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Lecture 1

The Hierarchy Problem

and

The Supersymmetric Standard Model

Standard Model Gauge Interactions

$$\begin{array}{ccccc} g_s & & g & & g' \\ SU(3)_c & \times & SU(2)_L & \times & U(1)_Y \end{array}$$

$$\underbrace{SU(2)_L \times U(1)_Y}_{\downarrow} \\ U(1)_Q$$

- Full Gauge Sym Hidden

- For $g, g' \ll 1$ — Useful to think of as Weakly Gauged Global Sym

Then Language of SB of Global Sym is Appropriate
Exact Symmetry - Vacuum not invariant

$\langle H \rangle$ - transforms non-trivially Under Sym
= Order Parameter = Higgs Field

Language of CM

[Aside: Should discuss Gauge inv Quantities - Weakly coupled gauge Sym This language Sufficient]

	Generators / Gauge Bosons
$SU(2)_L$	3
$U(1)_Y$	1
$-\ U(1)_Q$	-1
	3 $\longrightarrow$ massive Gauge Bosons = $W^{\pm}, Z^0$

- Know nothing About Dynamics of EWSB

- Can Deduce $\exists$ excitations of order Parameter
  $\equiv$ Higgs Boson(s) $\underline{m = ?}$

- Without Full Theory
  What Can we Learn About Higgs Sector Dynamics?

- In Low Energy Theory Can Calculate
  Q-Corrections to $V(H)$ by integrating out
  Known particles.

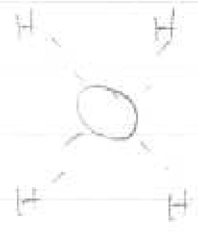
  Strength of Coupling to H ~ mass

  $\therefore$ Heaviest Particles most important

t	175	GeV
Z^0	91	GeV
$W^\pm$	80	GeV
$\vdots$		

- Focus on top: $\lambda_t \bar{t} t H$ $\quad \lambda_t \simeq 1$

  H Background Field.

$V(H) = V_0(H) +$  $+$  $+ \ldots$

$= m_0^2 H^\dagger H + \lambda (H^\dagger H)^4 + \ldots$

$m^2 H^* H + \lambda (H^* H)^4$

$\delta H: \quad m^2 + 2\lambda H^* H$

$$v^2 = -\frac{m^2}{2\lambda}$$

cutoff At which Couplings Defined

 $= \delta m_H^2 H^\dagger H = 3\lambda_t^2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} = \frac{3\lambda_t^2}{16\pi^2}\Lambda^2$

$\therefore \; m^2 = m_0^2 + \frac{3\lambda_t^2}{16\pi^2}\Lambda^2$ w/ Quadratic Div.

(others Similar including Self interaction)

Given The existence of Higher Mass Scales (Such as M_p) This is a Real Divergence!

Now:

$$v^2 = \frac{-m^2}{2\lambda} = \text{fixed}$$

- So in Eff Theory as $\Lambda \to$ Large
  m_0^2 must be "Tuned" to give Correct v

$$\underbrace{m^2}_{\text{Small}} = \underbrace{m_0^2}_{\text{Large}} + \underbrace{\frac{3\lambda^2}{16\pi^2}\Lambda^2}_{\text{Large}} + \cdots$$

Tuned: Near Cancellation between Contributions at Different orders!

Possible but looks unnatural: Sensitive to High orders in Perturbation Theory

Test!

Heavy Gauge Boson Masses Depend on $\langle H \rangle$ Rep.

For $H \in 2_{\frac{1}{2}}$ of $SU(2) \times U(1)_Y$

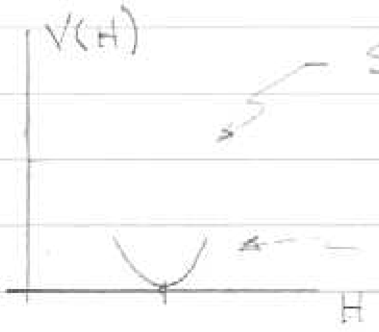
$$m_W = \frac{1}{2} g v \qquad m_Z = \frac{1}{2}\sqrt{g^2+g'^2}\, v \qquad \langle H \rangle = \frac{v}{\sqrt{2}}$$

→ Why Choose?

1. Allows Yukawa Couplings : Fermion Masses
2. No Other Simple Choices give $\frac{m_W}{m_Z} = \frac{g}{\sqrt{g^2+g'^2}}$

If $T \in 3$ of $SU(3)$ $\quad \frac{\langle T \rangle}{\langle H \rangle} \lesssim 0.02$

So $\exists$ Some Sector of SM (Higgs Sector)



Some Dynamics Minimizes Energy at non-Zero H

(Meta)-Stable

1. $H \in 2_{\frac{1}{2}}$ of $SU(2)$
2. $v = 246$ GeV (For last 25 yrs.)
3. $J = 0$

Have No Other Positive Information About Higgs Sector

- This is The Hierarchy Problem:

Why is Mass Parameter in Higgs Sector $\ll M_{(p)}$

↳ Discussion from Bottom up - in Eff Theory
No Symmetry Prevented Quadratically Div.
Correction to Higgs mass Parameter.

From Top Down: in Eff. Theory (c.f Fermion masses Gauge inv. forbid)
No Symmetry Protects / Forbids Higgs
mass Parameter:
So $m^2 \sim \Lambda^2$ most natural Expectation:

Terminology - Standard Model Higgs Sector
H: Elementary Scalar

$\rightarrow$ $m^2 \ll \Lambda^2$ Not Robust prediction of Framework

↳ Discussion in Eff Theory → Treated H as Fluctuating Order Parameter.

- If Theory of Higgs Sector Changes Above Some Scale Divergence may Not Exist in Full Theory.

In effective Theory:

** → $m^2 = m_0^2 + \frac{3\lambda^2}{16\pi^2}\Lambda^2$ ($\Lambda \lesssim 1$ TeV would Avoid tuning).

Possibilities for Higgs Sector Dynamics $\lesssim 1$ TeV.

1. Higgs Order Parameter = Fermion Composite

$H = \psi\psi$: Then $H^\dagger H \sim \psi^\dagger\psi^\dagger\psi\psi$ ← no Quad Div at High Energy

a) Technicolor $\langle Q\bar{Q}\rangle$

Analogous to χ-Sym Breaking Order Parameter of QCD

Global $SU(2)_A$ Spont. Broken

(In fact if Remove Higgs Sector from SM χ-Condensate of QCD would break $SU(2)_L \times U(1)_Y \to U(1)_Q$

$\pi^{\pm,0}$ Eaten by $W^\pm, Z^0$ . .)

Another QCD-like Sector with
$f_{\pi_T} = 246$ GeV not $f_\pi = 93$ MeV

b) Top Color $\langle t\bar{t} \rangle$
Large λ_t might imply Attractive Channel

- "Hard" to Build Theories of fermion Masses
- Generally inconsistent with precision Measurements
  (But Keep an Open Mind)

2. No Scale in nature Higher Than 1 TeV.
   Existence of Gravity → $M_p \sim$ TeV

↳ Why $G_N \ll 1/\text{TeV}^2$

$$G_{N,4} = \frac{G_{N,D}}{V_{D-4}}$$

Another Hierarchy Problem

↳ Why V_{D-4} large?

- Would be interesting . . . !!!
- No Complete Theory of EWSB
  Fermion masses
  ⋮
- No evidence pointing in This Direction.
  (But Keep an "open" mind) - - -

4. Supersymmetry

Higgs mass parameter Protected by SUSY
SUSY must be (Spontaneously) Broken

SUSY Scale → EWSB Scale

- Not Quite as Elegant as SM
- Indirect Evidence pointing in This Direction

↳ Topic of These Lectures.

3. Little Higgs - Extra Higgs, tops.

GIM

$$-\frac{3\lambda_t^2}{16\pi^2}(\Lambda^2 - m_t^2) + \frac{3\lambda_t^2}{16\pi^2}(\Lambda^2 - m_{t'}^2) = -\frac{3\lambda_t^2}{16\pi^2}(m_t^2 - m_{t'}^2)$$

Quad Div. Cancels 1-loop

Cancels 2-loops (No Sym)

Quartic Coupling, Only factor 2-3 Real Models, PEW

How SUSY Cutoff Quad Div?

$\psi \leftrightarrow \phi$ $\quad \forall \psi \; \exists \phi$ and Vice Versa.

$$-\frac{3\lambda_t^2}{16\pi^2}(\Lambda^2 - m_t^2) + \frac{3\lambda_t^2}{16\pi^2}(\Lambda^2 - m_{\tilde t}^2) = -\frac{3\lambda_t^2}{16\pi^2}(m_{\tilde t}^2 - m_t^2)$$

· Quad Divergences Cancel, B-F !!

↳ Quad Div in low Energy Cutoff by $m_{\tilde t}^2$

New Physics

$m_{\tilde t}^2 - m_t^2$ is a measure of The Magnitude of SUSY So See Already That Scale of SUSY will Determine mass Scale in Higgs Potential.

For all orders Cancellation:

Need Sym 1. Superpartners $\forall$ particle

2. Couplings Related

↳ $\Rightarrow$ SUSY.

$$\lambda^2\left(H^2 - H^2 + \Omega_0^2\right)$$

Supersymmetric Limit.

Minimal SuperSymmetric Standard Model (MSSM):

- Matter Fields ⊃ Quarks, Leptons (⇒ largest SUSY That Allows χ-Reps : And in D=4 Smallest SUSY : Unique)

Fermions in Chiral Reps:

For D=4 ⇒ N=1 SUSY (4 $\mathbb{R}$ Q_α's)

J = 0 ½

χ-SuperFields: $\Phi = \phi + \theta\psi + \theta^2 F$ $D_{\dot\alpha}\Phi = 0$

↳ Irred. Rep of SUSY ⊃ χ-fermion Arbitrary Gauge Rep.

$SU(3)_c \times SU(2)_L \times U(1)_Y$

		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
i=1,3	Q_i	3	2	1/3
Generations	u_i	$\bar{3}$	1	-4/3
	d_i	$\bar{3}$	1	2/3
	L_i	1	2	-1
$Q = T_3 + \frac{1}{2}Y$	e_i	1	1	+2
	H_u	1	2	+1
	H_d	1	2	-1

Upper Case = SU(2) Doublet

Lower Case = SU(2) Singlet

H_u ← to break EWS

(Aside: Note L scalar Has correct Q #s. But Then Lepton # Spont. Broken ⇒ Too Large a Viol of Lepton #.)

↳ Required for Anomaly Cancellation $\mathrm{Tr}\, Y = 0$

$\mathrm{Tr}\, L^2 Y = 0$

- Vector Fields:

$SU(3)_C \times SU(2)_L \times U(1)_Y$ Vector multiplets:

J: $\frac{1}{2}$ 1

$V \supset \bar{\theta}^2 \theta \lambda + \theta \sigma^\mu \bar{\theta} A_\mu + \cdots$ $V = V^\dagger$

$$W_\alpha = \frac{1}{4} \bar{D}^2 e^{-V} D_\alpha e^V \supset \lambda_\alpha + \sigma^{\mu\nu} \theta_\alpha F_{\mu\nu} + \cdots$$

Kinetic Terms:

$$\frac{1}{4g_a^2} \int d^2\theta \, W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \, \Phi^\dagger e^V \Phi$$

- <u>Interactions</u>:

  - Need Yukawa Couplings: $\psi \psi H$

$\int d^2\theta \, W(\Phi)$ → SUSY invariant
Product of χ-Super Fields : χ-Super Field :
Variation of Highest θ^2 Component is total Derivative

Holomorphic

$$\Phi_i \Phi_j \Phi_k \supset \theta^2 \psi_i \psi_j \phi_k + \theta^2 F_i \phi_j \phi_k$$

↳ Gives Quartic Coupling

So $W \supset \lambda^u_{ij} Q_i H_u \bar{u}_j + \lambda^d_{ij} Q_i H_d \bar{d}_j + \lambda^e_{ij} L_i H_d \bar{e}_j$

Yukawa Matrices

<u>Note</u>: $\Phi \Phi \Phi^*$ is not a χ-SuperField
and ∴ $\int d^2\theta \, \Phi \Phi \Phi^*$ not SUSY inv.
$W(\Phi)$ must be Holomorphic fcn.

So No $L_i H_u^* \bar{e}_j$ (Gauge inv but Not SUSY inv)

- So it's Good both H_u, H_d Required by Anomaly Cancellation.

- Other Allowed Renormalizable interactions? by Gauge inv. and Holomorphy?

<u>Aside</u>: <u>Accidental Global Symmetry</u>

↳ Symmetry Up to Some order in Interactions Which Arises as Result of Gauge Symmetries + Matter Reps.

<u>Example</u>: QED with $\psi_e, \psi_{\bar{e}}, \psi_\mu, \psi_{\bar{\mu}}$

$$\mathcal{L} \supset m_e \psi_e \psi_{\bar{e}} + m_\mu \psi_\mu \psi_{\bar{\mu}} + i\psi_i^* i\not{D}\psi_i$$

At <u>Renormalizable Level</u>: Seperate $U(1)_e, U(1)_\mu$ Conserved!

Non-Renorm Level $\frac{1}{M}\psi_e \psi_{\bar{\mu}} \psi_\mu \psi_{\bar{e}} + \ldots$ Breaks $U(1)_e \times U(1)_\mu \to U(1)_\ell$

Start ↳ *

<u>Example</u>: (Non-<u>Susy</u> SM)

$$\mathcal{L}_{Yuk} = \lambda^u_{ij} \psi_{Q_i} H \psi_{\bar{u}_j} + \lambda^d_{ij} \psi_{Q_i} H^* \psi_{\bar{d}_j} + \lambda^e_{ij} L_i H^* \bar{e}_j$$

$\lambda^u_{ij} \neq 0$
$\lambda^d_{ij} \neq 0$
$\lambda^e_{ij} \neq 0$

• $\lambda^u = \lambda^d = \lambda^e = 0$

Global Family Sym

$$\underbrace{U(3)_Q \times U(3)_{\bar{u}} \times U(3)_{\bar{d}}}_{\downarrow} \times \underbrace{U(3)_L \times U(3)_{\bar{e}}}_{\downarrow}$$

• $\lambda^u \neq 0$, $\lambda^d \neq 0$, $\lambda^e \neq 0$

$$U(1)_B \qquad U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$$

$\lambda^u_{ij}, \lambda^d_{ij}$ Not Sim. Diag.
$U(1)_{Q_i}$ not Conserved
(why $Q_i \to Q_j$ X Allowed)

Only One λ^e_{ij}
- Interactions Simultaneously Diag With Masses

∴ Renormalizable SM Has Accidental B, L_i Conservation (In Agreement with observation)

↳ Didn't Have to put in by Hand!
↳ Why SM is So Elegant/Beautiful!!

• Non-Renormalizable SM Does not Have Accidental B, L_i Conservation But in Non-Renorm Scale High - OK

Returning to MSSM:

There are other Gauge inv Holomorphic Renormalizable SuperPot Couplings

$$W = \lambda_{[ij]k} L_i L_j \bar{e}_k + \lambda'_{ijk} Q_i L_j \bar{d}_k + \lambda''_{i[jk]} \bar{u}_i \bar{d}_j \bar{d}_k$$

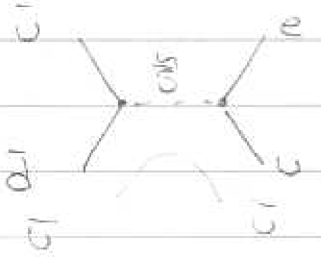
$U(1)_L$	-1	$+1$	0
$U(1)_B$	0	0	-1

↳ Very Dangerous: Proton Decay

Have Both

$p^+ \to e^+ X$ Requires Both $U(1)_B$, $U(1)_L$

For example λ' λ''



$p^+ \to \pi^0 e^+$

$$\Gamma(p^+ \to \pi^0 e^+) \sim \left(\frac{\lambda'_{\bar{u}d\bar{s}} \lambda''_{ue\bar{s}}}{m_{\tilde{s}}^4}\right)^2 \frac{m_p^5}{8\pi}$$

$$\tau > 10^{33}\,\mathrm{yr} \Rightarrow \lambda'_{uds} \lambda'' \lesssim 10^{-27}$$

So Small That Something must be Zero / Forbidden

• This is the worst (Aestetic) Feature of MSSM

Required (Meta) Stability of Proton is not enforced by an Accidental Sym of Renormalizable Theory.

⇒ [Existence of Scalars Allowed Couplings which were forbidden by Lorentz inv in SM]

• Possibilities to Avoid Very Rapid Proton Decay:

- Impose by Hand $U(1)_B$ And/or $U(1)_L$ Conservation

- But Don't Expect Global Symmetries Respected in Full Theory (Gravity).
c.f. Black Holes only Respect Gauge Q #s Not Global Q #s.

Skip

Perturbative String Theory -
Global Sym on WS → Gauge in ST

(Probably not Applicable Here)

(Exception PQ - Sym !
At All orders in Pert. Theory Dilaton Decouples at $p^\mu = 0$ ⇒ Axial Shift Sym $a \to a + c$ at all orders)

— Non-Anomalous Combination

- $U(1)_{B-L}$ Gauged :

  But No $m=0$ B-L Gauge Boson
  And if B-L Spont. Broken - No Reason
  for Exact Conservation

- Discrete (Gauge) Symmetries

  ↳ Either Remnant of Continuous Sym
  $U(1) \to \mathbb{Z}_K$ e.g.
  Or Exact Sym of Theory

Exact ⇒ Anomaly free

J^μ_x 

$Tr(XL^2) = 0$
$Tr(XC^2) = 0$
$Tr(X) = 0$ — L(Gravity)

Then no (non)perturbative process Should Viol. Sym. (And Can be Gauged)

{ Not Relevant for a Z_2 Since Massive Matter Can Contribute }

Operationally With Particles Only Consequence of Gauged is ⇒ Exact!
[If (Cosmic) Strings Carry Discrete Flux - Then Affects A-B phases.]

1 Discrete Symmetries (Gauge)

Forbid ....

Excites...

2. R parity

N=1 SUSY Theories Sometimes (Depending on W)
Have $U(1)_R$ Symmetries

Define: → ↳ By Def. Doesn't Commute with SUSY
So B, F members of Super-mplt
Don't Have Same Charge

If $R(\Phi) = R$

$\Phi = \phi + \theta\psi + \theta^2 F$ Then

	R
Φ	R
ϕ	R
ψ	$R-1$
F	$R-2$
θ	$+1$
$\int d\theta$	-1
$\int d^2\theta$	-2
W	$+2$
λ^α	$+1$
A^μ	0

Important later

In a Given Theory
If $W(\Phi)$ Allows Charge
Assignments so That $R(W(\Phi)) = 2$
$\exists$ $U(1)_R$ Symmetry

- Why Can There be a Sym That Doesn't Commute SUSY?
Can Think of as Accidental Sym
of Component Action (Not Generic)

Jumping Ahead $U(1)_R \to Z_2$ by $\cancel{SUSY}$
Gaugino Masses
So That's All That's Available

Define Discrete Z_K Sym Generators

$$e^{2\pi i R/K}$$

R-charge

- Note Discrete charge
is Multiplicative not Additive
Quantum # Since Only
Defined mod K.

- $\exists$ an R-parity which Forbids $QL\bar{d}$, $LL\bar{e}$, $\bar{u}\bar{d}\bar{d}$ terms:

  Anomaly Free ... Possible to Gauge...

- others ...?

See Table: → Define:

	Z_2	
Std Particle	+1	
Superpartner	−1	⇒ Lightest Superpartner Stable

{ Exercise: Are There Z_N Anomaly Free Discrete Symmetries which Allow Renormalizable $\cancel{B}$ or $\cancel{L}$ but not Both? That Commute with Family Symmetries?

(Can Add $SU(3)_c \times SU(2)_L \times U(1)_Y$ Singlet Matter to Cancel $Tr(X) \neq 0$)

Discrete Generator $e^{2\pi i X/K}$ (X only Defined mod K)

	Z_2 R-parity	Z_2 Matter Parity	Z_2 Baryon Parity	Z_2 Lepton Parity	Z_2 B-L Parity	Z_5 X	...
Q_i	-1	-1	-1	0	+1	3	
$\bar{u}_i$	+1	-1	-1	0	-1	3	
$\bar{d}_i$	+1	-1	-1	0	-1	-1	
L_i	-1	-1	0	-1	+1	-1	
$\bar{e}_i$	+1	-1	0	-1	-1	3	
H_u	0	0	0	0	0	2	
H_d	0	0	0	0	0	-2	
$LL\bar{e}$	-1	-1	0	-1	+1	1	
$QL\bar{d}$	-1	-1	0	-1	+1	1	
$\bar{u}\bar{d}\bar{d}$	+1	-1	-1	0	-1	1	
$QH_u\bar{u}$	0	0	0	0	0	0	
$QH_d\bar{d}$	0	0	0	0	0	0	
$LH_d\bar{e}$	0	0	0	0	0	0	
$\mathrm{Tr}(X)$						0	
$2\mathrm{Tr}(XL^2)$	0	-1	-1	-1	0	0	
$2\mathrm{Tr}(XC^2)$	0	0	0	0	0	0	

↳ Not Relevant if Spont. Broken

Accidental Z_2 R-parity

Z_2 R-parity $\equiv \begin{cases} \text{Particle} & + \\ \text{Sparticle} & - \end{cases}$

Most Natural Way These Discrete Sym might Arise?

1. Discrete R-Sym → String, M-Theory Compactifications Can Leave an Exact Discrete Subgroup (at least at points on M Space)

2. Discrete Gauge Sym

- Existence of $LL\bar{e}$, $QL\bar{d}$, or $\bar{u}\bar{d}\bar{d}$ in W
  Generally Referred to as R-parity Violation
  Since no R-parity Symmetry (Accidental or otherwise)

- With R-parity (Accidental or otherwise)
  SUSY MSSM Enjoys Same Global Sym as SM

  B, L_i $i = e, \mu, \tau$ Conserved (perturbatively)

  [Won't be true with SUSY]

→ Note: Even Though two Higgs: H_u, H_d
  - Holomorphy Restricted Couplings

$$W \supset \lambda^u_{ij} Q_i H_u \bar{u}_j + \lambda^d_{ij} Q_i H_d \bar{d}_j + \lambda^e_{ij} L_i H_d \bar{e}_j$$

Each type of Right Handed Quark, Lepton Couples to Only One Higgs Field

- Avoids Lepton Flavor Viol. Since Mass / Higgs interactions Simultaneously Diagonalized / Aligned

General Non-SUSY Theory $\lambda^e_{ij} L_i H_u^* \bar{e}$ would Not have Alignment - Viol. Lepton Flavor L_i
(SUSY more Than two Higgs - Also L_i)

- Another Possibility to Avoid R-parity terms

Enhance Gauge Group So That
$LL\bar{e}$, $QL\bar{d}$, $\bar{u}\bar{d}\bar{d}$ Not Allowed by Gauge Sym...

Requires Additional Matter to Cancel
Gauge Anomalies

- Remaining Term(s) in SUSY MSSM:

(One) More Term in Renormalizable W
Allowed by Gauge inv and Holomorphy

$W \supset \mu H_u H_d + \mu_i H_u L_i$

Violates $U(1)_L$

1. Forbid by Sym of Previous

2. Rotate/Redefine

$\begin{pmatrix} H_d \\ L_i \end{pmatrix}$ so That $\mu_i = 0$

4 of $SU(4)_{H_d - L_i}$

$\begin{pmatrix} \mu \\ \mu_i \end{pmatrix} = 4$ → Points in Some Direction
Call That H_d direction

Then get in general $LL\bar{e}$, $QL\bar{d}$ Couplings

$W = \mu H_u H_d$

Dirac Mass Term H_u, H_d Vector pair:

$\therefore$ MSSM is Not a ~~Chiral Theory~~
~~(Has χ-matter) but also Has a mass term~~

μ Must be order of SUSY Scale As see later

$\hookrightarrow$ How Superpotential Term Related to SUSY?

μ- Problem

~~- Another (mildly) Bad Feature of MSSM~~

- It is Possible to Enhance Gauge Group to Forbid $\mu H_u H_d$ by Gauge Sym

Exercise

- Write down a Chiral $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ SUSY Model which forbids $\mu H_u H_d$
What is minimal Additional Matter needed for Anomaly Cancellation?
Does Your Theory Have Accidental R-Sym protects p-decay? R-parity?