
120
115
110
105
100
300
500
700
1000
m_t (GeV)
m_h (GeV)
m_t=180 GeV
m_t=170 GeV

5. Third Generation Yukawas:

$\lambda_b - \lambda_\tau$ Unify in SUSY   c.f. Gauge Couplings

And in SU(5)   ← b, τ in Same multiplet

$$10 = \begin{pmatrix} & Q & \bar{u} \\ & & \bar{e} \end{pmatrix} \qquad \bar{5} = \begin{pmatrix} L \\ \bar{d} \end{pmatrix}$$

$$H_d = \begin{pmatrix} \\ H_d \end{pmatrix}$$

$$\lambda\, \bar{5}\, \bar{5}_d\, 10 = \lambda\left(Q H_d \bar{d} + \bar{e} H_d L\right)$$

So Suggestive of an SU(5) invariant Yukawa Coupling for Third Generation at M_U

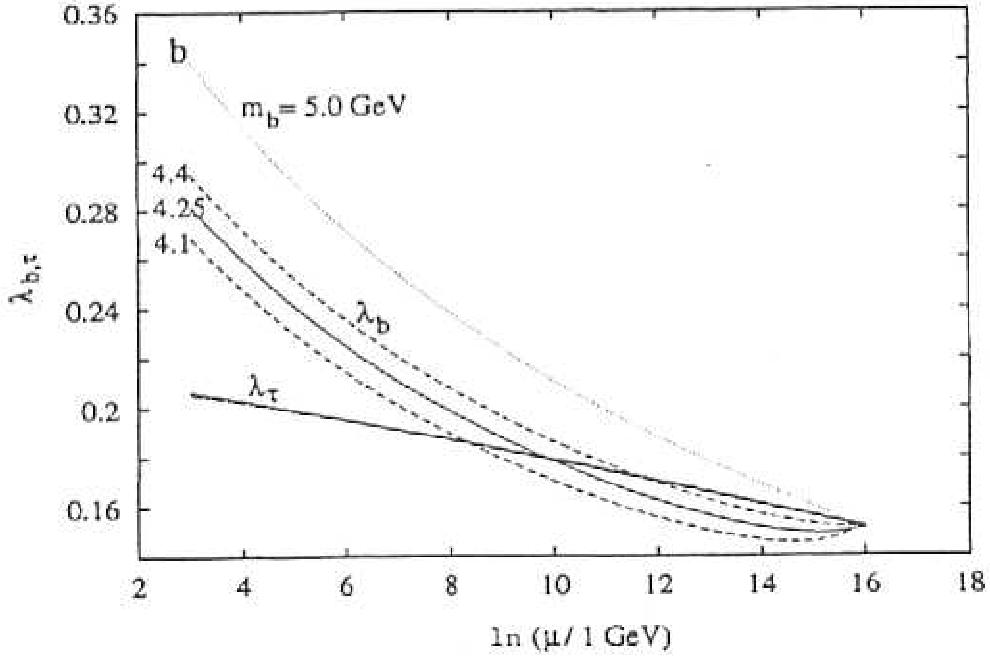
[1st 2nd Generation Yukawas Don't Unify But They are Small And may be Related to GUT Breaking]

- If $\tan\beta \sim 50$ $\lambda_b - \lambda_\tau - \lambda_t$ Unify (it's quite as well)

And in SO(10) or E_6 All Matter in Same multiplet:

$\lambda\, 16\, 10_H\, 16$

↳ Suggestive of SO(10) or E_6 invariant Yukawa in Third Gen.


b
m_b= 5.0 GeV
4.4
4.25
4.1
λ_b
λ_τ
$\lambda_{b,\tau}$
0.36
0.32
0.28
0.24
0.2
0.16
2
4
6
8
10
12
14
16
18
ln (μ/ 1 GeV)

G. Dark Matter Candidate

If (Accidental) R-parity:

Multiplicative Q# $\begin{cases} \text{Super Partners} & -1 \\ \text{SM Particles} & +1 \end{cases}$

└ Then lightest Superpartner Stable!
(At least at Renormalizable Level) - enough)

If neutral χ_1^0 = Combination $\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d$

└ Good Dark Matter Candidate!

Thermal Freeze Out!

$$n_{Eq} \sim T^3 e^{-m/T} \qquad T \ll m$$

Drops out of Eq when $\Gamma \lesssim H$

More: Weakly Interacting → Earlier → n Larger

Strongly . . . → Later → n Smaller!

For 100 GeV Dirac ν

$\sigma \sim G_F M_\nu^2$

n far too Small

For 100 GeV χ_1^0

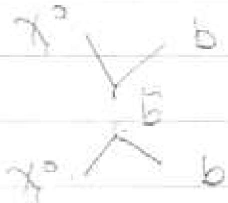
p-wave $\left(\frac{m_f}{m_\chi}\right)^2$

Majorana

↳ identical particle

Anti Sym statistics ⇒ Sym Spin $J=1$

In Addition if $\chi_1^0 = \tilde{B}$



↳ Heavy Also Suppresses

↳ All Goes in right Direction

to Get σ Down } Relative to

n up } G_F Cross Section

↳ Didn't put Any of These Features in By Hand