GRAND UNIFICATION :

a (THE ?) THEORY OF

FERMION MASSES and

MIXINGS ?

OR

CONSTRUCTING THE

MINIMAL STANDARD

GRAND UNIFIED THEORY :

MSGUT

B. Bajc, A. Melfo,

C. Aulakh, F. Vissani

HISTORICAL IRONY:

GUT - theory of proton decay and magnetic monopoles

p decay exp. $\Rightarrow$ ν OSCILLATIONS

Kamiokande

$\Rightarrow$ ν mass

THEORY: hard to compute τ_p (branching ratios)

A theory of fermion (neutrino) masses and mixings

$\Downarrow$

CONSTRUCTION OF THE THEORY (minimal)

$\sim$ SM

FERMION MASSES

and

MIXINGS

~ GeV : / GUT

$m_e = .5\ MeV$ — $m_d \simeq 10\ MeV$: 3

$m_\mu = 100\ MeV$ — $m_s \simeq 100\ MeV$: 3

$m_\tau = 1.8\ GeV$ — $m_b \simeq 4.5\ GeV$: 3

$meV = 10^{-3}\ eV$

$m_{\nu_1} \simeq (0 - 300)\ meV$ — $m_u = 5\ MeV$: 3

$m_{\nu_2} \simeq (10 - 300)\ meV$ — $m_c = 1.2\ GeV$: 3

$m_{\nu_3} \simeq (50 - 300)\ meV$ — $m_t = 180\ GeV$: 3

$\theta_{12} \simeq 30^\circ$ — $\theta_c \simeq 13^\circ$

$\boxed{\theta_{23} \simeq 45^\circ \quad \longleftrightarrow \quad \theta_{cb} = 2^\circ}$

$\theta_{13} \lesssim 15^\circ$ — $\theta_{ub} \simeq .2^\circ$

• NATURAL CONSEQUENCE •

OF GRANDUNIFICATION

$SO(10)$: TWO SIMPLE EXAMPLES

SU(5)

- Fermions not truly unified

generation : $10_F + \bar{5}_F$

minimal model: $m_\nu = 0$ (SM)

- Add ν_R (singlet) (as in SM)

or new Higgs (15_H)

$\Downarrow$

no connection between neutrinos

and charged fermions

$\Downarrow$

no information of m_ν

no connection between θ_ℓ and θ_q

MINIMAL THEORY

- G fixed: SO(10)
- matter fixed: 16_F $(i = 1,2,3)$
- Higgs: minimal set *

$\Downarrow$

EXTRACT ~~[illegible]~~ PREDICTIONS
(CONSISTENCY)

* depends on the realization of see-saw
(i.e. how you break B-L)

(spinor) 16_H $\qquad$ $\overline{126}_H$ (5-index AS)

within each: only SO(10)
\+ minimal Yukawa

Higgs : Yukawa sector

• SM (standard Model)

$$\mathcal{L}_Y = y_d \underbrace{\bar{Q}_L}_{\text{doublet}} \underbrace{\Phi}_{\text{doublet}} \underbrace{d_R}_{\text{singlet}} + \cdots$$

• SO(10)

$$16_F = \Psi_+$$

$$16 \times 16 = 10 + 120 + 126$$

$$10 \to 10_i \qquad \underbrace{120 \to 120_{[ijk]} \qquad 126 \to [126]_{ijklm}}_{\text{anti-symmetric}}$$

$$\mathcal{L}_Y = 16_F \, \underbrace{(y_{10} 10_H + y_{120} 120_H + y_{126} 126_H)}_{\text{how many needed ?}} \, 16_F$$

• $\underline{126_H}$ rather interesting (5-index AS)

$$126_H = \underline{(3,1,10)} + \underline{(1,3,\overline{10})} + (1,1,6)$$

$$+ \underline{(2,2,15)}$$

Higgs doublet:

$$3 m_l = - m_e$$

Mohapatra, G.S. Lazarides et al

• $(3,1,10)$: Δ_L triplet (II)

$$(l_L^T \Delta_L l_L + l_R^T \Delta_R l_R)$$

• $(1\ 3\ \overline{10})$: Δ_R – gives mass to ν_R (I)

both present

126 : complete ?
Both ordinary Higgs + see-saw } A MUST!!

II see-saw:

Lazarides, Shafi, Wetterich
Mohapatra, G.S.
'80

$$(3,1)\ (1,3)\ (2,2)^2$$

$$V = \dots + \Delta_L \Delta_R \Phi^2 + M_\Delta^2 \Delta_L^2$$

L-R doublet
(Li-doublet)

$$\Downarrow$$

$$\langle \Delta_L \rangle \neq 0$$

$$M_\Delta \gtrsim \langle \Delta_R \rangle$$

$$\frac{\partial V}{\partial \Delta_L} = 0 \Rightarrow \Delta_L = \frac{\Delta_R \Phi^2}{M_\Delta^2}$$

$$\Downarrow$$

$$\langle \Delta_L \rangle = \frac{M_R M_W^2}{M_\Delta^2} \leq \frac{M_W^2}{M_\Delta} \quad (M_\Delta \gtrsim M_R)$$

arbitrary

Needs unification

$$\Downarrow$$

SO(10)

· Minimal Higgs (Yukawa)

- one Higgs (one multiplet)

10_H or $\overline{126}_H$ or 120_H

$16_F \; Y \; \Phi \; 16_F$

$\Downarrow$

diagonalize Y

$\Downarrow$

all M_f diagonal

$\Downarrow$

- $\theta_q = \theta_\ell = 0$ · $M_u \propto M_d \propto \cdots$

$\Downarrow$

AT LEAST TWO MULTIPLETS

· Minimal : TWO

$\overline{126}_H$ (ν_R - see saw)

$+ 10_H$ (small)

- $\overline{126}_H$ CLASS: AT LENGTH

Clark, Kuo, Nakagawa
'82 Lazarides, Shafi, Wetterich
'92 Babu, Mohapatra
'02 Bajc, Vissani, G.S.

$$\boxed{\overline{126}_H + 10_H}$$

$$W_Y = 16_F \left(Y_{10}\, 10_H + Y_{126}\, \overline{126}_H \right) 16_F$$

OVERCONSTRAINED $\Rightarrow$ PREDICTIVE

Y_{10} : 3 real couplings (diagonalize)

Y_{126} : $6 \times 2 = 15$ real — " — (≥ 33 in $SU(5)$)

$Y_{10}^T = Y_{10}$, $Y_{126}^T = Y_{126}$

SUPERSYMMETRY (LOW ENERGY)

- Answer: all m_f, θ_f, δ_f from the same source: Y_{10}, Y_{126}

Lazarides, Shafi, Wetterich '81

Babu, Mohapatra '92

$$M_u = Y_{10}\, v_{10}^{u} + Y_{126}\, v_{126}^{u}$$

$$M_d = Y_{10}\, v_{10}^{d} + Y_{126}\, v_{126}^{d}$$

$$M_e = Y_{10}\, v_{10}^{d} - 3\, Y_{126}\, v_{126}^{d}$$

$$M_D = Y_{10}\, v_{10}^{u} - 3\, Y_{126}\, v_{126}^{u}$$

$$M_R = Y_{126} \langle \Delta_R \rangle$$

$$M_L = Y_{126} \langle \Delta_L \rangle$$

Mohapatra, G.S.

Δ_L, Δ_R : L, R triplets in 126_H

$$G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_c$$

$$10_H = (2, 2, 1) + \cancel{(1, 1, 6)}$$

$$126_H = (2, 2, 15) + (3, 1, \overline{10}) + (1, 3, 10) + \cancel{(1, 1, 6)}$$

$$M_d = Y_{10} \langle 221 \rangle_{10} + Y_{\overline{126}} \langle 2215 \rangle_{\overline{126}}$$

$$M_E = Y_{10} \langle 221 \rangle_{10} - 3\, Y_{\overline{126}} \langle 2215 \rangle_{\overline{126}}$$

(1, 1, 1, −3) DIRECTION in $SU(4)_c$

(type II) $$M_\nu = Y_{126} \langle 31\overline{10} \rangle$$

$$\Downarrow$$

$$M_\nu \propto M_E - M_D$$

basis: diagonal charged leptons

2nd – 3rd gen.

(θ_{ed} small)

$$\Downarrow$$

$$M_\nu \propto \begin{pmatrix} \epsilon & \epsilon \\ \epsilon & m_b - m_\tau \end{pmatrix} \qquad \epsilon \ll 1$$

SUSY running

AUTOMATIC:

b − τ unif. ⟺ large θ_{atm}

Bajc, Vissani, G.S.

- numerical study of the complete theory

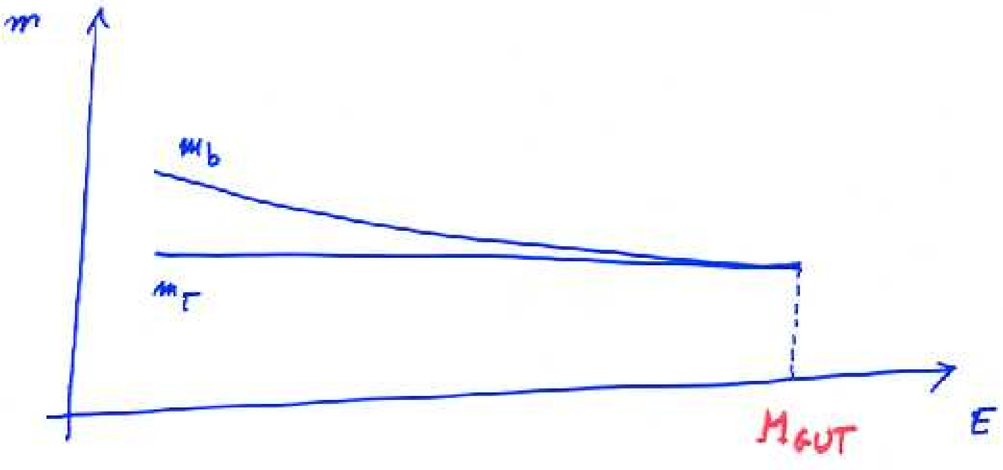
$\boxed{|U_{13}| \gtrsim .1}$

Mohapatra, Goh, Ng, ...
Bertolini, Malinski

$$\tan 2\theta_e = \frac{\theta_{bc}}{\theta_{bc}^2 - \varepsilon/2} \qquad \varepsilon \equiv \frac{y_b - y_\tau}{y_b}$$

$$\varepsilon \ll \theta_{bc}$$

$$\boxed{b-\tau \text{ unif.} \iff \theta_e \equiv \theta_A \simeq 45^\circ}$$



$\Lambda_{susy} = \text{TeV}$

MINIMAL SO(10) GUT

Aulakh, Bajc, Melfo, Vissani
G.S. (2003)

REQUIRES A NUMBER OF

HIGGS SUPERMULTIPLETS

Kuo, Love, Nakagawa ('82)
Lee '92

$210, \; 126 + \overline{126}, \; 10$

PLUS MATTER FIELDS :

16_F (three generations)

$$W = m_1 \, 126 \, \overline{126} + m_2 \, 210^2 + m_3 \, 10^2$$

$$+ \lambda_1 \, 210^3 + \lambda_2 \, 210 \, 126 \, \overline{126} + (\lambda_3 \, 126 \, 10 + \lambda_4 \, \overline{126} \, 10) \, 210$$

$7 \times 2 - 4 = 10$ real couplings

$$+ \; Y_{10} \; 16 \; 16 \; 10 + Y_{126} \; 16 \; 16 \; \overline{126}$$

3 real $\qquad$ $6 \times 12 =$ real $\qquad$ $= 15$

SIMPLE COUNTING

26 real parameters ($15 + 10 + 1 =$ gauge)

— BUT COVERS MANY MORE PHENOMENA (p decay, P-S, ...)

$\boxed{\text{PREDICTIVE}}$ $\qquad$ Bajc

NEUTRINO MIXINGS

Many in-depth studies

	Goh, Mohapatra, Ng
	Aulakh, Bajc, Melfo, Vissani, G.S.
Babu, Mohapatra	Bertolini, Malinski
Bajc, Vissani, G.S.	Mohapatra, Goh, Nasri
Bertolini, Malinski	Bajc, Melfo, Vissani, G.S.
	Aulakh, Girdhar
. PARTICLE SPECTRA	Fukuyama et al
. PROTON DECAY *	Mohapatra, Dutta, Mimura
. UNIF. CONSTRAINTS	Fukuyama et al
. MASSES and MIXINGS	
. LEPTOGENESIS *	Bertolini, Malinski
	Babu,
	⋮

Review: in SEESAW 25

G. S.

* not completed

PREDICTIONS

• $|U_{13}| > 0.1$ 1-3 mixing fairly large

exp. $|U_{13}| \lesssim .15$ – hope for $\not{CP}$

CP phase δ predicted

• neutrinos hierarchical: $m_{\nu_3} \simeq 5\, m_{\nu_2}$

$m_3 \simeq 1/20$ eV
$m_2 \simeq 1/100$ eV
$m_1 = ?$

well below cosmological limits

$(\beta\beta)^0$: hard to observe

neutrino-less double β decay

• R-parity exact at low energies
LSP stable (dark matter) Aulakh, Melfo, Rasin, G.S.

• recently:

Bajc, Melfo, Visconi; G.S.

take constraints of Higgs structure

on fermion mass, θ predictions

Aulakh '05

Bertolini, Malinsky, Schwetz '06

tension between

m, θ fit and unification or p decay

• ruled out

Question: fitting at M_{GUT} - error?

better: at M_W where error is known

(much harder)

Bajc, Nemevšek

- 16_H INSTEAD OF $\overline{126_H}$

$$16_H = (2\,1\,4) + \langle(1\,2\,\bar{4})\rangle \quad (\sim q, \ell)$$

$$\| $$

$$\boxed{\overline{16_H} \times \overline{16_H} = \overline{126_H} + \cdots} \qquad M_R$$

(a) HIGHER DIMENSIONAL OPERATORS

- $16_F \; 16_F \; \dfrac{\overline{16_H} \; \overline{16_H}}{M_{pe}}$

large # couplings

- $16_F 16_F 10_H$

$m_b \simeq m_\tau$

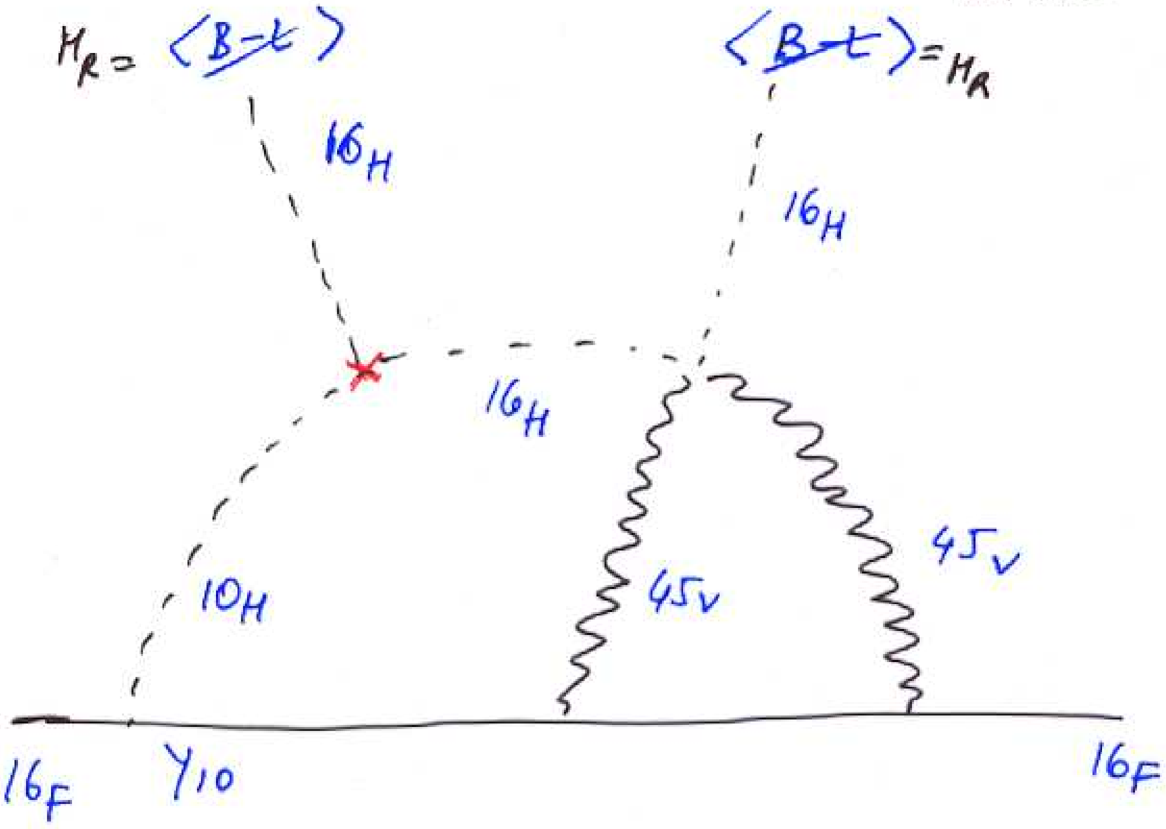
$$\Downarrow$$

TEXTURES (beyond SO(10))

$G = SO(10) \times \cdots$

(b) Radiative see-saw

Witten 1980



$\overline{126}_H$ = (5 index antisym.)

45 = 2 index antisym.

5 = 2 + 2 + 1

$\overline{16}_H \times \overline{16}_H = \overline{126}_H$

$$M_{\nu_R} = \left(\frac{\alpha}{\pi}\right)^2 Y_{10} \frac{M_R^2}{M_{GUT}} \quad \left(\frac{m_{SUSY}}{M_{GUT}}\right)$$

• a more minimal version

$10_H + 120_H$ Bajc, G.S. '05

$10_H = (221)_{10} + \dots$

$\hookrightarrow 120_H = (221)_{120} + (221\bar{5})_{120} + \dots$

$$W_Y \; (\mathcal{L}_Y) = 16_F \left(Y_{10} \, 10_H + Y_{120} \, 120_H \right) 16_F$$

$$Y_{10} = Y_{10}^T \,, \qquad Y_{120} = - Y_{120}^T$$

diagonal: 3 real

$\Downarrow$

$3 \times 2 = 6$ real

[illegible] only 9 real parameters in Yukawa

([illegible])

• 2-3 gen. ($m_2 \simeq 0$: second generation masses small)

$$M_d = M_0 + M_2 \qquad M_u = c_0 M_0 + c_2 M_2$$

$$M_e = M_0 + c_3 M_2 \qquad M_{\nu_D} = c_0 M_0 + c_4 M_2$$

$$M_0 = Y_{10} \, \langle 221 \rangle_{10}^d \,, \quad M_2 = Y_{126} \left[\langle 221 \rangle_{120}^d + \langle 221\bar{5} \rangle_{120}^d \right]$$

etc.

$$M_d = Y_{10} \langle 221 \rangle_{10} + Y_{120} \left[\langle 221 \rangle_{120} + \langle 2215 \rangle_{120} \right]$$

$$M_e = Y_{10} \langle 221 \rangle_{10} + Y_{120} \left[\langle 221 \rangle_{120} - 3 \langle 2215 \rangle_{120} \right]$$

$\langle 15 \rangle = \mathrm{diag}(1, 1, 1, -3)$ ← adjoint in $SU(4)_c$

diagonalize Y_{10}

$(Y_{120}^T = - Y_{120})$

$$\Downarrow$$

$$M_d = \begin{pmatrix} a & \alpha \\ -\alpha & b \end{pmatrix}, \quad M_e = \begin{pmatrix} a & c\alpha \\ -c\alpha & b \end{pmatrix}$$

$m_\mu \ll m_\tau \qquad m_s \ll m_b$

$$\Rightarrow \quad ab + \alpha^2 \simeq 0, \quad ab + c^2\alpha^2 \simeq 0$$

$$\Downarrow$$

$$c = \pm 1, \quad \alpha = i|\alpha|$$

$$\Rightarrow \quad m_\tau \simeq \mathrm{Tr}\, M_e = a + b = \mathrm{Tr}\, M_d \simeq m_b$$

$\boxed{m_\tau \simeq m_b}$ at M_{GUT}

$\boxed{10_H + 120_H}$ $\left.\begin{array}{ll} Y_{10}^T = Y_{10} & (3 - \text{diag.}) \\ Y_{120}^T = -Y_{120} & (3\times 2 = 6) \end{array}\right\}$ 9

- $$\frac{m_3^2 - m_2^2}{m_3^2 + m_2^2} = \frac{\cos 2\theta_A}{1 - \frac{1}{2}\sin^2 2\theta_A} \left(1 + m_\mu/m_\tau\right)$$

neutrino degeneracy measured
by the maximality of θ_A ($\sim 45°$)

Bajc, Melfo, Vissani, G.S.

- $|V_{cb}| \simeq m_s/m_b \, |\cos 2\theta_A|$ + small

small became $\theta_A \simeq 45°$

numerical study !

3 generations

- $|U_{13}| = ?$

Bajc, Nemevšek, G.S.

- degenerate ?

COSMOLOGY, NEUTRINO-LESS DOUBLE
β DECAY

CONCLUSION

- CONSTRUCTING MS GUT:

$$\left.\begin{array}{c}\text{theory of fermion masses}\\ \text{and}\\ \text{unification, } p \text{ decay}\end{array}\right\}\ \text{testable}$$

- MINIMAL THEORY: $SO(10)$

$$\left.\begin{array}{l}\text{small } \nu \text{ mass}\\ \text{large } \theta_\ell, \text{ small } \theta_q\end{array}\right\}\ \text{naturally connected}$$

$$\left.\begin{array}{c}\text{neutrino spectrum}\\ \theta_{13}\ ,\ \delta\\ p \text{ decay}\end{array}\right\}\ \text{predicted}$$

- one minimal version ruled out!?

Bertolini, Malinsky, Schwetz

- two more possible models: study going on