Quarks & Leptons:

Universality versus Complementarity

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Leptons vs Quarks

Understanding Fermion Masses

Unification of particles and forces GUT's, Strings...

Two fundamental issues

1. Confronting parameters of leptons and quarks

Quark-Lepton symmetry

Correspondence:

u_r , u_b , u_j <-> ν
d_r , d_b , d_j <-> e
color

Pati-Salam

Symmetry:

Leptons as 4th color

Unification:

form multiplet of the extended gauge group, in particular, 16-plet of SO(10)

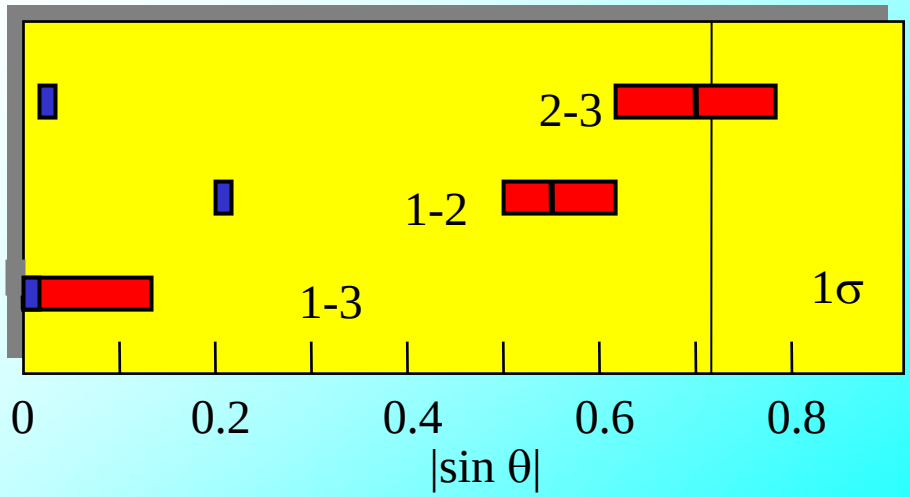
Accidental?

On the other hand …

Mixing

Mixing	Quarks	Leptons	Complementarity
1-2, θ_{12}	13°	34°	$\theta_{12} + \theta_C = 46.7° \pm 2.4°$
2-3, θ_{23}	2.3°	42°	$\theta_{23} + V_{cb} = 43.9° + 5.1/-3.6°$
1-3, θ_{13}	~ 0.5°	< 8°	

$\nu_\mu - \nu_\tau$ permutation symmetry?



2-3 symmetry: remark

$\nu_\mu - \nu_\tau$ permutation symmetry

$$\begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}$$

Matrix for the best fit values of parameters (in meV)

$$\begin{pmatrix} 3.2 & 6.0 & 0.6 \\ & 24.8 & 21.4 \\ & & 30.7 \end{pmatrix}$$

$\sin^2\theta_{13} = 0.01$ $\sin^2\theta_{23} = 0.43$ (Bari group)

substantial deviation from symmetric structure
even stronger deviations are possible within the error bars

Structure of mass matrix is sensitive to small deviations of 1-3 mixing from zero and 2-3 mixing from maximal

Tri/bimaximal mixing

L. Wolfenstein

P. F. Harrison
D. H. Perkins
W. G. Scott

$U_{tbm} = U_{23}(\pi/4)U_{12}$

$$U_{tbm} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ \sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

ν_3 is bi-maximally mixed
ν_2 is tri-maximally mixed

- maximal 2-3 mixing
- zero 1-3 mixing
- no CP-violation

$U_{tbm} = U_{mag}\, U_{13}(\pi/4)$

$\sin^2\theta_{12} = 1/3$ in agreement with 0.315

Mixing parameters - some simple numbers 0, 1/3, 1/2

Clebsch Gordan coefficients

Relation to group matrices?
S_3 group matrix

In flavor basis…
relation to masses?
No analogy in the
Quark sector?
Implies non-abelian
symmetry

Masses

- **Hierarchy of masses:**

Quarks

$|m_c / m_t| \sim 0.005$

$|m_s / m_b| \sim 0.02 - 0.03$

Leptons

$|m_2 / m_3| > 0.18$

$|m_\mu / m_\tau| = 0.06$

Neutrinos

Neutrino mass hierarchy is the weakest one

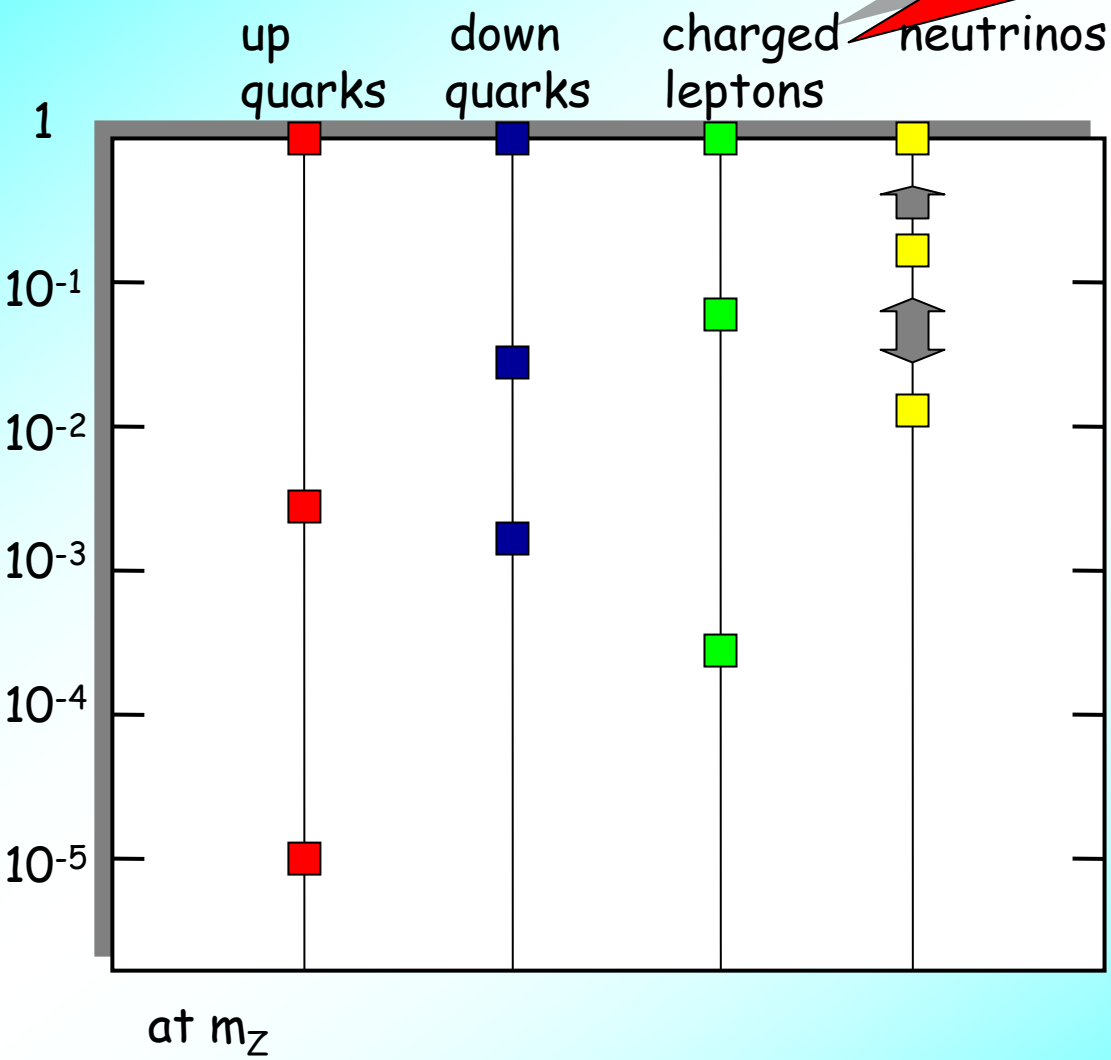
- $m_u : m_c : m_t = \lambda^4 : \lambda^2 : 1$

$m_d : m_s : m_b \sim \lambda^2 : \lambda : 1$

Upper and down fermions have different mass hierarchies

Mass Ratios

An interplay of regularities and Randomness ?



Regularities?

$m_u\, m_t = m_c^2$

$V_{us}\, V_{cb} \sim V_{ub}$

Gatto –Sartori-Tonin relation

$\sin\theta_C \sim \sqrt{m_d/m_s}$

Koide relation

Additional structure?

Similar gauge structure, correspondence

Very different mass and mixing patterns

Particular symmetries in leptonic (neutrino) sector?

Q-L complementarity?

Lepton sector ⟷ Symmetry correspondence ⟷ **Quark sector**

Additional structure exists which produces the difference.

Is this seesaw?
Something beyond seesaw?

2. Quark-Lepton universality

Can approximate quark lepton universality be realized
in spite of strong difference of mass and mixing patterns?

Perturbative approach

The idea is

$$M_f = M_0 + \delta M_f \qquad f = u, d, l, \nu$$

universal mass matrix

corrections depend on flavor

Weak universality:

two universal mass matrices for up and down fermions:

$M_0^{(up)}$ u, ν_D $M_0^{(down)}$ d, l

allows to explain different mass hierarchies

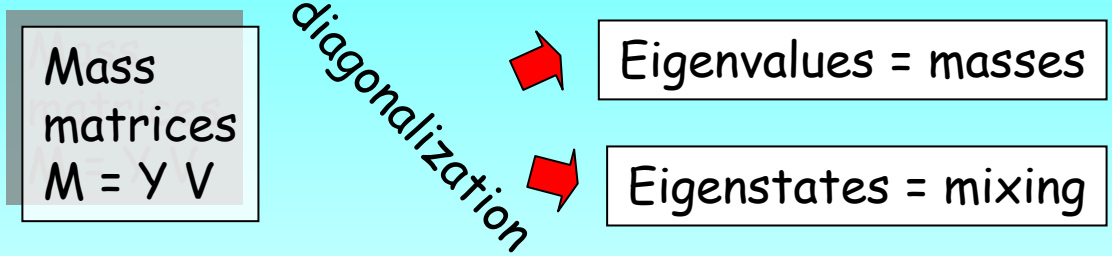
The Majorana mass matrix the RH neutrino can have different structure

Universality

approximate

I. Dorsner, A.S.
NPB 698 386 (2004)

- is realized in terms of the mass matrices (matrices of the Yukawa couplings) and not in terms of observables – mass ratios and mixing angles.



- Universal structure for mass matrices of all quarks and leptons in the lowest approximation:

$$Y_U = Y_D = Y_{\nu D} = Y_L = Y_0$$

- Small perturbations: $Y_f = Y_0 + \Delta Y_f$ $\quad (Y_0)_{ij} \gg (\Delta Y_f)_{ij}$

$f = u, d, L, D, M$

Singular mass matrices

Important example:

$$Y_0 = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}$$

Det M = 0

$\lambda \sim 0.2 - 0.3$

Unstable with respect to small perturbations

$$Y^f_{ij} = Y^0_{ij} (1 + \varepsilon^f_{ij})$$

$f = u, d, e, \nu$

Universal singular

Perturbations $\varepsilon \sim 0.2 - 0.3$

Small perturbations allow to explain large difference in mass hierarchies and mixings of quarks and leptons

Form of perturbations is crucial

Seesaw plays crucial role

Seesaw: m ~ 1/M

Nearly singular matrix of RH neutrinos leads to
- enhancement of lepton mixing
- flip of the sign of mixing angle,
  so that the angles from the charged leptons
  and neutrinos sum up

Origin of mass matrices

- Froggatt- Nielsen mechanism, U(1) family symmetry

$$a_{ij} f_i \, f_j^c \, H \left(\frac{\sigma}{M_F} \right)^{q(i) + q(j)}$$

q(i) is the U(1) charge of i-family, $a_{ij} = O(1)$,
σ is the F-N scalar whose VEV violates U(1)
M_F is the scale at which F-N operators are formed

$$\Rightarrow \quad (Y^f)_{ij} = a_{ij} \, \lambda^{q(i) + q(j)} \qquad \lambda = \langle \sigma \rangle / M_F$$

- $a_{ij} = a_0 (1 + \varepsilon_{ij})$ nearly universal

- For the simplest prescription $q(1) = 2$, $q(2) = 1$, $q(3) = 0$, $q(\sigma) = -1$ the required structure is reproduced

Universality of mixing

A Joshipura, A.S. hep-ph/0512024

In some (universality) basis in the first approximation all the mass matrices but M_l (for the charged leptons) are diagonalized by the same matrix V:

$$V^+ M_f V = D_f$$

For the charged leptons, the mass is diagonalized by V*

$$V^T M_l V^* = D_l$$

$$M_l = M_d^T$$

SU(5) type relation

Diagonalization:
- V for u, d, ν
- V* for l

In the first approximation

Quark mixing: $V_{CKM} = V^+ V = I$

Lepton mixing: $V_{PMNS} = V^T V$

Another version is when neutrinos have distinguished rotation:

V for u, d, l
V* for ν

In general

In general, up and down fermions can be diagonalized by different matrices V' and V respectively

$V_{CKM} = V'^{+} V$

$V_{PMNS} = V^{T} V'$

$V_{PMNS} = V^{T}V \; V_{CKM}^{+} = V^{0}_{PMNS} \; V_{CKM}^{+}$

$V_{PMNS} \, V_{CKM} = V^{T}V$

Quark and lepton rotations are complementary to VV^{T}

$V^{0}_{PMNS} = V^{T}V$

- symmetric, characterized by 2 angles;
- close to the observed mixing for $\theta/2 \sim \phi \sim 20 - 25^{o}$
- 1-3 mixing near the upper bound

V_{PMNS} (with CKM corr.)

- gives very good description of data
- predicts $\sin \theta_{13} > 0.08$

Features of zero order mixing

$V_{PMNS}^{0} = V^{T}V = P\,U\,P$ P = diag. matrix of phases

$$U = \begin{pmatrix} \cos\theta & -\cos\phi\sin\theta & -\sin\phi\sin\theta \\ \cdots & \sin^2\phi - \cos^2\phi\cos\theta & -\sin\phi\cos\phi\,(1+\cos\theta) \\ \cdots & \cdots & \cos^2\phi - \sin^2\phi\cos\theta \end{pmatrix}$$

symmetric matrix phase $\alpha = 0$

Origin of universal mixing

Universal mixing and universal matrices

$M_{u,\nu} \sim m\ D^*\ A\ D^*$

$M_d \sim m\ D^* A\ D$

$M_l \sim m\ D\ A\ D^*$

D = diag(1, i, 1)

A is the universal matrix:

$$A \sim \begin{pmatrix} \varepsilon_1^2\,\varepsilon_2^2 & \varepsilon_1^2\,\varepsilon_2 & \varepsilon_1\,\varepsilon_2 \\ \cdots & \varepsilon_1\,\varepsilon_2 & \varepsilon_1 \\ & & 1 \end{pmatrix}$$

$\varepsilon_i \sim 0.2 - 0.3$

Can be embedded in to SU(5) and SO(10)
with additional assumptions

3. Quark-Lepton complementarity

Complementarity relations

A.S.
M. Raidal
H. Minakata

$$\theta^{l}_{12} + \theta^{q}_{12} \sim \pi/4$$

$$\theta_{12} + \theta_{C} = 46.7^{o} \text{ +/- } 2.4^{o}$$

$$\theta^{l}_{23} + \theta^{q}_{23} \sim \pi/4$$

$$\theta_{23} + V_{cb} = 43.9^{o} + 5.1/-3.6^{o}$$

1σ

H. Minakata, A.S.
Phys. Rev. D70: 073009 (2004)
[hep-ph/0405088]

Difficult to expects exact equalities but qualitatively

- 2-3 leptonic mixing is close to maximal because 2-3 quark mixing is small
- 1-2 leptonic mixing deviates from maximal substantially because 1-2 quark mixing is relatively large

accidental?

Possible implications

``Lepton mixing = bi-maximal mixing – quark mixing"

Quark-lepton symmetry

Existence of structure which produces bi-maximal mixing

Mixing matrix weakly depends on mass eigenvalues

In the lowest approximation:

$V_{quarks} = I, \; V_{leptons} = V_{bm}$
$m_1 = m_2 = 0$

Appears in different places of theory

Bi-maximal mixing

F. Vissani
V. Barger et al

$U_{bm} = U_{23}{}^{m} U_{12}{}^{m}$

Two maximal rotations

$$U_{bm} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

As dominant structure?
Zero order?

$U_{PMNS} = U_{bm}$

- maximal 2-3 mixing
- zero 1-3 mixing
- maximal 1-2 mixing
- no CP-violation

⇐ Contradicts data at (5-6)σ level

QLC1: V_bm from neutrinos

In the symmetry basis

Leptons

- $V_\nu = V_{bm}$
- $V_l = V_{CKM}$

Seesaw?

q-l symmetry

Quarks

$V_u = I$

$V_d = V_{CKM}$

$V_{PMNS} = V_l^+ V_\nu = V_{CKM}^+ V_{bm}$

$V_{quarks} = V_u^+ V_d = V_{CKM}$

(or both rotations from neutrinos)

Predictions:

$\sin\theta_{12} = \sin(\pi/4 - \theta_C) + 0.5\sin\theta_C(\sqrt{2} - 1 - V_{cb})$ ⇒ $\sin^2\theta_{12} = 0.330$

$\sin\theta_{13} = \sin\theta_{23}\sin\theta_C = 0.16$ ⇒ large !

$D_{23} = 0.5\sin^2\theta_C + \cos^2\theta_C V_{cb}\cos\alpha = 0.02 +/- 0.04$

H. Minakata, A.S.

Coincidence?

QLC1: $\sin\theta_{12} = \sin(\pi/4 - \theta_C) + 0.5\sin\theta_C (\sqrt{2} - 1 - V_{cb}\cos\alpha)$

Tri-bimaximal: $\sin\theta_{12} = 1/\sqrt{3}$

QLC1

$V_{CKM}^{+}\ V_{bm}$

$\sim V(\theta_C)^{+}\ V_{23}(45^0)\ V_{12}(45^0)$

$\sin^2\theta_{12} = 0.330$

Tri-bimaximal

$V_{23}(45^0)\ \ V_{12}(\arcsin 1/\sqrt{3})$

$\sin^2\theta_{12} = 0.333$

RGE -> can reduce?

M. Schmidt, A.S.

Two independent combinations of matrices give the same value of 1-2 mixing

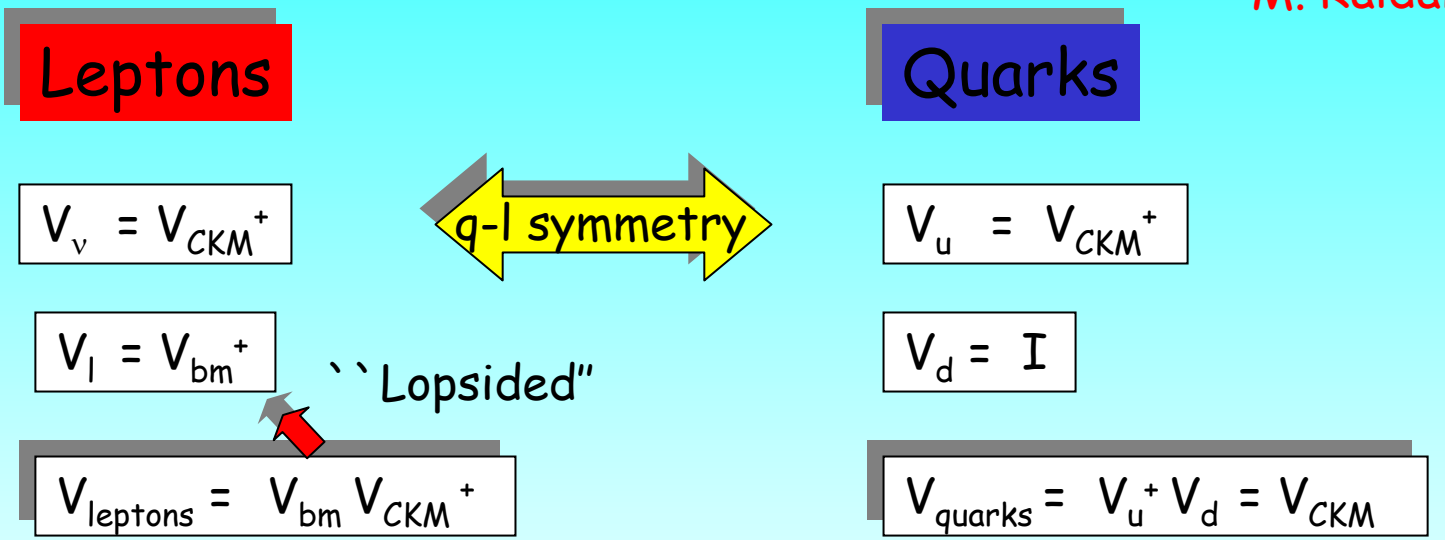
At least one of the approaches is wrong

or formula for the Cabibbo angle:

$\sin\theta_C = \sqrt{2}\cos\theta_C - 2/\sqrt{3}$

QLC2: V_bm from charged leptons

M. Raidal



Predictions:

$\sin\theta_{sun} \sim \sin(\pi/4 - \theta_C)$

$\sin\theta_{13} = -\sin\theta_{sun} V_{cb} = 0.03$

$D_{23} = \cos\theta_{sun} V_{cb} \cos\delta < 0.03$

H. Minakata, A.S.

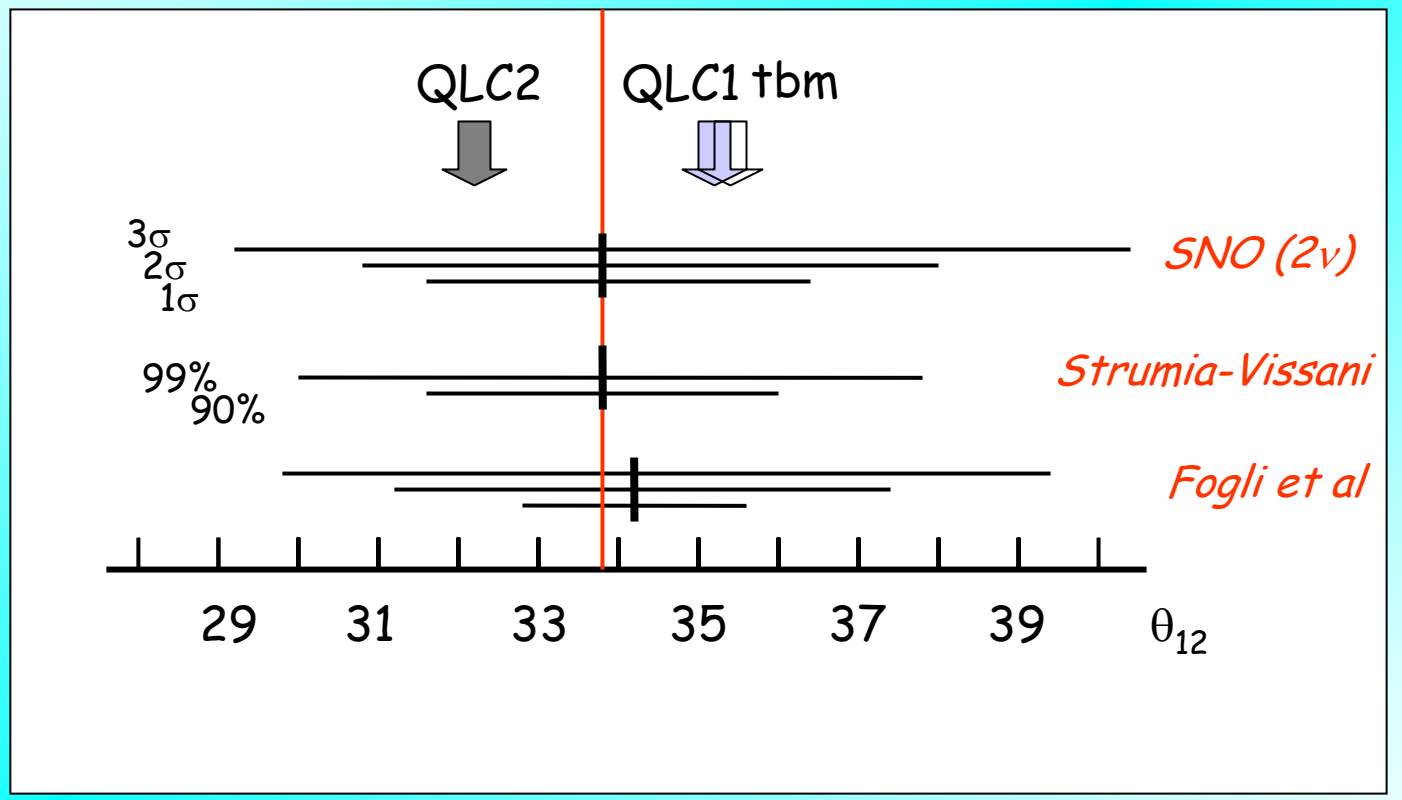
1-2 mixing

$U_{tbm} = U_{tm}\, U^{m}_{13}$

$U_{QLC1} = U_{C}\, U_{bm}$

Give the almost same 12 mixing

coincidence

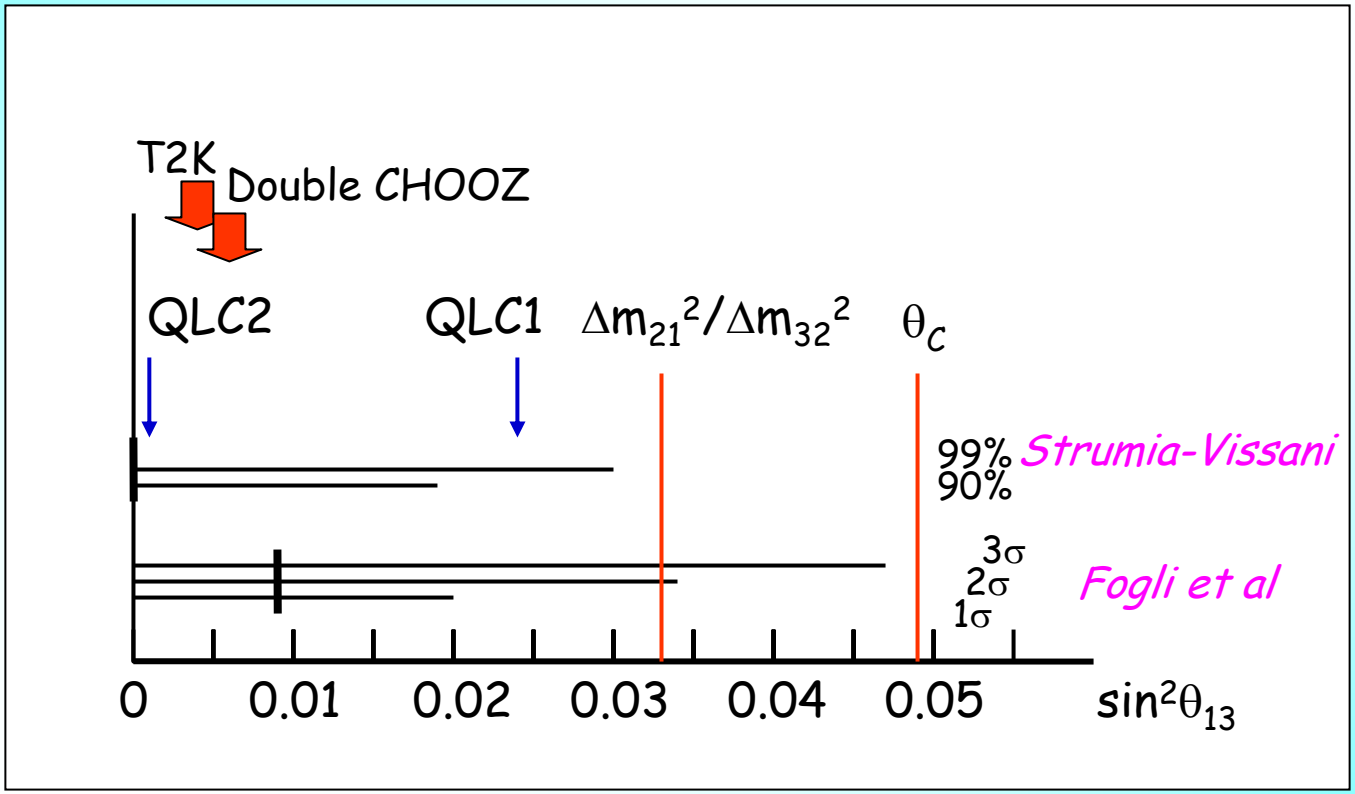


3ν - analysis does change bft but error bars become smaller

$\theta_{12} + \theta_{C} \sim \pi/4$

1-3 mixing

In agreement with 0 value

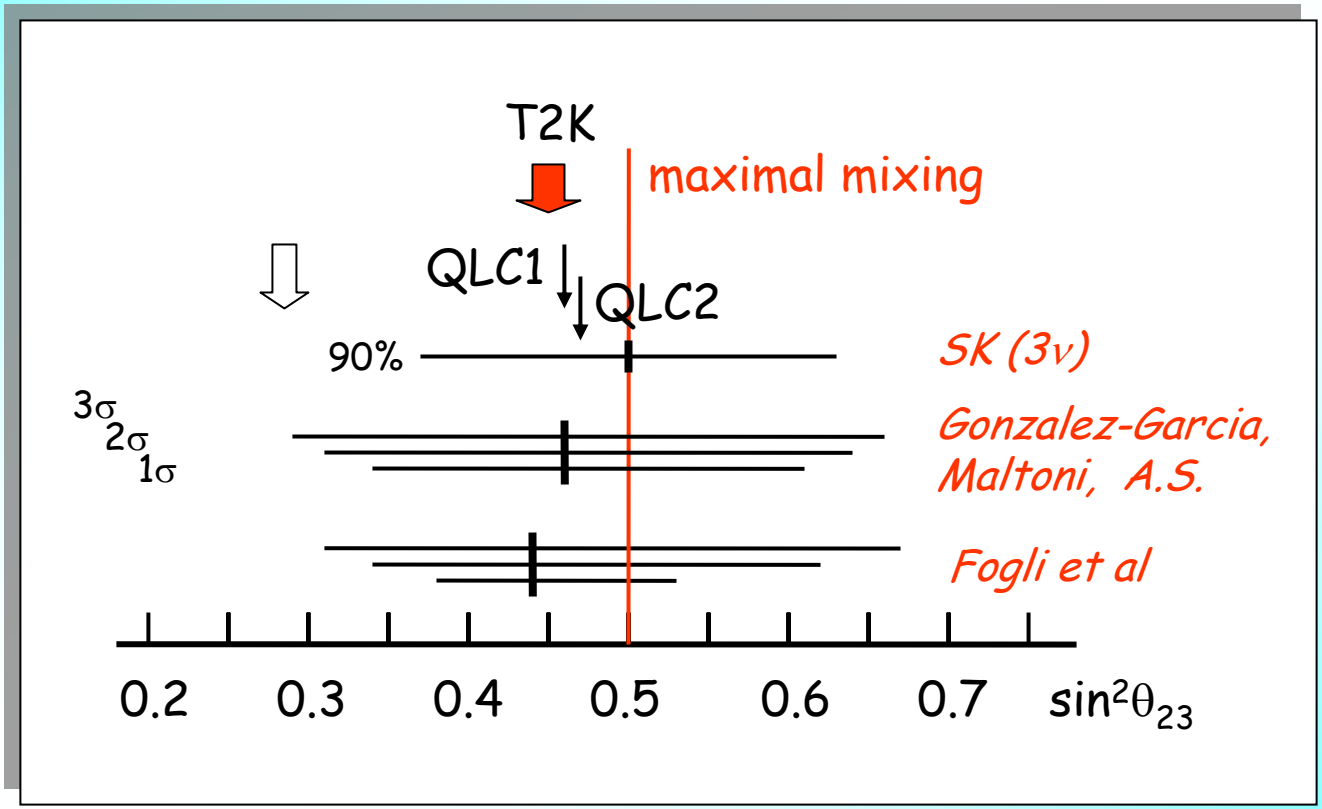


Non-zero central value (Fogli, et al): Atmospheric neutrinos, SK spectrum of multi-GeV e-like events

2-3 mixing

SK (3ν) - no shift from maximal mixing

$\sin^2 2\theta_{23} > 0.93$, 90% C.L.



1). in agreement with maximal

2). shift of the bfp from maximal is small

3). still large deviation is allowed: $(0.5 - \sin^2\theta_{23})/\sin\theta_{23} \sim 40\%$ 2σ

Summary

Q & L: strong difference of mass and mixing pattern;
- ti-bimaximal mixing;
- possible presence of the special leptonic (neutrino) symmetries

Still approximate quarks-leptons universality can be realized.
No additional structures apart from seesaw is required.

Data may indicate existence of the Q-L complementarity
which implies generation of bi-maximal mixing and Q-L symmetry

Coincidences:
real or accidental?

Empirical relations

Koide relation

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 2/3$$

Y. Koide, Lett. Nuov. Cim. 34 (1982), 201

with accuracy 10^{-5}

$$\tan\theta_C = \sqrt{3}\ \frac{\sqrt{m_\mu} - \sqrt{m_e}}{2\sqrt{m_\tau} - \sqrt{m_\mu} - \sqrt{m_e}}$$

Both relations can be reproduced if

$$m_i = m_0(z_i + z_0)^2$$

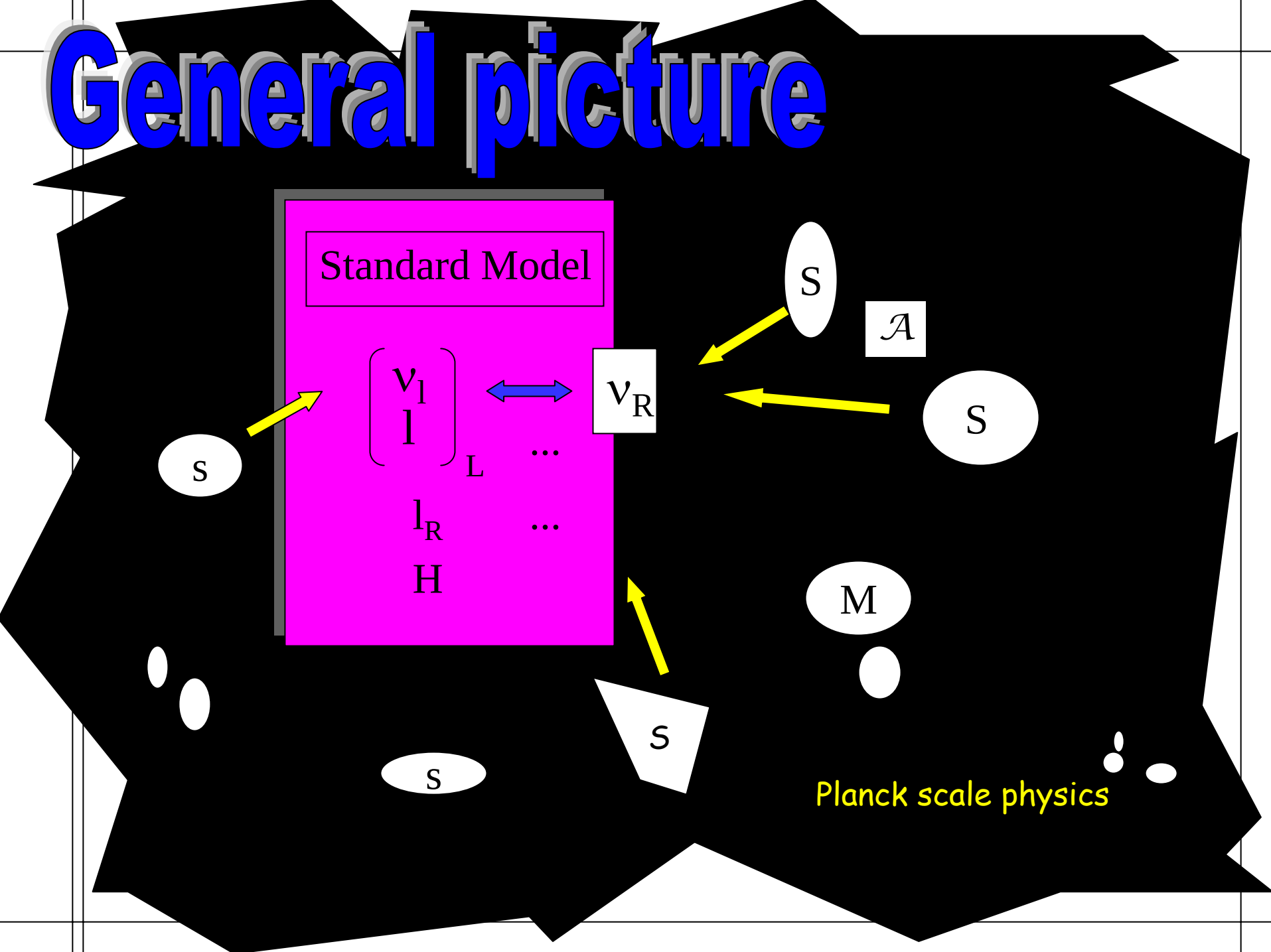
$$\Sigma_i\, z_i = 0$$

$$z_0 = \sqrt{\Sigma_i\, z_i^2 / 3}$$

C A Brannen Neutrinos

Another representation which is closely connected to circulant symmetry

General picture



Screening of Dirac structure

Double (cascade) seesaw

M. Lindner
M. Schmidt
A.S.
JHEP0507, 048 (2005)

$$m = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_D^T \\ 0 & M_D & M_S \end{pmatrix} \quad \begin{matrix} \nu \\ N \\ S \end{matrix}$$

Additional fermions

$m_D \ll M_D \ll M_S$

M_S – Majorana mass matrix of S

R. Mohapatra
PRL 56, 561, (1986)
R. Mohapatra.
J. Valle

$$M_R = - M_D^T M_S^{-1} M_D$$

$$m_\nu = m_D M_D^{-1} M_S M_D^{-1} m_D$$

If $M_D = A^{-1} m_D$ $\Rightarrow$ $m_\nu = A^2 M_S$

$A \sim v_{EW}/M_{GU}$

A.S.
PRD 48, 3264 (1993)

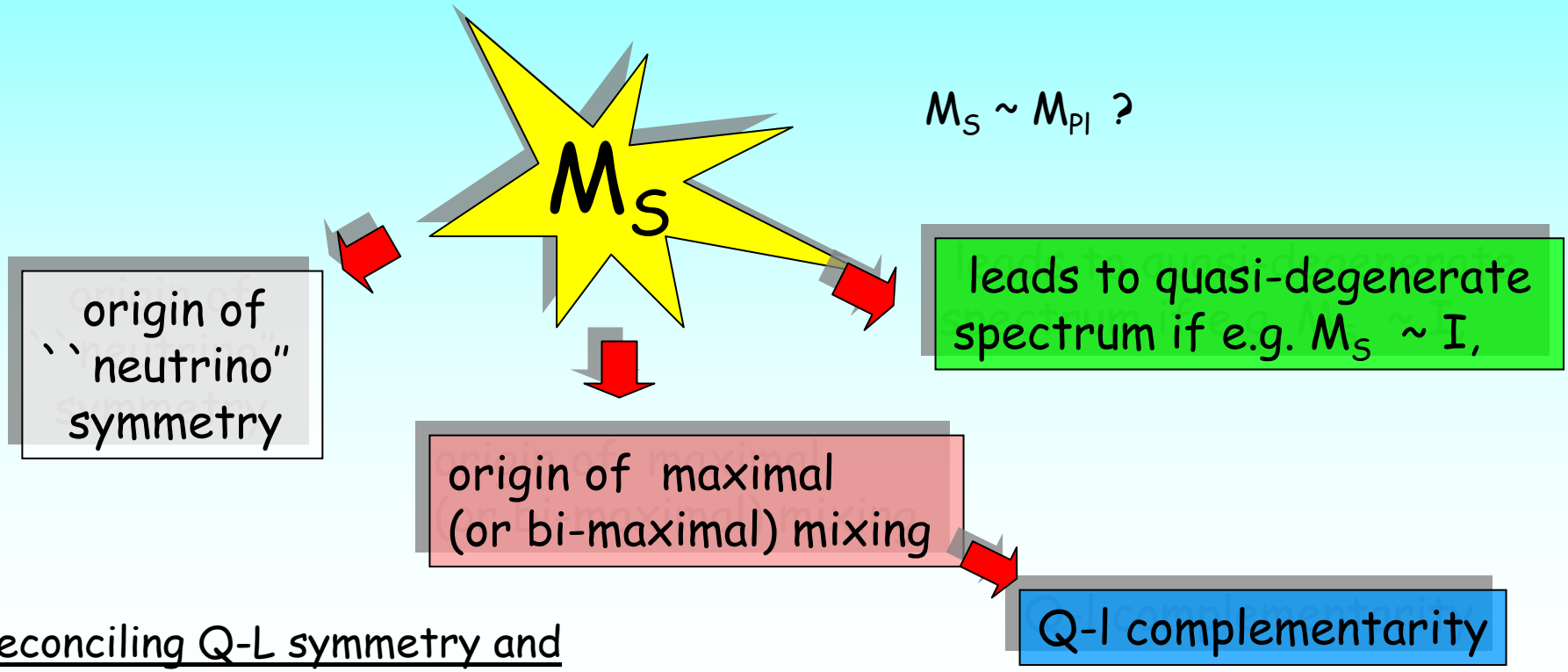
m_D similar (equal) to quark mass matrix - cancels

Structure of the neutrino mass matrix is determined by M_S
-> physics at highest (Planck?) scale immediately

Applications

- Seesaw provides scale and not the flavor structure of neutrino mass matrix

- Structure of the neutrino mass matrix is determined by



Reconciling Q-L symmetry and different mixings of quarks and leptons

Masses

P. Minkowski
T. Yanagida
M. Gell-Mann, P. Ramond, R. Slansky
S. L. Glashow
R.N. Mohapatra, G. Senjanovic

Smallness of neutrino masses

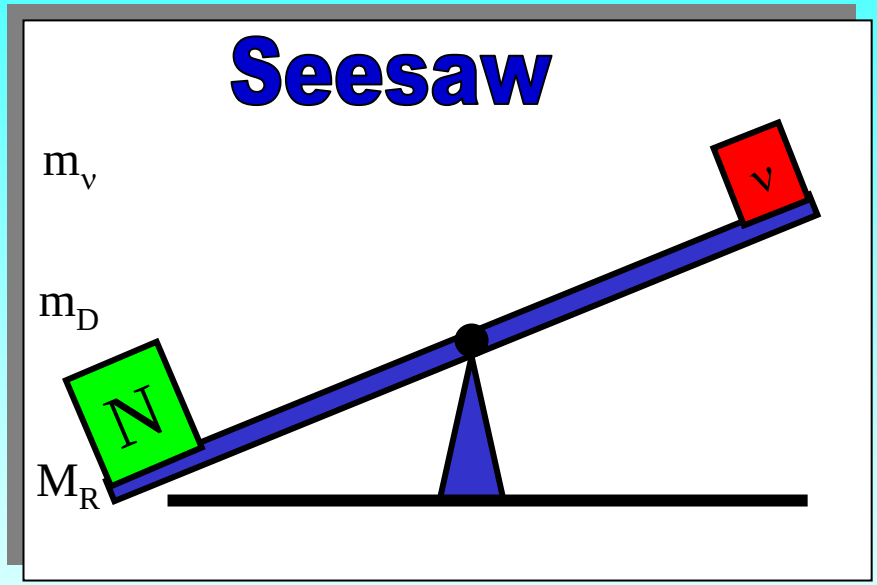
For the third generation:

$$\frac{m_3}{m_\tau} \sim (0.3 - 1)\, 10^{-10}$$

$$\frac{m_\tau}{m_t} \sim 10^{-2}$$

$$m_3 \sim m_t \frac{m_t}{M}$$

Scale of new physics



- ν N

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

$m_D = Y\langle H\rangle$

If $M_R >> m_D$

$$m_\nu = - m_D^T M_R^{-1} m_D$$

Neutrino symmetry?

Maximal 2-3 mixing

Can both features be accidental?

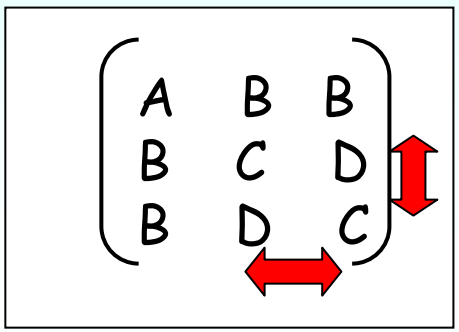
Zero 1-3 mixing

$\nu_\mu - \nu_\tau$ permutation symmetry

Neutrino mass matrix in the flavor basis: For charged leptons: D = 0

Often related to equality of neutrino masses

Discrete symmetries S_3, D_4



Can this symmetry be extended to quark sector?

Are quarks and leptons fundamentally different?

Universal 2-3 symmetry?

2-3 symmetry

Smallness of V_{cb}

Maximal (large) 2-3 leptonic mixing

A. Joshipura, hep-ph/0512252

2-3 symmetry does not contradict mass hierarchy

Universal mass matrices

$$\begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix} + \delta m$$

Quarks, charged leptons: $B \sim C,\ X \ll A \ll B$

Neutrinos: $B \gg C,\ X \sim B$

- Hierarchical mass spectrum
- Small quark mixing

- Degenerate neutrino mass spectrum;
- Large lepton mixing

Still additional symmetries are needed to explain hierarchies/equalities of parameters

Real or accidental?

Tri-bimaximal mixing

Small 1-3 mixing

Q-L-complementarity

Koide relation

Maximal 2-3 mixing

Plan

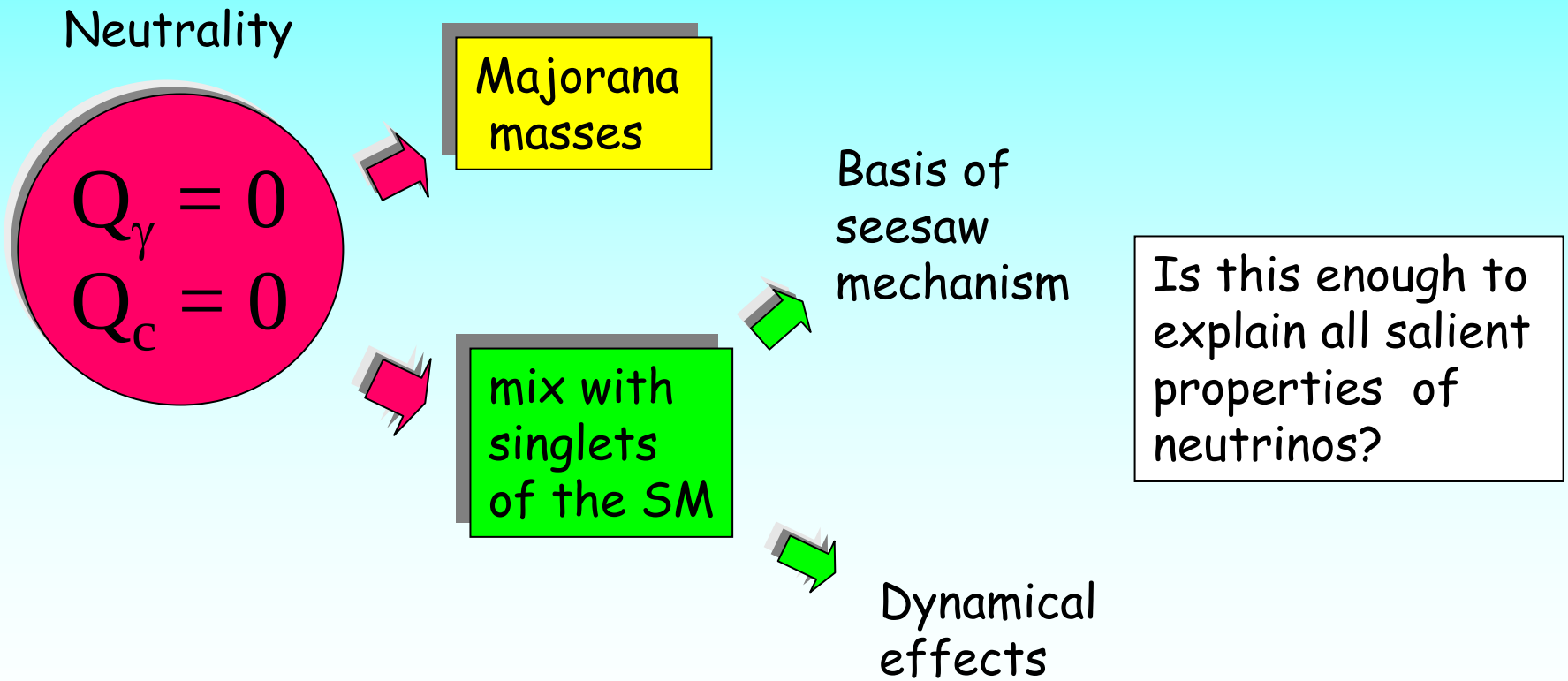
Quark-lepton

Comparing results

Universality

Complementarity

The hope is


Neutrality
$Q_\gamma = 0$
$Q_c = 0$
Majorana masses
Basis of seesaw mechanism
mix with singlets of the SM
Dynamical effects
Is this enough to explain all salient properties of neutrinos?